Covariant Faddeev calculation of N-$\Delta(1232)$ form factors

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Electromagnetic N-N* Transition Form Factors Workshop
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Themes

- Demonstrate utility of Nambu–Jona-Lasinio model
  - Nucleon, Delta, $N \rightarrow \Delta$ form factors
  - quark distributions, etc
- Highlight important challenges in modelling form factors
  - in particular the pion cloud
- Results: Nucleon, Delta form factors
- Preliminary Results: $N \rightarrow \Delta$ Transition Form Factors
- Examine Off-Shell Form Factors
Nambu–Jona–Lasinio Model

- Low energy chiral effective theory of QCD

\[
\mathcal{L}_{NJL} = \overline{\psi} \left( i \not{\! \! \! \partial - m \right) \psi + G \left( \overline{\psi} \Gamma \psi \right)^2
\]

- Investigate the role of quark degrees of freedom.

- Much in common with DSE

- Lagrangian has same symmetries as QCD:
  - Importantly chiral symmetry and $D\chi_{SB}$,
    \(\longrightarrow\) Dynamically generated quark masses,
    \(\longrightarrow\) Non-zero chiral condensate.

- Lagrangian
  \(\Gamma = \text{Dirac, colour, isospin matrices}\)
Baryons in the NJL model

- Baryons approximated as quark-diquark bound states.
- Use relativistic Faddeev approach:

  \[ P k = \frac{P}{P - k} P \]

- Diquark - bound state of two quarks:
- Solve Bethe-Salpeter equation for diquark.

  \[ = + \]

- We include scalar and axial-vector diquarks.
- Static Approximation: \( S_{ex}(k) \rightarrow -1/M \).
Proper-time regularization

\[
\frac{1}{X^n} = \frac{1}{(n-1)!} \int_0^\infty d\tau \tau^{n-1} e^{-\tau X}
\]

\[
\rightarrow \frac{1}{(n-1)!} \int_{1/(\Lambda_{IR})^2}^{1/(\Lambda_{UV})^2} d\tau \tau^{n-1} e^{-\tau X}.
\]

\(\Lambda_{IR}\) eliminates unphysical thresholds for the nucleon to decay into quarks: \(\rightarrow\) simulates confinement.


Needed for: nuclear matter saturation, \(\Delta\) baryon.
Model Parameters

- **Free Parameters:**
  \[ \Lambda_{IR}, \Lambda_{UV}, M_0, G_\pi, G_s, G_a, G_\omega \text{ and } G_\rho \]

- **Constraints:**
  - \( f_\pi = 93 \text{ MeV}, \ m_\pi = 140 \text{ MeV} \quad \& \quad M_N = 940 \text{ MeV} \)
  - \( \int_0^1 dx \ (\Delta u_v(x) - \Delta d_v(x)) = g_A = 1.267 \)
  - \( (\rho, E_B/A) = (0.16 \text{ fm}^{-3}, -15.7 \text{ MeV}) \)
  - \( a_4 = 32 \text{ MeV} \)
  - \( \Lambda_{IR} = 240 \text{ MeV} \)

- **We obtain [MeV]:**
  - \( \Lambda_{UV} = 644 \)
  - \( M_0 = 400, \ M_s = 690, \ M_a = 990, \ldots \)

- **Can now model a very large array of observables**
**Nucleon–Delta Transition Form Factors**

- **Themes**
  - NJL model
  - Baryons . . .
- **$N \rightarrow \Delta$ FFs**
  - Constituents
  - Nucleon FFs
  - Scalar Diquark FF
  - Axial-Vector FF
  - Nucleon FFs
  - Delta FFs
  - $N \rightarrow \Delta$ FFs
  - Off-Shell
  - Conclusion

- **Transition Form Factor Feynman diagrams**

\[ \gamma^\mu \, F_{1q}(Q^2) + \frac{i \sigma^{\mu\nu} q_\nu}{2M} \, F_{2q}(Q^2) \]

- **Approach is completely covariant**
- **No frame is assumed & Current is conserved**
- **Diagrams are expressed in form (Jones & Scadron):**

\[ J^\mu = \bar{u}_{\Delta,\alpha} \left[ H_1^{\alpha\mu} \, G_1(Q^2) + H_2^{\alpha\mu} \, G_2(Q^2) + H_3^{\alpha\mu} \, G_3(Q^2) \right] \, u_N \]

- **Many Model ingredients necessary**
**Constituent Quark Form Factors**

- **Themes**
  - NJL model
  - Baryons . . .
  - \( N \rightarrow \Delta \) FFs
- **Constituents**
  - Nucleon FFs
  - Scalar Diquark FF
  - Axial-Vector FF
  - Nucleon FFs
  - Delta FFs
  - \( N \rightarrow \Delta \) FFs
  - Off-Shell
  - Conclusion

- **Vector Meson Dominance – traditional view**
  \[ \omega, \rho, \ldots \propto \frac{M^2}{M^2 + Q^2} \]

- **We solve integral equation for vertex**
  \[
  \left[ \begin{array}{c}
  \end{array} \right]
  = \left[ \begin{array}{c}
  \end{array} \right] + \left[ \begin{array}{c}
  \end{array} \right]
  \]

- **Vertex becomes**
  \[
  \left( \frac{1}{6} + \frac{\tau_3}{2} \right) \gamma^\mu \rightarrow \left[ \frac{1}{6} F_\omega + \frac{\tau_3}{2} F_\rho \right] \gamma^\mu
  \]
Expanding about $Q^2 = 0$ gives

$$\left[ \frac{1}{6} F_\omega + \frac{\tau_3}{2} F_\rho \right] \gamma^\mu \sim \left[ \frac{1}{6} \frac{M_\omega^2}{M_\omega^2 + Q^2} + \frac{\tau_3}{2} \frac{M_\rho^2}{M_\rho^2 + Q^2} \right] \gamma^\mu$$
**Constituent Quarks – Pion**

- **Themes**
  - NJL model
  - Baryons . . .
  - \( N \to \Delta \) FFs

- **Constituents**
  - Nucleon FFs
  - Scalar Diquark FF
  - Axial-Vector FF
  - Nucleon FFs
  - Delta FFs
  - \( N \to \Delta \) FFs
  - Off-Shell
  - Conclusion

- **Probability for find bare quark:**
  \[
  Z_q = 1 + \frac{\partial \Sigma_q}{\partial \psi}
  \]

- **Pion cloud \( \to \) anomalous m.m for constituent quarks.**

\[
F_{1q}(Q^2) = Z_q \left( \frac{1}{6} F_\omega + \frac{1}{2} \tau_3 F_\rho \right) + (F_\omega - \tau_3 F_\rho) F_{1q}^{(q)} + \tau_3 F_\rho F_{1q}^{(\pi)}
\]

\[
F_{2q}(Q^2) = (F_\omega - \tau_3 F_\rho) F_{2q}^{(q)} + \tau_3 F_\rho F_{2q}^{(\pi)}
\]

- **Self-consistent pion cloud**
- **However no pion exchange between quarks**
- **Better to add pion at nucleon level**
Constituent Up Quark Results

Themes
NJL model
Baryons...
\( N \rightarrow \Delta \) FFs

Constituents
Nucleon FFs
Scalar Diquark FF
Axial-Vector FF
Nucleon FFs
Delta FFs
\( N \rightarrow \Delta \) FFs
Off-Shell
Conclusion

\[
\begin{align*}
F_{1u}^{\pi} & \quad F_{2u}^{\pi} \\
F_{1u}^{\nu\text{MD}+\pi} & \quad F_{2u}^{\nu\text{MD}+\pi}
\end{align*}
\]

Constituent Up Quark

\( Q^2 \) (GeV\(^2\))
Constituent Down Quark Results

Themes
- NJL model
- Baryons ...
- $N \rightarrow \Delta$ FFs

Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- Delta FFs
- $N \rightarrow \Delta$ FFs
- Off-Shell
- Conclusion

![Graph showing Constituent Down Quark](image-url)

Q$^2$ (GeV$^2$) vs. Constituent Down Quark
Nucleon Form Factors

- Form Factor Feynman diagrams

\[ \gamma^\mu \, F_1(q^2) + \frac{i \sigma^{\mu\nu} q_{\nu}}{2M} \, F_2(q^2) \]

- Approach is completely covariant
- No frame is assumed
- Charge is conserved automatically
- Diagrams are expressed in form:

\[ \langle J^\mu \rangle = \bar{u}_N(p') \left[ \gamma^\mu \, F_{1N}(Q^2) + \frac{i \sigma^{\mu\nu} q_{\nu}}{2M_N} \, F_{2N}(Q^2) \right] u_N(p) \]
Scalar Diquark & Pion Form Factors

- Themes
- NJL model
- Baryons . . .
- $N \rightarrow \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- Delta FFs
- $N \rightarrow \Delta$ FFs
- Off-Shell
- Conclusion

Scalar diquark: $(\gamma_5 C \tau_2 \beta A^I) (C^{-1} \gamma_5 \tau_2 \beta A)$, Pion: $(\tau \gamma_5)$

- Form Factor expressions same: $g_\pi \leftrightarrow g_s$, $m_\pi \leftrightarrow M_s$

- Two form factors in general

\[
\langle J^{\mu}_{\pi} \rangle = (p' + p)^{\mu} F_{\pi}(Q^2) + (p' - p)^{\mu} F_{\pi}^{OS}(Q^2) \rightarrow 0
\]

- Result charge radius: $\langle r_E^2 \rangle_\pi = 0.46 \text{ fm}^2$

- Experiment: $\langle r_E^2 \rangle_\pi = 0.45 \pm 0.01 \text{ fm}^2$
Pion Form Factor

\[ F_\pi(Q^2) = \left[1 + Q^2 / \Lambda^2\right]^{-1} \quad \Lambda^2 = 0.5 \text{GeV}^2 \]

- No pion dressing on quarks \(\leftrightarrow\) \(\rho\) excitation
### Pion Form Factor

#### Themes
- NJL model
- Baryons
- N → Δ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- Delta FFs
- N → Δ FFs
- Off-Shell
- Conclusion

#### Plot

![Graph showing pion form factor](image)

- $F_{\pi}^{\text{bare}}(Q^2)$
- $F_{\pi}^{\text{vmd}}(Q^2)$
- $Q^2 F_{\pi}(Q^2)$
- Empirical Experiment

#### References

#### Formulas
- $Q^2 F_{\pi}(Q^2) \rightarrow 16\pi f_{\pi}^2 \alpha_s(Q^2)$
- $\alpha_{NJL} = 0.94$ ↪ $Q^2 \sim 0.46 \text{ GeV}^2$
Axial-Vector Diquark & Rho Form Factors

- AV diquark: \((\gamma^\beta C \tau_i \tau_2 \beta A')(C^{-1} \gamma^\alpha \tau_2 \tau_j \beta A)\),  \(\text{Rho: } (\tau_j \gamma^\mu)\)
- Form Factor expressions same: \(g_\rho \leftrightarrow g_\alpha\), \(m_\rho \leftrightarrow M_a\)
- 3 on shell form factors

\[
J_\rho^\mu = \left[ g^{\alpha\beta} F_1(Q^2) - \frac{q^\alpha q^\beta}{2M_a^2} F_2(Q^2) \right] (p+p')^\mu - \left( q^\alpha g^{\mu\beta} - q^\beta g^{\mu\alpha} \right) F_3(Q^2)
\]

- Sachs Form Factors

\[
G_C(Q^2) = F_1(Q^2) + \frac{2}{3} \frac{Q^2}{4M^2} G_Q(Q^2), \quad G_M(Q^2) = F_3(Q^2)
\]

\[
G_Q(Q^2) = F_1(Q^2) + \left( 1 + \frac{Q^2}{4M^2} \right) F_2(Q^2) - F_3(Q^2)
\]

- NJL Results: \(\langle r_E^2 \rangle_\rho = 0.52\), \(\mu_\rho = 2.08\), \(Q_\rho = -0.52\)
- DSE Results: \(\langle r_E^2 \rangle_\rho = 0.54\), \(\mu_\rho = 2.01\), \(Q_\rho = -0.41\)

\[\downarrow\]

Axial-Vector Diquark Form Factors

- Themes
- NJL model
- Baryons . . .
- $N \rightarrow \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- Delta FFs
- $N \rightarrow \Delta$ FFs
- Off-Shell
- Conclusion

For large $Q^2$ we find $G_Q \rightarrow G_E$. 

![Graph showing Axial Diquark Form Factors](image)
Form Factor Feynman diagrams

\[ \gamma^\mu F_1(q) + \frac{i \sigma^{\mu\nu} q_\nu}{2M} F_2(q) \]

Diagrams are expressed in form:

\[ \langle J^{\mu} \rangle = \overline{u}_N(p') \left[ \gamma^\mu F_{1N}(Q^2) + \frac{i \sigma^{\mu\nu} q_\nu}{2M_N} F_{2N}(Q^2) \right] u_N(p) \]
Proton Form Factors: Results

- Themes
  - NJL model
  - Baryons . . .
  - $N \rightarrow \Delta$ FFs
  - Constituents
  - Nucleon FFs
  - Scalar Diquark FF
  - Axial-Vector FF
- Nucleon FFs
- Delta FFs
- $N \rightarrow \Delta$ FFs
- Off-Shell
- Conclusion


- **NJL**: $\kappa_p = 1.77$,  
  **Experiment**: $\kappa_p = 1.79$

- **NJL**: $\langle r_E^2 \rangle_p = 0.58 \text{ fm}^2$,  
  **Experiment**: $\langle r_E^2 \rangle_p = 0.72 \text{ fm}^2$

- **NJL**: $\langle r_M^2 \rangle_p = 0.56 \text{ fm}^2$,  
  **Experiment**: $\langle r_M^2 \rangle_p = 0.71 \text{ fm}^2$

- **NJL$_{\text{bare}}$**: $\kappa_p = 1.61$, $\langle r_E^2 \rangle_p = 0.36 \text{ fm}^2$, $\langle r_M^2 \rangle_p = 0.38 \text{ fm}^2$

- Need extra $\sim 1/Q^2$ factor $\leftrightarrow$ Static Approximation
Neutron Form Factors: Results

Themes
- NJL model
- Baryons . . .
- $N \to \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF

Nucleon FFs
- Delta FFs
- $N \to \Delta$ FFs
- Off-Shell
- Conclusion

NJL: $\kappa_n = 1.79$, Experiment: $\kappa_n = 1.91$

NJL: $\langle r^2_E \rangle_n = -0.15$ fm$^2$, Experiment: $\langle r^2_E \rangle_n = -0.12$ fm$^2$

NJL: $\langle r^2_M \rangle_n = 0.54$ fm$^2$, Experiment: $\langle r^2_M \rangle_n = 0.79$ fm$^2$

bare: $\kappa_n = -1.46$, $\langle r^2_E \rangle_n = -0.07$ fm$^2$, $\langle r^2_M \rangle_p = 0.38$ fm$^2$

Need extra $\sim 1/Q^2$ factor $\leftrightarrow$ Static Approximation
**Delta Form Factors**

- **Form Factor Feynman diagrams**

  ![Feynman diagrams](image)

- **Only axial-vector diquarks**

- **completely covariant, current conservation, no frame**

- **Diagrams are expressed in form:**

  \[
  \Gamma^{\mu,\alpha\beta} = H_1^{\alpha\beta,\mu} a_1 + H_2^{\alpha\beta,\mu} a_2 + H_3^{\alpha\beta,\mu} c_1 + H_4^{\alpha\beta,\mu} c_2
  \]

- **Multipole Form Factors:** \( G_{E0}, G_{E2}, G_{M1}, G_{M3} \)
Form Factors Results

- Themes
- NJL model
- Baryons...
- $N \rightarrow \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- Delta FFs
- $N \rightarrow \Delta$ FFs
- Off-Shell
- Conclusion

### $\Delta^{++}$ Form Factors Results

- $G_{E0\Delta^{++}}(Q^2)$
- $G_{E2\Delta^{++}}(Q^2)$
- $G_{M1\Delta^{++}}(Q^2)$
- $G_{M3\Delta^{++}}(Q^2)$

![Graphs of Form Factors](#)
\[ G_{\Delta^+} \sim \frac{1}{2} G_{\Delta^{++}} \]
Form Factors with Lattice Results

Themes
- NJL model
- Baryons
- $N \to \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs

Delta FFs
- $N \to \Delta$ FFs
- Off-Shell
- Conclusion

Delta Moments

- Largely unexplored experimentally: Summary
  - $\mu_{\Delta^{++}} = 4.52 \pm 0.51 \pm 0.45 \mu_N \text{ or } \mu_{\Delta^{++}} = 6.14 \pm 0.51 \mu_N$
    - Bosshard, 1991
    - Lopez Castro, 2002
  - $\mu_{\Delta^+} = 2.7^{+1.0}_{-1.3} \text{(stat.)} \pm 1.5 \text{(syst.)} \pm 3 \text{(theory)} \mu_N$
    - Kotulla, 2002

- Chiral Extrapolated Lattice QCD
  - Cloët, 2003
  - $\mu_{\Delta^{++}} = 4.99 \pm 0.56 \mu_N$, $\mu_{\Delta^+} = 2.49 \pm 0.29 \mu_N$
  - $\mu_{\Delta^0} \sim 0.06 \mu_N$, $\mu_{\Delta^-} = 2.45 \pm 0.27 \mu_N$

- NJL results ($\mu_N$)
  - Pion: $\mu_{\Delta^{++}} = 5.59$, $\mu_{\Delta^+} = 2.60$, $\mu_{\Delta^0} = -0.38$, $\mu_{\Delta^-} = -3.36$
  - Bare: $\mu_{\Delta^{++}} = 5.39$, $\mu_{\Delta^+} = 2.61$, $\mu_{\Delta^0} = 0.0$, $\mu_{\Delta^-} = -2.61$

- Lattice: $\mu_{\Delta^+} < \mu_p$, NJL agrees.
**N → Δ Transition Form Factors**

- **Form Factor Feynman diagrams**

- **Necessary ingredients are in place**

- **Scalar and axial-vector diquarks**

- **completely covariant, current conservation, no frame**

- **Diagrams are expressed in form:**

\[
J^\mu = \bar{u}_\Delta,\alpha \left[ H_1^{\alpha\mu} G_1(Q^2) + H_2^{\alpha\mu} G_2(Q^2) + H_3^{\alpha\mu} G_3(Q^2) \right] u_N
\]

- **Jones & Scadron multipole form factors:** \( G_E, G_M, G_C \)
Form Factor Results: $G_M$

- Themes
- NJL model
- Baryons . . .
- $N \rightarrow \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- Delta FFs
- $N \rightarrow \Delta$ FFs
- Off-Shell
- Conclusion


- Pion playing very important role

- We have a lot of missing strength at $G_M(Q^2) \sim 0.$
Form Factor Results: $G_E$ & $G_C$

- Themes
- NJL model
- Baryons . . .
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- Form factors are small
- Pion cloud playing important large role
- Significant pion cloud effects for $G_E$ at large $Q^2$. 
Form Factor Ratios

- Themes
- NJL model
- Baryons . . .
- $N \rightarrow \Delta$ FFs
- Constituents
- Nucleon FFs
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- Conclusion

- $R_{EM}$ is roughly a fact 10 too large
- $G_E$ is very sensitive to cancellations between diagrams
  - compare neutron charge radius
- Nice result for $R_{SM}$
  - Data from MAMI, LEGS, MIT-Bates and JLab
What’s Missing

- Need Faddeev calculation without static approx.
- Detailed description of pion cloud
- Need diagrams like:

\[ \text{important to quantify these effects} \]

- Possible to calculate diagrams self-consistently in NJL
Off-Shell Effects

- Largely unexplored in literature
- Relax constraint: $p'^2 = p^2 = M^2$.
- Very difficult, or impossible, in many model approaches
  - Dynamical model in terms of elementary d.o.f.
  - Solve dynamical equations.
- Potentially important: in-medium quark distributions, form factors, etc
- Pion has two off-shell form factors

\[
j^\mu_\pi = (p'^\mu + p^\mu) \ F_{\pi,1} + (p'^\mu - p^\mu) \ F_{\pi,2}
\]

- For $p'^2 = p^2 = M^2$, we have $F_{\pi,2} = 0$ & $F_{\pi,1} \to F_{\pi}$. 

Largely unexplored in literature

Relax constraint: $p'^2 = p^2 = M^2$.

Very difficult, or impossible, in many model approaches

Dynamical model in terms of elementary d.o.f.

Solve dynamical equations.

Potentially important: in-medium quark distributions, form factors, etc

Pion has two off-shell form factors

\[
j^\mu_\pi = (p'^\mu + p^\mu) \ F_{\pi,1} + (p'^\mu - p^\mu) \ F_{\pi,2}
\]

For $p'^2 = p^2 = M^2$, we have $F_{\pi,2} = 0$ & $F_{\pi,1} \to F_{\pi}$. 

- Define $\delta = p'^2 - p^2$, put final state on-shell, i.e. $p'^2 = m^2_\pi$
- Effects maybe large
- Off-shell effects may be important for experimental extraction of $N \rightarrow \Delta$ transition.
- Also for in-medium form factors
Conclusion & Outlook

- **Themes**
  - NJL model
  - Baryons . . .
  - $N \rightarrow \Delta$ FFs
  - Constituents
  - Nucleon FFs
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  - $N \rightarrow \Delta$ FFs
  - Off-Shell

**Conclusion**

- **Covariant, Confining, Faddeev formalism:**
  - Form Factors, Quark Distributions, etc

- **Good description of “$Q^2 = 0$” observables**
  - e.g. magnetic moments, quark distributions
  - has been long standing problem

- **Transition form factors require detailed understanding of pion contributions**
  - Important challenge for quark models

- **Size of off-shell effects may be studied**

- **Results so far suggest further investigation warranted.**
Gap Equation

Themes
- NJL model
- Baryons ...
- \( N \rightarrow \Delta \) FFs
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- Nucleon FFs
- Delta FFs
- \( N \rightarrow \Delta \) FFs
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- Conclusion

\begin{equation}
\frac{1}{p - m + i\epsilon} \rightarrow \frac{1}{p - M + i\epsilon}
\end{equation}

- Self-consistent solution – gives Quark Propagator

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{Dynamical Quark Mass (MeV) vs. \( G/G_{\text{crit}} \) for different quark masses.}
\end{figure}
$F_1$ Form Factor Diagrams

- Themes
- NJL model
- Baryons . . .
- $N \rightarrow \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- Delta FFs
- $N \rightarrow \Delta$ FFs
- Off-Shell
- Conclusion
Δ⁰ Form Factors Results

Themes
- NJL model
- Baryons . . .
- N → Δ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- Delta FFs
- N → Δ FFs
- Off-Shell
- Conclusion
Form Factors Results

- Themes
- NJL model
- Baryons . . .
- $N \rightarrow \Delta$ FFs
- Constituents
- Nucleon FFs
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- Axial-Vector FF
- Nucleon FFs
- Delta FFs
- $N \rightarrow \Delta$ FFs
- Off-Shell
- Conclusion
Pion Cloud contribution surprisingly small

NJL model consistent with this result

Lattice: $\mu_{\Delta^+} < \mu_p$, NJL agrees.

Nucleon quark distributions

- Associated with a Feynman diagram calculation.

\[ q(x), \Delta q(x), \Delta_T q(x) \]

\[ \rightarrow X = \delta(x - \frac{k^+}{p^+}) [\gamma^+, \gamma^+\gamma_5, \gamma^+\gamma^1\gamma_5] \]

- Satisfies baryon and momentum sum rules.
- Satisfies positivity constraints and Soffer bound.
- Covariant and gives correct support
- Model testing ground
$u_v(x)$ and $d_v(x)$ distributions

Themes
- NJL model
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$Q_0^2 = 0.16 \text{ GeV}^2$

$Q^2 = 5.0 \text{ GeV}^2$

MRST (5.0 GeV$^2$)

$\Delta u_v(x)$ and $\Delta d_v(x)$ distributions

Themes
- NJL model
- Baryons . . .
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\( \Delta_{T}u_{v}(x) \) and \( \Delta_{T}d_{v}(x) \) distributions

- Themes
  - NJL model
  - Baryons ...
  - From \( N \rightarrow \Delta \) FFs
  - Constituents
  - Nucleon FFs
  - Scalar Diquark FF
  - Axial-Vector FF
  - Nucleon FFs
  - Delta FFs
  - From \( N \rightarrow \Delta \) FFs
  - Off-Shell
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- Non-relativistic limit: \( \Delta_{T}q(x) = \Delta q(x) \)

- \( M \sim 400 \) MeV, large relat. corrections unexpected

- Potential problem for models based concept of “constituent quarks” – maybe running mass
Transversity: Reanalysis

Themes
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$Q^2 = 2.4 \text{ GeV}^2$

Anselmino et al DIS 08

Transversity Moments

- Themes
- NJL model
- Baryons...
- $N \rightarrow \Delta$ FFs
- Constituents
- Nucleon FFs
- Scalar Diquark FF
- Axial-Vector FF
- Nucleon FFs
- Delta FFs
- $N \rightarrow \Delta$ FFs
- Off-Shell
- Conclusion

![Graph showing models comparison](image)

- Anselmino et al DIS 08