Neutron and Proton Structure Functions and Duality

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Overview

Quark-hadron duality: non-trivial phenomenon (see Wally’s talk)

- Manifestation: insight into the dynamic of strong interactions
  Standard tests of quark-hadron duality
- Application: could be used to access kinematic regions otherwise inaccessible
  Use averaged resonance region data to constrain PDFs at large $x$?

Experimental tests of quark-hadron duality in:

- proton $F_2^p$ structure function
- neutron $F_2^n$ structure function

New method: extract $F_2^n$ from nuclear $F_2$

- Application of method to smooth curves
- Application of method to data + Quark-Hadron Duality in $F_2^n$
- Application of method to data: (a lot of) technical details
  S.P. Malace, Y. Kahn, W. Melnitchouk, in preparation

Plans for future
On average, the resonance region data **mimic** the twist-2 pQCD calculation (pQCD + Target Mass corrections)

This happens at a surprisingly low $Q^2$

“**The successful application of duality to extract known quantities suggests that it should also be possible to use it to extract quantities that are otherwise kinematically inaccessible.**”

(CERN Courier, 2004)

**Quark-Hadron Duality: needs to be verified and quantified**
Basic test of Duality: the $Q^2$ behavior of averaged resonance region data when compared to QCD calculations

Example: integrals over RES Region ($W^2 < 4 \text{ GeV}^2$); comparison of data to MRST+TM

Y. Liang et al., nucl-ex/0410027 (2004)

→ 2004: agreement better than 5% at $Q^2 = 0.5 \text{ GeV}^2$ but ~ 18% at $Q^2 = 3.5 \text{ GeV}^2$

→ 2009: deviation of data from MRST+TM increases with $Q^2$ up to $Q^2 \sim 4.5 \text{ GeV}^2$ then saturates
Kinematics: with increasing $Q^2$ resonances slide in regions of larger and larger $x$

PDFs (CTEQ, MRST, MSTW) poorly constrained at large $x$
It is not surprising then:

--- though RES data **DO** average to MSTW08+TM at $Q^2 = 0.9\ \text{GeV}^2$, $x \sim (0.25,0.7)$

--- RES data **DO NOT** average to MSTW08+TM at $Q^2 = 6.4\ \text{GeV}^2$, $x \sim (0.7,0.95)$

Not a violation of duality but very likely due to an underestimation of large-$x$ strength in the pQCD parametrization
Tests of Quark-Hadron Duality at large $x$

What should we use for quantitative tests of Duality at large $x$?

- **Leading Twist (LT) calculations** ↔ PDFs constrained up to $x \sim 0.65 - 0.7$: CTEQ, MRST (MSTW)...

- **Calculations beyond LT** ↔ PDFs constrained up to $x \sim 0.8 - 0.9$

**Alekhin et al.**


**CTEQ6X**

Accardi, Christy, Keppel, Melnitchouk, Monaghan, Morfín, Owens, Phys. Rev. D 81, 034016 (2010)

Accardi et al., in preparation
Tests of Quark-Hadron Duality at large $x$

Resonance region data average to the QCD (beyond LT) calculation

Quantitative Tests of Local Duality

1) Delimit W regions for duality tests

<table>
<thead>
<tr>
<th>Region</th>
<th>$W^2_{\text{min}}$</th>
<th>$W^2_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>1.3</td>
<td>1.9</td>
</tr>
<tr>
<td>2nd</td>
<td>1.9</td>
<td>2.5</td>
</tr>
<tr>
<td>3rd</td>
<td>2.5</td>
<td>3.1</td>
</tr>
<tr>
<td>4th</td>
<td>3.1</td>
<td>3.9</td>
</tr>
<tr>
<td>DIS</td>
<td>3.9</td>
<td>4.5</td>
</tr>
</tbody>
</table>

$x = Q^2 / (W^2 + Q^2 - M^2)$

2) $F_2$ from data and QCD calculation

3) Calculate:

$$\int_{x_m}^{x_M} F_2^{\text{data}}(x, Q^2) \, dx$$

$$\int_{x_m}^{x_M} F_2^{\text{QCD calc.}}(x, Q^2) \, dx$$
Quark-Hadron Duality in Proton $F_2^p$

$$\int_{x_m}^{x_M} F_2^{p,\text{data}}(x,Q^2)dx \Bigg/ \int_{x_m}^{x_M} F_2^{p,\text{param}}(x,Q^2)dx$$

QCD calculation of Alekhin

S. I. Alekhin, JETP Lett. 82, 628 (2005).

• Within 10%: globally, 4th, 3rd, 2nd
• 1st: special case
  • some models predict stronger violations of duality
  • calculation based on handbag diagram may break at such low $W$
  • sits at the largest $x$ (QCD fits poorly constrained) => difficult to test duality

**Quark-Hadron Duality in Neutron $F_2^n$**

- **Verify quark-hadron duality in $F_2^n$**
  - Need $F_2^n$ in the resonance region...
  - Could use proton $F_2^p$ and deuteron $F_2^d$ and

**New method to extract $F_2^n$ from $F_2^p$ and $F_2^d$: iterative procedure of solving integral convolution equations**

**Impulse Approximation:**

$$F_2^A(x, Q^2) = \sum_{N=p,n}^{M_A/M} dy f_0^{N/A}(y, \gamma) F_2^N \left( \frac{x}{y}, Q^2 \right)$$

- nuclear $F_2$
- light-cone momentum distribution of nucleons in nucleus (smearing function)
- nucleon $F_2$

Smearing Function for $F_2^d$

Smearing function evaluated in the weak binding approximation, including finite-$Q^2$ corrections


\[
\sqrt{1 + \frac{4M^2}{Q^2} x^2}
\]
We need $F^{n}_2$ from:

$$\tilde{F}^{n}_2 = F^d_2 - F^d_2(QE) - \delta^{(off)} F^d_2 - \tilde{F}^p_2$$

$$\tilde{F}^{n,p}_2 = \int_{x}^{M_d/M} dy f(y, \gamma) F^{n,p}_2 \left( \frac{x}{y} \right)$$

**Additive extraction method:** solve equation iteratively

Additive extraction method: solve equation iteratively

$f(y, \gamma) = N \delta(y - 1) + \delta f(y, \gamma)$

Normalization of smearing function

$\tilde{F}^{n}_2(x) = NF^{n}_2(x) + \int_{x}^{M_d/M} dy \delta f(y, \gamma) F^{n}_2 \left( \frac{x}{y} \right)$

Perturbation

$perturbation$

$F^{n(1)}_2(x) = \left[ F^{n(0)}_2(x) \right] + \frac{1}{N} \left[ \tilde{F}^{n}_2(x) - \int_{x}^{M_d/M} dy f(y, \gamma) F^{n(0)}_2 \left( \frac{x}{y} \right) \right]$
Application of Method to Smooth Curves

- Monotonic curves: $F_2^p$ and $F_2^n$ input from MRST; $F_2^d$ is simulated using the finite-$Q^2$ smearing function
  - Additive method applied with initial guess $F_2^{n(0)} = 0$

Fast convergence: extracted $F_2^{n(1)}$ almost indistinguishable from $F_2^n$ input after only 1 iteration (smearing function sharply peaked around $y = 1$)

Application of Method to Smooth Curves

Curves with resonant structures: $F_2^n$ input from MAID

- Additive method applied with initial guess $F_2^{n(0)} = 0$

After 1 or 2 iterations: resonant peaks clearly visible; after 5 iterations extracted result very close to “true” result

Application of Method to Smooth Curves

Essential to take into account $Q^2$ effects in the smearing function

- Additive method ($F_2^{n(0)} = 0$): $Q^2$-dependent smearing function and $Q^2$-independent smearing function

After 10 iterations: extraction with $Q^2$-dependent smearing function converges to the input; extraction with $Q^2$-independent smearing function does not

Application of Method to Data

Use proton and deuteron data at fixed $Q^2$ (matched kinematics)

$$
\tilde{F}_2^n(x) = F_2^d(x) - F_2^{d(QE)} - \delta^{(off-shell)}(x) F_2^d(x) - \tilde{F}_2^p(x)
$$

Data:

**SLAC** at $Q^2 = 0.6, 0.9, 1.7, 2.4$ GeV$^2$

**JLab** (Hall C E00-116) at $Q^2 = 4.5, 5, 5.5, 6.2, 6.4$ GeV$^2$

- data at fixed $Q^2$ =>
  bin-centering at cross section level using 2 different models

---

Application of Method to Data

\[ \tilde{F}_2^n(x) = F_2^d(x) - F_2^{d(QE)} - \delta^{(\text{off-shell})} F_2^d(x) - \tilde{F}_2^p(x) \]

QE: extracted from data using model (form factors + smearing function)

Off-shell corrections:
- upper limit from model \(~1.5\%
- subtract \(\frac{1}{2}\) of model prediction
- assign 100\% uncertainty to correction
- contributes \(< 2\%\) to total uncertainty on \(F_2^n\)

**Application of Method to Data**

- $F_2^n$ extraction: initial guess $F_2^{n(0)} = F_2^p$; number of iterations = 2

- $F_2^n$ in resonance region: 3 resonant enhancements (fall with $Q^2$ at ~ rate as for $F_2^p$)

- $F_2^d$ reconstructed from $F_2^p$(data) and $F_2^n$(extraction) ~ $F_2^d$(data) after 2 iterations

Study dependence of result on number of iterations: compare extractions with 2 and 3 iterations

Small change in $F_2^n$ between iteration 2 and 3

Extracted $F_2^n$ changes to bring $F_2^d$ reconstructed closer to $F_2^d$ data; small differences between iteration 2 and 3

$\frac{(F_2^n (\text{it.}=2) - F_2^n (\text{it.}=3))}{\sigma_{F_2^n}}$

$\frac{(F_2^d (\text{data}) - F_2^d (\text{recon.}))}{\sigma_{F_2^d}}$

S.P. Malace, Y. Kahn, W. Melnitchouk, in preparation
Application of Method to Data

- Study dependence of result on initial guess $F_2^{n(0)}$: compare $F_2^n$ extracted with 2 different inputs for initial guess: $F_2^{n(0)} = F_2^p$ vs $F_2^{n(0)} = F_2^p / 2$

- After 2 iterations: only 6% of all data lay outside a $2\sigma$ range

- Exercise caution with number of iterations: irregularities in data result in increased scattered in $F_2^n$ with increasing number of iterations

S.P. Malace, Y. Kahn, W. Melnitchouk, in preparation
Comparison to BoNuS Data

- Plots by Nathan Baillie

BoNuS: $F_2^n$ data

MALACE: $F_2^n$ extracted from $F_2^p$ and $F_2^d$

BOSTED

Thanks to Nate, Sebastian 😊 and the BoNuS Collaboration
Comparison: data to ABKM
S. Alekhin, J. Blumlein, S. Klein, S. Moch.

- 2nd and 3rd RES regions: agreement within 15-20%, on average

- 1st RES region: agreement worsens at the highest $Q^2$ (corresponds to the largest $x$)

- globally remarkable agreement: within 10%

S.P. Malace, Y. Kahn, W. Melnitchouk,
Confirmation of duality in both proton and neutron \( \Rightarrow \) phenomenon not accidental but a general property of nucleon structure functions

\[
\int F_2(\text{data}) dx / \int F_2(\text{QCD} = \text{Alekhin}) dx
\]


Use averaged resonance region data \((W^2 > 1.9 \text{ GeV}^2)\) to extend PDFs extraction to the largest \(x\)
Quark-Hadron Duality: Application

- **Stage 1 (last few decades):** LT calculations ⇔ PDFs constrained up to $x \sim 0.7$ (CTEQ, MRST(MSTW), GRV, etc.)

- **Stage 2 (last decade):** calculations beyond LT ⇔ PDFs constrained up to $x \sim 0.8-0.9$
  - Alekhin *et al.*
  - CTEQ6X
    - Accardi *et al.*, in preparation

- **Stage 3:** future
A. Accardi, S.P. Malace, in preparation

\[
\int_{x_m}^{x_M} F_2^{p,\text{data}}(x, Q^2) dx \bigg/ \int_{x_m}^{x_M} F_2^{p,\text{param}}(x, Q^2) dx
\]

- Study sensitivity of quark-hadron duality ratios to various prescriptions for inclusion of:
  - Higher Twist: additive vs multiplicative; HT(proton) same or different than HT(neutron)
  - Target Mass Corrections: OPE, CF...
  
  etc.

\[
W^2 = (1.3, 1.9) \text{ GeV}^2
\]

\[
W^2 = (1.9, 2.5) \text{ GeV}^2
\]

\[
W^2 = (2.5, 3.1) \text{ GeV}^2
\]

\[
W^2 = (3.1, 3.9) \text{ GeV}^2
\]
A. Accardi, S.P. Malace, in preparation

Study applicability of QCD calculation at low values of $W$; criterion: separation between target jet and current jet
Extend proton and deuteron $F_2$ structure function precision measurements to larger $x$ and $Q^2$ in the resonance region and beyond up to $W^2 \sim 9 \text{ GeV}^2$, $Q^2 \sim 17 \text{ GeV}^2$ and $x \sim 0.99$. 

![Graphs showing $M_{p,d}(\text{data})/M_{p,d}(\text{CTEQ6L})$ versus $Q^2$ for different $W^2$ values.](image_url)
Extra Slides
$Q^2: (4.52, 7.38) \text{ GeV}^2$

- E00-116
- CTEQX
- Alekhin09