GPDs and DVCS with Positrons

Matthias Burkardt
burkardt@nmsu.edu

New Mexico State University & Jefferson Lab
probing GPDs in DVCS

Ji-relation

GPDs: probabilistic interpretation as Fourier transforms of impact parameter dependent PDFs

\[ H(x, 0, -\Delta^2_{\perp}) \rightarrow q(x, b_{\perp}) \]
\[ \tilde{H}(x, 0, -\Delta^2_{\perp}) \rightarrow \Delta q(x, b_{\perp}) \]
\[ E(x, 0, -\Delta^2_{\perp}) \]
\[ \rightarrow \perp \text{ deformation of unpol. PDFs in } \perp \text{ pol. target} \]

more on DVCS \rightarrow GPDs

Summary
Generalized Parton Distributions (GPDs)

GPDs: decomposition of form factors at a given value of $t$, w.r.t. the average momentum fraction $x = \frac{1}{2} (x_i + x_f)$ of the active quark

\[
\int dx H_q(x, \xi, t) = F_1^q(t) \quad \int dx \tilde{H}_q(x, \xi, t) = G_A^q(t)
\]

\[
\int dx E_q(x, \xi, t) = F_2^q(t) \quad \int dx \tilde{E}_q(x, \xi, t) = G_P^q(t),
\]

$x_i$ and $x_f$ are the momentum fractions of the quark before and after the momentum transfer

$2\xi = x_f - x_i$

GPDs can be probed in deeply virtual Compton scattering (DVCS)
\[ \sigma = |A_{BH} + A_{DVCS}|^2 = |A_{BH}|^2 + |A_{DVCS}|^2 + 2 \Re \{A_{BH} A^*_{DVCS}\} \]

\[ \Im A_{DVCS}(\xi, t) \sim \int_{-1}^{1} dx \frac{GPD(x, \xi, t)}{x - \xi} \]

\[ \Re A_{DVCS}(\xi, t) \sim GPD(\xi, \xi, t) \] from beam spin asymmetry

\[ \Longleftrightarrow \text{clean separation of real part with beam charge asymmetry (} e^+ \text{ v. } e^- \) \]
Generalized Parton Distributions (GPDs)

- formal definition (unpol. quarks):

\[
\int \frac{dx^-}{2\pi} e^{ix^-p^+x} \left< p' \left| \bar{q} \left( -\frac{x^-}{2} \right) \gamma^+ q \left( \frac{x^-}{2} \right) \right| p \right> = H(x, \xi, \Delta^2) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, \Delta^2) \bar{u}(p') \frac{i\sigma^+ \Delta^\nu}{2M} u(p)
\]

- in the limit of vanishing \( t \) and \( \xi \), the nucleon non-helicity-flip GPDs must reduce to the ordinary PDFs:

\[
H_q(x, 0, 0) = q(x) \quad \tilde{H}_q(x, 0, 0) = \Delta q(x).
\]

- DVCS amplitude

\[
A(\xi, t) \sim \int_{-1}^{1} \frac{dx}{x - \xi + i\varepsilon} GPD(x, \xi, t)
\]
Interesting observation: X. Ji, PRL 78, 610 (1997)

\[ \langle J_q \rangle = \frac{1}{2} \int_0^1 dx \ x \ [H_q(x, 0, 0) + E_q(x, 0, 0)] \]

\[ \text{DVCS} \iff \text{GPDs} \iff \vec{J}_q \]

lattice QCD (LHPC, QCDSF)

- \( L^u + L^d \approx 0 \)  
  (disconnected diagrams?)
- \( L^u - L^d < 0 \)

But: what other “physical information” about the nucleon can we obtain by measuring/calculating GPDs?
### Form Factors vs. GPDs

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<th>position space</th>
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<td>$Q$</td>
<td>$F(t)$</td>
<td>$\rho(\vec{r}^*)$</td>
</tr>
<tr>
<td>$\int \frac{dx^- e^{ix^+ x^-}}{4\pi} \bar{q}\left(-\frac{x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right)$</td>
<td>$q(x)$</td>
<td>$H(x, \xi, t)$</td>
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## Form Factors vs. GPDs

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\[
\int \frac{dx^- e^{ix\cdot p^+} x^-}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right) = q(x)
\]

$q(x, b_\perp) = \text{impact parameter dependent PDF}$
Impact parameter dependent PDFs

- define $\perp$ localized state [D.Soper, PRD 15, 1141 (1977)]

$$|p^+, R_{\perp} = 0_{\perp}, \lambda\rangle \equiv N \int d^2 p_{\perp} |p^+, p_{\perp}, \lambda\rangle$$

Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has

$$R_{\perp} \equiv \frac{1}{p^+} \int dx^- d^2 x_{\perp} x_{\perp} T^{++}(x) = \sum_i x_i r_i = 0_{\perp}$$

(cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

$$q(x, b_{\perp}) \equiv \int \frac{dx^-}{4\pi} \langle p^+, R_{\perp} = 0_{\perp} | \bar{q} \left( -\frac{x^-}{2}, b_{\perp} \right) \gamma^+ q \left( \frac{x^-}{2}, b_{\perp} \right) | p^+, R_{\perp} = 0_{\perp} \rangle e^{ixp^+x^-}$$

$$q(x, b_{\perp}) = \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} e^{i\Delta_{\perp} \cdot b_{\perp}} H(x, 0, -\Delta_{\perp}^2),$$

$$\Delta q(x, b_{\perp}) = \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} e^{i\Delta_{\perp} \cdot b_{\perp}} \tilde{H}(x, 0, -\Delta_{\perp}^2),$$
Impact parameter dependent PDFs

- No relativistic corrections (Galilean subgroup!)
- Corollary: interpretation of 2d-FT of $F_1(Q^2)$ as charge density in transverse plane also free from relativistic corrections
- $q(x, b_\perp)$ has probabilistic interpretation as number density ($\Delta q(x, b_\perp)$ as difference of number densities)
- Reference point for IPDs is transverse center of (longitudinal) momentum $R_\perp \equiv \sum_i x_i r_{i,\perp}$
- For $x \to 1$, active quark ‘becomes’ COM, and $q(x, b_\perp)$ must become very narrow ($\delta$-function like)
- $H(x, 0, -\Delta^2_\perp)$ must become $\Delta_\perp$ indep. as $x \to 1$ (MB, 2000)
- Consistent with lattice results for first few moments
- Note that this does not necessarily imply that ‘hadron size’ goes to zero as $x \to 1$, as separation $r_\perp$ between active quark and COM of spectators is related to impact parameter $b_\perp$ via $r_\perp = \frac{1}{1-x} b_\perp$. 
Transversely Deformed Distributions and $E(x, 0, -\Delta^2 \perp)$


- So far: only unpolarized (or long. pol.) nucleon! In general ($\xi = 0$):

$$
\int \frac{dx^-}{4\pi} e^{ip^+ x^- x} \langle P+\Delta, \uparrow | \bar{q}(0) \gamma^+ q(x^-) | P, \uparrow \rangle = H(x, 0, -\Delta^2 \perp)
$$

$$
\int \frac{dx^-}{4\pi} e^{ip^+ x^- x} \langle P+\Delta, \uparrow | \bar{q}(0) \gamma^+ q(x^-) | P, \downarrow \rangle = -\frac{\Delta_x - i\Delta_y}{2M} E(x, 0, -\Delta^2 \perp).
$$

- Consider nucleon polarized in $x$ direction (in IMF)

$|X\rangle \equiv |p^+, R_\perp = 0_\perp, \uparrow \rangle + |p^+, R_\perp = 0_\perp, \downarrow \rangle$.

$\rightarrow$ unpolarized quark distribution for this state:

$$
q(x, b_\perp) = \mathcal{H}(x, b_\perp) - \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} E(x, 0, -\Delta^2 \perp) e^{-ib_\perp \cdot \Delta_\perp}
$$

- Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from $j^3$!

[X.Ji, PRL 91, 062001 (2003)]
Intuitive connection with $\vec{J}_q$

- DIS probes quark momentum density in the infinite momentum frame (IMF). Quark density in IMF corresponds to $j^+ = j^0 + j^3$ component in rest frame ($\vec{p}_{\gamma^*}$ in $-\hat{z}$ direction)

- $j^+$ larger than $j^0$ when quark current towards the $\gamma^*$; suppressed when away from $\gamma^*$

- For quarks with positive orbital angular momentum in $\hat{x}$-direction, $j^z$ is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side

- Details of $\perp$ deformation described by $E_q(x, 0, -\Delta_\perp^2)$

- Not surprising that $E_q(x, 0, -\Delta_\perp^2)$ enters Ji relation!

$$\langle J^i_q \rangle = S^i \int dx \left[ H_q(x, 0, 0) + E_q(x, 0, 0) \right] x.$$
Transversely Deformed PDFs and $E(x, 0, -\Delta^2_\perp)$

- $q(x, b_\perp)$ in $\perp$ polarized nucleon is deformed compared to longitudinally polarized nucleons!

- mean $\perp$ deformation of flavor $q$ ($\perp$ flavor dipole moment)

$$d^q_y \equiv \int dx \int d^2b_\perp q_X(x, b_\perp)b_y = \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa^p_q}{2M}$$

with $\kappa^p_{u/d} \equiv F^{u/d}_2(0) = \mathcal{O}(1 - 2) \implies d^q_y = \mathcal{O}(0.2 \text{ fm})$

- simple model: for simplicity, make ansatz where $E_q \propto H_q$

  $$E_u(x, 0, -\Delta^2_\perp) = \frac{\kappa^p_u}{2} H_u(x, 0, -\Delta^2_\perp)$$

  $$E_d(x, 0, -\Delta^2_\perp) = \kappa^p_d H_d(x, 0, -\Delta^2_\perp)$$

with $\kappa^p_u = 2\kappa_p + \kappa_n = 1.673 \quad \kappa^p_d = 2\kappa_n + \kappa_p = -2.033$.

- Model too simple but illustrates that anticipated deformation is very significant since $\kappa_u$ and $\kappa_d$ known to be large!
IPDs on the lattice (Hägler et al.)

- lowest moment of distribution of unpol. quarks in $\perp$ pol. proton (left) and of $\perp$ pol. quarks in unpol. proton (right):
**GPD ↔ SSA (Sivers)**

- **example:** $\gamma p \rightarrow \pi X$

- $u, d$ distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign “determined” by $\kappa_u$ & $\kappa_d$

- Attractive FSI deflects active quark towards the center of momentum

- FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

- Correlation between sign of $\kappa^p_q$ and sign of SSA: $f_{1T}^{q} \sim -\kappa^p_q$

- $f_{1T}^{q} \sim -\kappa^p_q$ confirmed by HERMES data (also consistent with COMPASS deuteron data $f_{1T}^{u} + f_{1T}^{d} \approx 0$)
GPD $\leftrightarrow \bar g_2$ (twist-3 polarized DIS)

- $\sigma_{LT}$ in polarized DIS $\rightarrow g_1 + g_2$
- $g_1$ $\rightarrow$ spin fraction carried by quark spin
- $g_2$ (after subtracting ‘Wandzura-Wilzcek piece’) sensitive to quark gluon correlations $d_2 \sim \int dx x^2 \bar g_2(x)$

$(MB, arXiv 0810.3589)$ perpendicular color Lorentz force acting on quark in DIS from perpendicular polarized target

- perpendicular deformation of $q(x, b_\perp)$ provides intuitive explanation for sign of $d_2$
\[ \mathcal{A}_{DVCS}(\xi, t) \sim GPD(\xi, \xi, t) \]
\[ \mathcal{R}A_{DVCS} \sim \int dx \frac{GPD(x, \xi, t)}{x-\xi} \]

Dispersion relation \( \Rightarrow \mathcal{R}A_{DVCS} \sim \int dx \frac{GPD(x, x, t)}{x-\xi} + \Delta(t) \)

\( \Rightarrow \) In addition to information along diagonal \( x = \xi \) that is also available from \( \mathcal{A}_{DVCS}(\xi, t) \) \( \mathcal{R}A_{DVCS} \) provides access to
- GPDs along diagonal that is not kinematically accessible through \( \mathcal{A}_{DVCS}(\xi, t) \)
- ‘D-form factor’ \( \Delta \) (Polyakov Weiss)

Ji relation requires \( GPDs(x, \xi, t) \) for \(-1 < x < 1 \) at fixed \( \xi \)
- \( q(x, b_\perp) \) requires \( GPDs(x, 0, t) \)
Information away from diagonal \((x = \xi)\):

- \(\mathcal{R}A_{DVCS}\) (positrons!) \(\Rightarrow\) \(D\)-form factor
- Polynomiality condition: \(n\)-th Mellin moment of \(GPD(x, \xi)\)
  must be even polynomial in \(\xi\) of order \(n\)
- \(GPD(x, \xi)\) cannot depend on variables \(x\) and \(\xi\) completely independently
- \(Q^2\) evolution: changes \(x\) distribution in a known way for fixed \(\xi\)
- Double Deeply Virtual Compton Scattering \(D^2VCS\) (lepton pair instead of real photon in final state)
DVCS $\rightsquigarrow$ GPD\( (x, \xi, t) \)

- Example: dispersion relations/polynomiality $\Rightarrow$

\[
\int_{-1}^{1} dx \frac{H^{(+)}(x, 0, t)}{x} = \int_{-1}^{1} dx \frac{H^{(+)}(x, x, t)}{x} + \Delta(t)
\]

$\leftrightarrow$ DVCS allows access to same generalized form factor

\[
\int_{-1}^{1} dx \frac{H^{(+)}(x, 0, t)}{x}
\]

also available in WACS (wide angle Compton scattering), but \( t \) does not have to be of order \( Q^2 \)

$\leftrightarrow$ after flavor separation, comparing \( \frac{1}{F_{1}(t)} \int_{-1}^{1} dx \frac{H^{(+)}(x, 0, t)}{x} \)

at large \( t \) provides information about the 'typical \( x \)' that dominates large \( t \) form factor
Summary

- Deeply Virtual Compton Scattering ⇒ GPDs
- beam charge asymmetry ⇒ clean separation of $\Re A_{DVCS}$
- GPDs $\leftrightarrow^{FT}$ IPDs (impact parameter dependent PDFs)
- $H(x, 0, -\Delta_{\perp}^2) \xrightarrow{FT} q(x, b_{\perp})$ for unpolarized target
- $\Delta_{\perp} E(x, 0, -\Delta_{\perp}^2) \xrightarrow{FT} $ deformation of PDFs for $\perp$ polarized target
- $\kappa^{q/p} \Rightarrow$ sign of deformation
- attractive FSI ⇒ $f_{1T}^{uu} < 0$ & $f_{1T}^{dd} > 0$
- Interpretation of sign of $M^2 d_2 \equiv 3M^2 \int dx x^2 \bar{g}_2(x)$ as sign of $\perp$ force on active quark in DIS on $\perp$ polarized target
- $\Im A_{DVCS}$ only sensitive to GPDs$(\xi, \xi, t)$
- use $\Re A_{DVCS}$/polynomiality/dispersion relations/$Q^2$-evolution/DDVCS to get information on GPDs$(x, \xi, t)$ for $x \neq \xi$