QCD Vacuum, Centre Vortices and Flux Tubes

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Visualizations of QCD vacuum structure

Action and Topological Charge densities

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- Role of Instantons and Anti-instantons
 - Dynamical mass generation
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 - What are they? Why are they interesting?
 - What happens if they are removed from QCD?
- Potential Energy between heavy quarks
 - Y versus Δ shape flux-tubes in baryons
 - Emphasize how the nature of the flux tube revolutionizes the concept of a constituent-quark.

CSSM Lattice Collaboration

Sundance Bilson-Thompson Francois Bissey Sharada Boinepalli **Frederic Bonnet** Patrick Bowman **Fu-Guang Cao Paul Coddington** Seungho Choe Patrick Fitzhenry John Hedditch Urs Heller Waseem Kamleh Adrian Kitson

Daniel Kusterer Kurt Langfeld Ben Lasscock Frank Lee Derek Leinweber Wally Melnitchouk Maria Parappilly **Tony Signal** Lorenz von Smekal Mark Stanford **Tony Williams** James Zanotti Jianbo Zhang

One-Loop $F_{\mu\nu}$ F. Bonnet *et.al*, Phys.Rev.D62:094509,2000



Two-loop Improvement



- Generalized to five-loop improvement in
 - S. O. Bilson-Thompson, D. B. Leinweber and A. G. Williams, arXiv:hep-lat/0203008.

Five-Loop Improvement

▲ An $\mathcal{O}(a^4)$ -improved field-strength tensor is given by the following sum of Clover contributions $C^{m \times n}{}_{\mu\nu}$ for $m \times n$ loops:

$$F_{\mu\nu}^{\rm Imp} = k_1 \, C_{\mu\nu}^{(1\times1)} + k_2 \, C_{\mu\nu}^{(2\times2)} + k_3 \, C_{\mu\nu}^{(1\times2)} + k_4 \, C_{\mu\nu}^{(1\times3)} + \mathbf{k_5} \, C_{\mu\nu}^{(3\times3)},$$

where

 $k_1 = \frac{19}{9} - \frac{55 \, k_5}{5}, \qquad k_2 = \frac{1}{36} - \frac{16 \, k_5}{5}, \\ k_3 = \frac{64 \, k_5}{5} - \frac{32}{45}, \qquad k_4 = \frac{1}{15} - \frac{6 \, k_5}{5},$

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where

- $k_1 = \frac{19}{9} 55 \, k_5 \,, \qquad k_2 = \frac{1}{36} 16 \, k_5 \,,$ $k_3 = \frac{64 \, k_5}{5} - \frac{32}{45} \,, \qquad k_4 = \frac{1}{15} - 6 \, k_5 \,,$
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- k_5 is a tunable free parameter.
- Governs $\mathcal{O}(a^6)$ errors.
 - When $k_5 = 1/90$, $k_3 = k_4 = 0$ providing a 3-loop $F_{\mu\nu}$.
 - Setting $k_5 = 0$, provides 4-loop $F_{\mu\nu}$.
 - We consider $k_5 = 1/180$, as the 5-loop $F_{\mu\nu}$.

2, 3, 4 and 5-Loop Improved $F_{\mu\nu}$



Consider the Action density

$$S(x) = \frac{1}{2} F^{ab}_{\mu\nu}(x) F^{ba}_{\mu\nu}(x) \,.$$

- S(x) is a scalar field in four dimensions.
- Consider a three-dimensional slice of the lattice.

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- To view the structure of typical vacuum gluon-field configurations
 - Render areas of intense action density in red.
 - Render areas of moderate action density in blue.
 - Low action-density regions are not rendered to allow us to see into the gluon field.

Exposing Long-Distance Physics

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 - Cabibbo-Marinari algorithm identifies the link which maximally reduces the local action.
 - Process is called "Cooling."
- Each frame in the <u>animation</u> follows one sweep of Cooling.
 - All links are updated to locally minimize the action.
 - Highly-improved lattice operators are utilized.
 - Both $\mathcal{O}(a^2)$ and $\mathcal{O}(a^4)$ errors are removed
 - $\mathcal{O}(a^6)$ errors tuned to stabilize nonperturbative phenomena

Consider the Topological Charge density

$$q(x) = \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{ab}_{\mu\nu}(x) F^{ba}_{\rho\sigma}(x) \,.$$



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 - Render areas of positive charge density in red.
 - Render areas of negative charge density in blue.
 - Low charge-density regions are not rendered to allow us to see into the gluon field.

Instanton Facts

- Gluon fields having nontrivial winding represented by the topological charge $Q = \sum_{x} q(x) = \pm 1$.
- Classical solutions to the QCD equations of motion
- They live in four dimensions.
- They have small finite action, $S_0 = 8\pi^2/g^2$
 - Approximately 10,000 times smaller than the action of a typical field configuration

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Revealed in lattice QCD only after extensive cooling.

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- At long distances, a rich structure in the QCD vacuum is revealed.
- Quarks and gluons do not propagate as free particles.
- The quark acquires an effective mass due to its interaction with the QCD vacuum.

Massless Quarks in the QCD Vacuum



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- **Solution** For small quark masses (like the u or d quarks in nature)
 - Eigenmodes having small eigenvalues, λ_i , dominate the nature of how quarks propagate in the QCD vacuum.

Eigenmodes of the Dirac Operator are defined by

The spectral representation of the quark propagator is

For λ_i small, the real scalar field

$$P_i(x) = \langle x \mid \psi_i \rangle \langle \psi_i \mid x \rangle = \psi(x) \psi^{\dagger}(x) ,$$

describes the probable locations of the quarks in the vacuum as they propagate.

Low-lying Eigenmode Density

Low-lying eigenmodes of the Dirac operator are located on the topological structures giving rise to them.



Each low-lying nondegenerate mode is associated with a single topological structure.

What are Centre Vortices?

- 1. Gauge fix gluon configurations to Maximal Centre Gauge
 - Bring the links $U_{\mu}(x)$ close to the centre elements of SU(3)

$$Z = \exp\left(2\pi i\,rac{m}{3}
ight){f I}\,,\,\,{
m with}\,\,m=-1,0,1$$

On the lattice, search for the gauge transformation Ω

$$\sum_{x,\mu} \left| \operatorname{tr} U_{\mu}{}^{\Omega}(x) \right|^2 \xrightarrow{\Omega} \max$$

What are Centre Vortices?

- 1. Gauge fix gluon configurations to Maximal Centre Gauge
- 2. Project the gluon field to the centre phase

$$U_{\mu}(x) \to Z_{\mu}(x)$$
 where $Z_{\mu}(x) = \exp\left(2\pi i \, \frac{m_{\mu}(x)}{3}\right), \ m_{\mu}(x) = -1, 0, 1$

Implemented by



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3. Vortices are identified by the centre charge

$$z = \prod_{\square} Z_{\mu}(x) = \exp\left(2\pi i \, \frac{n}{3}\right)$$

- If mod(n, 3) = 0 no vortex pierces the plaquette
- If mod(n, 3) = -1, or 1 a vortex with charge z pierces the plaquette
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4. Vortices are removed by removing the centre phase

$$U_{\mu}(x) \rightarrow U'_{\mu}(x) = Z^*_{\mu}(x) \cdot U_{\mu}(x),$$

Static Quark Potential



Gluon Propagator



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- Topologically non-trivial gauge fields (including instantons)
 - Give rise to zeromodes
- Hence, a link between centre vortices and topology implies a
 - Link between centre vortices and dynamical mass generation.

Gluon Action Evolution



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Gluon Action Evolution



Quark Propagator Decomposition

In a covariant gauge, Lorentz invariance allows

$$S^{aa}(\zeta;q) \equiv S(\zeta;q) = \frac{Z(\zeta;q^2)}{i\gamma \cdot q + M(q^2)},$$



 \square $Z(\zeta; q^2)$ is the Renormalization function

 \boldsymbol{P} is the renormalization point (3 GeV) with conditions

•
$$Z(\zeta;\zeta^2) \equiv 1$$

•
$$M(\zeta^2) \equiv m(\zeta)$$



Quark Renormalization Function at $m_q^0 = 78$ **MeV**



Quark Mass Function at $m_q^0 = 116$ **MeV**



Quark Mass Function at $m_q^0 = 116$ **MeV**



Quark Mass Function at $m_q^0 = 78$ **MeV**



Quark Mass Function at $m_q^0 = 58$ **MeV**



Infrared Mass Function



Infrared Mass Function



LCG Mass Function with $m_q^0 = 29$ **MeV**



Potential Energy between Heavy Quarks



Gluon Field Distribution in Mesons

How does the Vacuum respond to the presence of Quarks?

$$C(\vec{y}, \vec{d}) = \frac{\langle Q(\vec{x}) \ S(\vec{y} + \vec{x}) \ Q^{\dagger}(\vec{x} + \vec{d}) \rangle_{\vec{x}}}{\langle Q(\vec{x}) \ Q^{\dagger}(\vec{x} + \vec{d}) \rangle_{\vec{x}} \ \langle S(\vec{x}) \rangle_{\vec{x}}}$$

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- Sigma $S(\vec{y} + \vec{x})$ denotes the action density at position \vec{y} from the quark at \vec{x} .
- \checkmark $\langle \ldots \rangle_{\vec{x}}$ denotes average over \vec{x} and gluon field configurations.
- If there is no correlation between the <u>action density</u> and the locations of the <u>quark</u> and antiquark,

$$C(\vec{y}, d) = 1.$$

Baryonic Wilson Loop



Quark Coordinates

	(x, y) Coordinates (lattice units)				Distance (fm)	
#	Q_1	Q_2	Q_3	F	$\langle r_s \rangle$	$\langle d_{qq} \rangle$
1	(1, 0)	(-1, 1)	(-1, -1)	(-0.42, 0)	0.15	0.35
2	(2, 0)	(-1, 2)	(-1, -2)	(0.15,0)	0.27	0.54
3	(3,0)	(-1, 2)	(-1, -2)	(0.15, 0)	0.31	0.69
4	(3,0)	(-2, 3)	(-2, -3)	(-0.27, 0)	0.42	0.89
5	(4, 0)	(-3, 4)	(-3, -4)	(-0.69, 0)	0.57	1.24
6	(5,0)	(-4, 5)	(-4, -5)	(-1.11, 0)	0.72	1.58
7	(7,0)	(-4, 6)	(-4, -6)	(-0.54, 0)	0.88	1.93
8	(8, 0)	(-4,7)	(-4, -7)	(0.04, 0)	0.99	2.12

T- and Y-Shape Source Paths



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How does the Vacuum respond to the presence of Quarks?

$$C(\vec{y}; \vec{r_1}, \vec{r_2}, \vec{r_3}; \tau) = \frac{\left\langle W_{3Q}(\vec{r_1}, \vec{r_2}, \vec{r_3}; \tau) \, S(\vec{y}, \tau/2) \right\rangle}{\left\langle W_{3Q}(\vec{r_1}, \vec{r_2}, \vec{r_3}; \tau) \right\rangle \left\langle S(\vec{y}, \tau/2) \right\rangle}$$



If there is no correlation between the <u>action density</u> and the locations of the <u>quarks</u>,

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Similar results are observed for \vec{E}^2 and \vec{B}^2 separately.

30-sweep T-shape Source



30-sweep Y-shape Source



Effective Potential at $\tau = 1 \rightarrow 2$



Flux Tube Cross Section



Cross Section Fit



Baryonic Ground State Properties

Flux-tube radius is 0.38(3) fm .

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- Flux-tube radius is 0.38(3) fm .
- Vacuum-field action suppressed by 7.2(6)%.
- Flux-tube node is 25% larger at 0.47(2) fm.
- Vacuum-field action suppression is 15(3)% larger at 8.1(7)%.

The Heart of the Atom



The Structure of the Nucleon



Information on the Web

The software used to create the animations is

- Advanced Visual System's AVS/Express
- http://www.avs.com/

Many of these animations are available on the web.

http://www.physics.adelaide.edu.au/theory/staff/leinweber


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Flux Tubes in SU(2) Gauge Theory



G.S. Bali, K. Schilling and C. Schlichter (Wuppertal U.)
Phys. Rev. **D51** (1995) 5165

It is common to express the correlation as

$$C_S(\vec{y}, \vec{d}) = \frac{\langle Q(\vec{x}) \ S(\vec{y} + \vec{x}) \ Q^{\dagger}(\vec{x} + \vec{d}) \rangle_{\vec{x}}}{\langle Q(\vec{x}) \ Q^{\dagger}(\vec{x} + \vec{d}) \rangle_{\vec{x}}} - \langle S(\vec{x}) \rangle_{\vec{x}}$$

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In Euclidean space

$$S(\vec{x}) = \frac{1}{2} tr\left(\vec{B}_{\text{Eucl}}^2 + \vec{E}_{\text{Eucl}}^2\right) = \frac{1}{2} tr\left(\vec{B}^2 - \vec{E}^2\right) > 0$$

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But the Minkowski action is

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The sign of the correlation may be selected freely.

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