Baryon Spectroscopy with FLIC fermions

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Aims of this presentation

- The nucleon resonances in QLQCD at light quark masses.
  - We review the spectrum of excited states of $N$ and $\Delta$.
  - Consideration is given to extracting the Roper resonance.
  - Reveal chiral curvature in the $\Delta^{++}$.

- Search for the $\Theta^+$ pentaquark in QLQCD.
  - We review the spectrum of our 5-quark interpolators.
  - We present the details of our analysis.
  - Comparison with Doi et al. and our lattice simulations.
Outline

Nucleon resonances

Search for the $\Theta^+$ pentaquark

Form factors of spin-1 mesons, with FLIC fermions
Interpolating Fields

Nucleon interpolating fields,

\[ \chi_1(x) = \epsilon^{abc} (u^T a(x) C \gamma_5 d^b(x)) u^c(x) \]
\[ \chi_2(x) = \epsilon^{abc} (u^T a(x) C d^b(x)) \gamma_5 u^c(x) \]
\[ \chi_3^\mu(x) = \epsilon^{abc} (u^T a(x) C \gamma_5 \gamma^\mu d^b(x)) \gamma_5 u^c(x) \]

\( \Delta \) interpolating field,

\[ \chi_\Delta^{++}(x) = \epsilon^{abc} (u^T a(x) C \gamma^\mu u^b(x)) u^c(x) \]
2pt Functions at the Hadronic Level

The correlation function, $G$, of the interpolating field $\chi$ at time $t$ and momentum $\vec{p}$ as

$$G(t, \vec{p}) = \sum_{\vec{x}} \exp(-i\vec{p} \cdot \vec{x}) \langle 0 | T \chi(x) \bar{\chi}(0) | 0 \rangle$$

Inserting a complete set of momentum, energy and spin states, and using translational invariance,

$$G(t, \vec{p}) = \sum_{B} e^{-E_B t} \times \sum_{s} \langle 0 | \chi(0) | B, \vec{p}, s \rangle \langle B, \vec{p}, s | \bar{\chi}(0) | 0 \rangle$$
The correlation function, $\mathcal{G}$, of the interpolating field $\chi$ at time $t$ and momentum $\vec{p}$ as

$$\mathcal{G}(t, \vec{p}) = \sum_{\vec{x}} \exp(-i\vec{p} \cdot \vec{x}) \langle 0 | T \chi(x) \bar{\chi}(0) | 0 \rangle$$

Inserting a complete set of momentum, energy and spin states, and using translational invariance,

$$\mathcal{G}^{\mu\nu}(t, \vec{p}) = \sum_{B} \sum_{s} e^{-E_{B}t} \langle 0 | \chi^{\mu}(0) | B, \vec{p}, s \rangle \langle B, \vec{p}, s | \bar{\chi}^{\nu}(0) | 0 \rangle$$
Spin-1/2 Correlation Functions at the Hadronic Level


Phenomenology of spin-1/2 states,

\[
\langle \Omega | \chi_{1,2}(0) | B^+, \bar{p}, s \rangle = \lambda_{B^+} \sqrt{\frac{M_{B^+}}{E_{B^+}}} u_{B^+}(p, s)
\]

\[
\langle \Omega | \chi_{1,2}(0) | B^-, \bar{p}, s \rangle = \lambda_{B^-} \sqrt{\frac{M_{B^-}}{E_{B^-}}} \gamma_5 u_{B^-}(p, s)
\]

\[
\langle 0 | \chi_{3,\Delta}^\mu | N_{1/2}^+(p, s) \rangle = (\alpha_{1/2} + p^\mu + \beta_{1/2} + \gamma^\mu) \sqrt{\frac{M_{1/2}^+}{E_{1/2}^+}} \gamma_5 u(p, s)
\]

\[
\langle 0 | \chi_{3,\Delta}^\mu | N_{1/2}^-(p, s) \rangle = (\alpha_{1/2} - p^\mu + \beta_{1/2} - \gamma^\mu) \sqrt{\frac{M_{1/2}^-}{E_{1/2}^-}} u(p, s)
\]

\[
\sum_s u_B(p, s) \bar{u}_B(p, s) = \frac{(\gamma \cdot p + M_B)}{2M_B}
\]
Spin-3/2 Correlation Functions at the Hadronic Level


Phenomenology of spin-3/2 states,

\[
\langle 0 | \chi_\mu | N^{3+}_{3/2}(p, s) \rangle = \lambda_{3/2^+} \sqrt{\frac{M_{3/2^+}}{E_{3/2^+}}} u_\mu(p, s)
\]

\[
\langle 0 | \chi_\mu | N^{3-}_{3/2}(p, s) \rangle = \lambda_{3/2^-} \sqrt{\frac{M_{3/2^-}}{E_{3/2^-}}} \gamma_5 u_\mu(p, s)
\]

\[
\sum_s u^\mu(p, s) \bar{u}^\nu(p, s) = \frac{(\gamma \cdot p + m)}{2m} \left\{ \frac{1}{3} \gamma^\mu \gamma^\nu - g^{\mu\nu} \right\} \left\{ - \frac{2p^\mu p^\nu}{3m^2} + \frac{p^\mu \gamma^\nu - p^\nu \gamma^\mu}{3m} \right\}
\]
Projection Operators

States of specific parity extracted with,

$$\Gamma^\mp = \frac{1}{2} \left( 1 \pm \frac{M_{B^\pm}}{E_{B^\pm}} \gamma_0 \right)$$

States of specific spin extracted with,

$$P_{\mu\nu}^{\frac{3}{2}}(p) = g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{1}{3p^2}(\gamma \cdot p)\gamma_\mu p_\nu + p_\mu \gamma_\nu \gamma \cdot p$$

$$P_{\mu\nu}^{\frac{1}{2}}(p) = g_{\mu\nu} - P_{\mu\nu}^{\frac{3}{2}}(p)$$
Lattice simulation parameters

- Simulations done on a $20^3 \times 40$ lattice, $a = 0.128$ fm.
- 400 gauge field configurations.
- Smallest $m_\pi = 300$ MeV.
  We use the FLIC fermion action for improved scaling.
Summary of nucleon resonances
Summary of nucleon resonances
Summary of $\Delta$ resonances

![Graph showing $\Delta$ resonances](image-url)
Summary of $\Delta$ resonances

- $\Delta 3/2^+$
- $\Delta 3/2^-$
- $\Delta 1/2^-$
- $\Delta 1/2^+$

$M$ (GeV) vs. $m_{\pi}^2$ (GeV$^2$)

- S-Wave N+$\pi$
- P-Wave N+$\pi$
Summary of $\Delta$ resonances
Our $\chi_1, \chi_2, \chi_3$ Roper search

To construct the correlation matrix we evaluate the cross terms,

$$G^\mu(t, \vec{p}) = \sum_B e^{-E_B t} \times \sum_s \langle 0 | \chi_3^\mu(0) | B, \vec{p}, s \rangle \langle B, \vec{p}, s | \bar{\chi}_{1,2}(0) | 0 \rangle$$

$$G^\mu(t, \vec{p}) = \sum_B e^{-E_B t} \times \sum_s \langle 0 | \chi_{1,2}(0) | B, \vec{p}, s \rangle \langle B, \vec{p}, s | \bar{\chi}_3^\mu(0) | 0 \rangle$$
Nucleon resonances
Search for the $\Theta^+$ pentaquark
Form factors of spin-1 mesons, with FLIC fermions

$2 \times 2$ Correlation Matrix Analysis

$\chi_1$ and $\chi_3$

$\chi_2$ and $\chi_3$
Fierz transformation of $\chi_3$

Results suggested that $\chi_3$ has significant overlap with $\chi_1, \chi_2$. Using the Fierz identity,

$$\delta_{\alpha\alpha'}\delta_{\beta\beta'} = \frac{1}{4} \sum_J (\Gamma_J)_{\alpha\beta} (\Gamma_J^{-1})_{\beta'\alpha'}$$

$$\chi_3^\mu = -\frac{1}{4} \gamma^\mu \gamma^5 \chi_{\text{Ioffe}} - \frac{1}{8} \gamma^\mu \gamma^5 \chi_A + \frac{1}{2} \varepsilon^{abc} (u^a T C \gamma^\mu u^b) d^c - \frac{i}{2} \varepsilon^{abc} (u^a T C \sigma^\mu\alpha u^c) \gamma_\alpha d^c$$

D.B. Leinweber, Phys. Rev. D51, 6383-6393, 1995

$$\chi_{\text{Ioffe}} = \varepsilon^{abc} (u^a T C \gamma^\alpha u^b) \gamma^5 \gamma_\alpha d^c$$

$$\chi_A = \varepsilon^{abc} (u^a T C \sigma^\alpha\beta u^b) \gamma^5 \sigma_{\alpha\beta} d^c$$
3 × 3 Correlation Matrix Analysis
Summary

- Nucleon spectrum successfully simulated a light quark masses.
- Effects of the open decay channel important can be important.
- Chiral curvature observed in the $\Delta^{++}$
- We do not extract a state consistent with the Roper.
Outline

Nucleon resonances

Search for the $\Theta^+$ pentaquark

Form factors of spin-1 mesons, with FLIC fermions
**NK-type Interpolating Field**

- Csikor *et al.*, JHEP 0311:070,(2003)
- Spin-1/2, isospin 0,1

\[
\chi_{NK} = \frac{1}{\sqrt{2}} \epsilon^{abc} (u^T a C \gamma_5 d^b) \left\{ u^c (\bar{s}^e i \gamma_5 d^e) \mp (u \leftrightarrow d) \right\}
\]

\[
\chi_{\overline{NK}} = \frac{1}{\sqrt{2}} \epsilon^{abc} (u^T a C \gamma_5 d^b) \left\{ u^e (\bar{s}^e i \gamma_5 d^c) \mp (u \leftrightarrow d) \right\}
\]

The \( \mp \) for isospin \( I = 0 \) and \( 1 \) channels, respectively.
Diquark-Diquark Style Interpolating Fields

- Spin-1/2, isospin 0

\[ \chi_{PS} = \epsilon^{abc} \epsilon^{aef} \epsilon^{bgh} (u^T C d^f)(u^g C \gamma_5 d^h) C \bar{s} c^T \]

- Our SS-type interpolating field, Phys.Rev.D**72** 014502,(2005).
- Spin-1/2, isospin 1

\[ \chi_{SS} = \frac{1}{\sqrt{2}} \epsilon^{abc} (u^T a C \gamma_5 d^b)(u^T c C \gamma_5 d^e) C \bar{s} T e \]
Results, $I(J^P) = 0(1/2^-)$

- NK $I(J^P)=0(1/2^-)$ colour singlet
- NK $I(J^P)=0(1/2^-)$ colour fused
- PS $I(J^P)=0(1/2^-)$
- S–Wave N+K
Mass Splitting, \( I(J^P) = 0(1/2^-) \)
Results, $I(J^P) = 1(1/2^-)$
Summary, $I(J^P) = 0(1/2^-)$
Results, $I(J^P) = 0, 1(1/2^+)$
Mass Splitting, $I(J^P) = 0(1/2^+)$
Spin-3/2 Pentaquark Interpolating Field

- **Lasscock et al., Phys D72, 074507, 2005**
- Couples to both Spin-1/2 and Spin-3/2 states, isospin 0,1

\[
\chi_{NK^*} = \frac{1}{\sqrt{2}} \epsilon^{abc} (u^T a C \gamma_5 d^b) \left\{ u^c (\bar{s} e i \gamma^\mu d^e) \mp (u \leftrightarrow d) \right\},
\]

\[
\chi_{LY} = \epsilon^{abc} (u^T a C \gamma_5 \gamma^\mu d^b) \left\{ (u^T c C \gamma_5 d^e) \mp (u^T e C \gamma_5 d^c) \right\} C \bar{s} e^T
\]

The \( \mp \) for isospin \( I = 0 \) and \( 1 \) channels, respectively.

- Phenomenology and projection operators based on nucleon resonance study.

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Efron, An Introduction to the Bootstrap

The jackknife sample of the mean is calculated with one of the data points excluded,

$$\bar{x}_i = \frac{1}{n-1} \sum_{j \neq i} x_j$$  \hspace{1cm} (1)

The usual estimate of the variance is then,

$$\sigma^2 = \frac{n-1}{n} \sum_i \left( \bar{x}_i - \frac{1}{n} \sum_j \bar{x}_j \right)^2$$  \hspace{1cm} (2)
Jackknife Subensembles

Efron, An Introduction to the Bootstrap

Pion

0(3/2^+) pentaquark
Jackknife Subensembles

Efron, An Introduction to the Bootstrap

Pion

\[ 0(3/2^+) \text{ pentaquark} \]
Isoscalar odd parity states

Effective mass of the \(I(J^P) = 0(3/2^-)\) state extracted with the \(LY\) interpolator.
Isoscalar odd parity states

\[ \chi^2_{\text{dof}} \] at the largest quark mass.

Lower bound fixed at \( t=14 \)  
Upper bound fixed at \( t=21 \)
Isoscalar odd parity states

Effective mass of the $I(J^P) = 0(3/2^-)$ state extracted with the $LY$ interpolator.
Isoscalar odd parity states

$$\chi^2_{\text{dof}}$$ at the largest quark mass.

Lower bound fixed at \( t=14 \)

Upper bound fixed at \( t=21 \)
Isoscalar odd parity states

![Graph showing the relationship between $m$ (GeV) and $m^2_{\pi}$ (GeV²). The graph includes various symbols representing different states and their quantum numbers, such as LY $I(J^P)=0(1/2^-)$, LY $I(J^P)=0(3/2^-)$, NK* $I(J^P)=0(1/2^-)$, NK* $I(J^P)=0(3/2^-)$, and LY $I(J^P)=0(3/2^-)$, (t=14−16). The graph also shows two theoretical curves: S-Wave N+K* and S-Wave N+K.]
Isoscalar even parity states
Isoscalar even parity states

\[ \chi^2_{\text{dof}} \] at the third largest quark mass.

Lower bound at \( t=12 \) and \( t=13 \)

Upper bound fixed at \( t=22 \)
Isoscalar even parity states

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Isoscalar even parity states

\[ \chi^2_{\text{dof}} \] at the third largest quark mass.

Lower bound at \( t=12 \) and \( t=13 \)

Upper bound fixed at \( t=22 \)
Isoscalar even parity states

\[
M \text{ (GeV)}
\]

\[
\begin{align*}
\text{O} & \quad \text{NK}^* I(J^P)=0(1/2^+) \\
\text{●} & \quad \text{NK}^* I(J^P)=0(3/2^+) \\
\text{△} & \quad \text{LY} I(J^P)=0(1/2^+) \\
\text{▲} & \quad \text{LY} I(J^P)=0(3/2^+), (t\geq 19)
\end{align*}
\]

- P-Wave N+K
- P-Wave N+K*
- S-Wave N*+K

\[
m_\pi^2 \text{ (GeV}^2\text{)}
\]
Isoscalar even parity states

\[ M \text{ (GeV)} \]

-4.0
-3.5
-3.0
-2.5
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0

\[ t \]

10
15
20
25
30

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Summary of Spin-3/2 pentaquark calculations

Summary of Spin-3/2 pentaquark calculations
Summary of Spin-3/2 pentaquark calculations

\( \chi^2_{\text{dof}} \) at the third largest quark mass.

Lower bound at \( t=12 \) and \( t=13 \)

Upper bound fixed at \( t=22 \)
Summary of Spin-3/2 pentaquark calculations
Summary of Spin-3/2 pentaquark calculations

Summary of Spin-3/2 pentaquark calculations

Summary

- No clear evidence of attraction is found in the spin-1/2 channel.
- “Sort and Cut” estimate of the confidence interval is more robust for the analysis of noisy pentaquark data.
- The size of the error bar does not determine the goodness of the fit.
- The discrepancy between our calculation and Doi et al. is due to the fit window.
- Evidence of attraction is found in the even parity, spin-3/2 Isoscalar, channel.
Outline

Nucleon resonances

Search for the $\Theta^+$ pentaquark

Form factors of spin-1 mesons, with FLIC fermions
Electromagnetic Structure in Quenched LQCD

- Charge, magnetic and quadrupole form factors calculated in Quenched LQCD.
- Charge radii and magnetic moment derived.

- Quark Models:
  - Hyperfine interaction \( \frac{(\sigma_1 \cdot \sigma_2)}{(m_1 m_2)} \).
  - Magnetic moment of the \( \rho \) meson, \( \mu_\rho \approx 1.84 \mu_N \) at SU(3) flavour limit.

- Lattice QCD: Oblate \( \rho \)-meson?

- Environmental sensitivity of observables?
Form Factors, Formulas

Following,
For the pion and kaon,

$$
\langle p' s' | J^\alpha | p, s \rangle = \frac{1}{2 \sqrt{E_p E_{p'}}} \left[ p^\alpha + p'^\alpha \right] F_1(Q^2).
$$

For the $\rho$ and $K^*$,

$$
\langle p' s' | J^\mu | ps \rangle = \frac{1}{2 \sqrt{E_p E_{p'}}} \epsilon^\star_\alpha(p', s') \epsilon_\beta(p, s) J^{\alpha \mu \beta}(p', p),
$$

$$
J^{\alpha \mu \beta}(p', p) = -\left\{ G_1(Q^2) g^{\alpha \beta} \left[ p^\mu + p'^\mu \right] + G_2(Q^2) \left[ g^{\mu \beta} q^\alpha - g^{\mu \alpha} q^\beta \right] - G_3(Q^2) q^\beta q^\alpha \frac{p^\mu + p'^\mu}{2M^2} \right\}.
$$
Quark Sector Contributions to the Charge Radii $\langle r^2 \rangle$ (fm$^2$).

Pseudoscalar mesons

Vector mesons

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Environmental Sensitivity of Charge Radii

Ratio of the up-quark contributions to $<r^2>$ fm$^2$.

Pseudoscalar mesons

Vector mesons
Summary of the meson and proton charge radii

![Graph showing the summary of meson and proton charge radii](image-url)
The Magnetic Moment

\[ \mu (\mu_N) \]

\[ m^2 (\text{GeV}^2) \]

- \( \rho^+ \)
- \( K^{*+} \)
$\rho$-meson Quadrupole Form Factor

\[ G_Q(Q^2) \ (10^{-3} \text{ fm}^2) \]

\[ m_{\pi}^2 (\text{GeV}^2) \]

- $\rho^+$
- $K^{*+}$

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Summary

- Hyperfine repulsion in vector mesons on Quenched LQCD is significant.
- There is significant environmental sensitivity in the up quark contributions to vector meson charge radii.
- The charge radius of the proton is similar to that of the $\rho$-meson, but consistently larger than that of the pion and kaon.
- Vector mesons are oblate.
There are a number of differences in the lattice QCD setup between the current studies and Ref. [67], such as the gauge and the quark actions, and the implementation of the smeared source. However, we consider that, rather than being a consequence of these differences, the discrepancy mainly comes from the low statistics adopted in Ref. [67]. We emphasize again that spin-3/2 pentaquark correlators are quite noisy, and therefore require better statistics.