Coulomb distortion in the inelastic regime

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Work done in collaboration with
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Coulomb distortion and two-photon exchange

**OPE**

\[ e \rightarrow e' \]

\[ p \rightarrow p' \]

**TPE**

Exchange of 2 (hard) photons with a single nucleon

Exchange of one or more (soft) photons with the nucleus, in addition to the one hard photon exchanged with a nucleon

Incident (scattered) electrons are accelerated (decelerated) in the Coulomb well of the nucleus.

Opposite effect with positrons
How to correct for Coulomb distortion?

\[ \sigma_{tot}^{PWBA} = \sigma_{Mott} \cdot S_{tot}^{PWBA}(|\vec{q}|, \omega, \theta) \]

- Focusing of the electron wave function
- Change of the electron momentum

Effective Momentum Approximation (EMA)


\[ E \rightarrow E + \vec{V} \]
\[ E_p \rightarrow E_p + \vec{V} \]

\[ \begin{align*}
Q_{eff}^2 &= 4(E + \vec{V})(E_p + \vec{V}) \sin^2(\frac{\theta}{2}) \\
S_{tot}^{PWBA}(|\vec{q}|, \omega, \theta) &\rightarrow S_{tot}^{PWBA}(|\vec{q}_{eff}|, \omega, \theta)
\end{align*} \]

1\textsuperscript{st} method

2\textsuperscript{nd} method

\[ S_{tot}^{PWBA}(|\vec{q}|, \omega, \theta) \rightarrow S_{tot}^{PWBA}(|\vec{q}_{eff}|, \omega, \theta) \]
\[ \sigma_{Mott}^{eff} = 4\alpha^2 \cos^2(\theta/2)(E_p + \vec{V})^2/Q_{eff}^4 \]
\[ F_{foc}^i = \frac{E + \vec{V}}{E} \]
\[ \sigma_{tot}^{CC} = (F_{foc}^i)^2 \cdot \sigma_{Mott}^{eff} \cdot S_{tot}^{PWBA}(|\vec{q}_{eff}|, \omega, \theta) \]
How to correct for Coulomb distortion?

\[ \sigma_{tot}^{PWBA} = \sigma_{Mott} \cdot S_{tot}^{PWBA}(|\vec{q}|, \omega, \theta) \]

**DWBA**
- Focusing of the electron wave function
- Change of the electron momentum

Effective Momentum Approximation (EMA)


\[ Q_{eff}^2 = 4(E + \bar{V})(E_p + \bar{V}) \sin^2\left(\frac{\theta}{2}\right) \]

1st method

2nd method

One-parameter model depending only on the effective potential seen by the electron on average.

\[ \sigma_{tot}^{CC} = \sigma_{Mott} \cdot S_{tot}^{PWBA}(|\vec{q}_{eff}|, \omega, \theta) \]

\[ \sigma_{tot}^{CC} = (F_{foc}^i)^2 \cdot \sigma_{Mott}^{eff} \cdot S_{tot}^{PWBA}(|\vec{q}_{eff}|, \omega, \theta) \]
Coulomb distortion measurements in quasi-elastic scattering

\[ \tilde{k} = k - V(z) \]

\[ V(r) = -\frac{3\alpha(Z - 1)}{2R} + \frac{\alpha(Z - 1)}{2R} \left( \frac{r}{R} \right)^2 \]

\[ R = 1.1A^{1/3} + 0.86A^{-1/3} \]


Gueye et al., PRC60, 044308 (1999)
Coulomb distortion measurements in quasi-elastic scattering

\[ \tilde{k} = k - V(z) \]

\[ V(r) = -\frac{3\alpha(Z-1)}{2R} + \alpha(Z-1) \left( \frac{r}{R} \right)^2 \]

\[ R = 1.1A^{1/3} + 0.86A^{-1/3} \]


Coulomb potential established in Quasi-elastic scattering regime!
Physics sensitive to Coulomb distortion

Coulomb distortion:
- Not accounted for in typical radiative corrections
- Usually, not a large effect at high energy machines
- Important for $E_p \ll E$

- x>1 experiments
- L/T experiments
- EMC effect
- Color transparency
- In-medium modification on the nucleon FF

About every experiments that used nuclei with $A>12$
Physics sensitive to Coulomb distortion

Coulomb distortion:
- Not accounted for in typical radiative corrections
- Usually, not a large effect at high energy machines
- Important for $E_p << E$

About every experiments that used nuclei with $A > 12$
The EMC effect

Nucleus at rest

(A nucleons = Z protons + N neutrons)

Effects found in several experiments at CERN, SLAC, DESY

\[ e^- \gamma^* \rightarrow Z + N \]

\[ \sigma_A/\sigma_D \]

EMC (Cu)
BCDMS (Fe)
E139 (Fe)
SLAC E139 results on the EMC effect

SLAC E139:

- Most complete data set: $A=4$ to 197
- Most precise at large $x$
  - $Q^2$-independent
  - universal shape
  - magnitude dependent on $A$
Effect of Coulomb distortion on JLab E03-103 results

A(e,e’) at 5.0 and 5.8 GeV in Hall C

- Targets: H, $^2$H, $^3$He, $^4$He, Be, C, Al, Cu, Au

JLab is at lower energy than SLAC but the luminosity is much higher.

We obtain similar or larger $Q^2$ values in many cases by going to larger angles such that $E_p$ is smaller.

So Coulomb distortion effects are 'doubly' amplified: lower beam energy and lower fractional $E_p$. 
E03-103 heavy target results

no Coulomb corrections applied

\[ \frac{\sigma_A}{\sigma_D} \]

- SLAC E139 (Au)
- JLab E03-103 (Au)

Au Norm. (2.0%)

SLAC Norm. (2.5%)

Preliminary
E03-103 heavy target results

Coulomb corrections applied

\( \frac{\sigma_A}{\sigma_D} \) vs. \( x \)

- **SLAC E139 (Au)**
- **JLab E03-103 (Au)**

- **Au Norm. (2.0%)**
- **SLAC Norm. (2.5%)**

Preliminary
Extrapolation to nuclear matter

Exact calculations of the EMC effect exist for light nuclei and for nuclear matter.

\[
\frac{\sigma_A}{\sigma_d}\text{ is } 0.8 \text{ at } x=0.7
\]

No Coulomb corrections applied

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Argonne National Laboratory
Extrapolation to nuclear matter

Exact calculations of the EMC effect exist for light nuclei and for nuclear matter.

\[ \frac{\sigma_A}{\sigma_d} \] 

\( x = 0.7 \)

Coulomb corrections applied

Non-negligible effects on SLAC data
Extrapolation to nuclear matter

Exact calculations of the EMC effect exist for light nuclei and for nuclear matter.
\[ R(x, Q^2) \]

\[
\frac{d\sigma}{d\Omega dE'} = \Gamma \left[ \sigma_T(x, Q^2) + \varepsilon \sigma_L(x, Q^2) \right]
\]

\[ R(x, Q^2) = \frac{\sigma_L(x, Q^2)}{\sigma_T(x, Q^2)} \]

TPE can affect the \( \varepsilon \) dependence (talk of E. Christy on Thursday)

Coulomb Distortion could have the same kind of impact as TPE, but gives also a correction that is A-dependent.

[Graph with data points and lines]
Meaning of $R$

\[
\frac{d\sigma}{d\Omega dE'} = \Gamma \left[ \sigma_T(x, Q^2) + \varepsilon \sigma_L(x, Q^2) \right]
\]

\[
R(x, Q^2) = \frac{\sigma_L(x, Q^2)}{\sigma_T(x, Q^2)}
\]

In a model with:

a) spin-1/2 partons: $R$ should be small and decreasing rapidly with $Q^2$

b) spin-0 partons: $R$ should be large and increasing with $Q^2$

Dasu et al., PRD49, 5641(1994)
FIG. 13. The fits to the differential cross section ratio $\sigma_A/\sigma_D$ versus $\epsilon' = \epsilon/(1 + R^D)$ are shown for each $(x, Q^2)$ point. The errors on the cross section include statistical and point-to-point systematic contributions added in quadrature.

Nuclear higher twist effects and spin-0 constituents in nuclei: same as in free nucleons

$\Rightarrow R_A - R_D$
Access to nuclear dependence of $R$

Dasu et al., PRD49, 5641 (1994)

A non-trivial effect in $R_A - R_D$ arises after applying Coulomb corrections.
Access to nuclear dependence of $R$

New data from JLab E03-103: access to lower $\varepsilon$

Iron-Copper

![Graph showing the access to nuclear dependence of $R$ for Iron-Copper with and without Coulomb corrections applied.](image-url)

For $x=0.5$,
- No Coulomb corrections applied:
  - $R_x - R_d = -0.0354318 \pm 0.0214851$
- Coulomb corrections applied:
  - $R_x - R_d = -0.0888527 \pm 0.0216907$
Access to nuclear dependence of $R$

New data from JLab E03-103: access to lower $\varepsilon$

Gold

No Coulomb corrections applied

Coulomb corrections applied

$R_x - R_0 = 0.0673445 \pm 0.0288021$

$R_x - R_0 = -0.0388513 \pm 0.0299763$
Access to nuclear dependence of $R$

After taking into account the normalization uncertainties from each experiment

Hint of an A-dependence in R in Copper-Iron
dependence of the Coulomb distortion

The $\varepsilon$-dependence of the Coulomb distortion has effect on the extraction of $R$ in nuclei
Summary

- At present, corrections for Coulomb distortion in inelastic regime are done using a prescription for quasi-elastic scattering regime
  - need a measurement of the amplitude of the effect in the inelastic regime
  - need a prescription in the inelastic regime

- Coulomb distortion affects the extrapolation to nuclear matter which is key for comparison with theoretical calculations

- Coulomb distortion has a real impact on the A-dependence of R: clear $\epsilon$ -dependence
  - hint of an A-dependence of R: could impact many experiments which used $R_P$ or $R_D$ for $R_A$
  - could change our conclusion on the spin-0 constituent contents and higher twist effect in nuclei versus free nucleons.
Back-ups
Nucleon only model

Assumptions on the nucleon structure function:
- not modified in medium
- the same on and off the energy shell

\[
\frac{F_2^A(x_A)}{A} = \int_{x_A}^A dy \cdot f_N(y) F_2^N(x_A/y)
\]

Fermi momentum \(\ll M_{\text{nucleon}}\)

\(\Rightarrow\) \(f_N(y)\) is narrowly peaked and \(y \approx 1\)

\[
\frac{F_2^A}{A} \approx F_2^N \Rightarrow \text{no EMC effect}
\]

“… some effect not contained within the conventional framework is responsible for the EMC effect.”

Smith & Miller, PRC 65, 015211 (2002)
Nucleons and pions model

Pion cloud is enhanced and pions carry an access of plus momentum:

\[ P^+ = P_N^+ + P_{\pi}^+ = M_A \]

and using \( P_{\pi}^+ / M_A = 0.04 \) is enough to reproduce the EMC effect.

But excess of nuclear pions \( \Rightarrow \) enhancement of the nuclear sea.

But this enhancement was not seen in nuclear Drell-Yan reaction.

Another class of models

Interaction between nucleons

Model assumption:
- Nucleon wavefunction is changed by the strong external fields created by the other nucleons

Model requirements:
- Momentum sum rule
- Baryon number conservation
- Vanishing of the structure function at $x<0$ and $x>A$
- Should describe the DIS and DY data
EMC effect in nuclear matter

No Coulomb corrections applied

\[ \frac{\sigma_A}{\sigma_D} \]

Sick & Day, PLB274 (1992)

All world data
EMC effect in nuclear matter

Coulomb corrections applied

Sick & Day, PLB274 (1992)
All world data (cc)
EMC effect in nuclear matter

Coulomb corrections applied

- Sick & Day, PLB274 (1992)
- All world data (cc)
- Including E03-103 prel. (cc)
- Smith & Miller, PRL91, 212301 (2003)
# World data re-analysis

<table>
<thead>
<tr>
<th>Experiments</th>
<th>E (GeV)</th>
<th>A</th>
<th>x-range</th>
<th>Pub. 1st author</th>
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<td>CERN-EMC</td>
<td>280</td>
<td>56</td>
<td>0.050-0.650</td>
<td>Aubert</td>
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<td>12,63,119</td>
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<td>SLAC-E61</td>
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<td>0.2-0.5</td>
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<td>DESY-HERMES</td>
<td>27.5</td>
<td>3,14,84</td>
<td>0.013-0.35</td>
<td>Airapetian</td>
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</tbody>
</table>
A or density dependence?

Density calculated assuming a uniform sphere of radius: $R_e (r=3A/4\pi R_e^3)$
The structure of the nucleon

Deep inelastic scattering: probe the constituents of the nucleon, i.e. the quarks and the gluons

4-momentum transfer squared

\[ Q^2 = -q^2 = 4EE'\sin^2 \frac{\theta}{2} \]

Invariant mass squared

\[ W^2 = M^2 + 2M\nu - Q^2 \]

Bjorken variable

\[ x = \frac{Q^2}{2M\nu} \]

\[
\frac{d^2\sigma}{d\Omega dE'} = \sigma_{\text{Mott}} \left[ \frac{1}{\nu} F_2(x,Q^2) + \frac{2}{M} F_1(x,Q^2)\tan^2 \frac{\theta}{2} \right]
\]