Noise Estimator and Disconnected Diagrams in Lattice QCD

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OUTLINE

1. Disconnected Diagrams
   - What are these diagrams? How important are they?
2. How to calculate Disconnected Diagrams?
   - Difficulties : Trace of very large matrix
   - Solutions : Stochastic Noise Estimator
3. Stochastic Noise Method
4. Stochastic Noise Method and Unbiased Trace Estimator
5. What is more important? Number of noises or number of configurations?
   - A Toy Model Calculation.
6. Preliminary results
   - Scalar current
   - $< x >$ and $< x^2 >$
7. Improvement

How Important are Disconnected Diagrams?

- Since nucleon do not have any valence strange quark, the only way the strangeness of the nucleon can be explained via generation of $s\bar{s}$ pair from vacuum polarization. Therefore, to obtain strangeness content of the nucleon one must calculate disconnected diagrams.

- In order to calculate complete matrix element of a current, one should always consider disconnected diagrams along with the connected ones.

Various observables

- Strangeness content term $< N|s\bar{s}|N >$
- Disconnected diagrams for :
  - electromagnetic form factors (G_M^s and G_E^s).
  - flavor singlet axial current
  - $\pi N \sigma$ term
  - total angular momentum
  - $< x >$ and $< x^2 >$
- Flavor singlet mesons
- Hyperfine splitting in charmonium

Electromagnetic Form Factors and Moments

Electromagnetic vector current :

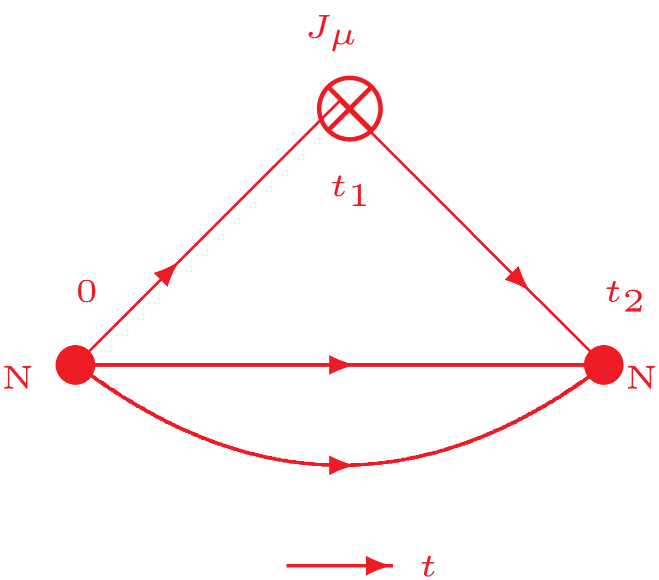
$$< \vec{p}s|j_\mu(0)|\vec{p}'s' >= \bar{u}(\vec{p},s)\left[\gamma_\mu F_1(q^2) - i\sigma^{\mu\nu}q_\nu \frac{F_2(q^2)}{2m}\right] u(\vec{p'},s')\,.$$

From these one can define following form factors

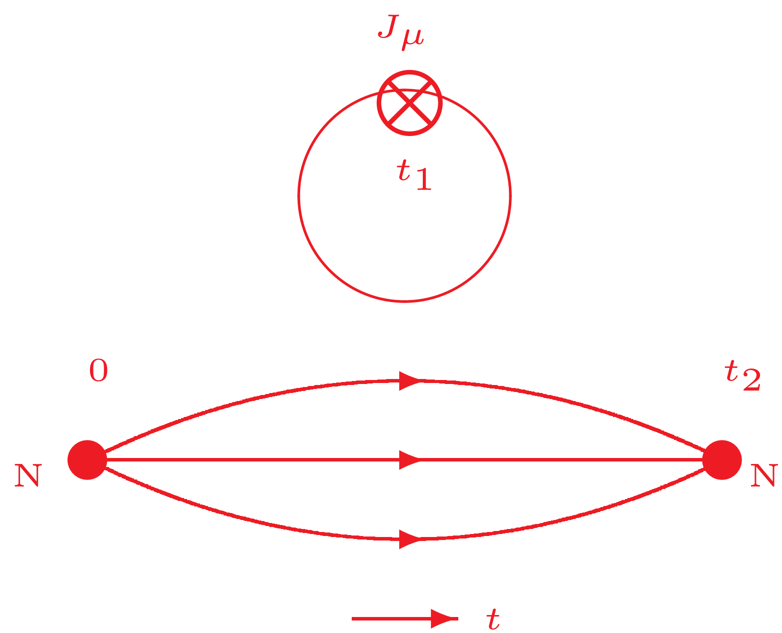
$$\begin{aligned} G_E(q^2) &= F_1(q^2) - \left(\frac{q}{2m}\right)^2 F_2(q^2)\,, \\ G_M(q^2) &= F_1(q^2) + F_2(q^2)\,, \end{aligned}$$

Connected and Disconnected Insertion

$$G^{\alpha\beta}_{PO_\mu P}(t_f,t,\vec{p},\vec{p'}) = \sum_{\vec{x_f},\vec{x}} e^{-i\vec{p}\cdot\vec{x_f}+i\vec{q}\cdot\vec{x}} < 0|\,\mathrm{T}\left(\chi^\alpha(x_f)\,O_\mu(x)\,\bar{\chi}^\beta(0)\right)\,|0> .$$



Connected



Disconnected

Disconnected Diagrams and Noise Method

Disconnected diagrams incorporates quark loops, and to simulate those we need to calculate traces of very big matrices :

$$\sum_x \mathrm{Tr}\left[M^{-1}(x+\mu,x)U_\mu\gamma_\mu\gamma_5\right],$$

which involve both diagonal and off-diagonal elements of M^{-1}.

For a moderate size lattice $16^3\times 24$, dimension of quark matrix

$$M^{-1ab}_{\alpha\beta}(x,y) \Rightarrow 16^3\times 24\times 2\times 3\times 4 \sim 10^6\times 10^6!!$$

Therefore, it will be extremely costly to invert the whole matrix M and then take all traces.

Instead, we use stochastic noise method to calculate these traces.

Noise Method

Noise method : projection of the signal using random noise vectors as input.

⋆ To account for the Brownian motion with the Langevin and Fokker-Planck equations and to compute time dependent correlation functions in statistical mechanics.... P.C. Hohenberg and B. Halperin, *Rev. Mod. Phys.* **49**, 435 (1977).

⋆ Stochastic formulation of quantum mechanics.... E. Nelson, *Phys. Rev.* **150**, 1079 (1966).

⋆ Stochastic formulation of quantum field theory.... G. Parisi and Wu Yongshi, *Scientia Sinica* **24**, 483 (1981).

Noise Method and Matrix Inversion

Need to calculate any inverse fermion matrix element M^{-1}_{ij} stochastically.

Introduce an ensemble of L column vectors $\eta \equiv \eta^1, ..., \eta^L$ (each of dimension $N \times 1$) with the properties of a white noise, i.e.,

$$\begin{aligned} \langle \eta_i \rangle &= 0, \\ \langle \eta_i \eta_j \rangle = \frac{1}{L} \sum_{n=1}^{L} \eta_i^n \eta_j^n &= \delta_{ij}, \end{aligned}$$

where η_i^n is the i-th entry in the noise vector n.

Noise Method and Matrix Inversion

Using these noise vectors (η) as source, we solve for the solution vector x

$$\sum_i M_{ki} x_i = \eta_k,$$

$$\Longrightarrow \quad x_i = \sum_k M^{-1}_{ik} \eta_k,$$

With this solution one can obtain any inverse matrix element as

$$< \eta_j x_i > = \sum_k M^{-1}_{ik} < \eta_j \eta_k > = M^{-1}_{ij}.$$

Noise Method and Trace Estimation

Let us consider a general complex matrix S.

Time requires to evaluate $S\eta \sim$ time needs to evaluate λ_i.

$$Trace \equiv \sum_{i=1}^{N} \lambda_i$$

If we can construct a trace estimator from the matrix-vector multiplication $S\eta$, we will save a time factor $\mathcal{O}(N)$.

$$\begin{aligned} S\eta &\Longrightarrow vector, \\ Trace &\Longrightarrow number, \\ Trace &\Longleftrightarrow \xi S\eta, \quad \xi \; is\, a\, row\, vector. \\ \xi S\eta &= \mathrm{Tr}\,(S) \end{aligned}$$

Noise Method and Trace Estimation

$$
\begin{aligned}
& & \xi S \eta &= \mathrm{Tr}\,(S) \\
\Rightarrow & & \sum_{m,n}^{N} S_{mn}\xi_m^* \eta_n &= \sum_{n}^{N} S_{nn} \\
& \Rightarrow & \xi_m^* \eta_n &= \delta_{mn} \qquad \forall\, m,n.
\end{aligned}
$$

It is impossible to choose the vectors ξ and η which satisfy previous $N \times N$ conditions simultaneously. However, if ξ and η vary randomly, it is possible that the condition may be satisfied in an expectation

$$E[\xi_m^* \eta_n] = \delta_{mn} \qquad \forall\, m,n.$$

This equality holds if η_n: an arbitrary random variables with mean zero and unit variance, and $\xi_m \equiv \eta_m$. With such a choice, it follows :

$$E[\eta^\dagger S \eta] = \mathrm{Tr}\,(S).$$

Noise Method and Trace Estimation

$$
\begin{aligned}
E\left[< \eta^{\dagger} M^{-1} \eta >\right] \quad &\equiv \quad E\left[\frac{1}{L}\sum_{j=1}^{L}\sum_{m,n=1}^{N} < \eta_m^{*j} M_{mn}^{-1} \eta_n^{j} >\right] \\
&= \quad \frac{1}{L}\sum_{j=1}^{L}\left(\sum_{n=1}^{N} M_{nn}^{-1}\right) E\left[< \eta_n^{*j} \eta_n^{j} >\right] \\
&\quad + \frac{1}{L}\sum_{j=1}^{L}\left(\sum_{m\neq n}^{N} M_{mn}^{-1}\right) E\left[< \eta_m^{*j} \eta_n^{j} >\right] \\
&= \quad \sum_{n}^{N} M_{nn}^{-1} + \left(\sum_{m\neq n}^{N} M_{mn}^{-1}\right) E\left[\frac{1}{L}\sum_{j=1}^{L} < \eta_m^{*j} \eta_n^{j} >\right] \\
&= \quad \mathrm{Tr}\left(M^{-1}\right).
\end{aligned}
$$

Therefore, instead of calculating the trace for M^{-1} directly, we use the estimator $E\left[< \eta^{\dagger} M^{-1} \eta >\right]$. In the infinite noise limit, this trace estimation will be exact.

Noise and Variance

CHOICE OF RANDOM NOISES :

Z_2 noise :

A set of noise vectors $\eta_N^1, \eta_N^2, \cdots\cdots, \eta_N^L$, where each vector has N components, and each component has one of the four values $\{\pm 1, \pm i\}$ chosen independently with equal probability.

Better than Gaussian noise:

S.J. Dong and Keh-Fei Liu, Phys. Lett. **B328**, 130 (1994).

VARIANCE : For a given L number of noises,

$$\begin{aligned} \sigma_M^2 &\equiv \mathrm{Var}\left[< \eta^\dagger M^{-1} \eta > \right] \\ &= E\left[| < \eta^\dagger M^{-1} \eta > - \mathrm{Tr}(M^{-1})|^2 \right] \\ &= \frac{1}{L} \sum_{m \neq n}^{N} \left| M_{mn}^{-1} \right|^2 . \end{aligned}$$

Unbiased Subtraction

$$\mathrm{Tr}A^{-1} = E\left[\left\langle \eta^\dagger \left(A^{-1} - \sum_{p=1}^{P} \lambda_p Q^{(P)} \right) \eta \right\rangle\right]$$

η : Z_2 noise vectors,
Q^P : set of traceless matrices,
λ_p: variational coefficients needed to be tuned to get the minimum variance.

$$\sigma_A^2(\lambda) = Var\left[\left\langle \eta^\dagger \left(A^{-1} - \sum_{p=1}^{P} \lambda_p Q^{(P)} \right) \eta \right\rangle\right]$$
$$= \frac{1}{L} \sum_{m \neq n} \left| A^{-1}_{m,n} - \sum_{p=1}^{P} \lambda_p Q^{(P)}_{m,n} \right|^2 .$$

....C. Thron et al., Phys. Rev. **D57**, 1642 (1998).

Unbiased Subtraction

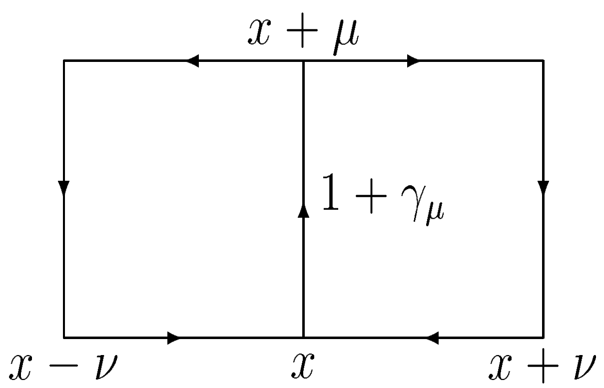
CHOICE OF Q^P

Any traceless matrix. However, it should match the off-diagonal behavior of the matrix A. We used a set of traceless matrices obtained from the hoping parameter expansion of the Wilson fermion matrix M.

$$M^{-1} = I + \kappa D + \kappa^2 D^2 + \kappa^3 D^3 + \cdots\cdots$$

Unbiased Subtraction

Need to subtract trace-full part like (for vector current) :



CH-Symmetry

By CH-symmetry we mean the charge conjugation (C) and Euclidean hermiticity (H).

$$M^{-1}(x, y, U) \quad = \quad C^{-1}\tilde{M}^{-1}(y, x, U^*)C.$$

Euclidean hermiticity :

$$\gamma_5 M^{-1}(x, x + a_\mu)\gamma_5 = M^{-1\dagger}(x + a_\mu, x),$$

Under CH-transformation,

$$M^{-1}(x, y, U) = C^{-1}\gamma_5 M^{-1\,*}(x, y, U^*)\gamma_5 C.$$

Using CH-symmetry one can prove that

$$O_{vc}(U, M^{-1}[U]) \quad = \quad O^*_{vc}(U^*, M^{-1}[U^*]),$$

i.e., the three-point function of the vector current is REAL.

Loop Reduction

Loop Part :

$$\mathrm{Tr}\left[M^{-1}(x, x+a_\mu)\,(1+\gamma_\mu)\,U_\mu^\dagger(x_1)\right]$$
$$-\mathrm{Tr}\left[M^{-1}(x+a_\mu, x)\,(1-\gamma_\mu)\,U_\mu(x)\right].$$

Now, using the identity, $\gamma_5\, M^{-1}(x,y)\,\gamma_5 = M^{-1\,\dagger}(y,x)$,

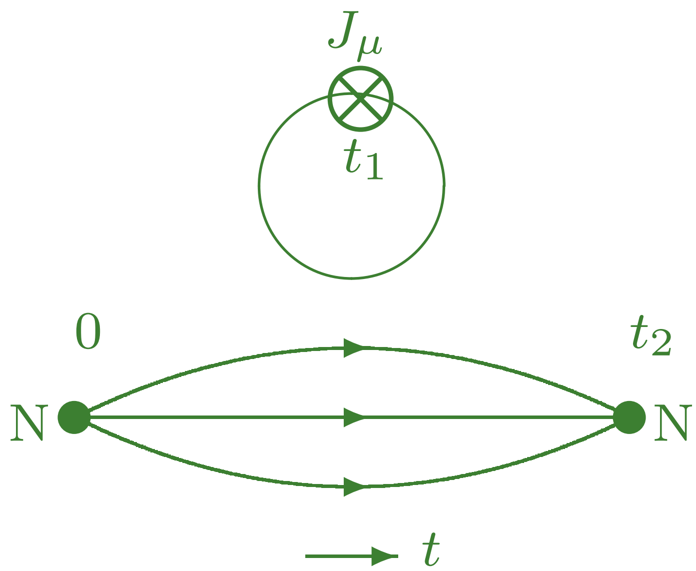
$$\begin{aligned}
&\mathrm{Tr}\left[M^{-1}(x+a_\mu, x)\,(1-\gamma_\mu)\,U_\mu(x)\right]\\
&= \mathrm{Tr}\left[\gamma_5\, M^{-1}(x+a_\mu, x)\,(1-\gamma_\mu)\,U_\mu(x)\,\gamma_5\right]\\
&= \mathrm{Tr}\left[M^{-1\,\dagger}(x, x+a_\mu)\,(1+\gamma_\mu)\,U_\mu(x)\right]\\
&= \mathrm{Tr}\left[M^{-1}(x, x+a_\mu)\,(1+\gamma_\mu)\,U_\mu^\dagger(x)\right]^\dagger
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\mathrm{Tr}\left[M^{-1}(x, x+a_\mu)\,(1+\gamma_\mu)\,U_\mu^\dagger(x_1)\right]\\
&\quad -\mathrm{Tr}\left[M^{-1}(x+a_\mu, x)\,(1-\gamma_\mu)\,U_\mu(x)\right]\\
&= 2Im\mathrm{Tr}\left[M^{-1}(x, x+a_\mu)\,(1+\gamma_\mu)\,U_\mu^\dagger(x_1)\right]
\end{aligned}$$

Thus, after Fourier transformation only REAL part of the loop exists.

Loop Reduction



Disconnected

$$\{cos(qx) + isin(qx)\}\{(Loop)_R + (Loop)_I\} \times \{(2Pt)_R + (2Pt)_I\}$$
$$= -sin(qx)(Loop)_I \times \{(2Pt)_R + (2Pt)_I\}$$
$$= -sin(qx)(Loop)_I \times \{(2Pt)_R\}$$

Error Calculation over Configurations and Noises

$\{x_1, x_2, \cdots\cdots x_N\}$: a set of configurations.

$$\text{Average, } a = \frac{1}{N}\sum_{i=1}^{N} x_i,$$

$$\text{Error, } \epsilon_1 = \frac{\sigma_1}{\sqrt{N}}.$$

Next, each x_i is stochastically estimated with n noises with mean x_i and variance σ_2:

$$x_i = \frac{1}{n}\sum_{j=1}^{n} x_{ij},$$

$$\text{Error, } \epsilon_2 = \frac{\sigma_2}{\sqrt{n}}.$$

The characteristic function (fourier transform of the probability distribution for the deviation of the compound noise and configuration average) is given by,

$$\begin{aligned}\Phi(k) &= \int e^{ik(a-\bar{x})} Q(a-\bar{x}) da \\ &= \prod_{ij} \int \int exp \frac{ik}{N} \left[\frac{1}{n}(x_{ij}-\bar{x}) + (x_i - \bar{x}) \right] \\ &\qquad \times\ q(x_{ij}) p(x_i) dx_{ij}\, dx_i \\ &\sim e^{-k^2(\sigma_1 + \bar{\sigma_2^2}/n)/2N},\end{aligned}$$

where p and q are the distribution functions for the Monte Carlo and noise estimator, respectively.

$$\bar{\sigma_2^2} = \langle \sum_{i,k\neq i} [M_{ik}^{-1}]^2 \rangle$$

is the configuration average of individual $\sigma_2^2(m)$. σ_1 is the standard deviation for $\mathrm{Tr}(M^{-1})$ in configuration averaging. Combined standard error squared is

$$\epsilon^2 = \frac{\sigma_1^2}{N} + \frac{\bar{\sigma_2^2}}{Nn}.$$

.......Private comm. R. W. Woloshyn

For $\mathrm{Tr}(M^{-1})$,

$$\epsilon^2 = \frac{\sigma_1^2}{N} + \frac{\langle \sum_{i,k\neq i} [M_{ik}^{-1}]^2 \rangle}{Nn}$$

Consider an extreme case of $n = 1$, for which,

$$\begin{aligned} E[Tr(M^{-1})] = \sum_i \eta_i X_i &= \sum_{i,k} M_{ik}^{-1} \eta_i \eta_k \\ &= \sum_i M_{ii}^{-1} + \sum_{i,k\neq i} M_{ik}^{-1} \eta_i \eta_k \\ &= \sum_i M_{ii}^{-1} \pm \sum_{i,k\neq i} M_{ik}^{-1}. \end{aligned}$$

However, one can change this $n = 1$ noise as one moves through the Markov chain for the gauge configurations.

Variance in this case involves the combined distribution of M_{ik}^{-1} and $\eta_i \eta_k$ which can be written as

$$P(x) \sim exp\left[-\frac{(x-\bar{x})^2}{2\sigma^2}\right] + exp\left[-\frac{(x+\bar{x})^2}{2\sigma^2}\right].$$

where $\bar{x} = M_{ik}^{-1}$ and $\sigma = \sigma_{ik}$ for the matrix element M_{ik}^{-1} over the configuration space. Standard error squared is

$$\epsilon^2 = \frac{\sigma_1}{N} + \frac{\langle \sum_{i,k\neq i} [M_{ik}^{-1}]^2 \rangle + \sum_{i,k\neq i} \sigma_{ik}^2}{Nn}.$$

Extra term σ_{ik}^2 will increase error.

Which one is more important?

More Configurations	Less Configurations
with	with
Less Noises	More Noises

A Toy Model Calculation

$\{X_1, X_2, \cdots\cdots X_N\}$: Gaussian Distribution with mean 0 and variance σ_G.

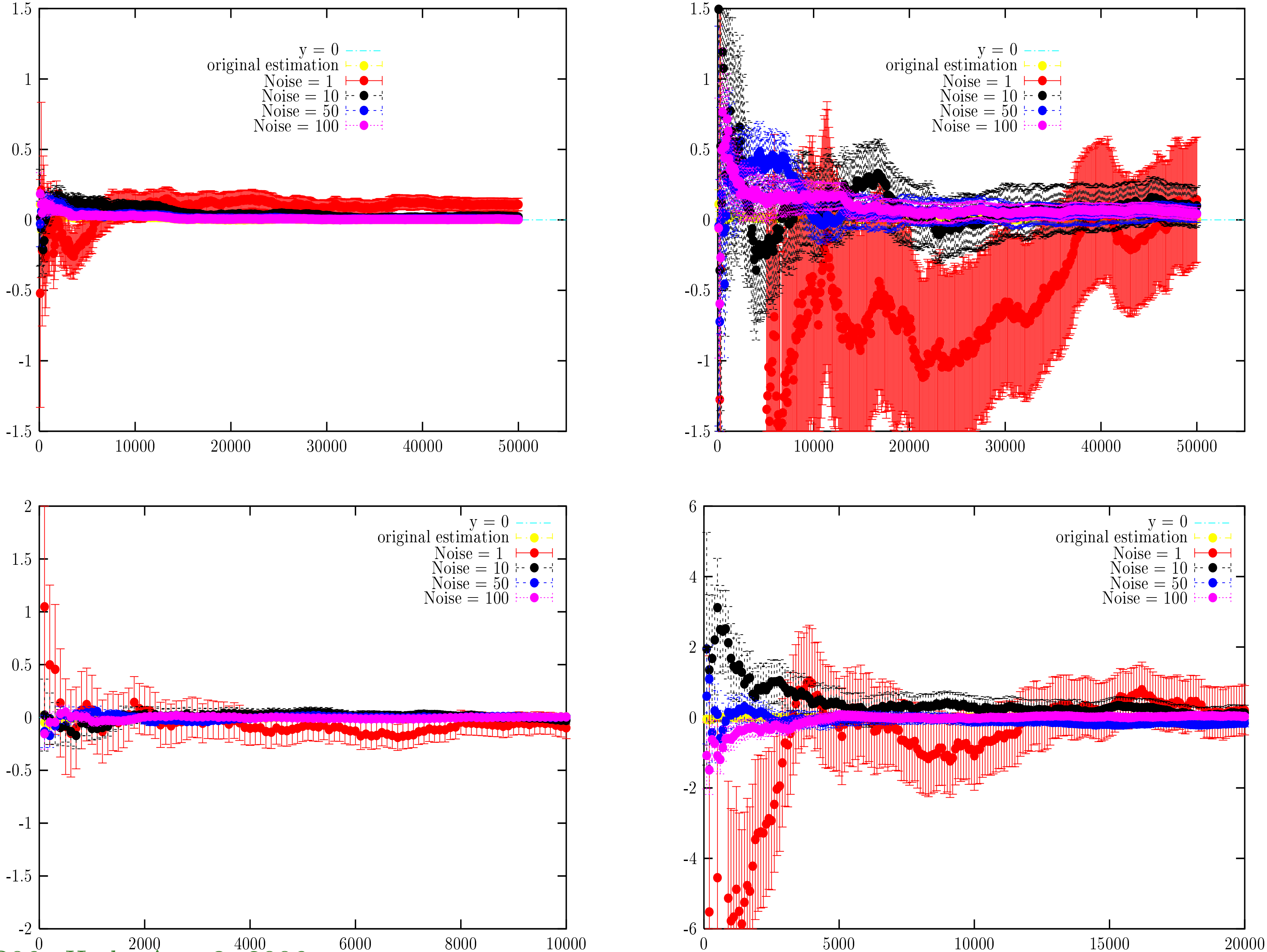
$$\text{Mean, } M_N = \frac{1}{N}\sum_{i=1}^{N} X_i,$$

$$\text{Error, } \epsilon = \frac{\sigma_N}{\sqrt{N}}.$$

Next, stochastically estimate each X_i :

For each X_i, generate n samples $(x_{ij},\ j = 1..n)$ from a Gaussian distribution with mean X_i and variance σ_n $(\sigma_n/\sigma_G = \lambda)$. Corresponding to each X_i, we get a noise average

$$\tilde{X}_i = \frac{1}{n}\sum_{j=1}^{n} x_{ij}.$$


y = 0
original estimation
Noise = 1
Noise = 10
Noise = 50
Noise = 100
1.5
1
0.5
0
-0.5
-1
-1.5
0
10000
20000
30000
40000
50000
y = 0
original estimation
Noise = 1
Noise = 10
Noise = 50
Noise = 100
1.5
1
0.5
0
-0.5
-1
-1.5
0
10000
20000
30000
40000
50000
y = 0
original estimation
Noise = 1
Noise = 10
Noise = 50
Noise = 100
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
2000
4000
6000
8000
10000
y = 0
original estimation
Noise = 1
Noise = 10
Noise = 50
Noise = 100
6
4
2
0
-2
-4
-6
0
5000
10000
15000
20000

A Toy Model Calculation

What is the fraction (f) of configurations whose means and errors touch $M_N \pm \sigma_N/\sqrt{N}$?

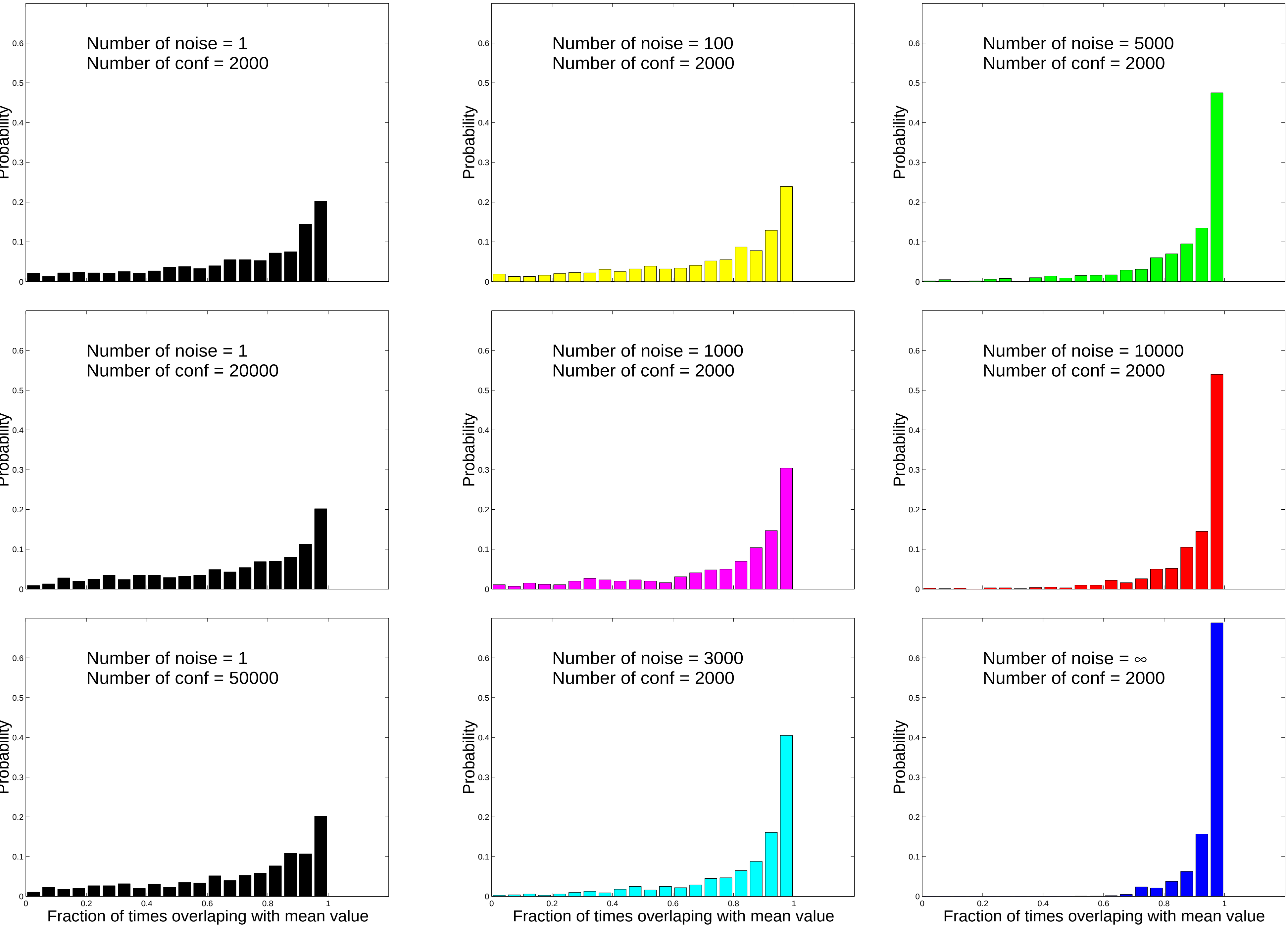
- Calculate cumulative mean and error

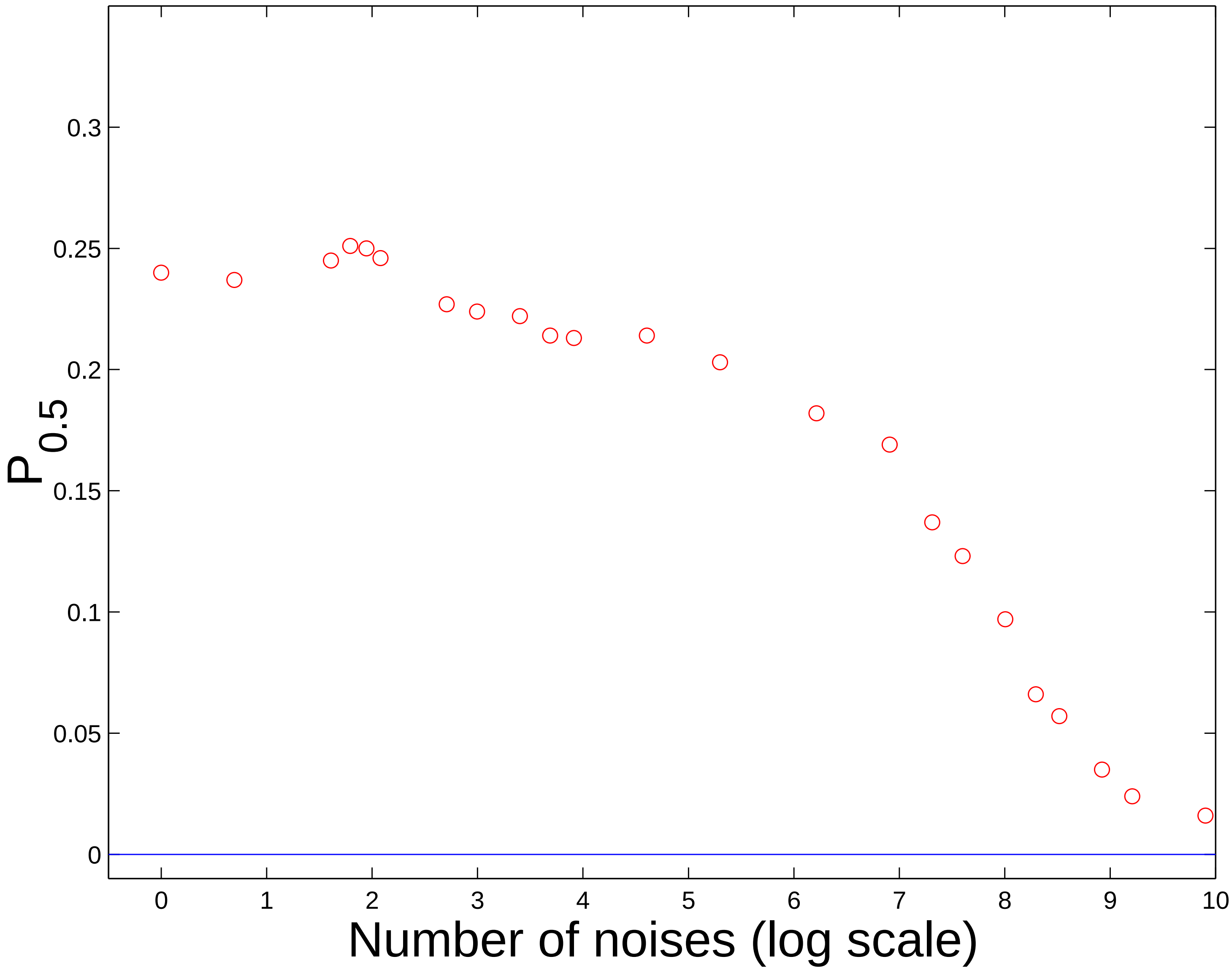
$$
\begin{aligned}
m_k &= \frac{1}{k}\sum_{i=1}^{k} \tilde{X}_i, \\
\epsilon_k &= \sigma_k/\sqrt{k},
\end{aligned}
$$

where k is the cumulative # of configurations starting from 2 to N.

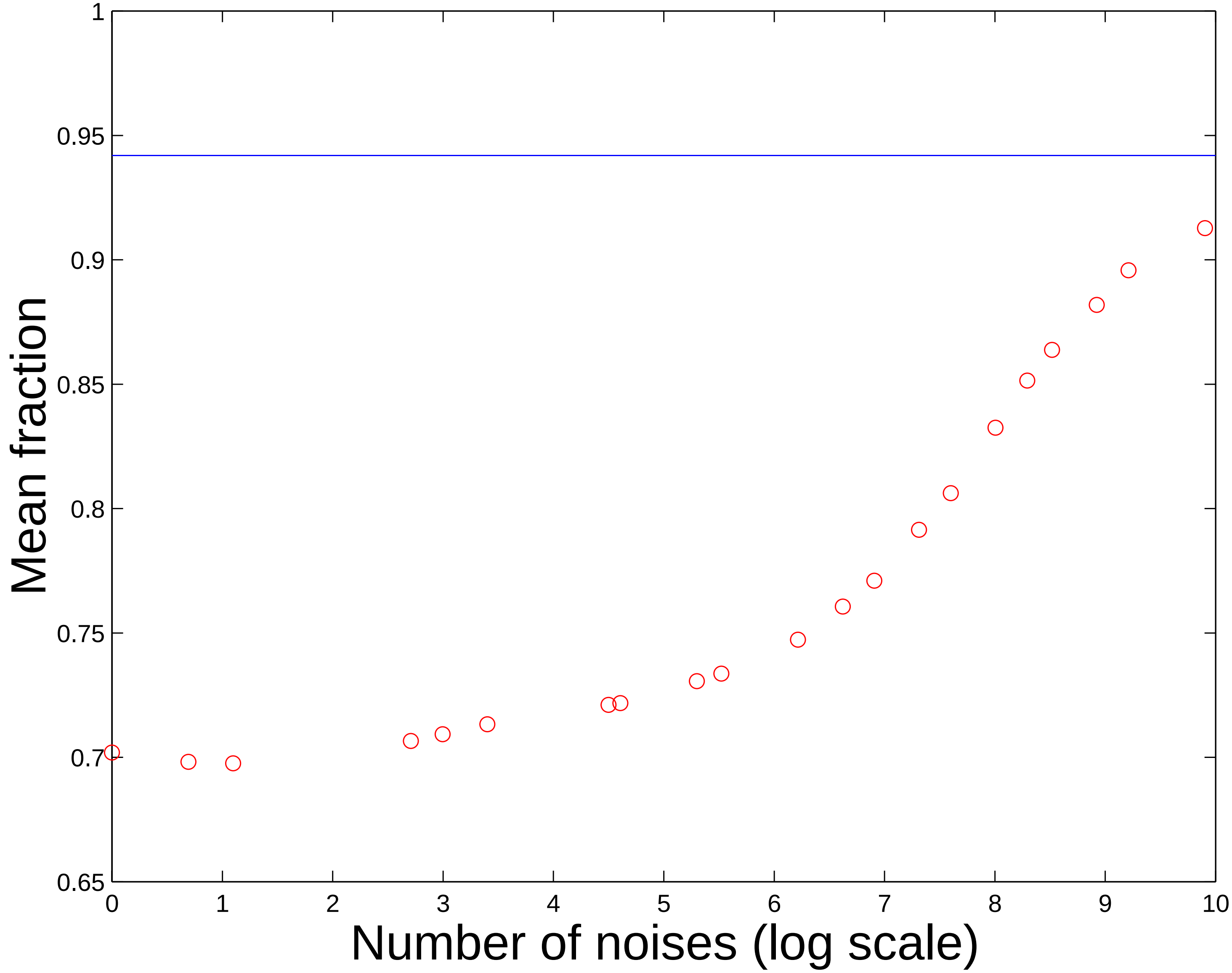
- Check for each k whether $m_k \pm \sigma_k/\sqrt{k}$ is consistent with $M_N \pm \sigma_N/\sqrt{N}$.
- If out of k ($k = 2$ to N) cumulative estimates, I estimates are consistent with $M_N \pm \sigma_N/\sqrt{N}$, then the corresponding fraction

$$f = I/(N-1).$$


Number of noise = 1
Number of conf = 2000
Number of noise = 100
Number of conf = 2000
Number of noise = 5000
Number of conf = 2000
Number of noise = 1
Number of conf = 20000
Number of noise = 1000
Number of conf = 2000
Number of noise = 10000
Number of conf = 2000
Number of noise = 1
Number of conf = 50000
Number of noise = 3000
Number of conf = 2000
Number of noise = ∞
Number of conf = 2000
Probability
Fraction of times overlaping with mean value



Probability (p) to be below fraction (f) 0.5 for different noise cases



Mean fraction ($p_i f_i$) for different noise cases.

A detail study for noise versus configuration

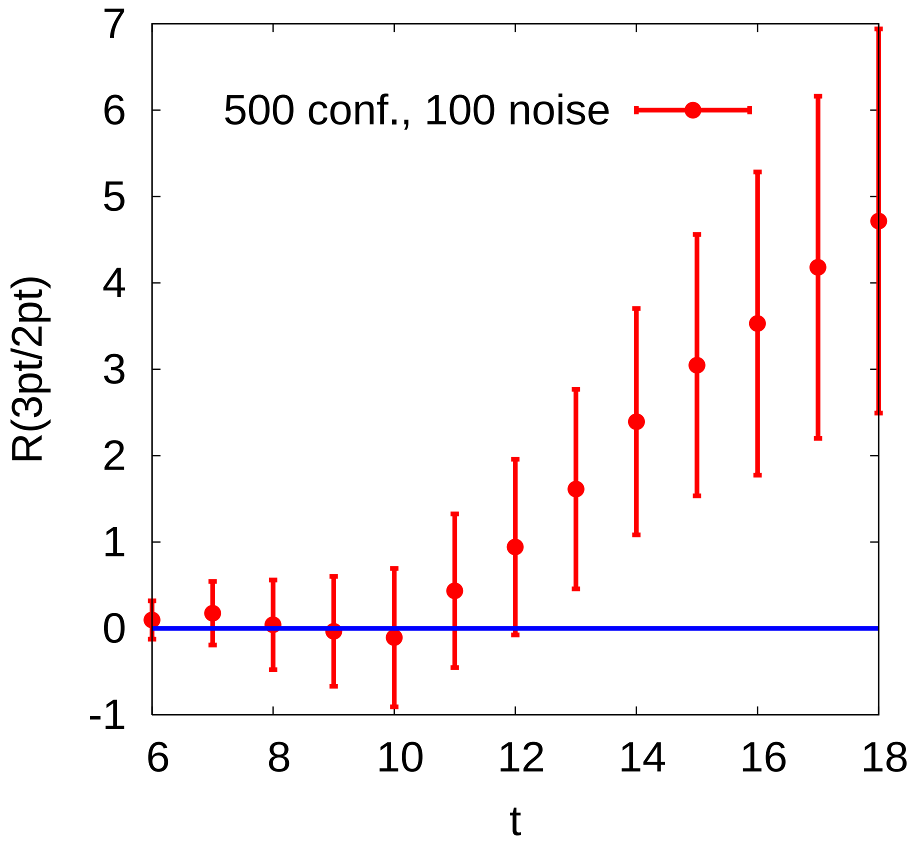
- Simulation details
  - Wilson action, $16^3 \times 24$, a = 0.1 fm
  - 500 configurations and 500 noises/configs, (in future 1000 configs and 500 noises/configs).
  - four loop masses
  - unbiased subtraction up to order 8 in D

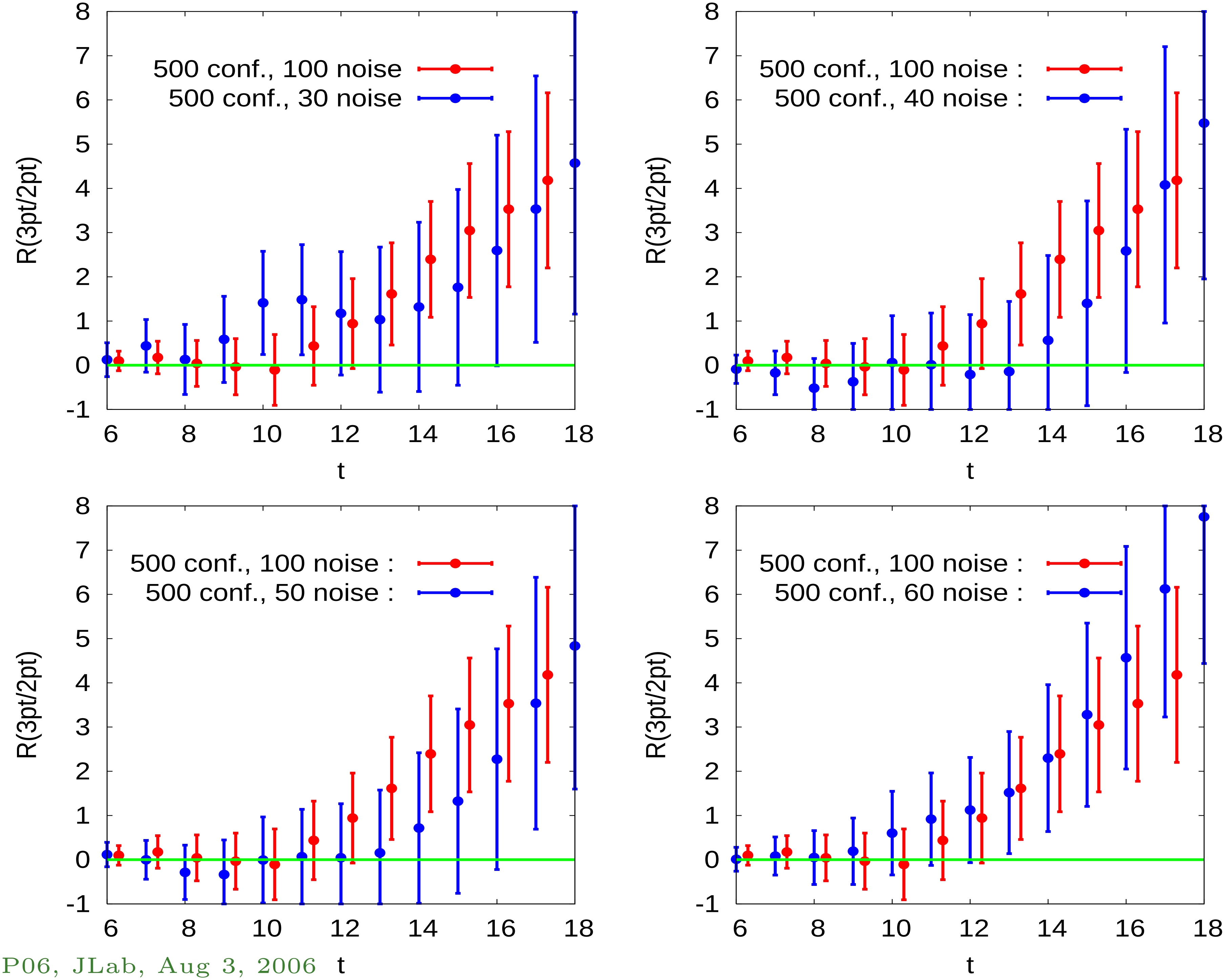
$$R(3pt/2pt) = \sum_t \frac{\Gamma_1^{\alpha\beta} G_{NSN}^{\beta\alpha}(t_f, \vec{p}, t, \vec{q}) \Gamma_2^{\alpha\beta} G_{NN}^{\beta\alpha}(t, \vec{p})}{\Gamma_2^{\alpha\beta} G_{NN}^{\beta\alpha}(t_f, \vec{p}) \Gamma_2^{\alpha\beta} G_{NN}^{\beta\alpha}(t, \vec{q})} \Longrightarrow \; const. + t_f g_{dis}(q^2)$$

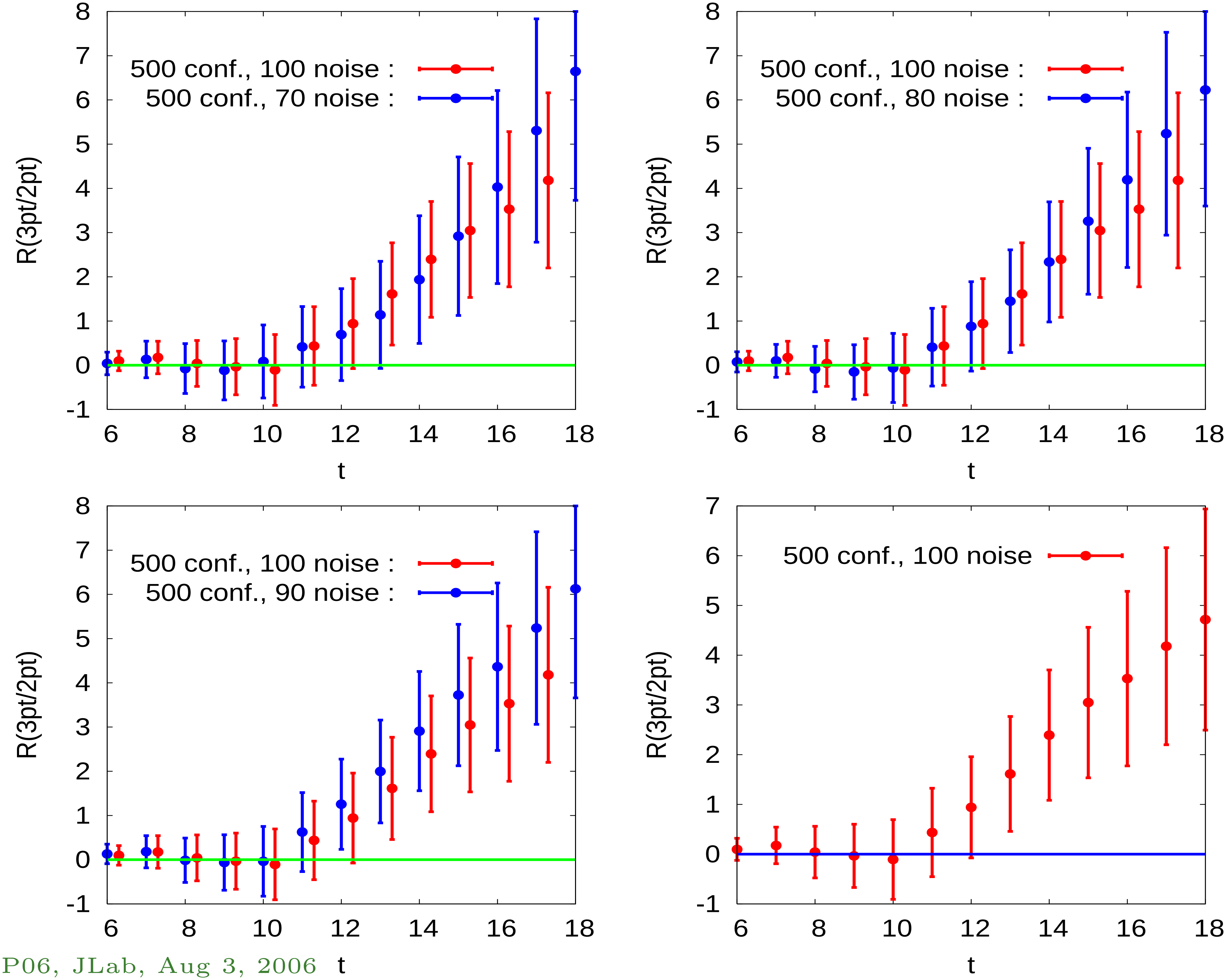
Slope will give disconnected contribution

One can also take difference of ratios to avoid fitting in slope ....Wilcox

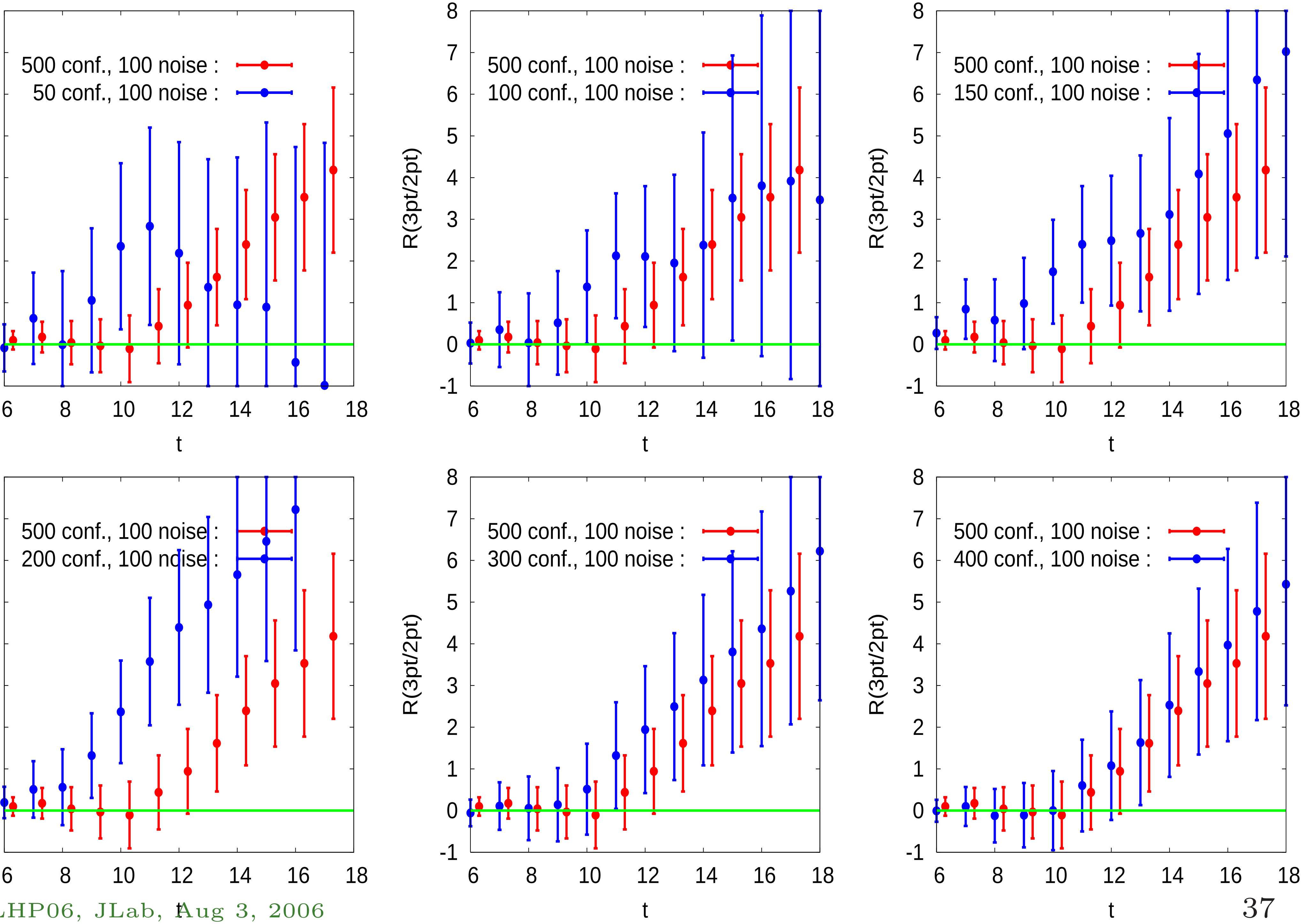
Preliminary results for scalar current



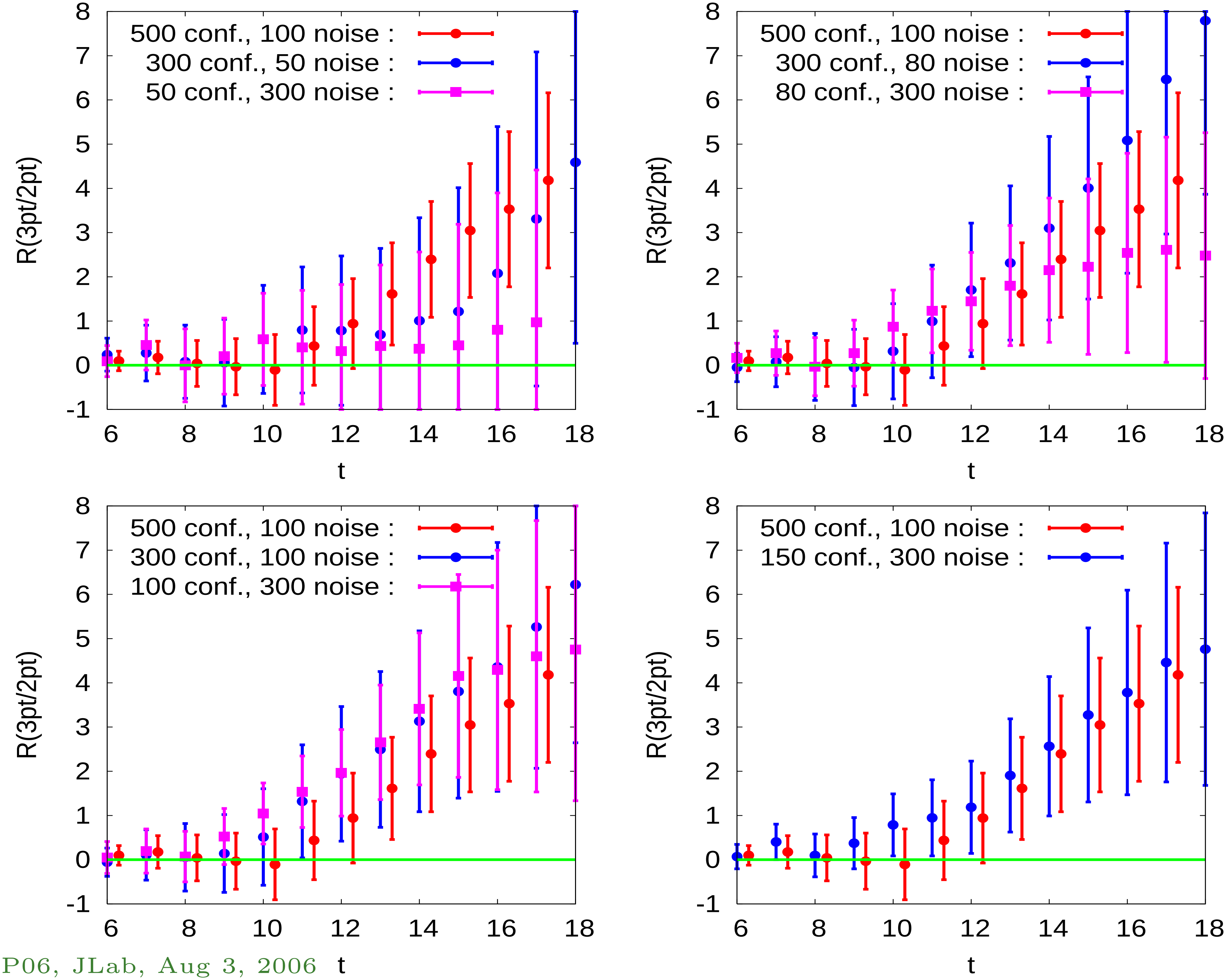

500 conf., 100 noise
500 conf., 30 noise
R(3pt/2pt)
t
500 conf., 100 noise :
500 conf., 40 noise :
R(3pt/2pt)
t
500 conf., 100 noise :
500 conf., 50 noise :
R(3pt/2pt)
t
500 conf., 100 noise :
500 conf., 60 noise :
R(3pt/2pt)
t



500 conf., 100 noise :
500 conf., 70 noise :
500 conf., 80 noise :
500 conf., 90 noise :
500 conf., 100 noise
R(3pt/2pt)
t



500 conf., 100 noise :
50 conf., 100 noise :
100 conf., 100 noise :
150 conf., 100 noise :
200 conf., 100 noise :
300 conf., 100 noise :
400 conf., 100 noise :
R(3pt/2pt)
t


500 conf., 100 noise :
300 conf., 50 noise :
50 conf., 300 noise :
500 conf., 100 noise :
300 conf., 80 noise :
80 conf., 300 noise :
500 conf., 100 noise :
300 conf., 100 noise :
100 conf., 300 noise :
500 conf., 100 noise :
150 conf., 300 noise :
R(3pt/2pt)
t

Strangeness content in parton distribution function

$$\langle P|\mathcal{O}_f^{(n)\{\mu_1\cdots\mu_n\}}|P\rangle = 2A_f^n P^{\mu_1}\cdots P^{\mu_n} \;- traces$$

where,

$$A_f^n = \int_0^1 dx\; x^{n-1}[\zeta_f(x) + (-1)^n \zeta_{\bar{f}}(x)]$$

$\zeta_f(x)$ is the parton distribution function.

- For s quark

$$A_s^2 =< x >= \int_0^1 dx\; x[s(x)+\bar{s}(x)], \qquad \text{is the first moment}$$

$$A_s^3 =< x^2 >= \int_0^1 dx\; x^2[s(x)-\bar{s}(x)), \qquad \text{is the second moment}$$

- Analytical expressions for $< x >$

for $\mathcal{O}_{\{4i\}}$ operator $(i = 1, 2, 3)$

$$R^{\rm dis}_{\{4i\}} = \sum_{t_1} \frac{{\rm Tr}\,[\ \Gamma\ G_{N\mathcal{O}_{\{4i\}}N}(t_2, t_1, p_i)\ ]}{{\rm Tr}\,[\ \Gamma\ G_{NN}(t_2, \vec{p})\ ]}\ \frac{2}{-ip_i} \to \text{const.} + t_2 < x >_{\rm dis}$$

$$R^{\rm con}_{\{4i\}} = \frac{{\rm Tr}\,[\ \Gamma\ G_{N\mathcal{O}_{\{4i\}}N}(t_2, t_1, p_i)\ ]}{{\rm Tr}\,[\ \Gamma\ G_{NN}(t_2, \vec{p})\ ]}\ \frac{2}{-ip_i} \to < x >_{\rm con.}$$

for $\mathcal{O}_{\{44\}} - \frac{1}{3}(\mathcal{O}_{\{11\}} + \mathcal{O}_{\{22\}} + \mathcal{O}_{\{33\}})$ operator

$$R^{\rm con}_{\{44\}} = -\frac{{\rm Tr}\,[\ \Gamma\ G_{N\mathcal{O}_{\{44\}}N}(t_2, t_1, \vec{p})\ ]}{{\rm Tr}\,[\ \Gamma\ G_{NN}(t_2, \vec{p})\ ]}\ \frac{2E_p}{E_p^2 + \frac{p^2}{3}} \to < x >_{\rm con.}$$

- Analytical expressions for $< x^2 >$

For $\mathcal{O}_{\{4ii\}} - \frac{1}{2}(\mathcal{O}_{\{4jj\}} + \mathcal{O}_{\{4kk\}})(i, j, k = 1, 2, 3$ and $i \neq j \neq k)$

$$R^{\rm dis}_{\{4ii\}} = \sum_{t_1} - \frac{\mathrm{Tr}\ [\ \Gamma\ G_{N\mathcal{O}_{\{4ii\}}N}(t_2, t_1, p_i)\ ]}{\mathrm{Tr}\ [\ \Gamma\ G_{NN}(t_2, \vec{p})\ ]} \frac{2}{p_i^2}$$

$$\rightarrow \mathrm{const.} + t_2 < x^2 >_{\rm dis}$$

$$R^{\rm con}_{\{4ii\}} = -\frac{\mathrm{Tr}\ [\ \Gamma\ G_{N\mathcal{O}_{\{4ii\}}N}(t_2, t_1, p_i)\ ]}{\mathrm{Tr}\ [\ \Gamma\ G_{NN}(t_2, \vec{p})\ ]} \frac{2}{p_i^2} \rightarrow < x^2 >_{\rm con}$$

The $\mathcal{O}_{\{4i\}}$ operator is defined as,

$$\mathcal{O}_{\{4i\}}(x) = \overline{\psi}(x)(\gamma)_{\{4}(-\frac{i}{2}\overleftrightarrow{\mathcal{D}})_{i\}}\psi(x)$$

The $\mathcal{O}_{\{4ii\}}$ operator is defined as,

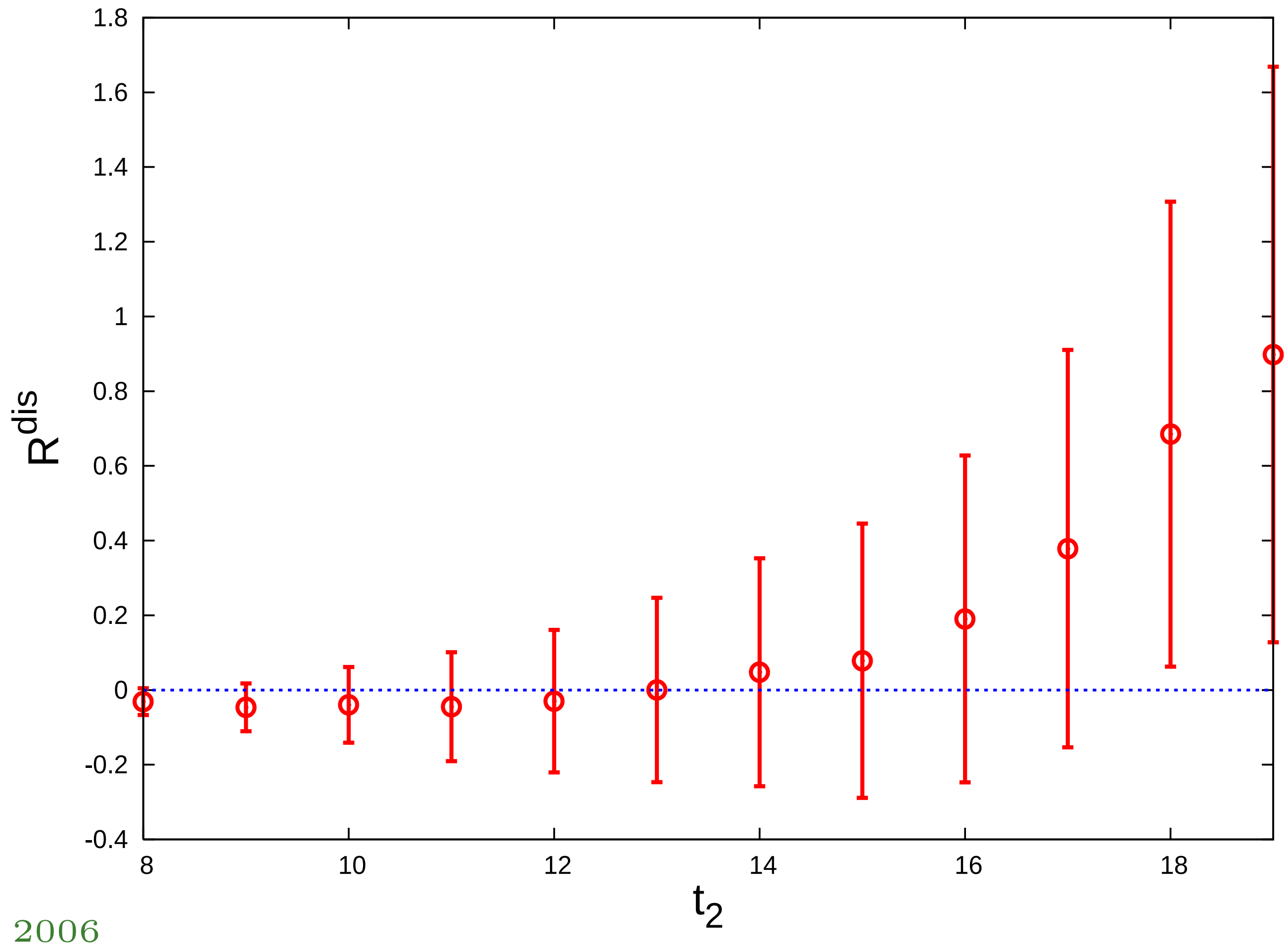
$$\mathcal{O}_{\{4ii\}}(x) = \overline{\psi}(x)(\gamma)_{\{4}(-\frac{i}{2}\overleftrightarrow{\mathcal{D}})_{i}(-\frac{i}{2}\overleftrightarrow{\mathcal{D}})_{i\}}\psi(x)$$

PRELIMINARY RESULTS

First Moment

$\kappa = 0.154$,

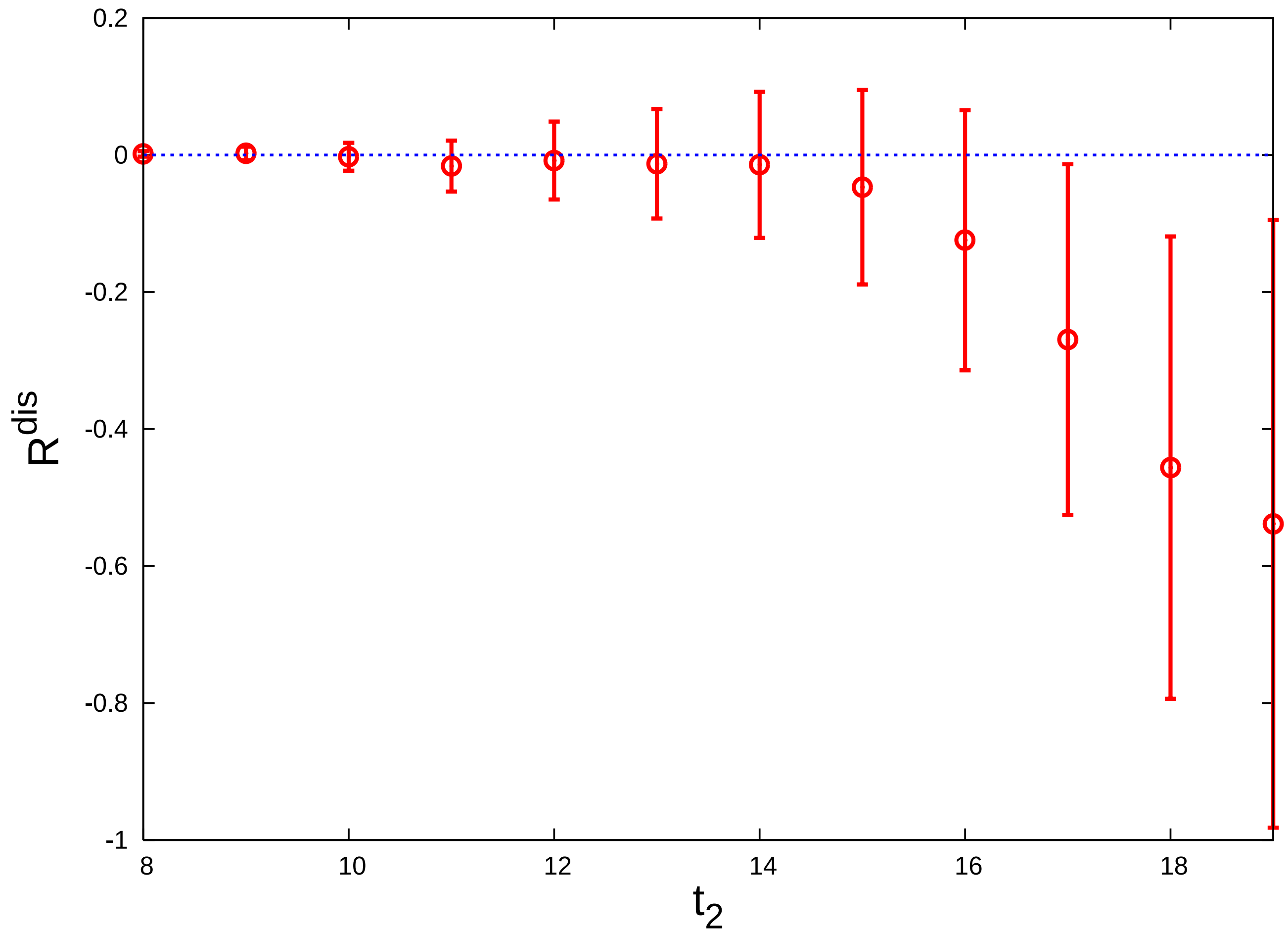
# of noise=100, # of conf = 500



Second Moment

κ = 0.154 ,

# of noise=100, # of conf = 500



Comments

- Preliminary results from scalar current indicate that
  - only a handful of noise vector and large number of configurations is not good
  - only a handful of configuration and large number of noise vectors is also not good
  - One needs a optimal set

- Our calculation with larger set of configurations and noise vectors with unbiased subtraction is ongoing.

Improvements

Use all-to-all propagators

...... M. Peardon's talk

- Use Dilution : **spin and color**
- Projection of a few eigenvalues
- Unbiased subtraction