Covariant models of Nucleon and $\Delta$
$N - N^*$ Transition Form Factors Workshop

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4. NΔ transition
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5. Conclusions
Covariant quark model to work at high $Q^2$ regime
Motivation

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- Can we describe the Nucleon Elastic form factor data with a simple model? [simple $\equiv$ S-wave] Yes
Motivation

- **Covariant quark model** to work at high $Q^2$ regime
- Can we describe the Nucleon Elastic form factor data with a **simple** model?  
  [simple $\equiv$ S-wave ]  **Yes**
- Can we extend the model (S-wave) to heavier baryons ($\Delta$)?
Motivation

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**Motivation**

- Covariant quark model to work at high $Q^2$ regime
- Can we describe the Nucleon Elastic form factor data with a simple model? [simple $\equiv$ S-wave] Yes
- Can we extend the model (S-wave) to heavier baryons ($\Delta$)? Almost
- Can we include systematically high angular momentum states? …
Constituent Quark Model view

- Quark dressed by gluons and $q\bar{q}$ interactions
- Gluon interactions between $q\bar{q}$ ⇒ quark form factors
- Quarks with anomalous magnetic moments $\kappa_u, \kappa_d$
- Nucleon FF can be explained without high angular momentum components

Light Front view

- Baryon states as a sum of Fock states:
  $qqq, qqqg, qqq(q\bar{q})$, ...
- Pointlike quarks
- No anomalous magnetic moments $\kappa_u, \kappa_d = 0$
- High angular momentum required to explain $\kappa_N \neq 0$
Formalism (Wave functions)

Construction of a baryon wave function:

\[ \text{Baryon} = \text{quark} \oplus \text{diquark} \]

- Non Relativistic structure; baryon rest frame: \( P = 0 \)  
  \( \Rightarrow \text{Relativistic form} \)
- Consider a boost in the \text{z-direction} \  
  fixed-axis polarization states
- Initial and final state wave functions defined in a \text{collinear frame} \  
  diquark constraint
- Arbitrary Lorentz transformation \( \Lambda \)  
  \( \Rightarrow \text{wave function defined in an arbitrary frame} \)
Construction of a baryon wave function:

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$\Rightarrow$ **Axial diquark** with **positive parity**: S or D; NO P-states  
$\Rightarrow$
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Construction of a baryon wave function:

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- **Arbitrary** Lorentz transformation \( \Lambda \)
  \( \Rightarrow \) wave function defined in an **arbitrary** frame

\( \Rightarrow \) Axial diquark with **positive parity**: S or D; NO P-states
\( \Rightarrow \) All states satisfies the **Dirac equation**
Spectator Quark Model

Hadronic current

\[ J^\mu = 3 \sum_\lambda \int_k \bar{\psi}_f(P_+, k)j^\mu_f \psi_i(P_-, k) \]

Quark current

\[ j^\mu_i = j_1 \left( \gamma^\mu - \frac{q_\mu q^\mu}{q^2} \right) + j_2 \frac{i\sigma^{\mu\nu}q_\nu}{2M} \]

\[ j_i = \frac{1}{6} f_{i+} + \frac{1}{2} f_{i-\tau_3} \]

Vector Meson Dominance quark ff

Two poles: \( m_v = m_\rho \), \( M_h \sim 2M \)

\( \kappa_\pm \) fixed by \( G_M(0) \)

3-4 parameter to adjust

Nucleon $J = 1/2$: superposition of mixed symmetry states:

$$\psi_N = \frac{1}{\sqrt{2}} \left[ \Phi_I^0 \Phi_S^0 + \Phi_I^1 \Phi_S^1 \right] \psi_N(P, k)$$

$\Phi_I^I$: isospin; $\phi_S^{Sz}$: spin; $\psi_N$ scalar wave function [PRC 77, 015202 (2008)]
S-state Wave Functions

- **Nucleon** $J = 1/2$: superposition of mixed symmetry states:

  $$\psi_N = \frac{1}{\sqrt{2}} \left[ \Phi_I^0 \Phi_S^0 + \Phi_I^1 \Phi_S^1 \right] \psi_N(P, k)$$

  $\Phi_I^I$: isospin; $\phi_S^{sz}$: spin; $\psi_N$ scalar wave function [PRC 77, 015202 (2008)]

- **Delta** $J = 3/2$: pure symmetric states

  $$\psi_\Delta = \Phi_I^1 \Phi_S^1 \psi_\Delta(P, k)$$

  $\psi_\Delta$: $\Delta$ scalar wave function [EPJ A36, 329 (2008)]
N and $\Delta$ spin wave functions

$$\{ \Phi^1_s, \bar{\Phi}^1_s \} \implies \Phi_S(\lambda, \lambda_s) \quad S = 1/2, 3/2$$

$\lambda =$ diquark polarization; $\lambda_s =$ N or $\Delta$ spin projections

$$\Phi_{1/2}(\lambda, \lambda_s) = - (\varepsilon^*_{\lambda P})_{\alpha} V_{1/2}^{\alpha}(P, \lambda_s)$$  
$$\Phi_{3/2}(\lambda, \lambda_s) = - (\varepsilon^*_{\lambda P})_{\alpha} V_{3/2}^{\alpha}(P, \lambda_s) [RS]$$

3-quark spin state given by ($B = N, \Delta$):

$$V_{S}^{\alpha}(P, \lambda_s) = \sum_{\lambda} \langle \frac{1}{2}\lambda; 1\lambda' | S_{\lambda s} \rangle \varepsilon^\alpha_{\lambda P} u_B(P, \lambda)$$

$$\varepsilon^\alpha_{\lambda P} = \text{fixed-axis polarization states}$$
Diquark polarization states

- Helicity states defined in terms of the \( \mathbf{k} = (E_k, k \sin \theta, 0, k \cos \theta) \)

\[ \varepsilon_k(\lambda) \] dependent of \( \theta \)

- Fixed-axis: vector particle is bound to a system with \( \mathbf{P} = (P_0, 0, 0, P) \):

\[
\varepsilon(0) = \frac{1}{M} \begin{bmatrix} P \\ 0 \\ 0 \\ P_0 \end{bmatrix}, \quad \varepsilon(\pm) = \mp \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{bmatrix}
\]

\( \rightarrow \) wave functions with No angular dependence;

Description of Elastic data with a S-wave [Rest frame]

\[ \psi_N = \frac{N_0}{(\beta_1 + (P-k)^2) (\beta_2 + (P-k)^2)} \]

Few parameters - - - Model II (3+2)
No explicit pion cloud ... but VMD
N\Delta transition: S-state


- S-states:

\[ G_E^* = G_C^* = 0 \]

\[ G_M^*(Q^2) = \frac{4}{3\sqrt{3}} \frac{M}{M + M_\Delta} f_v \int \psi_\Delta \psi_N \]
$N\Delta$ transition: S-state


- **S-states:**
  
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- **Cauchy-Schwartz inequality** \( \Rightarrow G_M^*(0) \leq 2.07 \)

  Spectator QM can explain only 70% of the experimental \( G_M^*(0) \)
**NΔ transition: S-state**


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- **Cauchy-Schwartz inequality** $\Rightarrow G_M^*(0) \leq 2.07$
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  - Characteristic of Constituent Quark models
N$\Delta$ transition: S-state


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  Spectator QM can explain only 70% of the experimental \( G_M^*(0) \)

- **Characteristic of Constituent Quark models**
  
  \( G_M^* = G_M^{\text{Bare}} + G_M^\pi \)

- **Quark Model** \( \Rightarrow G_M^{\text{Bare}} \)
  
  **Dynamical Model** \( \Rightarrow G_M^\pi \) (Sato-Lee & DMT)
NΔ transition: $G_M^*$ (Bare + Total)
N\Delta transition: $G_M^*$ (Bare + Total)

Bare data: Diaz et al, PRC 75, 015205 (2007)
N$\Delta$ transition: $G_M^*$ (Bare predictions)

![Graph showing data and model predictions for $G_M^*/(3G_D)$ vs $Q^2(\text{GeV}^2)$]

- **Data**
- **Bare Data**
- **Model II (Bare)**

Bare data from Diaz et al
$N\Delta$ transition: $G^*_M$ (Full predictions)
N$\Delta$ transition: S-state (Valence + Pion cloud)

**Valence Quarks (Bare):**
\[ G_M^B \] set the scale
\[ \psi_\Delta = \frac{N_\Delta}{(\alpha_1 + (P-k)^2)(\alpha_2 + (P-k)^2)^2} \]

**Sea quarks (Pion Cloud):**
\[ \frac{G_M^\pi}{3G_D} = \lambda_\pi \left( \frac{\Lambda_\pi^2}{\Lambda_\pi^2 + Q^2} \right)^2 \]
N$\Delta$ transition: High Angular Momentum States

Core spin = quark spin + diquark spin

\[ S = S_q + S_{dq} \Rightarrow \begin{cases} V_{1/2}^{\alpha}(P, \lambda_s) \\ V_{3/2}^{\alpha}(P, \lambda_s) \end{cases} \]

\[ V_S^{\alpha}(P, \lambda_s) = \sum_\lambda \langle \frac{1}{2}; 1 \lambda' | S \lambda_s \rangle \epsilon_{\lambda' P}^{\alpha} u_B(P, \lambda) \quad [S\text{-states}] \]

Total angular momentum \( J = 3/2 \):

\[ J = L + S \Rightarrow \begin{cases} S \quad (0, \frac{3}{2}) \\ D3 \quad (2, \frac{3}{2}) \\ D1 \quad (2, \frac{1}{2}) \end{cases} \]
**N\Delta transition: D-states** \((L = 2)\)

**D-state operator:**

\[
D^{\alpha\beta} = \tilde{k}^\alpha \tilde{k}^\beta - \frac{\tilde{k}^2}{3} \left( g^{\alpha\beta} - \frac{P^\alpha P^\beta}{M_B^2} \right) \\
\approx Y_2^m \text{ (Rest frame)}
\]

**Core-spin projectors**

\[
P_{1/2}^{\alpha\beta} + P_{3/2}^{\alpha\beta} = g^{\alpha\beta} - \frac{P^\alpha P^\beta}{M_B^2} \xrightarrow{NR} -\delta^{ij}
\]

[M. Benmerrouche et al PRC 39, 2339 (1989)]

**D-state:**

\[
W_D^\alpha = D_\beta^{\alpha}(P, k) V_{3/2}^\beta(P) \leftarrow \text{S-state} \\
= (P_{1/2})_{\beta}^{\alpha} W_D^\beta + (P_{3/2})_{\beta}^{\alpha} W_D^\beta \\
\left\{ \begin{array}{c}
D1\text{–state} \\
D3\text{–state}
\end{array} \right. 
\]
NΔ transition: States vs Form Factors

Simple current

\[ J^\mu = 3j_1 \sum_\lambda \int_k \bar{\Psi}_\Delta \gamma^\mu \psi_N + 3j_2 \sum_\lambda \int_k \bar{\Psi}_\Delta \frac{i \sigma^{\mu\nu} q^\nu}{2M} \psi_N \]

Modified current

\[ J_R^\mu = 3j_1 \sum_\lambda \int_k \bar{\Psi}_\Delta \left( \gamma^\mu - \frac{\dot{q} q^\mu}{q^2} \right) \psi_N + 3j_2 \sum_\lambda \int_k \bar{\Psi}_\Delta \frac{i \sigma^{\mu\nu} q^\nu}{2M} \psi_N \]

Equivalent prescriptions if \( \sum_\lambda \int_k \bar{\Psi}_\Delta \psi_N = 0 \) (all \( Q^2 \))

[orthogonal states]

Discuss \( S, D3 \) and \( D1 \) states
NΔ transition: States S and D3

States \((0, \frac{3}{2})\) and \((2, \frac{3}{2})\) are orthogonal to \((0, \frac{1}{2}) \equiv N\)

Current:

\[
J^\mu = 3j_1 \sum_\lambda \int_k \bar{\Psi}_\Delta \gamma^\mu \psi_N + 3j_2 \sum_\lambda \int_k \bar{\Psi}_\Delta \frac{i\sigma^{\mu\nu} q_\nu}{2M} \psi_N
\]

Using the Dirac equation:

\[
q_\mu J^\mu = 3j_1 \sum_\lambda \int_k \bar{\Psi}_\Delta \partial^I \psi_N = 3(M_\Delta - M) j_1 \sum_\lambda \int_k \bar{\Psi}_\Delta \psi_N
\]

Current conserved

It can also be shown that

\[
q_\mu J^\mu \propto G^*_C(Q^2)
\]

Conclusion: S and D3 states \(\Rightarrow G^*_C = 0\)
NΔ transition: State D1

State \((2, \frac{1}{2})\) is not orthogonal to \((0, \frac{1}{2})\)

In principle: 
\[ q_\mu J^\mu = 3(M_\Delta - M) \sum_\lambda \int_k \bar{\Psi}_\Delta \psi_N \neq 0. \]

There is a chance that \(G_C^* \neq 0\); but \(q_\mu J^\mu \neq 0\)

Imposing current conservation

\[ J^\mu_R = 3j_1 \sum_\lambda \int_k \bar{\Psi}_\Delta \left( \gamma^\mu - \frac{\partial q^\mu}{q^2} \right) \psi_N + 3j_2 \sum_\lambda \int_k \bar{\Psi}_\Delta \frac{i\sigma^{\mu\nu} q_\nu}{2M} \psi_N \]

\[ q_\mu J^\mu_R = 0, \quad G_C^* \propto \frac{1}{Q^2} \sum_\lambda \int_k \bar{\Psi}_\Delta \psi_N \]

To avoid divergence as \(Q^2 \rightarrow 0\):

\[ \sum_\lambda \int_k \bar{\Psi}_\Delta \psi_N \sim Q^2 \quad [\text{Orthogonality}] \]
N\Delta transition (S+ D states)

Adding all angular momentum components:

Configuration: \((L, S)\)

\[
\begin{align*}
\Psi_N & \rightarrow \Psi_\Delta \\
S (0, \frac{3}{2}) & \rightarrow G_M^* \\
D3 (2, \frac{3}{2}) & \rightarrow G_M^*, G_E^* \\
D1 (2, \frac{1}{2}) & \rightarrow \bar{G}_M^*, \bar{G}_E^*, G_C^*
\end{align*}
\]

\(\bar{G}_M^*, \bar{G}_E^* = 0 \quad \text{when} \quad Q^2 = 0\)
### N$\Delta$ transition: S+D3+D1

#### S-state

<table>
<thead>
<tr>
<th>Term</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_M^S$</td>
<td>$4\eta I_S$</td>
</tr>
<tr>
<td>$G_E^S$</td>
<td>$0$</td>
</tr>
<tr>
<td>$G_C^S$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

$$I_S = \int_k \phi_N \phi_S$$

$$\eta = \frac{2}{3\sqrt{3}} \frac{M}{M + M_\Delta} f_v$$

$$f_v = f_{1-} + \frac{2M}{M + M_\Delta} f_{2-}$$

$$f_C = f_{1-} - \frac{Q^2}{2M(M + M_\Delta)} f_{2-}$$

#### D3-state

<table>
<thead>
<tr>
<th>Term</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_M^{D3}$</td>
<td>$-2\eta I_{D3}$</td>
</tr>
<tr>
<td>$G_E^{D3}$</td>
<td>$-2\eta I_{D3}$</td>
</tr>
<tr>
<td>$G_C^{D3}$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

#### D1-state

<table>
<thead>
<tr>
<th>Term</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_M^{D1}$</td>
<td>$\eta I_{D1}$</td>
</tr>
<tr>
<td>$G_E^{D1}$</td>
<td>$-\eta I_{D1}$</td>
</tr>
<tr>
<td>$G_C^{D1}$</td>
<td>$\frac{4MM_\Delta}{\sqrt{3}} f_C \frac{I_{D1}}{Q^2}$</td>
</tr>
</tbody>
</table>

$$I_{D1} = \int_k b \phi_N \phi_{D1}$$

$$b \approx \sqrt{\frac{4\pi}{5}} k^2 Y_2(\hat{k})$$

Orthogonality between Nucleon (S-state) and $\Delta$ D1 state:

$$I_{D1} \sim Q^2 \text{ as } Q^2 \rightarrow 0$$
\( R_{EM} = - \frac{G_E^*(Q^2)}{G_M^*(Q^2)}, \quad R_{SM} = - \frac{|q|_{\Delta} G_C^*(Q^2)}{2M_{\Delta} G_M^*(Q^2)} \)
\[ R_{EM} = - \frac{G_E^*(Q^2)}{G_M^*(Q^2)}, \]
\[ R_{SM} = - \frac{|q|_\Delta G_C^*(Q^2)}{2M_\Delta G_M^*(Q^2)} \]

Valence Quark insufficient to explain \( G_C^* \) data
\[ R_{EM} = - \frac{G_E^*(Q^2)}{G_M^*(Q^2)}, \quad R_{SM} = -\frac{|q|_\Delta G_C^*(Q^2)}{2M_\Delta G_M^*(Q^2)} \]

Valence Quark insufficient to explain $G_C^*$ data

⇒ Include Sea Quark effects [Pion Cloud]
N\Delta transition: Pion Cloud - Simple Model

Pion Cloud effects in $G_{E}^*$ and $G_{C}^*$?

Large $N_c$ limit, low $Q^2$:

$$G_{C}^\pi(Q^2) = \sqrt{\frac{2M}{M_\Delta}} MM_{\Delta} \frac{G_{En}(Q^2)}{Q^2}$$

$$G_{E}^\pi(Q^2) = \left(\frac{M}{M_\Delta}\right)^{3/2} \frac{M^2_\Delta - M^2}{2\sqrt{2}} \frac{G_{En}(Q^2)}{Q^2}$$

[Buchmann et al; Pascalutsa and Vanderhaeghen]

No adjustable parameters

Nucleon: Pion Cloud $\Rightarrow G_{En} \neq 0$

N\Delta: $G_{C}^*, G_{E}^* \propto G_{En}$: represents Pion Cloud
N\Delta transition (S+D3+D1): Pion Cloud
N\Delta transition (S+D3+D1): Valence Quarks

\begin{align*}
\text{Valence Pion Cloud} \\
\text{Valence} & \quad \text{Pion Cloud}
\end{align*}
$N\Delta$ transition (S+D3+D1): Valence Q + Pion Cloud
N\Delta transition (S+D3+D1): Valence Q + Pion Cloud (2)

Model consistent with MAID analysis of CLAS data (Jlab)
Drechsel et. al., EPJA 34, 69 (2007)
Pion cloud dominant
Pion cloud dominant

Small D-state mixture improves the description of the data
(1% D3-state; 4% D1-state)
**NΔ transition (S+D3+D1): Valence Q + Pion Cloud**

- **Pion cloud dominant**
- **Small D-state mixture improves the description of the data**
  (1% D3-state; 4% D1-state)

Limitations?
Does it make any sense to use the pion cloud parametrization for $Q^2 \sim 3$ GeV$^2$?

Conclusion:
We need to consider consistent description of the pion cloud.
N\Delta transition (S+D3+D1): Valence parametrization

Is the valence quark model appropriated (for $G_C^*$)?

The answer depends of the contribution of the pion cloud
But... valence QM calibrated by the data
Can we trust in the data?
NΔ transition: Data analysis (low $Q^2$)

Is the Data consistent at low $Q^2$?
Is the Data consistent at low $Q^2$?
NΔ transition: Data analysis (high $Q^2$)

It is important to know the $Q^2$ dependence of the data for high $Q^2$.

![Graph showing $R_{SM}$ vs $Q^2$ for different models: Valence, Pion Cloud, MAID analysis, and Predictions for CLAS.](graph.png)
Quark models
- Consistent (correct normalization)
- Incomplete (no pion cloud)
N\Delta transition: comparing QM with DM

- **Quark models**
  - Consistent (correct normalization)
  - Incomplete (no pion cloud)

- **Dynamical Models**
Quark models
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- Hadrons as degrees of freedom (pion included)
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Quark Models should be used as input of Dynamical Models
$G_M^B$ dominates over $G_M^\pi \Rightarrow$ QM should be used as input
Conclusions

- Covariant spectator S-state wave functions for N and $\Delta$
  - Explains Nucleon data
  - Main contribution of N$\Delta$ ($\sim 60\%$ of $G_M^*$)
  - Consistent with CQM and Dynamical Models
Conclusions

- Covariant **spectator S-state** wave functions for N and Δ
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- **D-states in Δ**: NΔ transition
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  - Predicts non-zero contributions for E2 and C2
    Insufficient to explain data
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    - [Valence Quarks D-states improves description of $G_C^*$]
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  - Clarify the model dependence of the Data analysis
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  - Quark Models should be used as input of Dynamical Models
GR, M.T. Peña and F. Gross,

**D-state effects in the electromagnetic NΔ transition**

to be submitted ... !!!!
References

- GR, M.T. Peña and F. Gross, D-state effects in the electromagnetic NΔ transition to be submitted ... !!!!


