Simulating at Realistic Quark Masses: some recent QCDSF results on Hadronic Structure

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– QCDSF-UKQCD Collaboration –

[LHP 2006, Jefferson Lab]

Contents:

Other aspects: D. Pleiter, Ph. Hägler, J. M. Zannotti

- Introduction
  - The Problem
  - Our present Situation

Contents:

Contents:

Introduction

- Lattice simulations for QCD give first principle results

Introduction

- Lattice simulations for QCD give first principle results
- but need to have control of [ideally in this order]: 'Limit'
  - Statistical errors, $N_{conf} \sim O(1000)$ — $N_{conf} \to \infty$
  - Volume: $L \sim 1.5\,\text{fm} \to 3\,\text{fm}$ — $L \to \infty$
  - Scaling violations: $a \sim 0.1\,\text{fm} \to 0.04\,\text{fm}$ — $a \to 0$
  - Chiral extrapolation: $m_{ps} \sim 500\,\text{MeV} \to 200\,\text{MeV}$ — $m_{ps} \to m_\pi = 140\,\text{MeV}$
- difficult, need Tflop++ machines to approach the theoretical goal

Wilson-type fermions

As emphasised by Lüscher — Lat05, hep-lat/0509152

Wilson fermions are:

- Well understood
- Non-perturbative improvement/renormalisation exists
  - 'Clover' variation, discretisation errors are $O(a^2)$
  - (Some) NP Zs known (Schrödinger functional or $\mathrm{RI}' - \mathrm{MOM}$)
- Much experience with quenched QCD

Problem is that simulations for light quark masses are very costly

Recent advances:

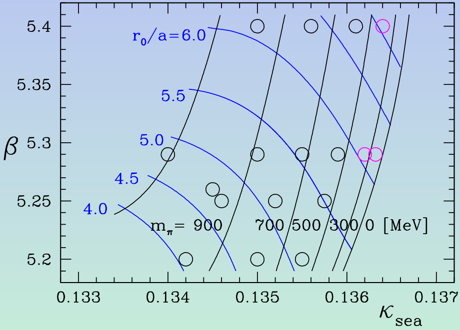
- Faster machines have become available
  - Bluegenes ($\sim$ Tflop):
    - ZAM (Jülich)
    - KEK (Tsukuba)
    - EPCC (Edinburgh)
- Improvements in HMC algorithm
  - Hasenbusch (introduce auxilliary mass)
  - 3 time scales (one for Wilson glue, two for Wilson fermions)

Unquenched [$n_f = 2$] Fermions

$O(a)$ improved fermions: 5.20, 5.25, 5.26, 5.29, 5.40 data sets

2005 → 2006 status

β	κ_{sea}	Volume	Trajectories	Group
5.20	0.1342	$16^3 \times 32$	5100	QCDSF
5.20	0.1350	$16^3 \times 32$	8000	UKQCD
5.20	0.1355	$16^3 \times 32$	8100	UKQCD
5.25	0.1346	$16^3 \times 32$	5800	QCDSF
5.25	0.1352	$16^3 \times 32$	7300	UKQCD
5.25	0.13575	$24^3 \times 48$	6000	QCDSF
5.26	0.1345	$16^3 \times 32$	4100	UKQCD
5.29	0.1340	$16^3 \times 32$	3900	UKQCD
5.29	0.1350	$16^3 \times 32$	5700	QCDSF
5.29	0.1355	$24^3 \times 48$	2100	QCDSF
5.29	0.1359	$24^3 \times 48$	4900	QCDSF
5.29	0.1362	$24^3 \times 48$	3400	QCDSF
5.29	0.13632	$32^3 \times 64$	1200	QCDSF
5.40	0.1350	$24^3 \times 48$	3800	QCDSF
5.40	0.1356	$24^3 \times 48$	3400	QCDSF
5.40	0.1361	$24^3 \times 48$	3600	QCDSF
5.40	0.1364	$24^3 \times 48$	2800	QCDSF



$m_{ps} \sim$ 1000 MeV ⟶ 500 MeV ⟶ 350↓ MeV

$a \sim$ 0.011 fm ⟶ 0.07 fm

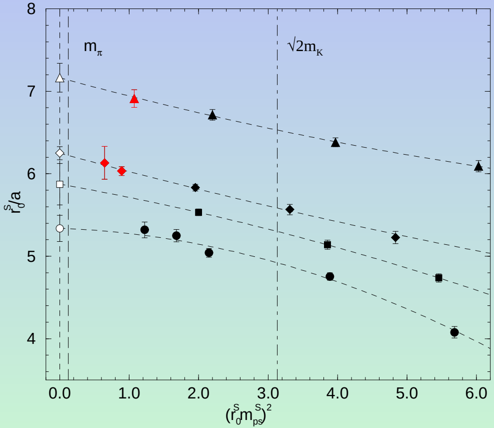
Force between static quarks – 'force unit/scale', r_0

[Sommer, eg hep-ph/9711243]

$$r^2 F(r)\Big|_{r=r_0} = 1.65$$

- unit: r_0/a can be obtained with good precision on the lattice

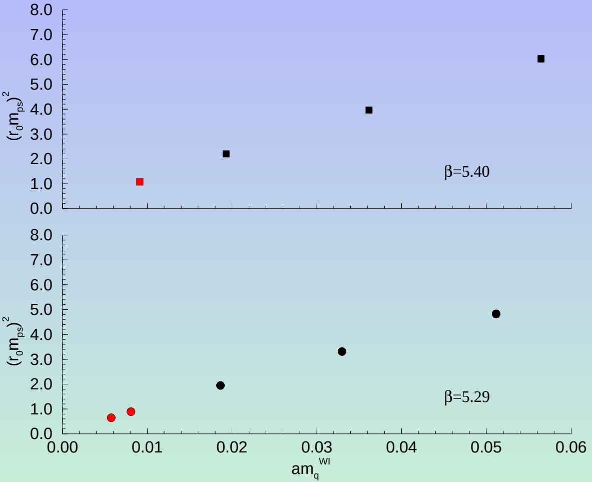
$m_q/m_s \sim \frac{1}{4}$



- scale: r_0 experimental value less well known
  - From Cornell potential: $r_0 = 0.5$ fm $\equiv (394.6\text{ MeV})^{-1}$
  - From nucleon mass: $r_0 = 0.467(33)$ fm $\equiv (422.5(29.9)\text{ MeV})^{-1}$

[Ali Khan et al., hep-lat/0312030]

Pseudoscalar masses



- Consistent linear behaviour: $(r_0 m_{ps})^2 \propto am_q^{WI}$

Pseudoscalar decay constants

$[M^{q_1 q_2} \equiv ps]$

$[\mathcal{A}_\mu^{q_1 q_2} \equiv \bar{q_1} \gamma_\mu \gamma_5 q]$

$$\langle 0 | \mathcal{A}_\mu(0) | ps(\vec{p}) \rangle = i\sqrt{2} f_{ps} p_\mu$$

$$\Gamma(ps^\pm \to l^\pm \overline{\nu}_l) = \frac{G_F^2}{8\pi} |V_{q_1 q_2}|^2 f_{ps}^2 m_{ps} m_l^2 \left(1 - \frac{m_l^2}{m_{ps}^2}\right)^2$$

Including radiative corrections give $\pi^+ \sim u\bar{d}$, $K^+ \sim u\bar{s}$

$$\pi^+ \to \mu^+ \bar{\nu}_\mu \qquad f_{\pi^+} = 92.42 \pm 0.07 \pm 0.25\,\text{MeV}$$
$$K^+ \to \mu^+ \bar{\nu}_\mu \qquad f_{K^+} = 113.0 \pm 1.0 \pm 0.3\,\text{MeV}$$

(Partially Quenched) Chiral Perturbation Theory

$$\frac{f_{ps}^{AB}}{f_0} = 1 + \left(\frac{1}{2}n_f\alpha_4 - \frac{1}{2n_f}\right)\chi_S + \left(\frac{1}{2}\alpha_5 + \frac{1}{2n_f}\right)\chi_{AB}$$
$$+\frac{1}{2n_f}\left(\frac{\chi_A\chi_B - \chi_S\chi_{AB}}{\chi_B - \chi_A}\ln\frac{\chi_A}{\chi_B}\right) - \frac{1}{4}n_f\left(\chi_{AS}\ln\chi_{AS} + \chi_{BS}\ln\chi_{BS}\right)$$

$$\left(\frac{m_{ps}^{AB}}{4\pi f_0}\right)^2 = \chi_{AB}\left[1 + n_f(2\alpha_6 - \alpha_4)\chi_S + (2\alpha_8 - \alpha_5)\chi_{AB}\right.$$
$$\left.+\frac{1}{n_f}\frac{\chi_A(\chi_S - \chi_A)\ln\chi_A - \chi_B(\chi_S - \chi_B)\ln\chi_B}{\chi_B - \chi_A}\right]$$

with $\qquad\qquad \chi_A \equiv \chi_{AA}$

$$\chi_{AB} = \frac{B_0^{\mathcal{S}}(m_A + m_B)^{\mathcal{S}}}{(4\pi f_0)^2} \qquad A, B \in \{V_1, V_2, S\}$$

- n_f [= 2] mass degenerate sea, S, quarks; valence, V, quarks
- LO + NLO Bernard et al., hep-lat/9306005; Sharpe, hep-lat/9707018 [NNLO – Bijnens hep-lat/0506004]
- α_i are LECs evaluated at scale $\mu = \Lambda_\chi = 4\pi f_0 \sim 1160\,\mathrm{MeV}$

Practically

- In f_{ps}^{AB} eliminate χ_{AB} in favour of m_{ps}^{AB} (LO result sufficient)
- Re-scale

$$\frac{m_{ps}^{AB}}{4\pi f_0} = c_m^S M_{ps}^{AB} \qquad \frac{f_{ps}^{AB}}{f_0} = c_f^S F_{ps}^{AB}$$

for example

$$M_{ps}^{AB} = r_0^S m_{ps}^{AB} \quad \leftrightarrow \quad c_m^S = \frac{1}{4\pi r_0^S} \qquad F_{ps}^{AB} = r_0^S m_{ps}^{AB} \quad \leftrightarrow \quad c_f^S = \frac{1}{f_0 r_0^S}$$

where r_0^S is the (force)-scale

For example to give (on expanding to $O(\chi^2)$) the [fit] function

$$\begin{aligned} F_{ps}^V &= f_a + f_b (M_{ps}^S)^2 + f_c (M_{ps}^V)^2 \\ &\quad + f_d \left((M_{ps}^S)^2 + (M_{ps}^V)^2\right) \ln\left((M_{ps}^S)^2 + (M_{ps}^V)^2\right) \end{aligned}$$

for degenerate quark masses $A = V$, $B = V$

Enhanced Chiral Logarithms

[Sharpe, hep-lat/9707018]

Potential problem

$$\text{term} \propto \left((M_{ps}^S)^2 + (M_{ps}^V)^2\right) \ln\left((M_{ps}^S)^2 + (M_{ps}^V)^2\right)$$

which for fixed $(M_{ps}^S)^2$ does not vary much with $(M_{ps}^V)^2$

- Wish for a term

$$\text{term} \propto (M_{ps}^S)^2 \ln(M_{ps}^V)^2$$

- Construct ratio

disadvantage: $A = V \neq B = S$ required

$$R = \frac{F_{ps}^{VS}}{\sqrt{F_{ps}^V F_{ps}^S}} - 1$$
$$= c\left((M_{ps}^S)^2 \ln\frac{(M_{ps}^V)^2}{(M_{ps}^S)^2} + (M_{ps}^S)^2 - (M_{ps}^V)^2\right)$$

for example with r_0 scale

$M_{ps} = r_0 m_{ps}$

$$c = -\frac{1}{4n_f}\left(\frac{1}{4\pi f_0 r_0}\right)^2$$

Pion and Kaon decay constants

Degenerate valence quark masses ($A = V = B$) are sufficient

did not have to be the case

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- Have
  - Sea: $m_q^S \equiv m_{ud} = \frac{1}{2}(m_u + m_d)$ – up/down quarks
  - Valence: $K^+ \sim u\bar{s}$, $\pi^+ \sim u\bar{d}$
    m_q^V – 3 possible valence quarks
    - m_s – strange quark
    - $m_u = m_{ud} - \Delta m_{ud}$, $m_d = m_{ud} + \Delta m_{ud}$ – up/down quarks

Pion and Kaon decay constants

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- Have
  - Sea: $m_q^S \equiv m_{ud} = \frac{1}{2}(m_u + m_d)$ – up/down quarks
  - Valence: $K^+ \sim u\bar{s}$, $\pi^+ \sim u\bar{d}$
    m_q^V – 3 possible valence quarks
    - ⋆ m_s – strange quark
    - ⋆ $m_u = m_{ud} - \Delta m_{ud}$, $m_d = m_{ud} + \Delta m_{ud}$ – up/down quarks
- Again manipulate structural form of LO + NLO equations to give

$$
\begin{aligned}
F_{\pi^+} &= f_a + (f_b + f_c + 2f_d \ln 2) M_{\pi^+}^2 + 2f_d M_{\pi^+}^2 \ln M_{\pi^+}^2 + O((\Delta m_{ud})^2) \\
F_{K^+} &= f_a + \left(f_b + f_d \left(\ln 2 - \frac{2}{n_f^2} \right) \right) M_{\pi^+}^2 + \left(f_c + f_d \frac{2}{n_f^2} \right) M_{K^+}^2 \\
& \quad + f_d \left(1 - \frac{1}{n_f^2} \right) M_{\pi^+}^2 \ln M_{\pi^+}^2 + f_d \left(M_{K^+}^2 + \frac{1}{n_f^2} M_{\pi^+}^2 \right) \ln \left(2M_{K^+}^2 - M_{\pi^+}^2 \right) \\
& \quad + O(\Delta m_{ud})
\end{aligned}
$$

The Lattice Approach

$\mathcal{A}_\mu$ is an $O(a)$ improved operator:

$[A_\mu = \bar{q}_1 \gamma_\mu \gamma_5 q_2,\ P = \bar{q}_1 \gamma_5 q_2]$

$$\mathcal{A}_\mu = Z_A \mathcal{A}_\mu^{IMP} \qquad \mathcal{A}_\mu^{IMP} = \left(1 + \tfrac{1}{2} b_A (am_{q_1} + am_{q_2})\right)\left(A_\mu + c_A a \partial_\mu P\right)$$

$$\langle 0 | \mathcal{A}_4 | ps \rangle = \frac{f_{ps}}{\sqrt{2}} m_{ps}$$

Compute

$$C_{\mathcal{O}_1\mathcal{O}_2}(t) = \langle \mathcal{O}_1(t) \mathcal{O}_2^\dagger(0) \rangle \equiv A_{\mathcal{O}_1\mathcal{O}_2} \left[e^{-m_{ps} t} + \tau_1 \tau_2 e^{-m_{ps}(T-t)} \right]$$

where

$$A_{\mathcal{O}_1\mathcal{O}_2} = \frac{1}{2m_{ps}} \langle 0 | \mathcal{O}_1 | ps \rangle \langle 0 | \mathcal{O}_2 | ps \rangle^*$$

giving

$$f_{ps} = Z_A \left(1 + \tfrac{1}{2} b_A (am_{q_1} + am_{q_2})\right) \left(f_{ps}^{(0)} + c_A a f_{ps}^{(0)} \right)$$

with

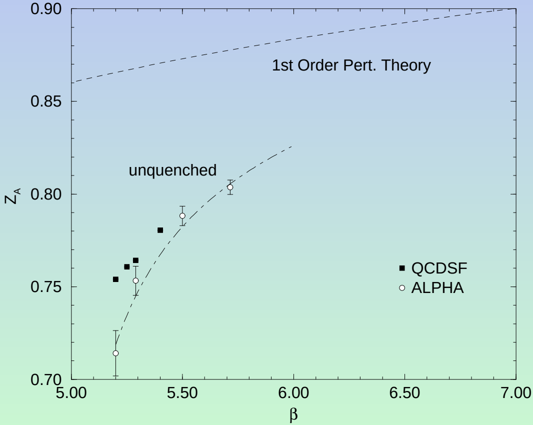
$$f_{ps}^{(0)} = -\sqrt{\frac{2}{m_{ps}} \frac{A_{A_4 P}^{LS}}{\sqrt{A_{PP}^{LS}}}} \qquad \frac{a f_{ps}^{(1)}}{f_{ps}^{(0)}} = a m_{ps} \frac{A_{PP}^{LS}}{A_{A_4 P}^{LS}}$$

Renormalisation, Improvement Coefficients

- c_A NP determination, ALPHA: hep-lat/0505026
- b_A only known perturbatively, use TI-BPT but expect $b_A a m_q \ll 1$
- Z_A

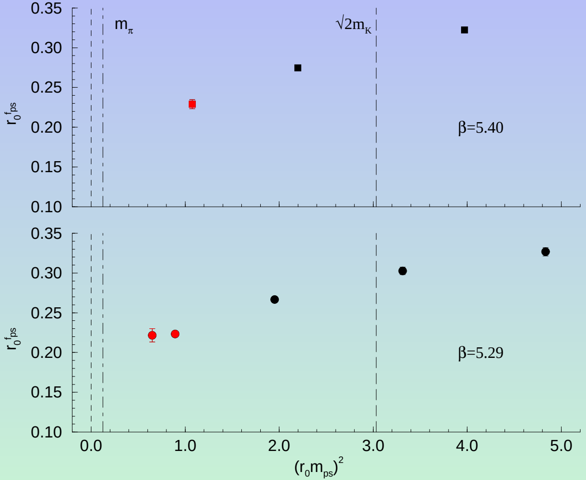
ALPHA - Schrödinger Functional – hep-lat/0505026

QCDSF - RI′-MOM – hep-lat/0603028



but $O(a^2)$ differences $\longrightarrow 0$

A first peek at the sea quark results:



Enhanced Chiral Logarithms

Recall:

$$R = \frac{(r_0^S f_{ps}^{VS})}{\sqrt{(r_0^S f_{ps}^V)(r_0^S f_{ps}^S)}} - 1$$

giving

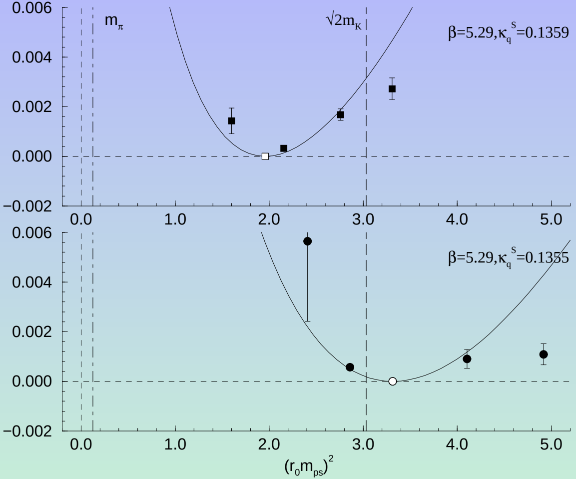
$$R = c\left((r_0^S m_{ps}^S)^2 \ln \frac{(r_0^S m_{ps}^V)^2}{(r_0^S m_{ps}^S)^2} + (r_0^S m_{ps}^S)^2 - (r_0^S m_{ps}^V)^2 \right)$$

with

$$c = -\frac{1}{4n_f}\left(\frac{1}{4\pi f_0 r_0}\right)^2 = \begin{cases} -0.0144 & r_0 = 0.5\,\mathrm{fm} \\ -0.0165 & r_0 = 0.467\,\mathrm{fm} \end{cases}$$

$f_0 \approx 92.4$ MeV

R:



- Some signs of activity for smaller quark mass $\lesssim m_s$
- But note *y*-axis scale – have subtracted 1, so really a very small effect

Recall:

$$r_0^S f_{ps}^V = f_a + f_b (r_0^S m_{ps}^S)^2 + f_c (r_0^S m_{ps}^V)^2$$
$$+ f_d \left((r_0^S m_{ps}^S)^2 + (r_0 m_{ps}^V)^2\right) \ln\left((r_0^S m_{ps}^S)^2 + (r_0^S m_{ps}^V)^2\right)$$

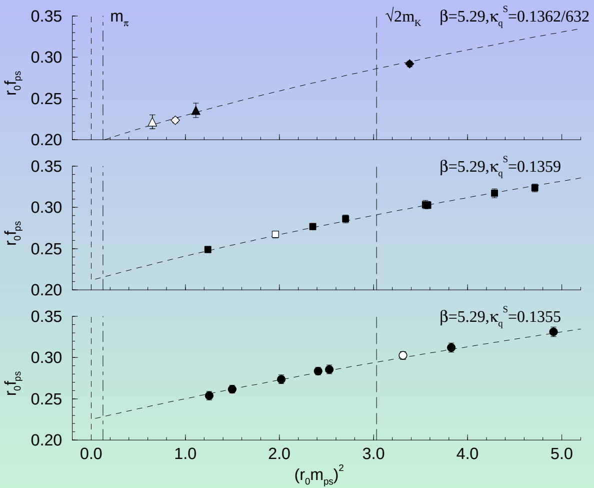
- Do not expect much influence from the chiral logarithms
- f_a, f_b, f_c, f_d taken as fit coefficients (and then extrapolated to $a^2 \to 0$)
- Then gives $r_0 f_{\pi^+}$, $r_0 f_{K^+}$:

$$r_0 f_{\pi+} = f_a + (f_b + f_c + 2 f_d \ln 2)(r_0 m_{\pi+})^2 + 2 f_d (r_0 m_{\pi+})^2 \ln (r_0 m_{\pi+})^2$$

$$r_0 f_{K+} = f_a + \left(f_b + f_d \left(\ln 2 - \frac{2}{n_f^2}\right)\right)(r_0 m_{\pi+})^2 + \left(f_c + f_d \frac{2}{n_f^2}\right)(r_0 m_{K+})^2$$
$$+ f_d \left(1 - \frac{1}{n_f^2}\right)(r_0 m_{\pi+})^2 \ln (r_0 m_{\pi+})^2 + f_d \left((r_0 m_{K+})^2 + \frac{1}{n_f^2}(r_0 m_{\pi+})^2\right) \ln\left(2 (r_0 m_{K+})^2 - (r_0 m_{\pi+})^2\right)$$

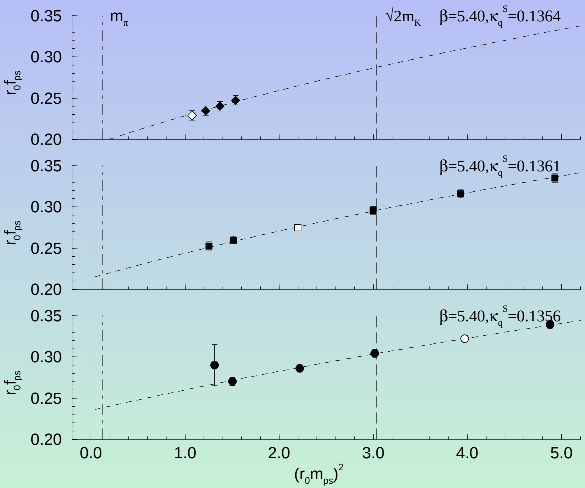
- Practically eliminate f_a in terms of $r_0 f_{\pi^+}$ or $r_0 f_{K^+}$

$\beta = 5.29$



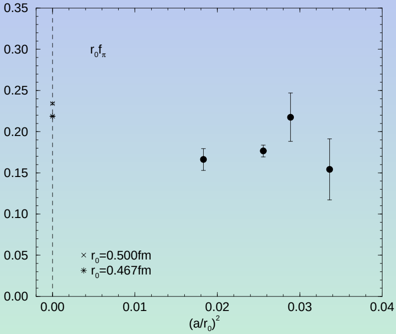
m$_\pi$
√2m$_K$
β=5.29,κ$_q^S$=0.1362/632
β=5.29,κ$_q^S$=0.1359
β=5.29,κ$_q^S$=0.1355
r$_0$f$_{ps}$
0.35
0.30
0.25
0.20
0.0
1.0
2.0
3.0
4.0
5.0
(r$_0$m$_{ps}$)^2

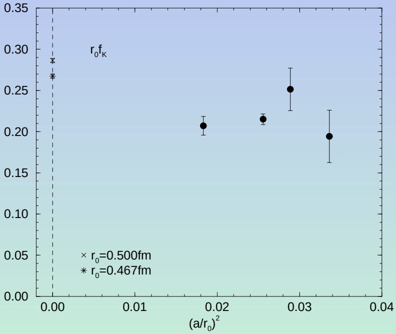
$\beta = 5.40$



0.35
0.30
0.25
0.20
r0fps
mπ
√2mK
β=5.40,κq S=0.1364
β=5.40,κq S=0.1361
β=5.40,κq S=0.1356
0.0
1.0
2.0
3.0
4.0
5.0
(r0mps)2

'Continuum Limit' for $r_0 f_{\pi^+}$, $r_0 f_{K^+}$





f_{π^+}, f_{K^+}:

Taking as an example the $\beta = 5.29$ results gives

$r_0 = 0.5$ fm

$$f_{\pi^+} = 70 \pm 3\,\text{MeV}$$
$$f_{K^+} = 85 \pm 3\,\text{MeV}$$

Compare

$$f_{\pi^+} = 92.42 \pm 0.07 \pm 0.25\,\text{MeV}$$
$$f_{K^+} = 113.0 \pm 1.0 \pm 0.3\,\text{MeV}$$

so about a 20% discrepancy

- Some evidence of chiral logarithms, influence is problematic
- Values of decay constants too small in comparison with experiment
  Perhaps partially due to scale setting:

$$\frac{f_{K^+}}{f_{\pi^+}} = 1.219 \qquad \left(\frac{f_{K^+}}{f_{\pi^+}}\right)_{expt} = 1.223$$

Moments of Unpolarised Nucleon Structure Functions

$$\int_0^1 dx x^{n-2} F^{NS}(x, Q^2) = f\, E^{\mathcal{S}}_{F;v_n}\left(\frac{M^2}{Q^2}, g^{\mathcal{S}}(M)\right) v_n^{\mathcal{S}}(g^{\mathcal{S}}(M))$$

- Matrix elements v_n are given by

$$\langle N(\vec{p})| \left[\mathcal{O}_q^{\{\mu_1 \cdots \mu_n\}} - \mathrm{Tr}\right] |N(\vec{p})\rangle^{\mathcal{S}} \equiv 2 v_n^{(q)\mathcal{S}} [p^{\mu_1} \cdots p^{\mu_n} - \mathrm{Tr}]$$

- Matrix elements can be measured on the lattice
- Results have to be renormalised either to RGI form or in a scheme (e.g. $\overline{MS}$) and scale (e.g. 2 GeV)
  Use NP RI′-MOM scheme and then convert via RGI to $\overline{MS}$ scheme
- Extrapolation to chiral and continuum limit

Operators

for $\vec{p} = (1, 0, 0)$

$$
\begin{aligned}
\mathcal{O}_{v_{2a}} &= \mathcal{O}^{\gamma}_{\{14\}} \\
\mathcal{O}_{v_{2b}} &= \mathcal{O}^{\gamma}_{\{44\}} - \tfrac{1}{3}\left(\mathcal{O}^{\gamma}_{\{11\}} + \mathcal{O}^{\gamma}_{\{22\}} + \mathcal{O}^{\gamma}_{\{33\}}\right) \\
\mathcal{O}_{v_3} &= \mathcal{O}^{\gamma}_{\{441\}} - \tfrac{1}{2}\left(\mathcal{O}^{\gamma}_{\{221\}} + \mathcal{O}^{\gamma}_{\{331\}}\right) \\
\mathcal{O}_{v_4} &= \mathcal{O}^{\gamma}_{\{1144\}} + \mathcal{O}^{\gamma}_{\{2233\}} - \tfrac{1}{2}\left(\mathcal{O}^{\gamma}_{\{1133\}} + \mathcal{O}^{\gamma}_{\{1122\}} + \mathcal{O}^{\gamma}_{\{2244\}} + \mathcal{O}^{\gamma}_{\{3344\}}\right)
\end{aligned}
$$

where

$$
\mathcal{O}^{\Gamma}_{q;\mu_1\cdots\mu_n} = \overline{q}\Gamma_{\mu_1\cdots\mu_i} \overleftrightarrow{D}_{\mu_{i+1}} \cdots \overleftrightarrow{D}_{\mu_n} q
$$

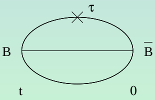
v_{2a}, v_{2b} are different representations of the same continuum operator

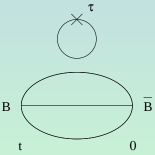
Determining the matrix element

$$R_\Gamma(t,\tau;\vec{p};\mathcal{O}) = \frac{C_{\Gamma^{unpol}}(t,\tau;\vec{p};\mathcal{O})}{C_{\Gamma^{unpol}}(t;\vec{p})}$$

For $0 \ll \tau \ll t \ll \frac{1}{2}N_T$ gives bare matrix elements v_n:

$$\begin{aligned}
R_{\Gamma unpol}(t,\tau;\vec{p}_1;\mathcal{O}_{v_{2a}}) &= i p_1 v_{2a} \\
R_{\Gamma unpol}(t,\tau;\vec{p};\mathcal{O}_{v_{2b}}) &= -\frac{E_{\vec{p}}^2 + \frac{1}{3}\vec{p}^2}{E_{\vec{p}}} v_{2b} \\
R_{\Gamma unpol}(t,\tau;\vec{p}_1;\mathcal{O}_{v_3}) &= -p_1^2 v_3 \\
R_{\Gamma unpol}(t,\tau;\vec{p}_1;\mathcal{O}_{v_4}) &= E_{\vec{p}_1} p_1^2 v_4
\end{aligned}$$

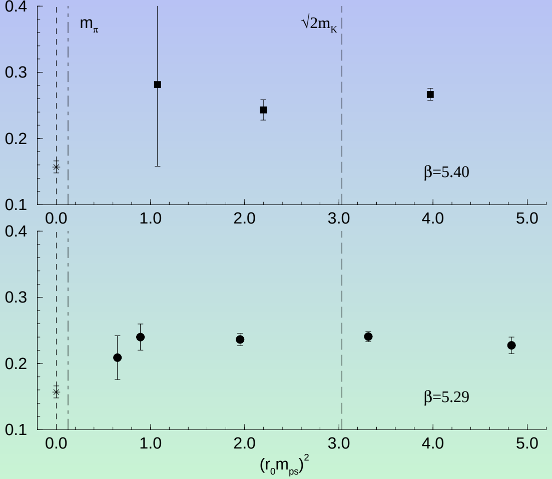




Non-singlet: $v_{n;NS} \equiv v_n^{(u)} - v_n^{(d)}$

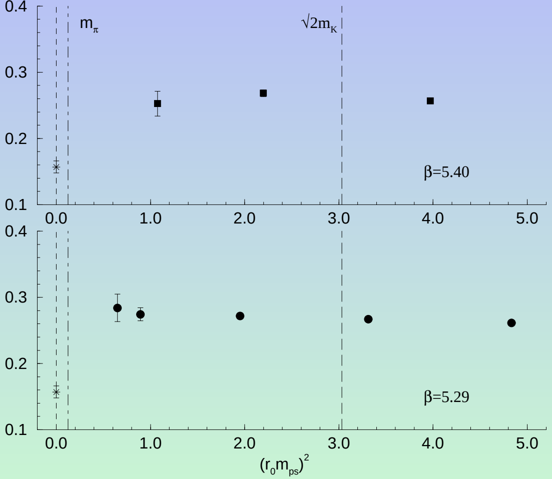
Results: v_{2a}

[$\overline{MS}$ scheme at 2 GeV]

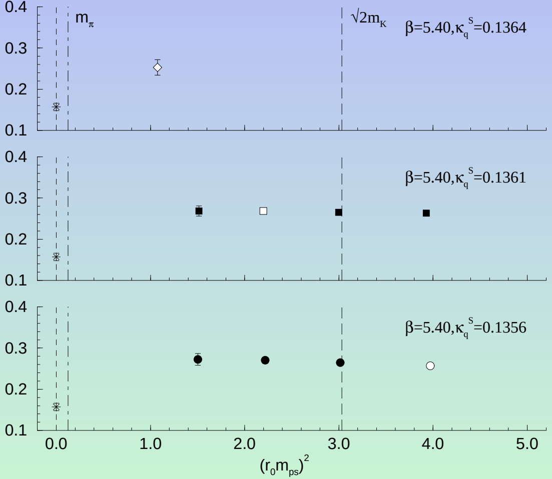


Results: v_{2b}

[$\overline{MS}$ scheme at 2 GeV,]

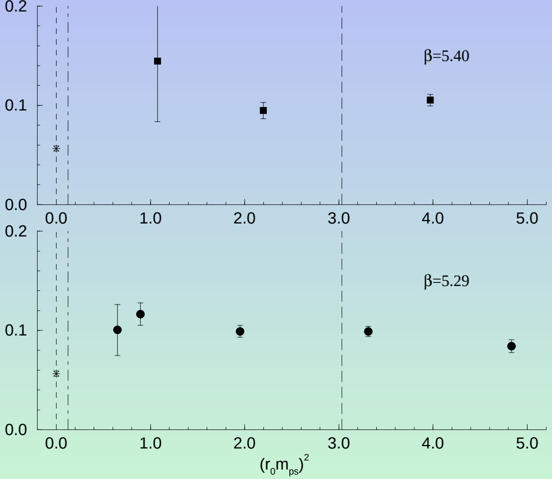


Results: v_{2b} – Partially Quenched



Results: v_3

[$\overline{MS}$ scheme at 2 GeV,]



- $\star$ is the MRS phenomenological result
- v_{2a}, v_{2b} in reasonable agreement
  - can have $O(a^2)$ differences
- $\vec{p} \neq 0$
  - v_{2a}, v_3 require $\vec{p} \neq \vec{0}$, so more noisy [This is worse for v_4]

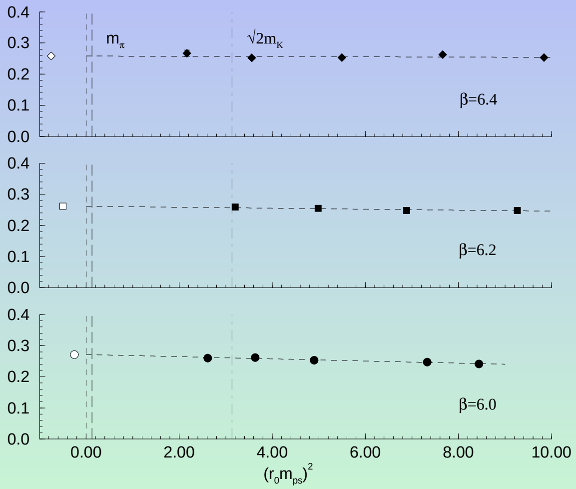
No real bending down to the phenomenological result

– for 400 – 350 MeV pseudoscalar masses [$n_f = 2$ clover fermions]

How does this compare with other results?

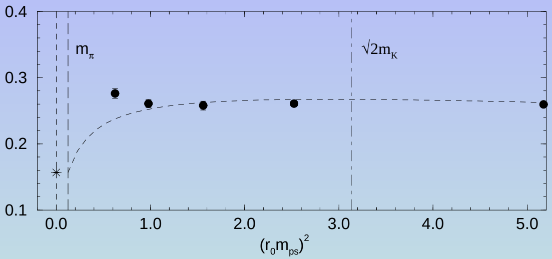
Results: v_{2b} for quenched clover ($\gtrsim m_s$)

[$\overline{MS}$ scheme at 2 GeV,]



Results: v_{2b} for quenched Wilson ($\beta = 6.0$)

[$\overline{MS}$ scheme at 2 GeV,]



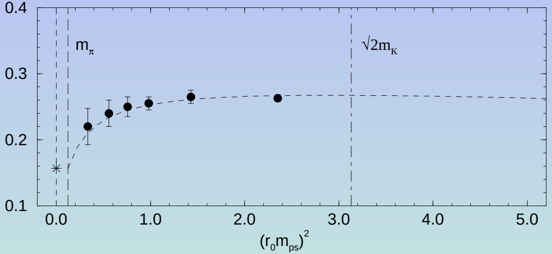
- χPT e.g. Detmold et al: [hep-lat/0103006] [$x = r_0 m_{ps}$]

$$v_{2;NS}(x) = a_2 x^2 + b_2 \left(1 - c x^2 \ln \frac{x^2}{(x^2 + (r_0 \Lambda_\chi)^2)}\right)$$

- Expect: $c = (3g_A^2 + 1)/(4\pi r_0 f_\pi)^2 \sim 0.28$, qu, or 0.67, QCD, $\Lambda_\chi \sim 1\text{GeV}$
- Find: $c \sim 4.4$, $\Lambda_\chi \sim 300\,\text{MeV}$

Results: v_{2b} for (quenched) overlap

[$\overline{MS}$ scheme at 2 GeV,]



- $\beta \approx 8.0 \sim$ Wilson $\beta \sim 5.9$
- $O(1000)$ configurations, $16^3 \times 32$ lattice (Wilson up to $32^3 \times 48$ lattice)

Tentative conclusion:

- For $(r_0 m_{ps})^2 \sim 1$ or $m_{ps} \sim 400\,\mathrm{MeV}$ flat
- For $(r_0 m_{ps})^2 \sim 0.7$ or $m_{ps} \sim 350\,\mathrm{MeV}$ no real evidence of bending
- Last two overlap fermion results:
  - $(r_0 m_{ps})^2 \sim 0.56$ or $m_{ps} \sim 300\,\mathrm{MeV}$
  - $(r_0 m_{ps})^2 \sim 0.34$ or $m_{ps} \sim 250\,\mathrm{MeV}$

  some bending (?)

Need to get below $m_{ps} \sim 300\,\mathrm{MeV}$ (and perhaps use chiral fermions)

Axial Charge - Bjorken Sum rule

$$\int_0^1 dx g_1^{p-n}(x,Q^2) = \tfrac{1}{6} E_{g_1;a_0;NS}(Q^2)(\underbrace{\Delta u - \Delta d}_{g_A})$$

- Matrix element given by

$$\langle \vec{p}, \vec{s} | \underbrace{\overline{q}\gamma^\mu \gamma_5 q}_{\mathcal{A}^\mu} | \vec{p}, \vec{s} \rangle = 2 s^\mu \Delta q^{\mathcal{S}}(M)$$

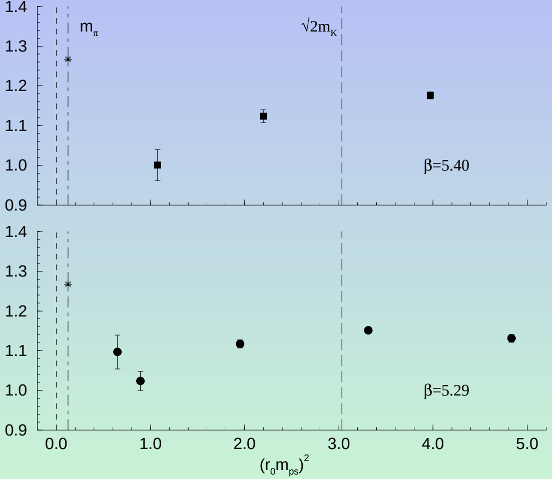
Can be found from ratio: [$\vec{p} = \vec{0}$ possible]

$$R_{\Gamma^{pol}}(t,\tau;\vec{p};\mathcal{A}_\mu) = \begin{cases} -\frac{\vec{p}\cdot\vec{s}}{m_N E_{\vec{p}}} \Delta q_{bare} & \mu = 4 \\ \frac{i}{m_N}\left(\frac{m_N}{E_{\vec{p}}}\vec{s} + \frac{\vec{p}\cdot\vec{s}}{E_{\vec{p}}(E_{\vec{p}}+m_N)}\vec{p}\right)_i \Delta q_{bare} & \mu = i \end{cases}$$

- Renormalization requires Z_A (cf f_{ps})

Results: g_A

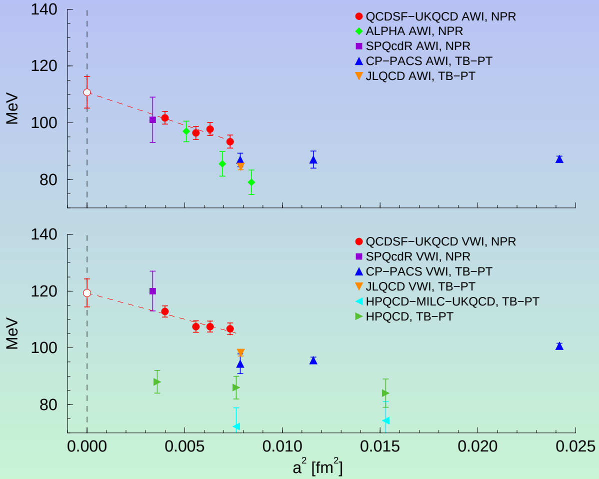
[$\overline{MS}$ scheme at 2 GeV]



- Certainly no sign of an upward trend

Strange Quark Mass - a potential problem

New result from HPQCD at Lat07



Conclusions

Use of

- 2-flavour QCD [$O(a)$ improved]
- Light quarks down to $m_q/m_s \sim \frac{1}{4}$
- a down to $\sim 0.07\,\text{fm}$

gives

- f_{ps}:
  - Some evidence of chiral logarithms, influence is problematic
  - Values of decay constants too small in comparison with experiment [perhaps partially due to scale setting (?)]
- v_2, v_3, g_A:
  - Still no sign of approach to phenomenological values
  - Little difference to quenched QCD
  - Apparently need $m_{ps} \lesssim 300\,\text{MeV}$
  - Need large data sets

'New Horizons':

- $\beta = 5.70$, $32^3 \times 64$, $48^3 \times 64$ lattices
- $m_{ps} \sim 400\,\text{MeV} - 250\,\text{MeV}$
- $a \lesssim 0.05\,\text{fm}$