Pseudoscalar Flavor-Singlet Physics with Staggered Fermions

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Overview

Quick review of singlet physics

Singlets as a probe of validity of the $\det^{1/4}$ trick in staggered formulation

Very preliminary results

Future plans
The $\eta'$ propagator

$$G_{\eta'}(x', x) = \langle \sum_i \bar{q}_i(x')(\gamma_5 \otimes 1)q_i(x') \sum_j \bar{q}_j(x)(\gamma_5 \otimes 1)q_j(x) \rangle$$
The $\eta'$ propagator

\[ G_{\eta'}(x', x) = \langle \sum_i \overline{q}_i(x')(\gamma_5 \otimes 1)q_i(x') \sum_j \overline{q}_j(x)(\gamma_5 \otimes 1)q_j(x) \rangle \]

$N_f$ terms from

\[ \langle \sum_i \overline{q}_i(x')(\gamma_5 \otimes 1)q_i(x') \sum_j \overline{q}_j(x)(\gamma_5 \otimes 1)q_j(x) \rangle \]
The $\eta'$ propagator

\[ G_{\eta'}(x', x) = \left\langle \sum_i \overline{q}_i(x')(\gamma_5 \otimes 1)q_i(x') \sum_j \overline{q}_j(x)(\gamma_5 \otimes 1)q_j(x) \right\rangle \]

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$N_f^2$ terms from

\[ \left\langle \sum_i \overline{q}_i(x')(\gamma_5 \otimes 1)q_i(x') \sum_j \overline{q}_j(x)(\gamma_5 \otimes 1)q_j(x) \right\rangle \]
Singlet propagator

$\mu^2$ represents the coupling between 2 octet PS mesons.

$\tilde{G}_{\eta'} = \frac{1}{p^2 + m^2_\pi}$
Singlet propagator

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\[
\tilde{G}_{\eta} = \frac{1}{p^2 + m^2_\pi} G_\pi
\]
Singlet propagator

$\mu^2$ represents the coupling between 2 octet PS mesons.

$$\tilde{G}_{\eta'} = \frac{1}{p^2 + m^2_\pi} - \frac{1}{p^2 + m^2_\pi} \mu^2 \left( \frac{1}{p^2 + m^2_\pi} \right)$$
Singlet propagator

\( \mu^2 \) represents the coupling between 2 octet PS mesons.

\[ \tilde{G}_{\eta'} = \]

\[ \frac{1}{p^2 + m_\pi^2} G_\pi \]

\[ - \frac{1}{p^2 + m_\pi^2} \mu^2 \frac{1}{p^2 + m_\pi^2} \]

Quenched prop stops
Singlet propagator

$\mu^2$ represents the coupling between 2 octet PS mesons.

$\tilde{G}_{\eta'} = \frac{1}{p^2+m_{\pi}^2} \rightarrow G_{\pi}$

$- \frac{1}{p^2+m_{\pi}^2} \mu^2 \frac{1}{p^2+m_{\pi}^2} \rightarrow$ Quenched prop stops

$+ \frac{1}{p^2+m_{\pi}^2} \mu^2 \frac{1}{p^2+m_{\pi}^2} \mu^2 \frac{1}{p^2+m_{\pi}^2}$
Singlet propagator

$\mu^2$ represents the coupling between 2 octet PS mesons.

$$ \tilde{G}_{\eta'} = \frac{1}{p^2 + m_\pi^2} G_\pi - \frac{1}{p^2 + m_\pi^2} \mu^2 \frac{1}{p^2 + m_\pi^2} + \frac{1}{p^2 + m_\pi^2} \mu^2 \frac{1}{p^2 + m_\pi^2} \mu^2 \frac{1}{p^2 + m_\pi^2} - \frac{1}{p^2 + m_\pi^2} \mu^2 \frac{1}{p^2 + m_\pi^2} \mu^2 \frac{1}{p^2 + m_\pi^2} \mu^2 \frac{1}{p^2 + m_\pi^2} + \ldots $$

Quenched prop stops
Singlet propagator

$\mu^2$ represents the coupling between 2 octet PS mesons.

$$\tilde{G}_{\eta'} =$$

\[
\frac{1}{p^2 + m_{\pi}^2} \quad \leftarrow \quad G_{\pi}
\]

\[
- \frac{1}{p^2 + m_{\pi}^2} \mu^2 \frac{1}{p^2 + m_{\pi}^2} \quad \leftarrow \quad \text{Quenched prop stops}
\]

\[
+ \frac{1}{p^2 + m_{\pi}^2} \mu^2 \frac{1}{p^2 + m_{\pi}^2} \mu^2 \frac{1}{p^2 + m_{\pi}^2} \quad \leftarrow \quad \text{Quenched prop stops}
\]

\[
- \frac{1}{p^2 + m_{\pi}^2} \mu^2 \frac{1}{p^2 + m_{\pi}^2} \mu^2 \frac{1}{p^2 + m_{\pi}^2} \quad \leftarrow \quad \text{Quenched prop stops}
\]

\[+ \ldots
\]

$$\tilde{G}_{\eta'} = \frac{1}{p^2 + m_{\pi}^2 + \mu^2}$$
For $N_f$ flavors

\[ G_\pi(t) = N_f \quad \Rightarrow \quad N_f C(t) \]

\[ G_{\eta'}(t) = N_f - N_f^2 D(t) \]

In full QCD we expect

\[ G_{\eta'}(t) \sim e^{-m_{\eta'} t} \quad \text{and} \quad G_\pi(t) \sim e^{-m_\pi t}. \]

So

\[ R(t) = \frac{N_f^2 D(t)}{N_f C(t)} = \frac{N_f C(t) - G_{\eta'}(t)}{N_f C(t)} = 1 - A \exp \left[ -(m_\eta - m_\pi) t \right] \]

Whereas in quenched QCD

\[ R(t) = \frac{N_f^2 D(t)}{N_f C(t)} = A + Bt \]

Or something else if $(\det M)^{1/4}$ introduces some strange pathology like $m_{\text{val}} \neq m_{\text{sea}}$ or $N_{\text{val}} \neq N_{\text{sea}}$. 
In fact, since the staggered formulation is a theory of four valence tastes, $D$ is too large by a factor of 4.

Rescaling of $D$ by a factor of $\frac{1}{4}$ is implicit in the rest of the talk.
2+1 flavors, \( m_u = m_d \neq m_s \)

Two connected correlators represent three diagrams:

\[
C_{uu} = C_{dd} \equiv C_{qq} \quad \text{and} \quad C_{ss}
\]

Three disconnected correlators represent nine diagrams:

\[
D_{uu} = D_{dd} = D_{ud} = D_{du} \equiv D_{qq} \\
D_{ss} \\
D_{us} = D_{ds} = D_{su} = D_{sd} \equiv D_{qs}
\]

So construct:

\[
R(t) = \frac{N_f^2 D(t)}{N_f C(t)} \rightarrow \frac{4D_{qq} + 4D_{qs} + D_{ss}}{2C_{qq} + C_{ss}}
\]
Staggered PS Singlets

\[ \gamma_4 \gamma_5 \otimes 1 \]

\[ \gamma_5 \otimes 1 \]
Measuring Correlators

- Measure connected correlators with standard point sources.
- Measure disconnected correlators with gaussian stochastic volume sources $\eta$.
- Invert and solve for

$$O(t) = \text{Tr} \eta^\dagger \Delta_{\gamma^5 \otimes 1} M^{-1} \eta,$$

(summed over timeslice $t$) where $\Delta_{\gamma^5 \otimes 1}$ effects the shifts and phasing appropriate to the staggered $\gamma^5 \otimes 1$ operator.
- We are currently using $N_s = 40$ stochastic sources per gauge configuration.
Noise dependence of errors

G5x1
16^3x32, Nf=2, beta=7.2, 32 configs

![Graph showing noise dependence of errors with different values of Ns (10, 40, 160, 640) for Z2 noise and Gaussian noise. The graph plots error against 1/Ns.](image-url)
CG residual dependence of PS operators

$\beta = 6.26 \ am = 0.010, 0.050 \ (\gamma_5 \times 1)$
Very preliminary results

To date have run small test ensembles:

- $N_f = 2, \beta = 7.2 \ 16^3 \times 32, \ am = 0.02$
- $N_f = 0, \beta = 8.0 \ 16^3 \times 32, \ am_{\text{val}} = 0.02$

and two larger $N_f = 2 + 1$ MILC ensembles:

- $N_f = 2 + 1, \beta = 6.76 \ 20^3 \times 64, \ am = 0.007, 0.05 \ 422 \ \text{cфgs}$
- $N_f = 2 + 1, \beta = 6.76 \ 20^3 \times 64, \ am = 0.010, 0.05 \ 644 \ \text{cфgs}$
\[ \beta = 6.76 \text{ } am = 0.007, 0.050 \]
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Obviously some normalization issues!
$\beta = 6.76 \ am = 0.010, 0.050 \ (\gamma_5 \times 1)$

$\beta = 6.26 \ am = 0.010, 0.050 \ (\gamma_5 \times 1)$
\( \beta = 6.76 \ am = 0.010, 0.050 \ (\gamma_5 \times 1) \)

\[
\frac{D(t)}{C(t)} = R(t) = 6 \left[1 - \exp(-(0.066)t)\right]
\]
$N_f = 2$ test run

Interesting quirk:

\[ \langle \tilde{q}(\gamma_5 \otimes 1)q \rangle \approx 0, \text{ but long range correlations.} \]

Stay tuned for results on larger \((20^3 \times 64)\) lattices.
Wold expect mixing, so to fit masses diagonalize

\[
C(t) = \begin{pmatrix}
  C_{qq}(t) - 2D_{qq}(t) & \sqrt{2}D_{qs}(t) \\
  \sqrt{2}D_{sq}(t) & C_{ss}(t) - D_{ss}(t)
\end{pmatrix}
\]

(properly normalized!)
**Bag of tricks**

- Dilution: Define noise source only on some subset of the lattice (e.g., one timeslice, one color, one hyper-cube corner ....)

- “Kilkup & Venkataraman method”
  Instead of $\mathcal{O} = \langle \phi_x \eta_y \rangle$, measure

  $$\mathcal{O} = m \langle \phi_x^\dagger \phi_y \rangle = (\not{D} + m) \langle \phi_x^\dagger \phi_y \rangle = \langle \phi_x^\dagger \eta_y \rangle,$$

  where $(\not{D} + m) \phi = \eta$. (Good only for 4-link ($\gamma_5 \otimes 1$).)

- eigen modes?
Future plans

- Continue measurements on $N_f = 0, 2, 2 + 1$ lattices
- Understand normalizations
- Optimizations
- Fit for $\eta'$ mass
- Need long time series ensembles to be produced this fall on QCDOC

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<th>trajectories</th>
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