Exploring some issues for twisted mass QCD

1. options for including the strange quark
   - *an extension of Abdel-Rehim, Lewis, Woloshyn and Wu, Phys. Rev. D74, 014507 (2006)*

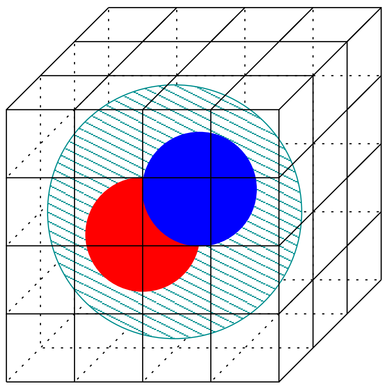
2. flavour splittings among baryon masses
   - *Abdel-Rehim, Lewis, Petry and Woloshyn, poster at LAT2006.*

3. hadron operators and group theory
   - *Harnett, Lewis and Petry, poster at LAT2006.*

twisted mass lattice QCD

Consider a hypercubic lattice in 4-dimensional spacetime, and a two flavour system:

$$\psi(x) = \begin{pmatrix} u(x) \\ d(x) \end{pmatrix}$$



The fermion action is

$$S_{\text{fermion}} = a^4 \sum_x \bar{\psi}(x) \left[D(r,\omega) + m_q \right] \psi(x)$$

with Dirac and Wilson terms,

$$\begin{aligned} D(r,\omega) &= \gamma \cdot \nabla + \exp(-i\omega\gamma_5\tau_3) W(r) \\ W(r)\psi(x) &\equiv M_{cr}(r)\psi(x) - \frac{ar}{2}\Box\,\psi(x) \end{aligned}$$

As usual, the covariant derivative ∇_μ and d'Alembertian $\Box$ contain the gauge field $U_\mu(x) \equiv \exp(iagT^b A_\mu^b(x))$.

Two key features of twisted mass lattice QCD

- Eigenvalues of $D(r,\omega) + m_q$ are bounded below for $m_q, \omega \neq 0$.

Frezzotti, Grassi, Sint and Weisz, J. High Energy Phys. 08, 058 (2001)

- $O(a)$ improvement is essentially automatic at $\omega = \pm\pi/2$.

Frezzotti and Rossi, J. High Energy Phys. 08, 007 (2004)

Symmetry consequences

- Parity and flavour symmetries are broken at $a \neq 0$, but get restored as $a \to 0$.

1. options for including the strange quark

The physics of (u,d,s) quarks is phenomenologically important.

The twisted mass action relies on quark doublets.

How should the strange quark be accommodated?

One option is $(\mathrm{u,d})_{\mathrm{tm}}$ + s, ie. strange is not twisted.

- ▷ *s is not protected from zero modes. (No practical problem for physical m_s.)*
- ▷ *The s has no automatic $O(a)$ improvement. (A clover term could be added.)*
- ▷ *Kaon operators will appear as parity mixtures, requiring extra effort.*

Another option is $(\mathrm{u,d})_{\mathrm{tm}}$ + $(\mathrm{c,s})_{\mathrm{tm}}$.

For degenerate u and d, the quark mass terms (in the twisted basis) are

$$\mathcal{L} = \bar{\psi}_l \left(m_l + i\gamma_5\mu_l\tau_3\right)\psi_l + \bar{\psi}_h \left(m_h + i\gamma_5\mu_h\tau_3 + \epsilon\tau_a\right)\psi_h$$

Should we choose $\tau_a = \tau_3$ or $\tau_a \neq \tau_3$?

(u,d)$_0$ + (c,s)$_\parallel$

Pena, Sint and Vladikas, JHEP ***0409****, 069 (2004)*

Advantage:

$\tau_a = \tau_3 \Rightarrow$ flavours do not mix.

Disadvantage:

$\tau_a = \tau_3 \Rightarrow$ the fermion determinant is not real.
These quarks could be used as valence quarks but not as sea quarks.

(u,d)$_0$ + (c,s)$_\perp$

Frezzotti and Rossi, Nucl. Phys. B Proc. Suppl. ***128****, 193 (2004)*
Farchioni et al., PoS ***LAT2005****, 072 (2005)*

Advantage:

$\tau_a \neq \tau_3 \Rightarrow$ the fermion determinant is real.
These quarks could be used as valence quarks and as sea quarks.

Disadvantage:

$\tau_a \neq \tau_3 \Rightarrow$ flavours mix.

To elucidate the flavour/isospin structure, consider kaons.

kaon mass splittings and lattice artifacts

Consider χPT for two twisted doublets, $\mathcal{L}_{\chi PT} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \ldots$, with

$$\begin{aligned} \mathcal{L}^{(2)} &= \frac{f^2}{4}\mathrm{Tr}(D_\mu\Sigma D_\mu\Sigma^\dagger) - \frac{f^2}{4}\mathrm{Tr}(\chi^\dagger\Sigma + \Sigma^\dagger\chi) - \frac{f^2}{4}\mathrm{Tr}(\hat{A}^\dagger\Sigma + \Sigma^\dagger\hat{A}) \\ \mathcal{L}^{(4)} &= -W_8'\mathrm{Tr}\left[(\hat{A}^\dagger\Sigma + \Sigma^\dagger\hat{A})^2\right] + \ldots \end{aligned}$$

$$\Sigma = \begin{pmatrix} e^{i\omega_l\tau_3/2} & 0 \\ 0 & e^{i\omega_h\tau_3/2} \end{pmatrix} e^{i\Phi/f} \begin{pmatrix} e^{i\omega_l\tau_3/2} & 0 \\ 0 & e^{i\omega_h\tau_3/2} \end{pmatrix}$$ for meson matrix Φ.

$$\chi = 2B \begin{pmatrix} m_l + i\mu_l\tau_3 & 0 \\ 0 & m_h + i\mu_h\tau_3 + \epsilon_h\tau_a \end{pmatrix}$$ is proportional to the quark mass matrix.

$\hat{A} \longrightarrow 2W_0 a(\mathbb{1})$ is proportional to the lattice spacing.

Twisted mass χPT with a strange quark has been discussed here:

Abdel-Rehim, Lewis, Woloshyn and Wu, Phys. Rev. D74, 014507 (2006).

Münster and Sudmann, hep-lat/0603019.

These are based on many previous 2-flavour studies. (See references therein.)

To interpret $(\mathsf{u,d})_0 + (\mathsf{c,s})_\perp$, diagonalize the quark mass matrix,

$$
\begin{aligned}
m_l + i\mu_l\tau_3 &= \sqrt{m_l^2+\mu_l^2}\,e^{i\tau_3\omega_l} \\
m_h + i\mu_h\tau_3 + \epsilon_h\tau_1 &= e^{i\tau_3\omega_h/2}Y^\dagger \begin{pmatrix} \sqrt{m_h^2+\mu_h^2}+\epsilon_h & 0 \\ 0 & \sqrt{m_h^2+\mu_h^2}-\epsilon_h \end{pmatrix} Y e^{i\tau_3\omega_h/2}
\end{aligned}
$$

where $Y = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ is a standard $\pi/4$ rotation.

The resulting mass terms for kaons and D mesons have the form

$$
\mathcal{L}_{K,D} = \begin{pmatrix} \mathbb{K}^+ & \bar{\mathbb{D}}^0 & \mathbb{K}^0 & \mathbb{D}^- \end{pmatrix} \begin{pmatrix} m-\delta & \alpha & 0 & 0 \\ \alpha & m+\delta & 0 & 0 \\ 0 & 0 & m-\delta & -\alpha \\ 0 & 0 & -\alpha & m+\delta \end{pmatrix} \begin{pmatrix} \mathbb{K}^- \\ \mathbb{D}^0 \\ \bar{\mathbb{K}}^0 \\ \mathbb{D}^+ \end{pmatrix}
$$

where m, δ and α contain various χPT parameters.

Here, mesons are labelled by their identities in the untwisted limit.

They are clearly not mass eigenstates for nonzero twist, $\alpha \neq 0$.

To determine meson eigenstates for general twist, we diagonalize,

mass eigenstates	*mass eigenvalues*
$\psi_{u-} = \mathbb{K}^+ \cos\theta - \bar{\mathbb{D}}^0 \sin\theta$	
$\psi_{d-} = \mathbb{K}^0 \cos\theta + \mathbb{D}^- \sin\theta$	$m_{u-} = m_{d-} = m - \sqrt{\delta^2 + \alpha^2}$
$\psi_{u+} = \mathbb{K}^+ \sin\theta + \bar{\mathbb{D}}^0 \cos\theta$	
$\psi_{d+} = \mathbb{K}^0 \sin\theta - \mathbb{D}^- \cos\theta$	$m_{u+} = m_{d+} = m + \sqrt{\delta^2 + \alpha^2}$

NOTE: for general θ, none of the mass eigenstates are isospin partners.

The rotation (not twist!) angle, $0 \leq \theta < \pi/2$, is given by

$$\tan\theta = -\left|\frac{\delta}{\alpha}\right| + \sqrt{\left(\frac{\delta}{\alpha}\right)^2 + 1}$$

$\alpha \sim \sin\omega_l \sin\omega_h$ vanishes if either (u,d) or (c,s) is not twisted.

$\delta \sim \epsilon_h$ vanishes when (c,s)$_{\rm tm}$ has no explicit mass splitting term.

Which eigenstates correspond to which physical states?

$(\psi_{u-}, \psi_{d-}, \psi_{u+}, \psi_{d+})$ $\qquad\qquad$ $(K^+, K^0, \bar{D}^0, D^-)$

$\alpha = 0 \Rightarrow \theta = 0 \Rightarrow$ **(u,d)$_0$ + s + c**

Quark flavours do not mix.
The physical kaons are a (lighter) degenerate isospin doublet.
The physical D mesons are a (heavier) degenerate isospin doublet.

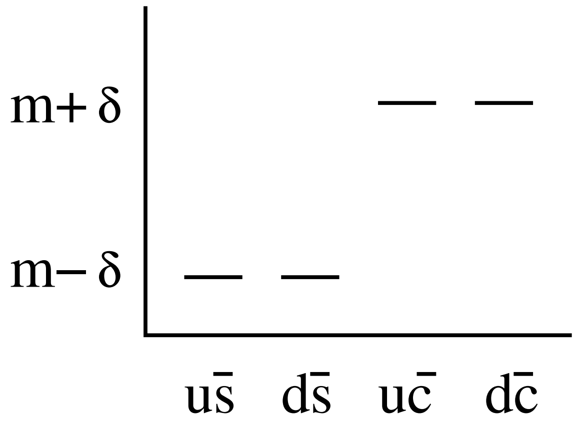
$\delta = 0 \Rightarrow \theta = \pi/4 \Rightarrow$ **(u,d)$_0$ + (c,s)$_0$**

Quark flavours can be diagonalized, and mixings are then avoided.
This becomes identical to (u,d)$_0$ + (c,s)$_{||}$ with $m_c = m_s$.
Mass-degenerate doublets are (K^+, D^-) and $(K^0, \bar{D}^0)$.
The physical kaons only become degenerate in the continuum limit.

$\alpha, \delta \neq 0 \Rightarrow$ **(u,d)$_0$ + (c,s)$_\perp$**

(u,d) has no mixing, but (c,s) does.
Interestingly, none of the mass eigenstates form isospin doublets unless isospin is somehow defined to involve (c,s) as well as (u,d).
The discretization errors then split kaon masses from D masses.

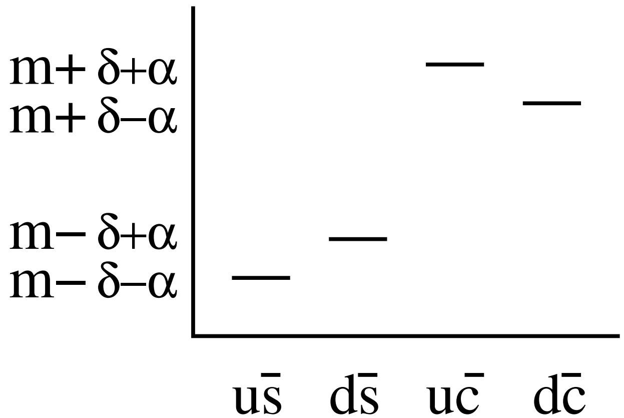
Meson mass eigenstates



$(u,d)_0$ + s + c

These masses have no isospin splitting.

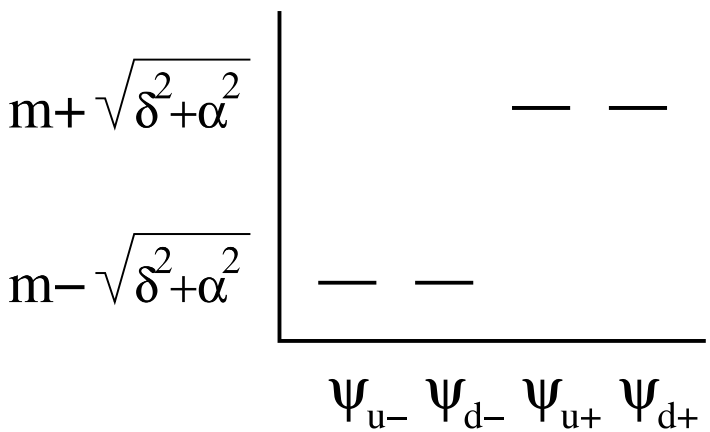
For simulations, see *Abdel-Rehim, Lewis, Woloshyn and Wu, Phys. Rev. D74, 014507 (2006).*



$(u,d)_0$ + $(c,s)_\parallel$

There are $O(a^2)$ isospin splittings, but flavours do not mix.

For simulations, see *Abdel-Rehim, Lewis, Woloshyn and Wu, Phys. Rev. D74, 014507 (2006).*



$(u,d)_0$ + $(c,s)_\perp$

Flavours mix. There are two degenerate pairs of eigenstates. The symmetry relating the degenerate states involves both quark doublets.

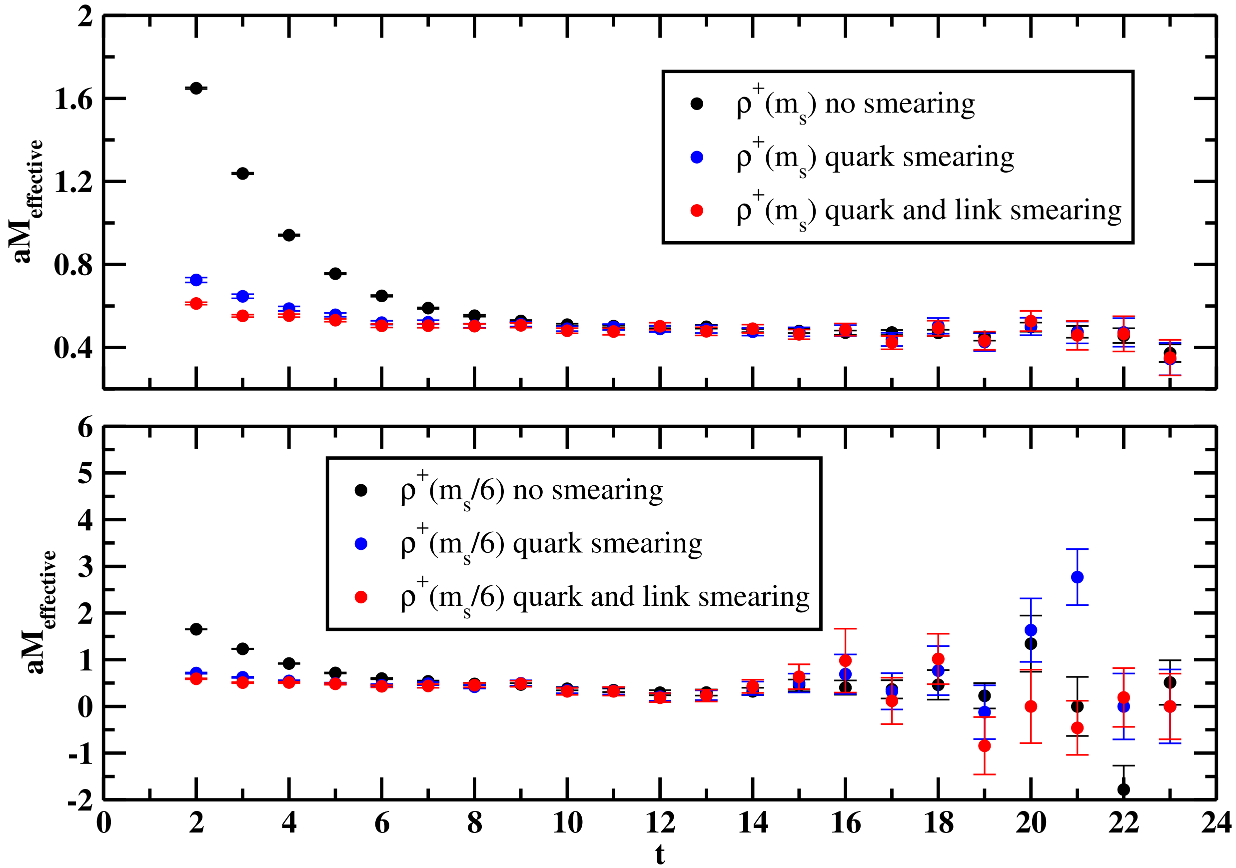
For simulations, see *Farchioni et al., PoS* ***LAT2005****, 072.*

2. flavour splittings among baryon masses

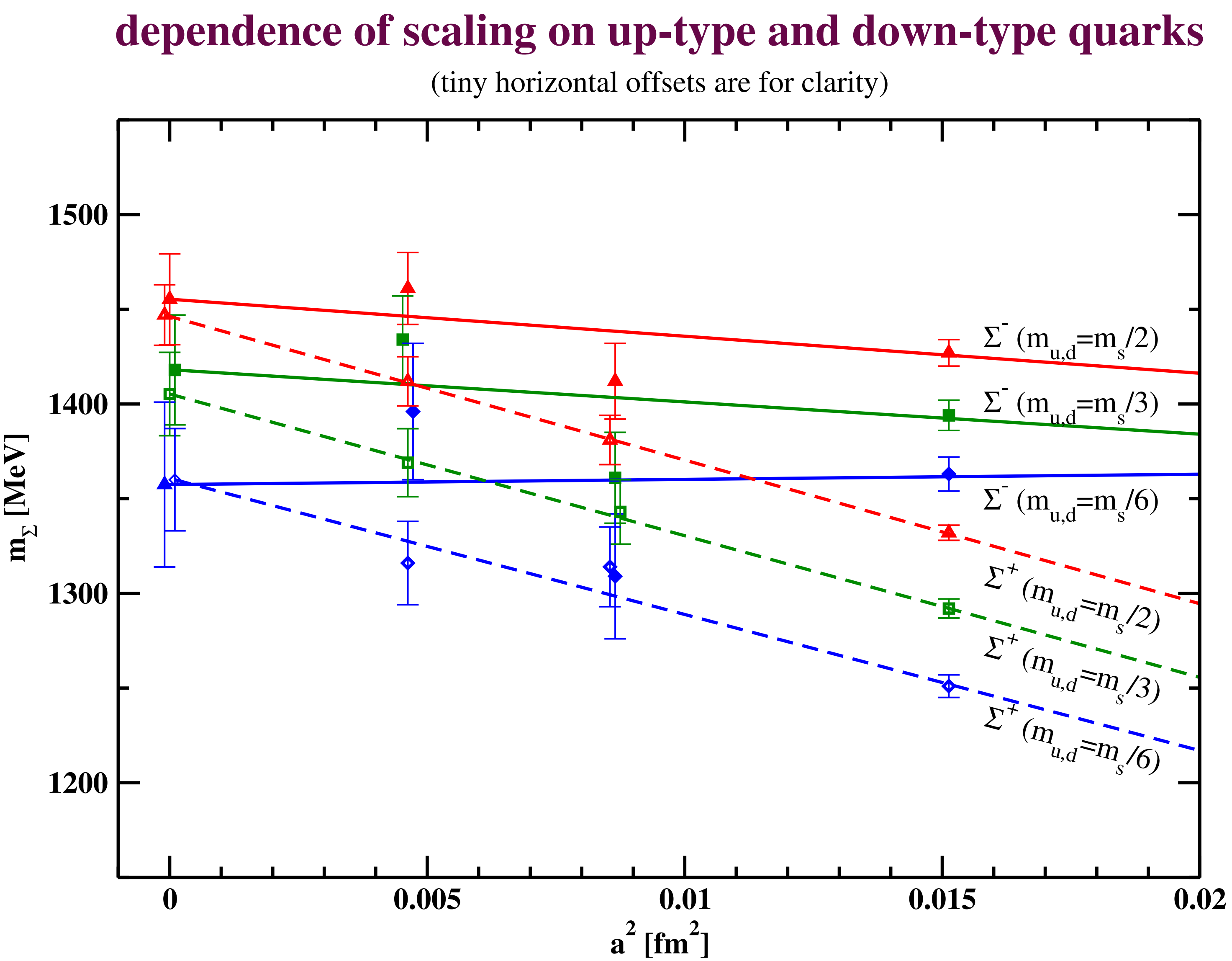
Gaussian quark smearing with stout links are used at the sink, tuned with the method of

Basak, Sato, Wallace, Edwards, Richards, Fleming, Heller, Lichtl and Morningstar, PoS **LAT2005**, *076.*

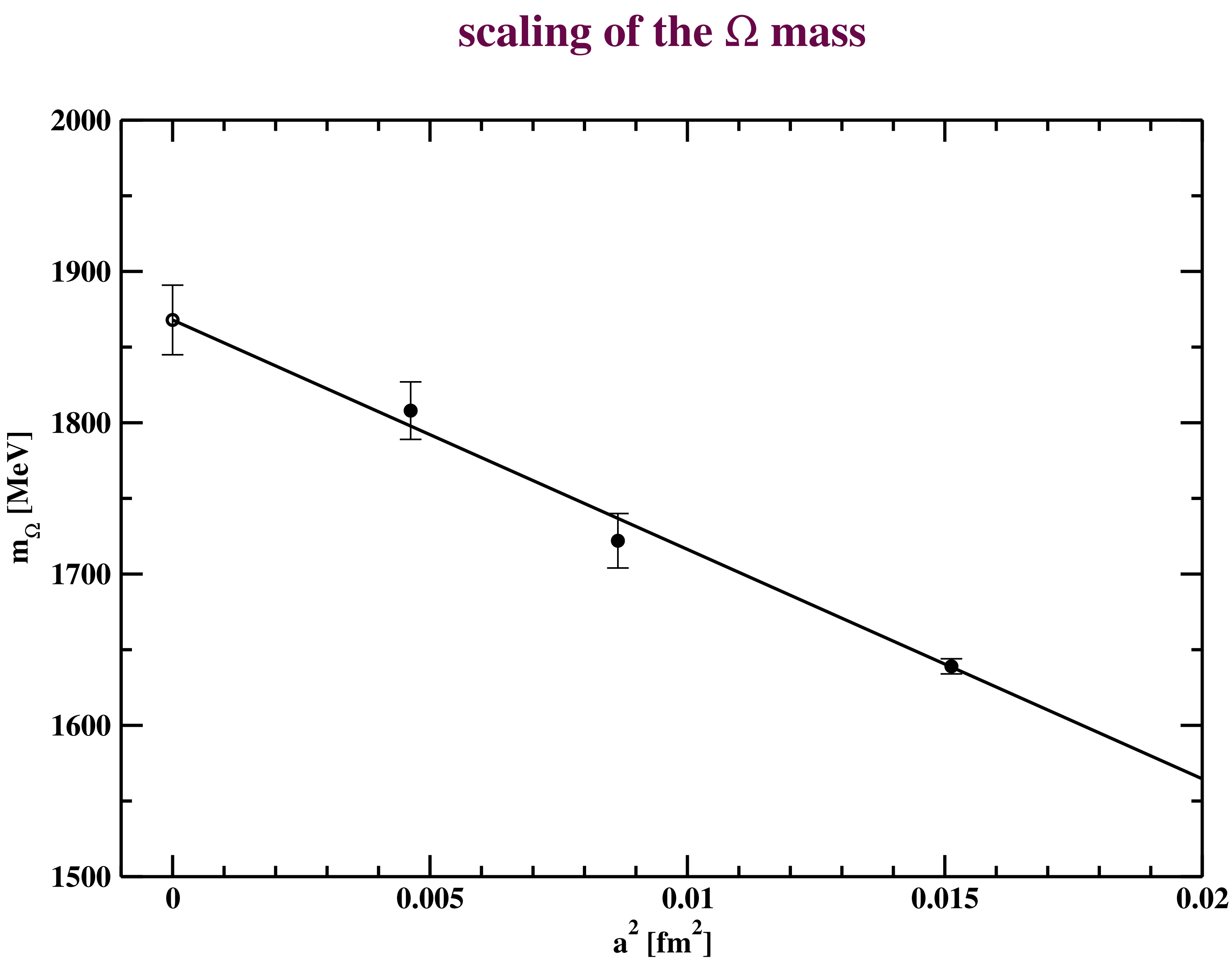
β=6.0, 20^3x48



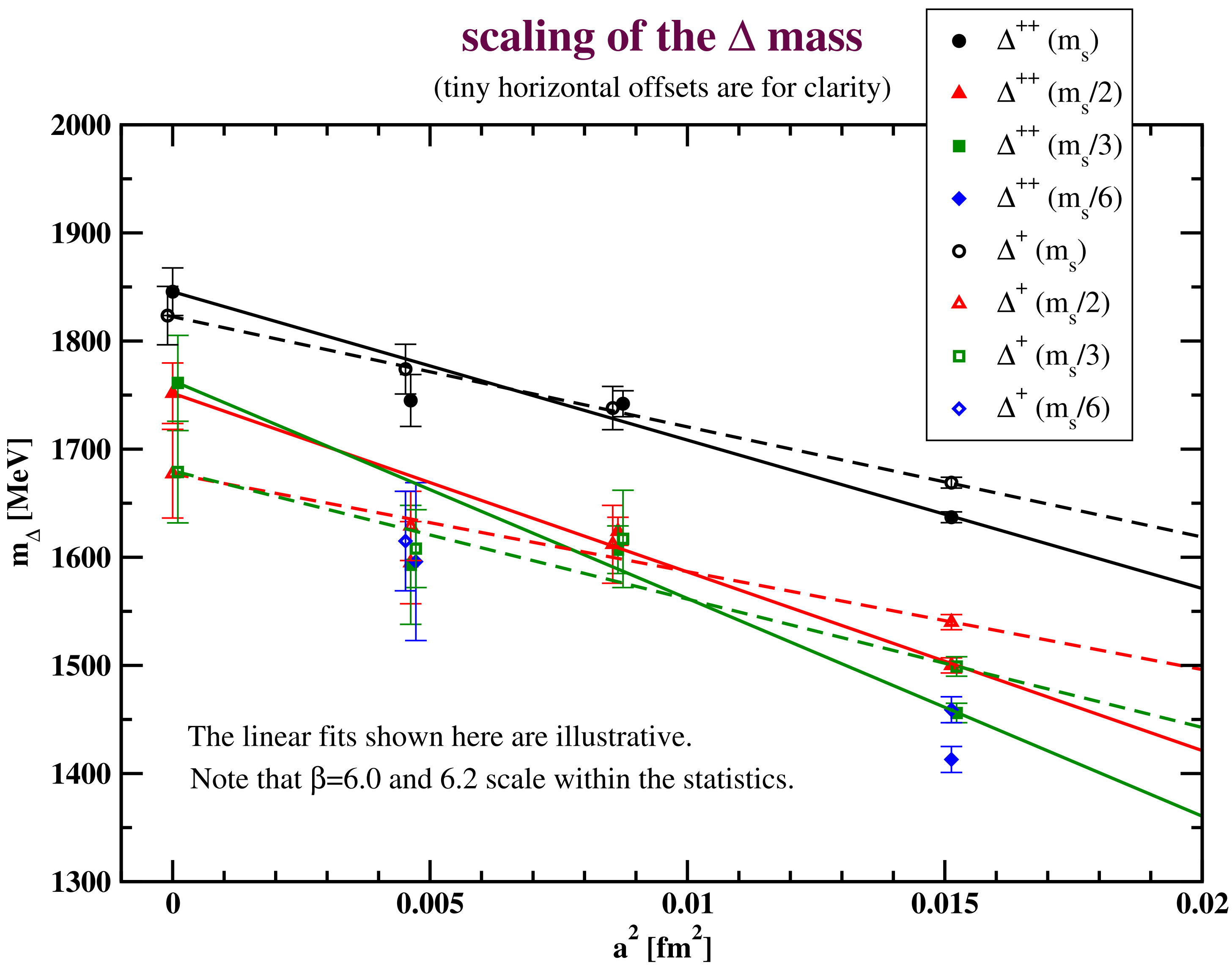
$O(a^2)$ flavour breaking is seen within the octet.



Even with smearing, spin-3/2 ground states are difficult.
Best case (no u,d quarks):



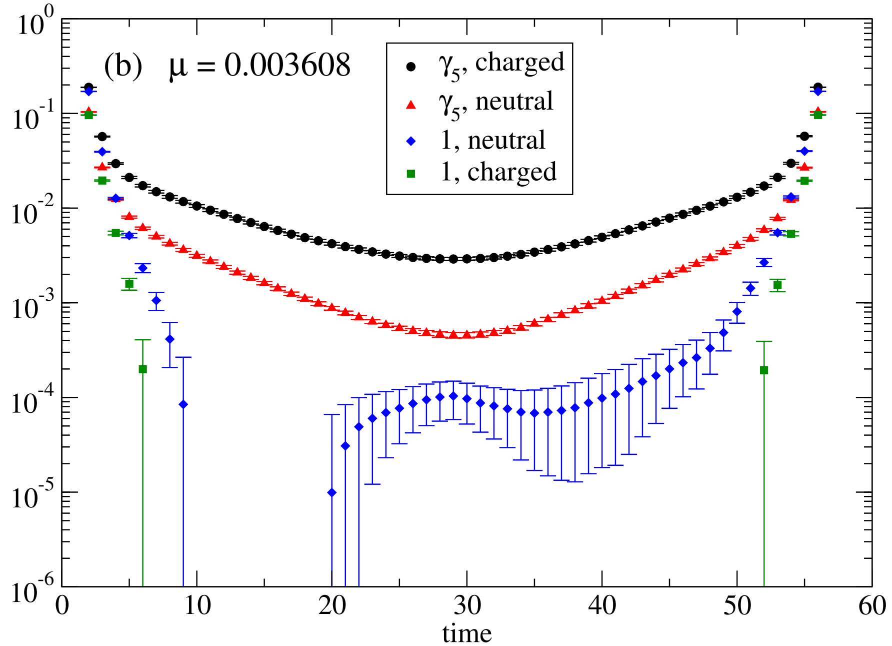
Worst case (all u,d quarks):

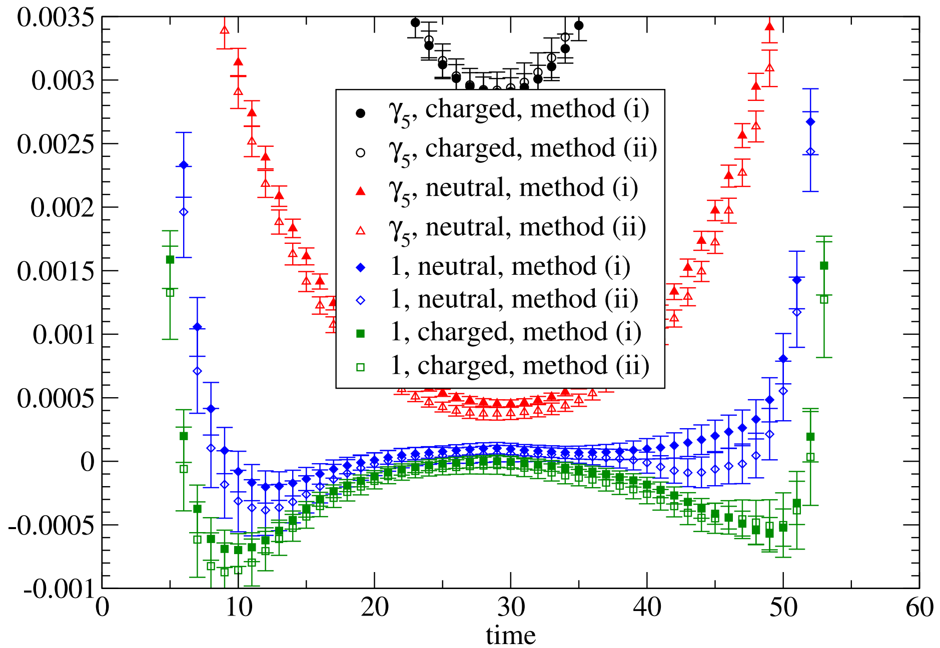


3. hadron operators and group theory

tmLQCD violates parity.
It is preferable to not rely on a numerical tuning for separating parities.

Example: For standard definitions of maximal twist, neutral scalar and pseudoscalar correlators still show some contamination (charged cannot).





tmLQCD violates parity, P, but it preserves a ‘‘twisted parity’’, $\tilde{P}$, defined by the product of P and $(\omega \to -\omega)$.

Let’s use creation/annihilation operators that are independent of ω, making P equivalent to $\tilde{P}$ and thus conserved.

Example:

A change of basis, by twist angle ω, gives

$$\begin{pmatrix} u' \\ d' \end{pmatrix} = \exp(i\omega\gamma_5\tau_3) \begin{pmatrix} u \\ d \end{pmatrix}$$

Therefore

$$\bar{u}'\gamma_i d' = \bar{u}\gamma_i d \cos\omega + \bar{u}\gamma_i\gamma_5 d \sin\omega$$

couples to both vector and axial mesons unless ω is carefully tuned, but

$$\bar{u}'\sigma_{4i}d' = \bar{u}\sigma_{4i}d$$

couples to the vector and never to the axial.

On a lattice we have the octahedral group, with Λ^{PC} related to J^{PC} via

$\Lambda \backslash J$	0	1	2	3	4	5	...
A_1	1	0	0	0	1	0	...
A_2	0	0	0	1	0	0	...
E	0	0	1	0	1	1	...
T_1	0	1	0	1	1	2	...
T_2	0	0	1	1	1	1	...

Using the 16 local fermion bilinears, one finds

Γ	I	γ_5	γ_4	$\gamma_4\gamma_5$	γ_i	$\gamma_i\gamma_5$	σ_{4i}	$\epsilon_{ijk}\sigma_{jk}$
Λ^{PC}	A_1^{++}	A_1^{-+}	A_1^{+-}	A_1^{-+}	T_1^{--}	T_1^{++}	T_1^{--}	T_1^{+-}

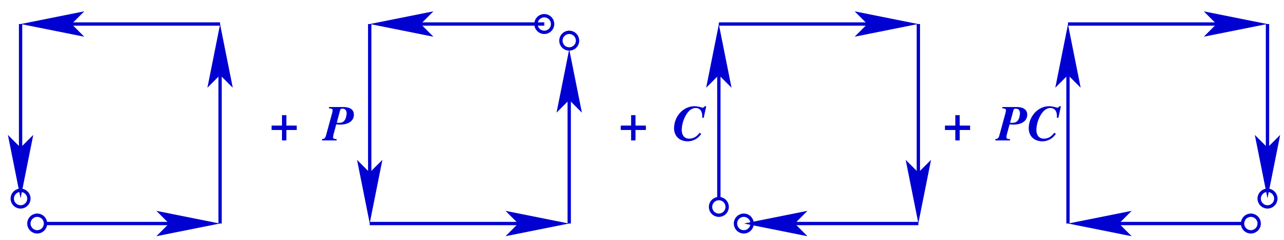
For a given flavour content, only half are independent of twist angle:

Red structures are independent of twist angle for charged mesons.

Green structures are independent of twist angle for neutral mesons.

To get all Λ^{PC} combinations, insert gauge links.

Example: elementary 4-sided paths, summed for definite P and C.



Combining this 24-dim gauge rep

with the 16-dim fermion rep produces. . .

$\Lambda \backslash PC$	++	+-	-+	--
A_1	4	4	6	2
A_2	4	4	2	6
E	8	8	8	8
T_1	12	12	10	14
T_2	12	12	14	10

For tmLQCD, this splits into

charged mesons*

$\Lambda \backslash PC$	++	+-	-+	--
A_1	3	1	3	1
A_2	1	3	1	3
E	4	4	4	4
T_1	5	7	5	7
T_2	7	5	7	5

neutral mesons

$\Lambda \backslash PC$	++	+-	-+	--
A_1	1	3	3	1
A_2	3	1	1	3
E	4	4	4	4
T_1	7	5	5	7
T_2	5	7	7	5

(No zeros in tables!)

*For charged mesons, recall $G = C(-1)^I$.

Summary

1. There are multiple methods for combining the strange quark with tmLQCD. Some characteristics of each have been identified.

2. With smeared operators, unphysical flavour splittings have observed among octet baryons. The splittings vanish with a^2 are expected.

3. Meson operators with all Λ^{PC} quantum numbers have been constructed, without adverse affects from the parity violation in tmLQCD.