QCD Vacuum, Centre Vortices and Flux Tubes

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Overview

- Visualizations of QCD vacuum structure
- Action and Topological Charge densities
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  - Action and Topological Charge densities
- Role of Instantons and Anti-instantons
  - Dynamical mass generation
  - Accurate algorithms required to reveal these configurations
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- Centre Vortices
  - What are they? Why are they interesting?
  - What happens if they are removed from QCD?
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- **Visualizations of QCD vacuum structure**
  - Action and Topological Charge densities

- **Role of Instantons and Anti-instantons**
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  - Accurate algorithms required to reveal these configurations

- **Centre Vortices**
  - What are they? Why are they interesting?
  - What happens if they are removed from QCD?

- **Potential Energy** between heavy quarks
  - Y versus $\Delta$ shape flux-tubes in baryons
  - Emphasize how the nature of the flux tube revolutionizes the concept of a constituent-quark.
One-Loop $F_{\mu\nu}$ F. Bonnet et al, Phys.Rev.D62:094509,2000
Two-loop Improvement

\[ O_{\mu\nu}^{(2)}(x) = c_1 U_{\mu}(x) + c_2 U_{\mu}(x) \]

Generalized to five-loop improvement in

S. O. Bilson-Thompson, D. B. Leinweber and A. G. Williams,
arXiv:hep-lat/0203008.
An $\mathcal{O}(a^4)$-improved field-strength tensor is given by the following sum of Clover contributions $C_{\mu\nu}^{m \times n}$ for $m \times n$ loops:

$$F_{\mu\nu}^{\text{Imp}} = k_1 C_{\mu\nu}^{(1 \times 1)} + k_2 C_{\mu\nu}^{(2 \times 2)} + k_3 C_{\mu\nu}^{(1 \times 2)} + k_4 C_{\mu\nu}^{(1 \times 3)} + k_5 C_{\mu\nu}^{(3 \times 3)},$$

where

$$k_1 = 19/9 - 55 k_5, \quad k_2 = 1/36 - 16 k_5,$$

$$k_3 = 64 k_5 - 32/45, \quad k_4 = 1/15 - 6 k_5.$$
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$k_5$ is a tunable free parameter.
Five-Loop Improvement

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$k_5$ is a tunable free parameter.

Governs $\mathcal{O}(a^6)$ errors.

- When $k_5 = 1/90$, $k_3 = k_4 = 0$ providing a 3-loop $F_{\mu\nu}$.
- Setting $k_5 = 0$, provides 4-loop $F_{\mu\nu}$.
- We consider $k_5 = 1/180$, as the 5-loop $F_{\mu\nu}$. 
2, 3, 4 and 5-Loop Improved $F_{\mu\nu}$
Revealing the Structure of Gluon Fields

Consider the Action density

\[ S(x) = \frac{1}{2} F_{\mu \nu}^{ab}(x) F_{\mu \nu}^{ba}(x). \]

- \( S(x) \) is a scalar field in four dimensions.
- Consider a three-dimensional slice of the lattice.
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Consider a three-dimensional slice of the lattice.

To view the structure of typical vacuum gluon-field configurations

- Render areas of intense action density in red.
- Render areas of moderate action density in blue.
- Low action-density regions are not rendered to allow us to see into the gluon field.
Short-distance physics in QCD is well understood.
- Interactions are weak at short distances.
- Quarks behave as approximately free particles.
- Corrections may be estimated via perturbation theory.
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This physics may be removed by locally minimizing the action.

- Cabibbo-Marinari algorithm identifies the link which maximally reduces the local action.
- Process is called “Cooling.”
Exposing Long-Distance Physics

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- This physics may be removed by locally minimizing the action.
  - Cabibbo-Marinari algorithm identifies the link which maximally reduces the local action.
  - Process is called “Cooling.”

- Each frame in the animation follows one sweep of Cooling.
  - All links are updated to locally minimize the action.
  - Highly-improved lattice operators are utilized.
  - Both $O(a^2)$ and $O(a^4)$ errors are removed
  - $O(a^6)$ errors tuned to stabilize nonperturbative phenomena
Consider the Topological Charge density

\[ q(x) = \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu}(x) F_{\rho\sigma}^{ab}(x). \]

\( q(x) \) is a measure of the winding of the gluon field lines.
Consider the **Topological Charge density**

\[
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- \( q(x) \) is a scalar field in four dimensions.
- Consider a three-dimensional slice of the lattice.

To view the structure of typical vacuum gluon-field configurations:
- Render areas of positive charge density in red.
- Render areas of negative charge density in blue.
- Low charge-density regions are not rendered to allow us to see into the gluon field.
Gluon fields having nontrivial winding represented by the topological charge \( Q = \sum_x q(x) = \pm 1 \).

Classical solutions to the QCD equations of motion

They live in four dimensions.

They have small finite action, \( S_0 = \frac{8\pi^2}{g^2} \)

Approximately 10,000 times smaller than the action of a typical field configuration
Gluon fields having nontrivial winding represented by the topological charge \( Q = \sum_x q(x) = \pm 1 \).

Classical solutions to the QCD equations of motion.

They live in four dimensions.

In isolation, the topological charge density is spherically symmetric

\[
q(x) = \pm \frac{6}{\pi^2} \frac{\rho^4}{(x^2 + \rho^2)^4},
\]

where \( \rho \) is a size parameter.

Similarly for the action.
Instanton Facts

- Gluon fields having nontrivial winding represented by the topological charge $Q = \sum_x q(x) = \pm 1$.
- Classical solutions to the QCD equations of motion
- They live in four dimensions.
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$$q(x) = \pm \frac{6}{\pi^2} \frac{\rho^4}{(x^2 + \rho^2)^4},$$

where $\rho$ is a size parameter.
- Similarly for the action.
- Revealed in lattice QCD only after extensive cooling.
The Effective Mass of Quarks

- Short-distance physics in QCD is well understood.
  - Interactions are weak at short distances.
  - Quarks behave as approximately free particles.
  - Corrections may be estimated via perturbation theory.
- This property of QCD is called asymptotic freedom.
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At long distances, a rich structure in the QCD vacuum is revealed.
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- Quarks and gluons do not propagate as free particles.
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- This property of QCD is called asymptotic freedom.
- At long distances, a rich structure in the QCD vacuum is revealed.
- Quarks and gluons do not propagate as free particles.
- The quark acquires an effective mass due to its interaction with the QCD vacuum.
Massless Quarks in the QCD Vacuum
Eigenmodes of the Dirac Operator are defined by

$$\mathcal{D} | \psi_i \rangle = \lambda_i | \psi_i \rangle.$$
Eigenmodes of the **Dirac Operator** are defined by

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The spectral representation of the quark propagator is

\[ S = \frac{1}{\mathcal{D} + m_q} = \sum_i | \psi_i \rangle \frac{1}{\lambda_i + m} \langle \psi_i |. \]
Eigenmodes of the Dirac Operator are defined by

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$$S = \frac{1}{\mathcal{D} + m_q} = \sum_i | \psi_i \rangle \frac{1}{\lambda_i + m} \langle \psi_i |.$$ 

For small quark masses (like the $u$ or $d$ quarks in nature)

- Eigenmodes having small eigenvalues, $\lambda_i$, dominate the nature of how quarks propagate in the QCD vacuum.
Eigenmodes of the Dirac Operator are defined by

\[ \mathcal{D} \psi_i = \lambda_i \psi_i. \]

The spectral representation of the quark propagator is

\[ S = \frac{1}{\mathcal{D} + m_q} = \sum_i \psi_i \frac{1}{\lambda_i + m} \langle \psi_i \rangle. \]

For \( \lambda_i \) small, the real scalar field

\[ P_i(x) = \langle x | \psi_i \rangle \langle \psi_i | x \rangle = \psi(x) \psi^\dagger(x), \]

describes the probable locations of the quarks in the vacuum as they propagate.

QCD Vacuum, Centre Vortices and Flux Tubes – p.14/50
Low-lying Eigenmode Density

- Low-lying eigenmodes of the Dirac operator are located on the topological structures giving rise to them.

- Each low-lying nondegenerate mode is associated with a single topological structure.
What are Centre Vortices?

1. Gauge fix gluon configurations to Maximal Centre Gauge
   - Bring the links $U_\mu(x)$ close to the centre elements of $SU(3)$
     \[
     Z = \exp \left( 2\pi i \frac{m}{3} \right) I, \text{ with } m = -1, 0, 1
     \]
   - On the lattice, search for the gauge transformation $\Omega$
     \[
     \sum_{x,\mu} \left| \text{tr} \ U_\mu^{\Omega}(x) \right|^2 \xrightarrow{\Omega} \text{max}
     \]
What are Centre Vortices?

1. Gauge fix gluon configurations to Maximal Centre Gauge
2. Project the gluon field to the centre phase

\[ U_\mu(x) \rightarrow Z_\mu(x) \text{ where } Z_\mu(x) = \exp \left( 2\pi i \frac{m_\mu(x)}{3} \right), \quad m_\mu(x) = -1, 0, 1 \]

Implemented by

\[ \frac{1}{3} \text{tr} \ U_\mu^\Omega(x) = r_\mu(x) \exp \left( i \varphi_\mu(x) \right), \]

\[ \cos \left( \varphi_\mu(x) - \frac{2\pi}{3} m_\mu(x) \right) \quad m_\mu \rightarrow \text{max}, \]

close to zero
What are Centre Vortices?

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3. Vortices are identified by the centre charge

\[ z = \prod_{\Box} Z_\mu(x) = \exp \left( 2\pi i \frac{n}{3} \right) \]

- If \( \text{mod}(n, 3) = 0 \) no vortex pierces the plaquette
- If \( \text{mod}(n, 3) = -1, \text{ or } 1 \) a vortex with charge \( z \) pierces the plaquette
What are Centre Vortices?

1. Gauge fix gluon configurations to Maximal Centre Gauge

2. Project the gluon field to the centre phase

\[ U_\mu(x) \to Z_\mu(x) \] where

\[ Z_\mu(x) = \exp \left( 2\pi i \frac{m_\mu(x)}{3} \right), \quad m_\mu(x) = -1, 0, 1 \]

3. Vortices are identified by the centre charge

\[ z = \prod_{\Box} Z_\mu(x) = \exp \left( 2\pi i \frac{n}{3} \right) \]

4. Vortices are removed by removing the centre phase

\[ U_\mu(x) \to U_\mu'(x) = Z_\mu^*(x) \cdot U_\mu(x), \]
Static Quark Potential

Fit to Full SU(3) potential

Vortex $b = 4.80$

No-Vortex $b = 4.80$

Vortex $b = 4.60$

No-Vortex $b = 4.60$

Vortex $b = 4.38$

No-Vortex $b = 4.38$
Gluon Propagator

\[ q^2 D(q^2) \]

\[ q \ (\text{GeV}) \]

\[ q^2 \]
Dynamical mass generation is associated with Dynamical Chiral Symmetry Breaking $\langle \bar{q}q \rangle \neq 0$. 
Centre Vortices and Mass Generation

- Dynamical mass generation is associated with Dynamical Chiral Symmetry Breaking $\langle \bar{q}q \rangle \neq 0$.

- Dynamical Chiral Symmetry Breaking is associated with a Finite density of zeromodes of the Dirac Operator, $\rho(0)$, via the Casher-Banks relation

$$\langle \bar{q}q \rangle = -\pi \rho(0), \text{ as } m_q \to 0$$
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\[
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Topologically non-trivial gauge fields (including instantons) give rise to zeromodes.
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Topologically non-trivial gauge fields (including instantons) Give rise to zeromodes

Hence, a link between centre vortices and topology implies a Link between centre vortices and dynamical mass generation.
Gluon Action Evolution

![Graph showing the evolution of S/S₀ or |ψ| with Cooling Sweep.](image)

- The graph illustrates the change in the ratio of the gluon action to its initial value, $S/S₀$, or the absolute value of the wave function, $|ψ|$, as the cooling sweep progresses.
- The x-axis represents the Cooling Sweep, ranging from 0 to 100.
- The y-axis represents $S/S₀$ or $|ψ|$, with values ranging from 0 to 10.
- The data shows a decrease in $S/S₀$ or $|ψ|$ with increasing Cooling Sweep, indicating a decrease in the gluon action or wave function magnitude.

This graph is crucial for understanding the behavior of gluons in the QCD vacuum, particularly in the context of center vortices and flux tubes.
Gluon Action Evolution

![Graph showing Gluon Action Evolution](chart.png)
Quark Propagator Decomposition

- In a covariant gauge, Lorentz invariance allows

\[ S^{aa}(\zeta; q) \equiv S(\zeta; q) = \frac{Z(\zeta; q^2)}{i \gamma \cdot q + M(q^2)}, \]

- \(M(q^2)\) is the Mass function
- \(Z(\zeta; q^2)\) is the Renormalization function
- \(\zeta\) is the renormalization point (3 GeV) with conditions
  - \(Z(\zeta; \zeta^2) \equiv 1\)
  - \(M(\zeta^2) \equiv m(\zeta)\)
- For sufficiently large \(\zeta\), \(m(\zeta) \to m_\zeta\), the current quark mass.
Quark Renormalization Function at $m_q^0 = 78$ MeV
Quark Mass Function at $m_0^q = 116$ MeV
Quark Mass Function at $m_q^0 = 116$ MeV
Quark Mass Function at $m_q^0 = 78$ MeV
Quark Mass Function at $m_q^0 = 58$ MeV
Infrared Mass Function

The diagram shows a plot of $M(q^2_{\text{min}})$ (GeV) versus $m_q^0$ (GeV). The data points with error bars are shown in black, while the solid red line represents a fit to the data. The data points follow a linear trend, indicating a relationship between the mass function and the mass parameter.
Infrared Mass Function

\[ M(q^2_{\text{min}}) \text{ (GeV)} \]

\[ m_q^0 \text{ (GeV)} \]

![Graph showing the relationship between \( M(q^2_{\text{min}}) \) and \( m_q^0 \).]
LCG Mass Function with $m^0_q = 29$ MeV
Potential Energy between Heavy Quarks

![Graph showing potential energy $V(r)$ vs. distance $r$. The graph plots the energy in GeV against the distance in fm (femtometers). The data points form a linear trend, indicating a direct relationship between the potential energy and the distance between heavy quarks.]
Gluon Field Distribution in Mesons

How does the Vacuum respond to the presence of Quarks?

\[ C(\vec{y}, \vec{d}) = \frac{\langle Q(\vec{x}) \, S(\vec{y} + \vec{x}) \, Q^\dagger(\vec{x} + \vec{d}) \rangle_{\vec{x}}}{\langle Q(\vec{x}) \, Q^\dagger(\vec{x} + \vec{d}) \rangle_{\vec{x}} \, \langle S(\vec{x}) \rangle_{\vec{x}}} \]

- \( Q(\vec{x}) \) denotes a quark at position \( \vec{x} \).
- \( Q^\dagger(\vec{x} + \vec{d}) \) denotes an antiquark at position \( \vec{x} + \vec{d} \).
- \( S(\vec{y} + \vec{x}) \) denotes the action density at position \( \vec{y} \) from the quark at \( \vec{x} \).
- \( \langle . . . \rangle_{\vec{x}} \) denotes average over \( \vec{x} \) and gluon field configurations.
- If there is no correlation between the action density and the locations of the quark and antiquark,

\[ C(\vec{y}, d) = 1. \]
Baryonic Wilson Loop
## Quark Coordinates

<table>
<thead>
<tr>
<th>#</th>
<th>$Q_1$</th>
<th>$Q_2$</th>
<th>$Q_3$</th>
<th>$F$</th>
<th>$\langle r_s \rangle$</th>
<th>$\langle d_{qq} \rangle$</th>
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<tr>
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<td>(−1, −2)</td>
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<td>(−1, −2)</td>
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<td>(−4, −6)</td>
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<td>(−4, −7)</td>
<td>(0.04, 0)</td>
<td>0.99</td>
<td>2.12</td>
</tr>
</tbody>
</table>
T- and Y-Shape Source Paths
How does the **Vacuum** respond to the presence of **Quarks**?

\[
C(\vec{y}; \vec{r}_1, \vec{r}_2, \vec{r}_3; \tau) = \frac{\langle W_{3Q}(\vec{r}_1, \vec{r}_2, \vec{r}_3; \tau) S(\vec{y}, \tau/2) \rangle}{\langle W_{3Q}(\vec{r}_1, \vec{r}_2, \vec{r}_3; \tau) \rangle \langle S(\vec{y}, \tau/2) \rangle}
\]

If there is no correlation between the **action density** and the locations of the **quarks**,

\[
C(\vec{y}, \vec{r}_1, \vec{r}_2, \vec{r}_3; \tau) = 1.
\]
Gluon Field Distribution in Baryons

How does the \textit{Vacuum} respond to the presence of \textit{Quarks}?

\[
C(\vec{y}; \vec{r}_1, \vec{r}_2, \vec{r}_3; \tau) = \frac{\left< W_{3Q}(\vec{r}_1, \vec{r}_2, \vec{r}_3; \tau) \right> S(\vec{y}, \tau/2)}{\left< W_{3Q}(\vec{r}_1, \vec{r}_2, \vec{r}_3; \tau) \right> \left< S(\vec{y}, \tau/2) \right>}
\]

If there is no correlation between the action density and the locations of the quarks,

\[
C(\vec{y}, \vec{r}_1, \vec{r}_2, \vec{r}_3; \tau) = 1.
\]

Similar results are observed for $\vec{E}^2$ and $\vec{B}^2$ separately.
30-sweep T-shape Source
30-sweep Y-shape Source
Effective Potential at $\tau = 1 \rightarrow 2$
Cross Section Fit

![Graph showing data points and a fit curve]

- Data points
- Fit

$C(y)$ vs $y$ with points and a fitted curve.
Baryonic Ground State Properties

- Flux-tube radius is 0.38(3) fm.
- Vacuum-field action suppressed by 7.2(6)%. 

QCD Vacuum, Centre Vortices and Flux Tubes – p.43/50
Baryonic Ground State Properties

- Flux-tube radius is 0.38(3) fm.
- Vacuum-field action suppressed by 7.2(6)%.
- Flux-tube node is 25% larger at 0.47(2) fm.
- Vacuum-field action suppression is 15(3)% larger at 8.1(7)%.
The Heart of the Atom

Structure within the Atom

Quark
Size $< 10^{-19}$ m

Nucleus
Size $= 10^{-14}$ m

Electron
Size $< 10^{-18}$ m

Neutron and Proton
Size $= 10^{-15}$ m

Atom
Size $= 10^{-10}$ m

If the protons and neutrons in this picture were 10 cm across, then the quarks and electrons would be less than 0.1 mm in size and the entire atom would be about 10 km across.
The Structure of the Nucleon
Information on the Web

- The software used to create the animations is Advanced Visual System’s AVS/Express, http://www.avs.com/
- Many of these animations are available on the web.
Flux Tubes in SU(2) Gauge Theory

G.S. Bali, K. Schilling and C. Schlichter (Wuppertal U.)

Enhancement or Expulsion?

It is common to express the correlation as

\[
C_S(\vec{y}, \vec{d}) = \frac{\langle Q(\vec{x}) S(\vec{y} + \vec{x}) Q^\dagger(\vec{x} + \vec{d}) \rangle_{\vec{x}}}{\langle Q(\vec{x}) Q^\dagger(\vec{x} + \vec{d}) \rangle_{\vec{x}}} - \langle S(\vec{x}) \rangle_{\vec{x}}
\]
Enhancement or Expulsion?

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$$C_S(\vec{y}, \vec{d}) = \frac{\langle Q(\vec{x}) S(\vec{y} + \vec{x}) Q^\dagger(\vec{x} + \vec{d}) \rangle_{\vec{x}}}{\langle Q(\vec{x}) Q^\dagger(\vec{x} + \vec{d}) \rangle_{\vec{x}}} - \langle S(\vec{x}) \rangle_{\vec{x}}$$

- In Euclidean space

$$S(\vec{x}) = \frac{1}{2} tr \left( \vec{B}_{\text{Eucl}}^2 + \vec{E}_{\text{Eucl}}^2 \right) = \frac{1}{2} tr \left( \vec{B}^2 - \vec{E}^2 \right) > 0$$
Enhancement or Expulsion?

It is common to express the correlation as

$$C_S(\vec{y}, \vec{d}) = \frac{\langle Q(\vec{x}) S(\vec{y} + \vec{x}) Q^\dagger(\vec{x} + \vec{d}) \rangle_{\vec{x}}}{\langle Q(\vec{x}) Q^\dagger(\vec{x} + \vec{d}) \rangle_{\vec{x}}} - \langle S(\vec{x}) \rangle_{\vec{x}}$$

In Euclidean space

$$S(\vec{x}) = \frac{1}{2} \text{tr} \left( \vec{B}_{\text{Eucl}}^2 + \vec{E}_{\text{Eucl}}^2 \right) = \frac{1}{2} \text{tr} \left( \vec{B}^2 - \vec{E}^2 \right) > 0$$

But the Minkowski action is

$$S_M(\vec{x}) = \frac{1}{2} \text{tr} \left( \vec{E}^2 - \vec{B}^2 \right) < 0$$
Enhancement or Expulsion?

It is common to express the correlation as

$$C_S(\vec{y}, \vec{d}) = \frac{\langle Q(\vec{x}) S(\vec{y} + \vec{x}) Q^\dagger(\vec{x} + \vec{d}) \rangle_{\vec{x}}}{\langle Q(\vec{x}) Q^\dagger(\vec{x} + \vec{d}) \rangle_{\vec{x}}} - \langle S(\vec{x}) \rangle_{\vec{x}}$$

In Euclidean space

$$S(\vec{x}) = \frac{1}{2} tr \left( \vec{B}_{\text{Eucl}}^2 + \vec{E}_{\text{Eucl}}^2 \right) = \frac{1}{2} tr \left( \vec{B}^2 - \vec{E}^2 \right) > 0$$

But the Minkowski action is

$$S_M(\vec{x}) = \frac{1}{2} tr \left( \vec{E}^2 - \vec{B}^2 \right) < 0$$

The sign of the correlation may be selected freely.
Gluon Field Distribution in Mesons

How does the **Vacuum** respond to the presence of **Quarks**?

\[ C(y, d) = \frac{\langle Q(x) \ S(y + x) \ Q^+(x + d) \rangle_x}{\langle Q(x) \ Q^+(x + d) \rangle_x \langle S(x) \rangle_x} \]

- \( Q(x) \) denotes a quark at position \( x \).
- \( Q^+(x + d) \) denotes an antiquark at position \( x + d \).
- \( S(y + x) \) denotes the action density at position \( y \) from the quark at \( x \).
- \( \langle \ldots \rangle_x \) denotes average over \( x \) and gluon field configurations.

If there is no correlation between the **action density** and the locations of the **quark** and **antiquark**,

\[ C(y, d) = 1. \]