Moments of parton distribution functions from lattice QCD

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lattice calculation of $\langle x \rangle^{u-d}$ close to physical pion mass by QCDSF

there appears to be a real puzzle in the momentum fraction

QCDSF LAT2010
Outline

- provocation
  - hadronic matrix elements more broadly
  - just a few details about lattice calculations of moments of PDFs
  - axial coupling, lowest moment of polarized distribution
  - average $x$, lowest non-trivial moment of unpolarized distribution
  - $\langle x^2 \rangle$, next moment of unpolarized distribution
- outlook
Hadronic matrix elements

- moments of PDFs are just one component of a very big effort that
- includes form factors (FFs) and generalized parton distributions (GPDs)
- also transition FFs and transition GPDs, $N \rightarrow N^*$ and $N \rightarrow \Delta$
- not just nucleon but also pion, delta and other hadrons
Moments of parton distributions

- the $x$ dependence in principle follows from an operator definition

$$q(x, \mu^2) = \frac{1}{2} \int \frac{dy^-}{2\pi} e^{ixp^+y^-} \langle p, s| \bar{q}(-y^-/2) \gamma^+ q(y^-/2) |p, s\rangle|_{\mu^2}$$

- light-cone expansion relates moments in $x$,

$$\langle x^n \rangle_{q, \mu^2} = \int_{-1}^{1} dx x^n q(x, \mu^2) = \int_{0}^{1} dx x^n \{q(x, \mu^2) - (-1)^n \bar{q}(x, \mu^2)\}$$

- to nucleon matrix elements of local operators

$$\langle p, s| \bar{q} \gamma^{\mu_1} iD^{\mu_2} \cdots iD^{\mu_n} q |p, s\rangle|_{\mu^2} = 2\langle x^n \rangle_{q, \mu^2} p^{\{\mu_1 \cdots \mu_n\}}$$

- bare matrix elements and renormalization are calc. non-perturbatively
High $x$

- high moments $n$ are dominated by high $x$ (i.e. $x \to \pm 1$)

\[ \langle x^n \rangle_{q,\mu^2} = \int_{-1}^{1} dx \ x^n q(x, \mu^2) \]

- the operators for low moments are multiplicatively renormalizable

\[
\bar{q} \, \gamma^{\{\mu_1 \ iD\mu_2 \ldots \ iD\mu_n\}}_{q} \bigg|_{\overline{MS},\mu} = Z_n(\mu a) \ \bar{q} \, \gamma^{\{\mu_1 \ iD\mu_2 \ldots \ iD\mu_n\}}_{q} \bigg|_{\text{lat},a}
\]

- reduced spatial symmetries allows mixing for higher moments

\[
\mathcal{O}_i^{\overline{MS},\mu} = \sum_j Z_{ij}(\mu a) \ a^{-(n+2-d_j)} \ \mathcal{O}_j^{\text{lat},a}
\]

- power divergent mixing for high $n$ and low dimension $d_j$
• axial coupling presents one of the "better" nucleon observables

• mild $m_\pi$ dependence can still reconcile lattice and exp. result
• $\langle x \rangle$ is currently a real challenge for lattice QCD

- scatter between lattice results hints at various problems
Momentum fraction again

- Flatness of $\langle x \rangle$ has been a problem since early quenched calculations.

- Lack of any $m_\pi$ dependence seems to preclude the use of chiral p.t.
• calc. at a single lattice spacing shows near agreement in $\langle x^2 \rangle_{u-d}$

• mild cutoff effects could easily account for this small discrepancy
Outlook

- QCDSF $\langle x \rangle^{u-d}$ is showing a strikingly flat behavior down to almost the physical pion mass but overestimates the results from global fits.

- Other moments, $g_A$ and $\langle x^2 \rangle$ shown as examples, are also very flat but the discrepancies are typically less severe than for $\langle x \rangle$.

- In the near term, expect careful studies of $a \to 0$, $L \to \infty$ and $m_\pi \to 140$ MeV and other systematics (already underway).

- Precise reproduction of well-measured nucleon properties will bolster confidence in a rich program of hadron structure from lattice QCD.