Extraction of the properties of nucleon resonances by means of a Genetic Algorithm

César Fernández-Ramírez

Center for Theoretical Physics and Laboratory for Nuclear Science
Massachusetts Institute of Technology

In collaboration with: E. Moya de Guerra (UCM), A. Udías (URJC), J.M. Udías (UCM)
Our pion photoproduction model

- Nucleons, pions, photons [Born terms]
- Vector mesons ($\rho$ and $\omega$)
- Nucleon resonances
  - Up to 1.8 GeV
  - Up to spin-3/2
  - $\Delta(1232)$, $\Delta(1620)$, and $\Delta(1700)$
  - N(1440), N(1520), N(1535), N(1650), and N(1720)

Fernández-Ramírez, Moya de Guerra, Udías, AP(NY) 321 (2006) 1408

The underlying physics is embedded in the constants of the model → obtained fitting the data
Optimization

- Gradient-based routines are the usual optimization tools (MINUIT, NAG)

  CERN, MINUIT 95.03, CERN Library D506 Edition, 1995
  Numerical Algorithms Group Ltd., http://www.nag.co.uk

- In complex optimization problems these routines may get easily stuck in local minima
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- Alternative: Stochastic optimization → Genetic algorithms

- Example: E04FCF from NAG by itself is useless for our problem: **gradient based methods alone fail**
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- Hybrid optimization: combine GA with gradient based routine E04FCF from NAG libraries

  GA + E04FCF → Parameters

- GA provides E04FCF the initial value
Evolution (optimization)

- Evolution as an optimization scheme
- The different kinds of species evolve to the optimal adaptation to the surrounding environment. Thus, evolution is an ’algorithm’ that searches for the best solution creating a set of individuals (a generation), it decides which individuals are the best ones, and, by means of crossover, keeps the good genetic characteristics for the next generation – that will be closer to the optimal solution – and removes the individuals with worst genetic content
Biology ↔ GA

Environment ↔ Objective function (v.g. $\chi^2$)
Individual ↔ Set of parameters
Generation (set of individuals) ↔ Set of possible solutions
How a GA works

- We start with a first generation randomly generated ($N$ individuals)
- Each individual encodes a complete set of parameters
  
  \[
  G_E \, G_M \, M_\Delta \, \ldots
  \]
  
  each parameter is a "gene"
- Scale population, v.g. using the $\chi^2$, to assign a survival and mating probability to each individual
- Generate the offspring (fight, crossover, and mutation of individuals)
- Repeat process until a given number of generations is reached
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Simulate evolution in a computer!
• We perform several optimizations to obtain a set of minima
Particularities of GAs

- Whereas most methods employ a single solution which evolves to reach the local optimum, GAs work on a population of many possible solutions simultaneously.

- GAs only need the objective function to determine how fit an individual is. Neither derivatives nor other auxiliary knowledge are required.

- GAs use probabilistic rules to evolve (randomness does not mean directionless!)
Evolution of the optimization

Evolution of the $\chi^2$ normalized to the final $\chi^2$
The best individual of each generation is plotted
We have performed several optimizations (20), for each run (x-axis) we get a different minimum (y-axis). We normalize all of the minima to the best one and we plot the minima given by the GA alone and the improvement achieved by the NAG routine for each run.
$\Delta(1232)$ parameters and E2/M1 ratio

![Plot of $A^{\Delta}_{1/2}$ vs. $A^{\Delta}_{3/2}$]
Δ(1232) parameters and E2/M1 ratio
$\Delta(1700)$ parameters and model and database effects
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2005 SAID database
Model up to 1 GeV

2006 SAID database
Model up to 1.2 GeV
\( \Delta(1700) \) tail fully covered
Fits to electromagnetics multipoles
GAs in experimental nuclear physics at JLab

- Hall A experiment E06-007: "Impulse approximation limitations to the \((e, e'p)\) on \(^{208}\text{Pb}\), identifying correlations and relativistic effects in the nuclear medium"
- Optics calibration using GAs
- Allows to get all the parameters of the optics database at the same time
- More efficient procedure (unattended optimization)

J.L. Herraiz, PhD Thesis (UCM, expected in 2009)
Conclusions (I)

- Optimization is not a trivial problem
- Traditional optimization tools are often useless for this kind of multi-parameter optimizations when the parameter space is large and the function to fit presents many local minima
- If the parameters of a resonance want to be assessed its energy range has to be fully covered, tail included. If not, misleading results may be obtained
Conclusions (II)

- The hybrid optimization procedure presented in this talk is a powerful and versatile optimization tool that can be applied to many problems in physics that involve the determination of a set of parameters from data.

- It is a promising method for extracting both reliable physical parameters as well as their confidence intervals, probably more meaningful than the simple covariance matrices returned by gradient based optimization routines.

- Not only the error bars have to be considered when quoting the uncertainty in the determination of a parameter, but also whether the minima are concentrated into one single region or split into several ones, and the possible physical implications of such situation.
References

- **GAs:**
  Fernández-Ramírez, Moya de Guerra, Udías, Udías, PRC 77 (2008) 065212

- **Pion photoproduction model:**

- **Extension to nuclei:**
  Fernández-Ramírez, Martínez, Vignote, Udías, PLB 664 (2008) 57

  RPWIA asymmetry prediction for $^{16}$O($\gamma,\pi^- p$) compares well to data from Hicks et al. PRC 61 (2000) 054609