# Hadron Structure in the Chiral Regime with Domain Wall Quarks on an Improved Staggered Sea

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# Collaborators

## Lattice Hadron Physics Collaboration

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# Outline

- Introduction
- Form factors
  - $F_1$ , $F_2$ , $G_A$ , $G_P$
- Generalized form factors
  - Transverse structure
  - Origin of nucleon spin
  - Comparison with phenomenology
- N-Delta transition form factor
- Summary and future challenges

# Goals

- Quantitative calculation of hadron observables from first principles
  - Agreement with experiment
  - Credibility for predictions and guiding experiment
- Insight into how QCD works
  - Mechanisms
    - Paths that dominate action - instantons
    - Variational wave functions
  - Dependence on parameters
    - $N_c$, $N_f$, gauge group
    - $m_q$

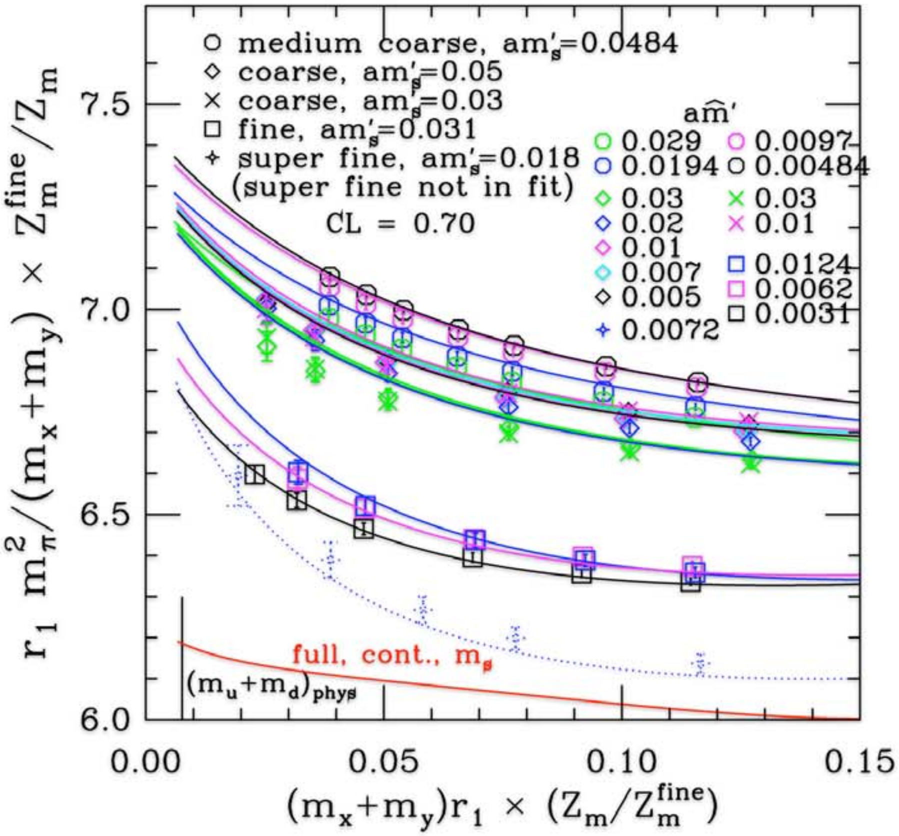
# The case for using an improved staggered sea

- Fourth root appears manageable
  - RG indicates coefficient of nonlocal term → 0
  - Partially quenched staggered XPT accounts well for ugly properties
- Successful predictions and accurate agreement with expt.
- Availability of lattices with large L, 3 lattice spacings

# The case for improved staggered quarks

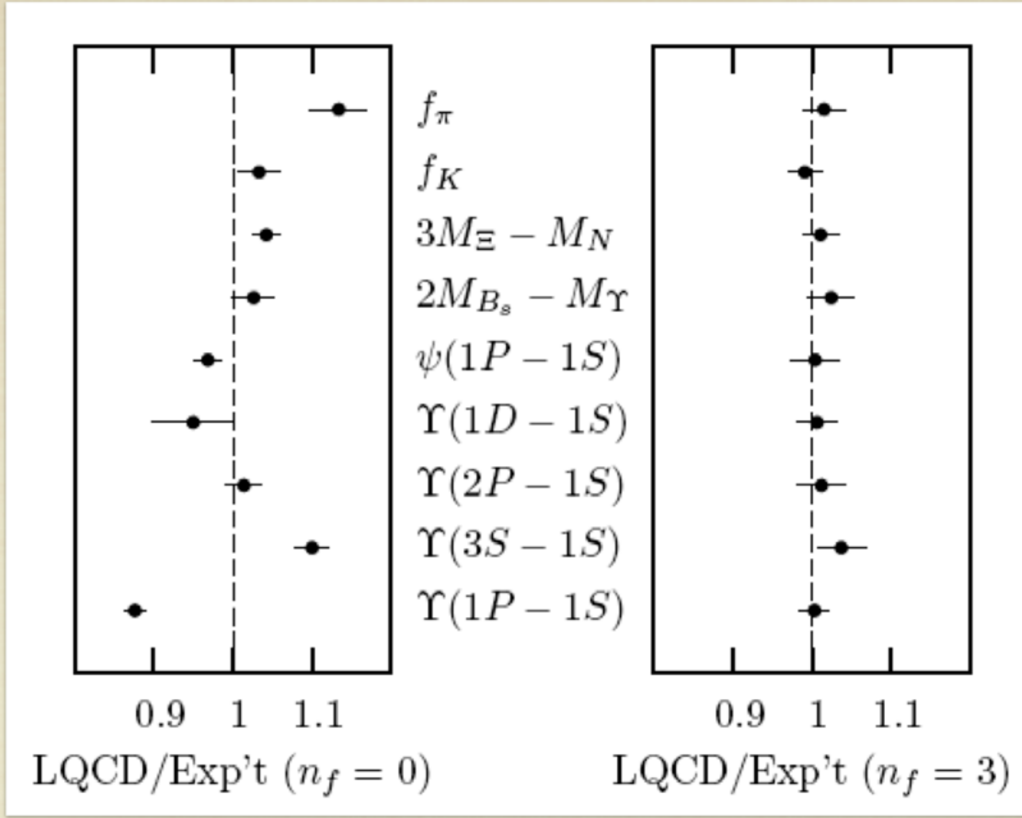
## MILC results for $m_\pi^2/(m_x + m_y)$ [Sugar]

- Results for $m_\pi^2/(m_x + m_y)$
- Part of global fit to PGB properties
- Super-fine lattice results agree with predictions
- Partially quenched staggered chiral perturbation theory describes data well



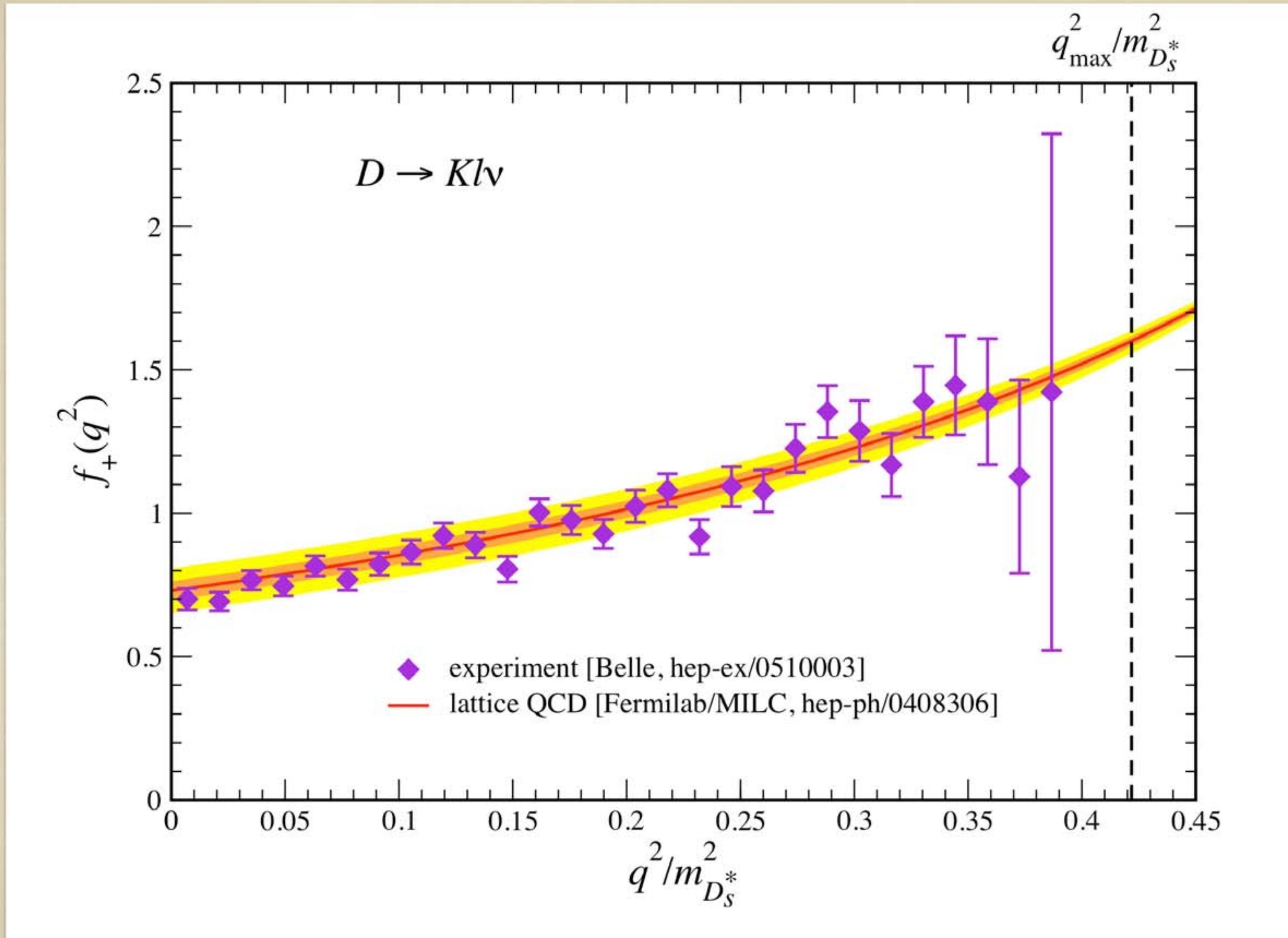

S. Sharpe, "Rooted Staggered Fermions: Good, Bad or Ugly", Lattice 2006, 7/26/2006 – p.14/50

# Precision agreement in heavy quark systems



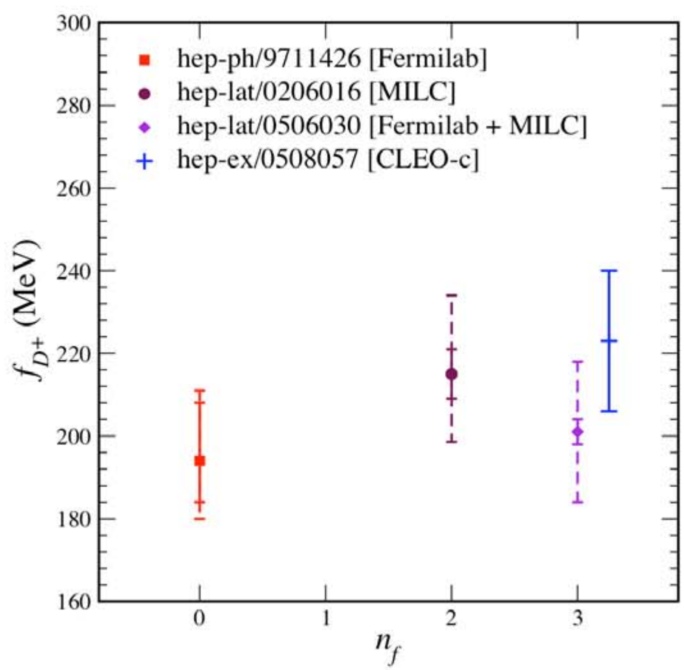

- "Gold Plated Observables" (Davies et. al. hep-lat/0304004)
  - Staggered quarks
  - Asqtad improved action
  - a = 0.13, 0.09 fm

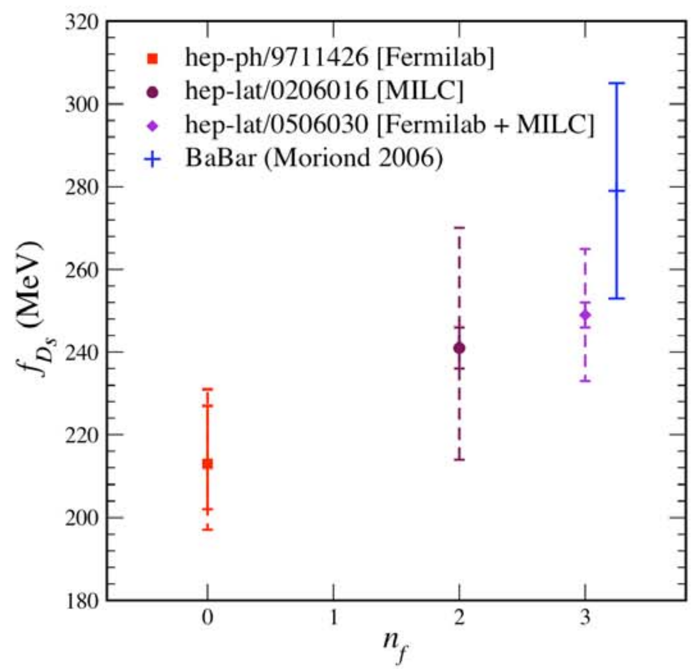
# Lattice QCD Predictions



# Lattice QCD Predictions

D meson decay constants




$$\frac{\sqrt{m_D} f_D}{\sqrt{m_{D_s}} f_{D_s}} = 0.786 \pm 0.042 \quad \text{lattice}$$

$$= 0.779 \pm 0.093 \quad \text{CLEO/BaBar}$$

Mass of $B_c$ meson

# The case for domain wall valence quarks

- Chiral symmetry avoids operator mixing
- Order $a^2$
- Conserved axial current facilitates renormalization
- Hybrid valence and sea actions are manageable
  - Hybrid XPT available
  - One-loop results have simple chiral behavior
  - Perturbative calculation of ratios of renormalization constants works well

# Hadron structure revealed by high energy scattering

- High energy scattering measures correlation functions along light cone
  - Asymptotic freedom: reaction theory perturbative
  - Unambiguous measurement of operators in light cone frame
  - Must think about physics on light cone
- Parton distribution q(x) gives longitudinal momentum distribution of light-cone wave function
- Generalized parton distribution $q(x, r_\perp)$ gives transverse spatial structure of light-cone wave function

# Parton and generalized parton distributions

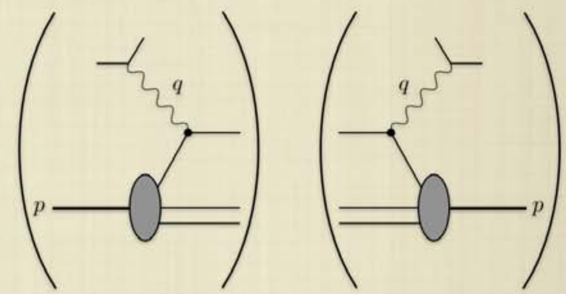
High energy scattering: light-cone correlation function $(\lambda = p^+ x^-)$

$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}(-\frac{\lambda}{2}n) \; \not{n} \mathcal{P} e^{-ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(\alpha n)} \psi(\frac{\lambda}{2}n)$$

Deep inelastic scattering: diagonal matrix element

$$\langle P|\mathcal{O}(x)|P\rangle = q(x)$$

$$[\not{n} \to \not{n}\gamma_5 : \quad \Delta q(x)]$$

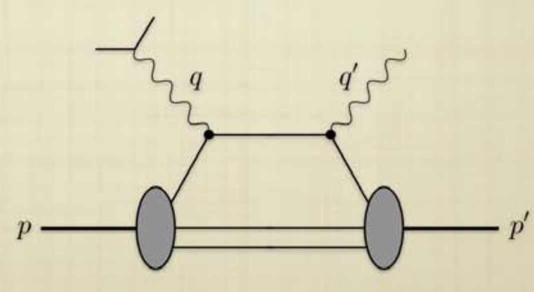



Deeply virtual Compton scattering: off-diagonal matrix element

$$\langle P'|\mathcal{O}(x)|P\rangle = \langle\gamma\rangle H(x,\xi,t) + \tfrac{i\Delta}{2m}\langle\sigma\rangle E(x,\xi,t)$$

$$\Delta = P' - P, \quad t = \Delta^2, \quad \xi = -n\cdot\Delta/2$$

$$[\not{n} \to \not{n}\gamma_5 : \quad \tilde{E}(x,\xi,t), \tilde{H}(x,\xi,t)]$$



# Moments of parton distributions

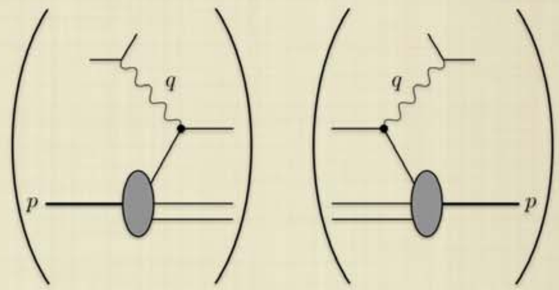
Expansion of $\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}(-\frac{\lambda}{2}n) \not{n} \mathcal{P} e^{-ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(\alpha n)} \psi(\frac{\lambda}{2}n)$

Generates tower of twist-2 operators

$$\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}} = \overline{\psi}_q \gamma^{\{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n\}} \psi_q$$

Diagonal matrix element

$$\langle P|\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}}|P\rangle \sim \int dx\, x^{n-1} q(x)$$



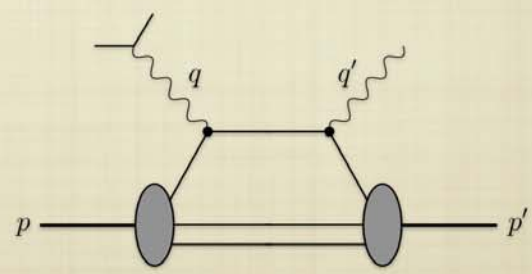

Off-diagonal matrix element

$$\langle P'|\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}}|P\rangle \to A_{ni}(t), B_{ni}(t), C_{n0}(t)$$

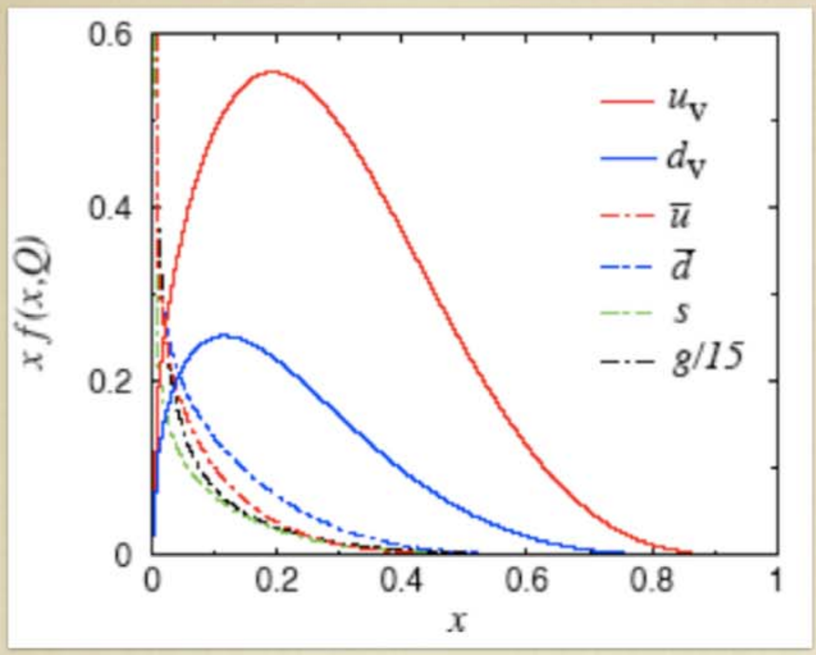
$$\int dx\, x^{n-1} H(x,\xi,t) \sim \sum \xi^i A_{ni}(t) + \xi^n C_{n0}(t)$$

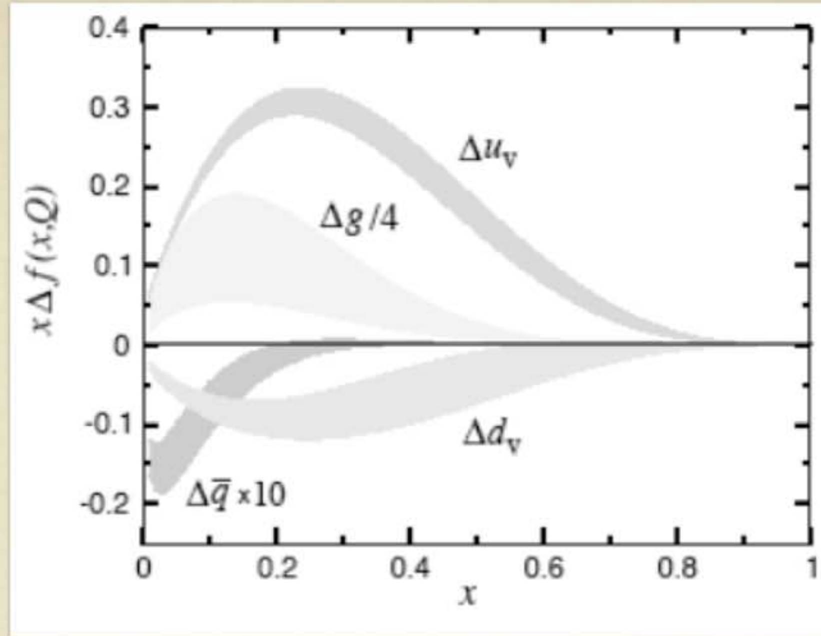
$$\int dx\, x^{n-1} E(x,\xi,t) \sim \sum \xi^i B_{ni}(t) - \xi^n C_{n0}(t)$$

$[\not{n} \to \not{n}\gamma_5 : \quad \tilde{A}_{ni}(t), \tilde{B}_{ni}(t)]$



# Moments of parton distributions




$$\langle p|\bar{\psi}\gamma_\mu D_{\mu_1}\cdots D_{\mu_n}\psi|p\rangle \rightarrow \quad \langle x^n\rangle_q \quad = \int_0^1 dx x^n[q(x)+(-1)^{(n+1)}\bar{q}(x)]$$

$$\langle p|\bar{\psi}\gamma_5\gamma_\mu D_{\mu_1}\cdots D_{\mu_n}\psi|p\rangle \rightarrow \quad \langle x^n\rangle_{\Delta q} \quad = \int_0^1 dx x^n[\Delta q(x)+(-1)^{(n)}\Delta\bar{q}(x)]$$

$$\langle p|\bar{\psi}\gamma_5\sigma_{\mu\nu} D_{\mu_1}\cdots D_{\mu_n}\psi|p\rangle \rightarrow \quad \langle x^n\rangle_{\delta q} \quad = \int_0^1 dx x^n[\delta q(x)+(-1)^{(n+1)}\delta\bar{q}(x)]$$

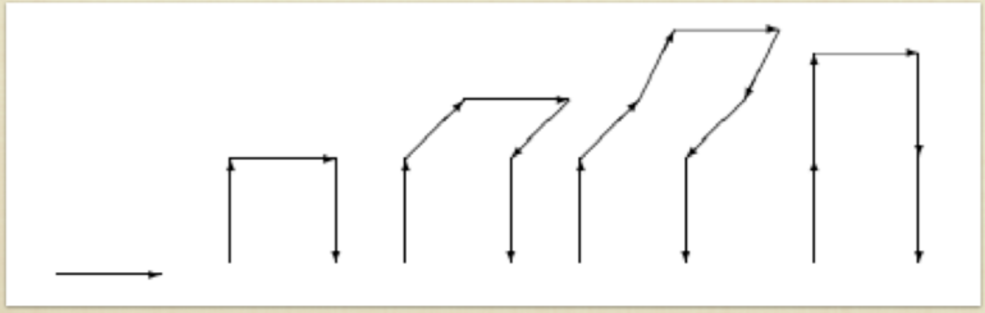
where $\quad q = q_\uparrow + q_\downarrow, \quad \Delta q = q_\uparrow - q_\downarrow, \quad \delta q = q_\top + q_\perp,$

Lattice operators: irreducible representations of hypercubic group with minimal operator mixing and minimal non-zero momentum components

| | | |
|---|---|---|
| $\langle x \rangle_q^{(a)}$ | $6_3^+$ | $\bar{\psi}\gamma_{\{1}\overleftrightarrow{D}_{4\}}\psi$ |
| $\langle x \rangle_q^{(b)}$ | $3_1^+$ | $\bar{\psi}\gamma_4\overleftrightarrow{D}_4\psi - \frac{1}{3}\sum_{i=1}^{3}\bar{\psi}\gamma_i\overleftrightarrow{D}_i\psi$ |
| $\langle x^2 \rangle_q$ | $8_1^-$ | $\bar{\psi}\gamma_{\{1}\overleftrightarrow{D}_1\overleftrightarrow{D}_{4\}}\psi - \frac{1}{2}\sum_{i=2}^{3}\gamma_{\{i}\overleftrightarrow{D}_i\overleftrightarrow{D}_{4\}}\psi$ |
| $\langle x^3 \rangle_q$ | $2_1^+$ | $\bar{\psi}\gamma_{\{1}\overleftrightarrow{D}_1\overleftrightarrow{D}_4\overleftrightarrow{D}_{4\}}\psi + \bar{\psi}\gamma_{\{2}\overleftrightarrow{D}_2\overleftrightarrow{D}_3\overleftrightarrow{D}_{3\}}\psi - \{3 \leftrightarrow 4\}$ |
| $\langle 1 \rangle_{\Delta q}$ | $4_4^+$ | $\bar{\psi}\gamma^5\gamma_3\psi$ |
| $\langle x \rangle_{\Delta q}^{(a)}$ | $6_3^-$ | $\bar{\psi}\gamma^5\gamma_{\{1}\overleftrightarrow{D}_{3\}}\psi$ |
| $\langle x \rangle_{\Delta q}^{(b)}$ | $6_3^-$ | $\bar{\psi}\gamma^5\gamma_{\{3}\overleftrightarrow{D}_{4\}}\psi$ |
| $\langle x^2 \rangle_{\Delta q}$ | $4_2^+$ | $\bar{\psi}\gamma^5\gamma_{\{1}\overleftrightarrow{D}_3\overleftrightarrow{D}_{4\}}\psi$ |
| $\langle 1 \rangle_{\delta q}$ | $6_1^+$ | $\bar{\psi}\gamma^5\sigma_{34}\psi$ |
| $\langle x \rangle_{\delta q}$ | $8_1^-$ | $\bar{\psi}\gamma^5\sigma_{3\{4}\overleftrightarrow{D}_{1\}}\psi$ |
| $d_1$ | $6_1^+$ | $\bar{\psi}\gamma^5\gamma_{[3}\overleftrightarrow{D}_{4]}\psi$ |
| $d_2$ | $8_1^-$ | $\bar{\psi}\gamma^5\gamma_{[1}\overleftrightarrow{D}_{\{3]}\overleftrightarrow{D}_{4\}}\psi$ |

# Asqtad Action: O($a^2$) perturbatively improved

- Symansik improved glue
  - $S_g(U) = C_0 W^{1\times1} + C_1 W^{1\times2} + C_2 W^{cube}$
- Smeared staggered fermions $S_f(V,U)$
  - Fat links remove taste changing gluons
  - Tadpole improved



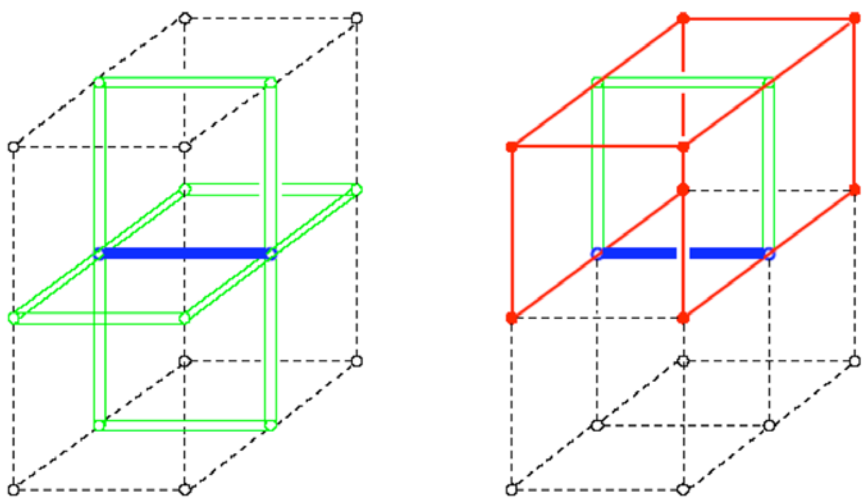
# HYP Smearing

- Three levels of SU(3) projected blocking within hypercube
- Minimize dislocations - important for DW fermions

$$V_{i,\mu} = Proj_{SU(3)}\Big[(1-\alpha_1)U_{i,\mu} + \frac{\alpha_1}{6}\sum_{\pm\nu\neq\mu} \tilde{V}_{i,\nu;\mu}\tilde{V}_{i+\hat{\nu},\mu;\nu}\tilde{V}^{\dagger}_{i+\hat{\mu},\nu;\mu}\Big],$$

$$\tilde{V}_{i,\mu;\nu} = Proj_{SU(3)}\Big[(1-\alpha_2)U_{i,\mu} + \frac{\alpha_2}{4}\sum_{\pm\rho\neq\nu,\mu} \bar{V}_{i,\rho;\nu\,\mu}\bar{V}_{i+\hat{\rho},\mu;\rho\,\nu}\bar{V}^{\dagger}_{i+\hat{\mu},\rho;\nu\,\mu}\Big],$$

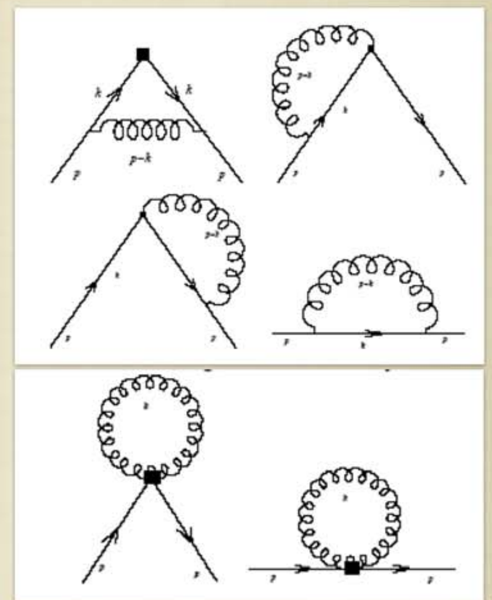
$$\bar{V}_{i,\mu;\nu\,\rho} = Proj_{SU(3)}\Big[(1-\alpha_3)U_{i,\mu} + \frac{\alpha_3}{2}\sum_{\pm\eta\neq\rho,\nu,\mu} U_{i,\eta}U_{i+\hat{\eta},\mu}U^{\dagger}_{i+\hat{\mu},\eta}\Big].$$



# Perturbative renormalization

## HYP smeared domain wall fermions - B. Bistrovic

| operator | $H(4)$ | NOS | HYP | APE |
|---|---|---|---|---|
| $\bar{q}[\gamma_5]q$ | $1_1^{\pm}$ | 0.792 | 0.981 | 1.046 |
| $\bar{q}[\gamma_5]\gamma_\mu q$ | $4_4^{\mp}$ | 0.847 | 0.976 | 0.994 |
| $\bar{q}[\gamma_5]\sigma_{\mu\nu}q$ | $6_1^{\mp}$ | 0.883 | 0.992 | 0.993 |
| $\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu\}}q$ | $6_3^{\pm}$ | 0.991 | 0.979 | 0.954 |
| $\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu\}}q$ | $3_1^{\pm}$ | 0.982 | 0.975 | 0.951 |
| $\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$ | $8_1^{\mp}$ | 1.134 | 0.988 | 0.934 |
| $\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$ | mixing | $5.71 \times 10^{-3}$ | $1.88 \times 10^{-3}$ | $8.21 \times 10^{-4}$ |
| $\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$ | $4_2^{\mp}$ | 1.124 | 0.987 | 0.934 |
| $\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_\alpha D_{\beta\}}q$ | $2_1^{\pm}$ | 1.244 | 0.993 | 0.919 |
| $\bar{q}[\gamma_5]\sigma_{\mu\{\nu}D_{\alpha\}}q$ | $8_1^{\pm}$ | 1.011 | 0.994 | 0.964 |
| $\bar{q}[\gamma_5]\gamma_{[\mu}D_{\nu]}q$ | $6_1^{\mp}$ | 0.979 | 0.982 | 0.989 |
| $\bar{q}[\gamma_5]\gamma_{[\mu}D_{\{\nu]}D_{\alpha\}}q$ | $8_1^{\pm}$ | 0.955 | 0.959 | 0.965 |



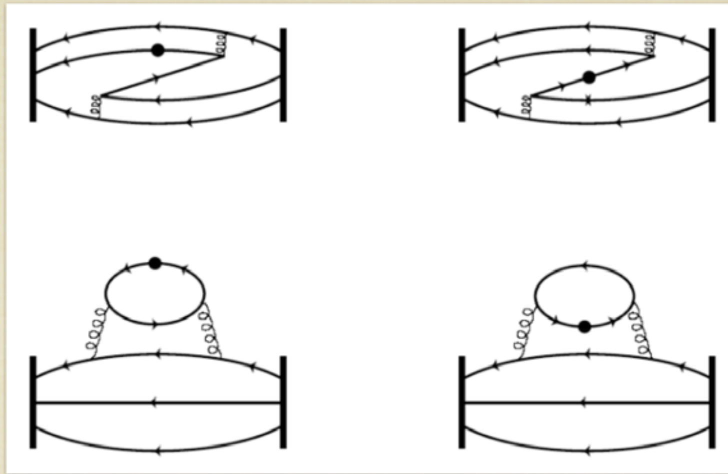
$$O_i^{\overline{MS}}(Q^2) = \sum_j \left( \delta_{ij} + \frac{g_0^2}{16\pi^2} \frac{N_c^2 - 1}{2N_c} \left( \gamma_{ij}^{\overline{MS}} \log(Q^2 a^2) - (B_{ij}^{LATT} - B_{ij}^{\overline{MS}}) \right) \right) \cdot O_j^{LATT}(a^2)$$

# Numerical calculations

- Improved staggered sea quarks (MILC configurations)
  - $N_F = 3$, $a = 0.125$ fm
- Domain wall valence quarks
  - $L_S = 16$, $M = 1.7$
  - Masses and volumes:

| $m_\pi$ | configs | Vol | L (fm) |
|---|---|---|---|
| 761 | 425 | $20^3$ | 2.5 |
| 693 | 350 | $20^3$ | 2.5 |
| 544 | 564 | $20^3$ | 2.5 |
| 486 | 498 | $20^3$ | 2.5 |
| 354 | 655 | $20^3$ | 2.5 |
| 354 | 270 | $28^3$ | 3.5 |

# Hadron matrix elements on the lattice



- Measure $\langle \mathcal{O} \rangle$ for $m_q, a, L$
- Connected diagrams
- Disconnected diagrams (cancel for $\langle \mathcal{O} \rangle_u - \langle \mathcal{O} \rangle_d$)
- Extrapolate $m_q : m_\pi \rightarrow 140 \text{ MeV}$
  $a \rightarrow \sim 0.05 \text{ fm}$
  $L \rightarrow \sim 5 \text{ fm}$

# Overdetermined system for form factors

Calculate ratio

$$R_{\mathcal{O}}(\tau, P', P) = \frac{C^{3\text{pt}}_{\mathcal{O}}(\tau, P', P)}{C^{2\text{pt}}(\tau_{\text{snk}}, P')} \left[ \frac{C^{2\text{pt}}(\tau_{\text{snk}} - \tau + \tau_{\text{src}}, P)\, C^{2\text{pt}}(\tau, P')\, C^{2\text{pt}}(\tau_{\text{snk}}, P')}{C^{2\text{pt}}(\tau_{\text{snk}} - \tau + \tau_{\text{src}}, P')\, C^{2\text{pt}}(\tau, P)\, C^{2\text{pt}}(\tau_{\text{snk}}, P)} \right]^{1/2}$$

Schematic form

$$\begin{aligned} \langle \mathcal{O}_i^{cont} \rangle &= \sum_j a_{ij} \mathcal{F}_j \\ \langle \mathcal{O}_i^{cont} \rangle &= \sqrt{E'E} \sum_j Z_{ij} \overline{R}_j \\ \overline{R}_i &= \frac{1}{\sqrt{E'E}} \sum_{jk} Z^{-1}_{ij} a_{jk} \mathcal{F}_k \\ &\equiv \sum_j a'_{ij} \mathcal{F}_j \,. \end{aligned}$$

# Nucleon axial charge in full lattice QCD

- Why $g_A$?

- Matrix element of axial current $A_\mu = \bar{q}\gamma_\mu\gamma_5 \frac{\vec{\tau}}{2} q$

$$\langle N(p+q)|A_\mu|N(p)\rangle = \bar{u}(p+q)\frac{\vec{\tau}}{2}\left[g_A(q^2)\gamma_\mu\gamma_5 + g_P(q^2)q_\mu\gamma_5\right]u(p)$$

$$g_A(0) = 1.2695 \pm 0.0029$$

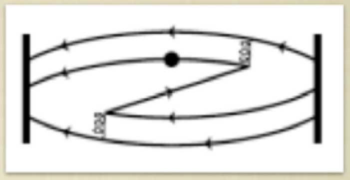
- Adler Weisberger $g_A^2 - 1 \sim \int(\sigma_{\pi^+p} - \sigma_{\pi^-p})$

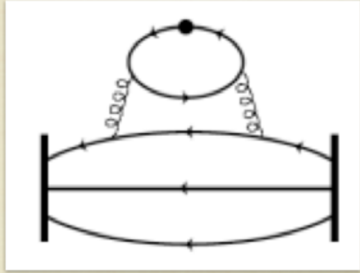
- Goldberger Treiman $g_A \rightarrow f_\pi g_{\pi NN}/M_N$

- Spin content $\langle 1\rangle_{\Delta q} = \int_0^1 dx[\Delta q(x) + \Delta\bar{q}(x)]$

$$g_A = \langle 1\rangle_{\Delta u} - \langle 1\rangle_{\Delta d} \qquad \Sigma = \langle 1\rangle_{\Delta u} + \langle 1\rangle_{\Delta d} + \langle 1\rangle_{\Delta s}$$

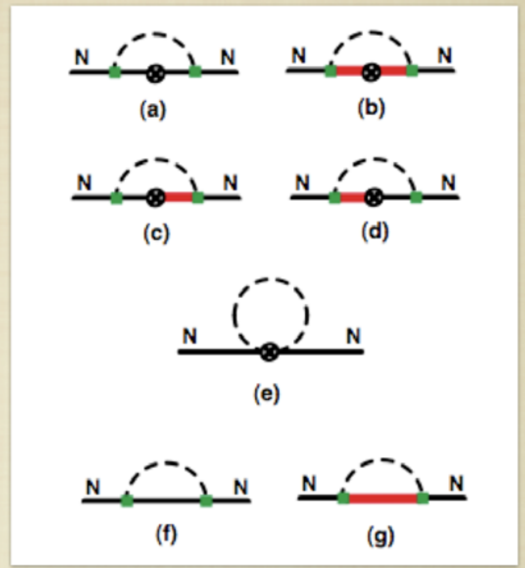
# Nucleon axial charge





- Gold-Plated observable
  - Accurately measured
  - No disconnected diagrams
  - Chiral perturbation theory for $g_A(m_\pi^2, V)$
  - Renormalization - 5-d conserved current

# Nucleon Axial Charge



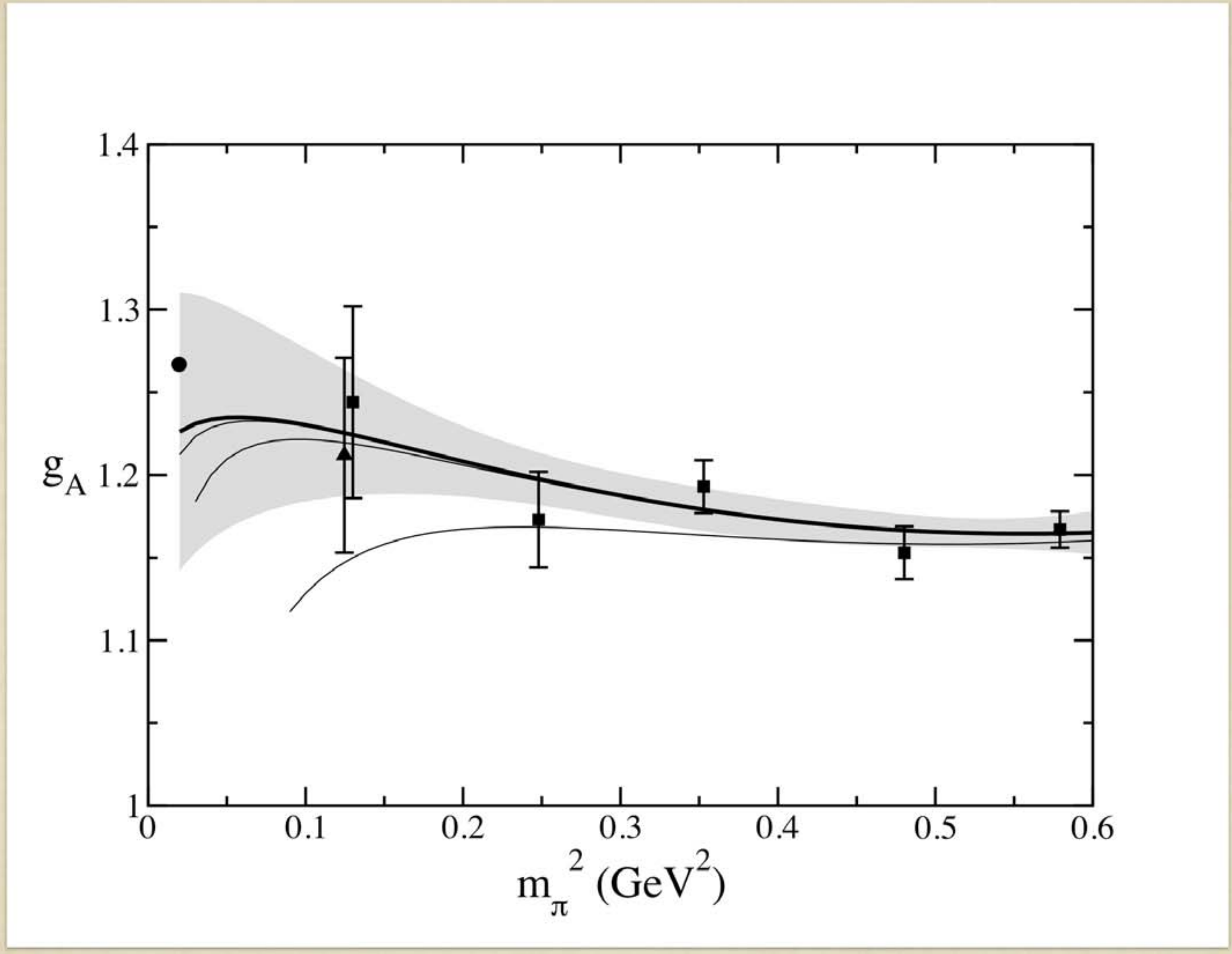

- Chiral perturbation theory $g_A(m_\pi^2, V)$
  - Beane and Savage hep-ph/0404131
  - Detmold and Lin hep-lat/0501007
- 1-loop theory has 6 parameters
  - Fix $f_\pi, m_\Delta - m_N, g_{\Delta N}$ (0.3% error)
  - Fit $g_A, g_{\Delta\Delta}, C$
  - Result $g_A(m_\pi = 140) = 1.212 \pm 0.084$

# Chiral expansion of axial charge

$$
\begin{aligned}
\Gamma_{NN} = g_A \quad & - \quad i\frac{4}{3f^2}[4g_A^3 J_1(m_\pi, 0, \mu) \\
& + \quad 4(g_{\Delta N}^2 g_A + \frac{25}{81} g_{\Delta N}^2 g_{\Delta\Delta}) J_1(m_\pi, \Delta, \mu) \\
& + \quad \frac{3}{2} g_A R_1(m_\pi, \mu) \\
& - \quad \frac{32}{9} g_{\Delta N} g_A N_1(m_\pi, \Delta, \mu)] \\
& + \quad C m_\pi^2
\end{aligned}
$$

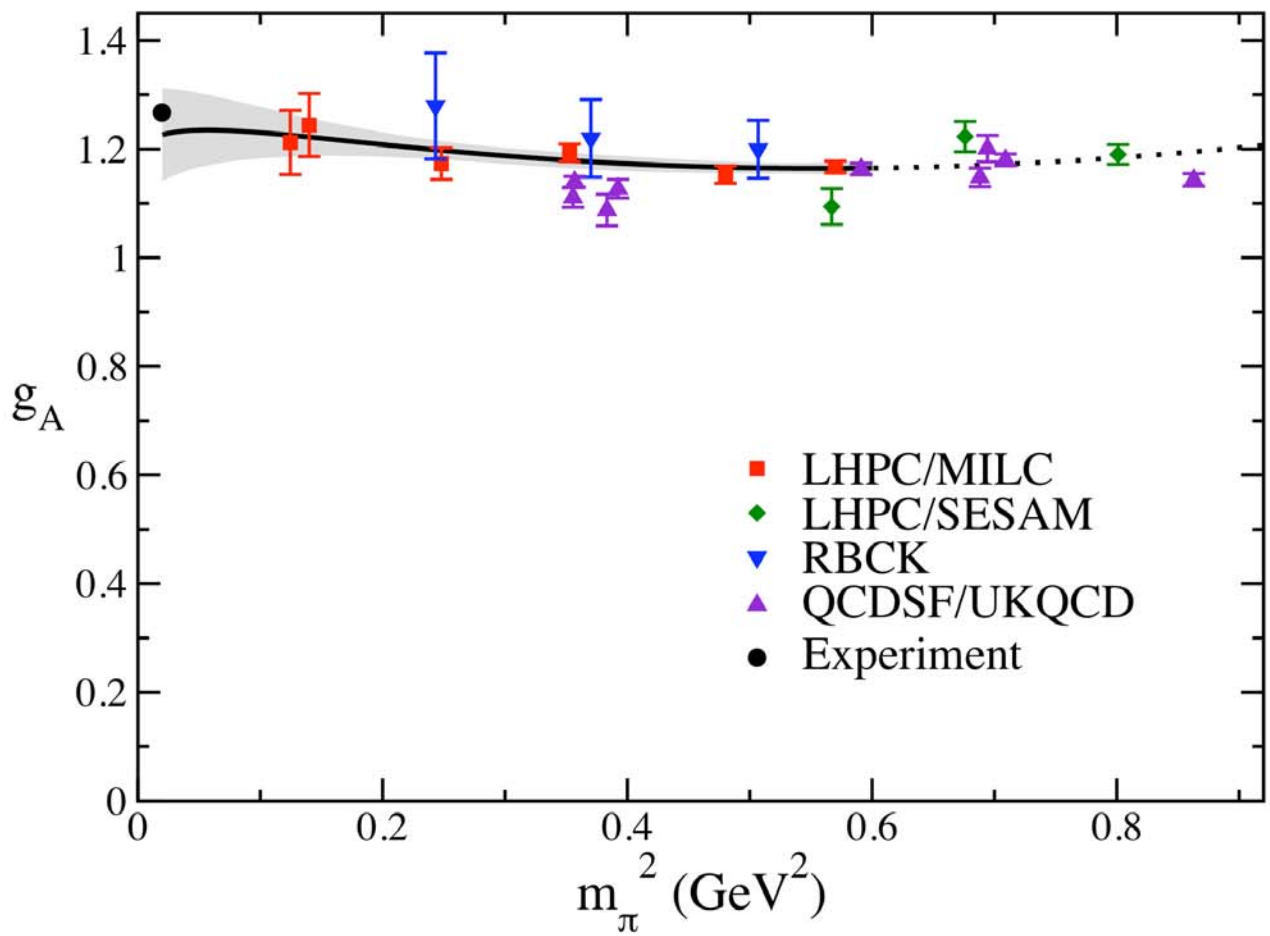
$$
\begin{aligned}
J_1(m, \Delta, \mu) &= -\frac{3}{4}\frac{i}{16\pi^2}\left[(m^2 - 2\Delta^2)\log\frac{m^2}{\mu^2} + 2\Delta F(m, \Delta)\right] \\
R_1(m, \mu) &= \frac{i}{16\pi^2} m^2 \left[\Gamma(\epsilon) + 1 - \log\frac{m^2}{\mu^2}\right] \\
N_1(m, \Delta, \mu) &= -\frac{3}{4}\frac{i}{16\pi^2}\left[(m^2 - \frac{2}{3}\Delta^2)\log\frac{m^2}{\mu^2} + \frac{2}{3}\Delta F(m, \Delta) + \frac{2}{3}\frac{m^2}{\Delta}[\pi m - F(m, \Delta)]\right] \\
f(m, \Delta) &= \sqrt{\Delta^2 - m^2 - i\epsilon}\,\log\left(\frac{\Delta - \sqrt{\Delta^2 - m^2 - i\epsilon}}{\Delta + \sqrt{\Delta^2 - m^2 - i\epsilon}}\right)
\end{aligned}
$$

# Nucleon axial charge $g_A$ $\langle 1 \rangle_{\Delta q}^{u-d}$




Fit: Beane and Savage hep-ph/0404131

# Nucleon axial charge $g_A$ $\langle 1 \rangle_{\Delta q}^{u-d}$



# Chiral Extrapolation of Moments

- for example, unpolarized moments

$$\langle x^n \rangle_{u-d} = a_n \left(1 - \frac{(3g_{A,0}^2 + 1)}{(4\pi f_{\pi,0})^2} m_\pi^2 \ln\left(\frac{m_\pi^2}{\mu^2}\right)\right) + b_n'(\mu) m_\pi^2$$

- choose $\mu = f_{\pi,0}$, and at one loop $g_{A,0} \to g_{A,m_\pi}$ and $f_{\pi,0} \to f_{\pi,m_\pi}$

$$\langle x^n \rangle_{u-d} = a_n \left(1 - \frac{(3g_{A,m_\pi}^2 + 1)}{(4\pi)^2} \frac{m_\pi^2}{f_{\pi,m_\pi}^2} \ln\left(\frac{m_\pi^2}{f_{\pi,m_\pi}^2}\right)\right) + b_n \frac{m_\pi^2}{f_{\pi,m_\pi}^2}$$

- self consistently $g_A \to g_{A,\text{lat}}$, $f_\pi \to f_{\pi,\text{lat}}$, $m_\pi \to m_{\pi,\text{lat}}$

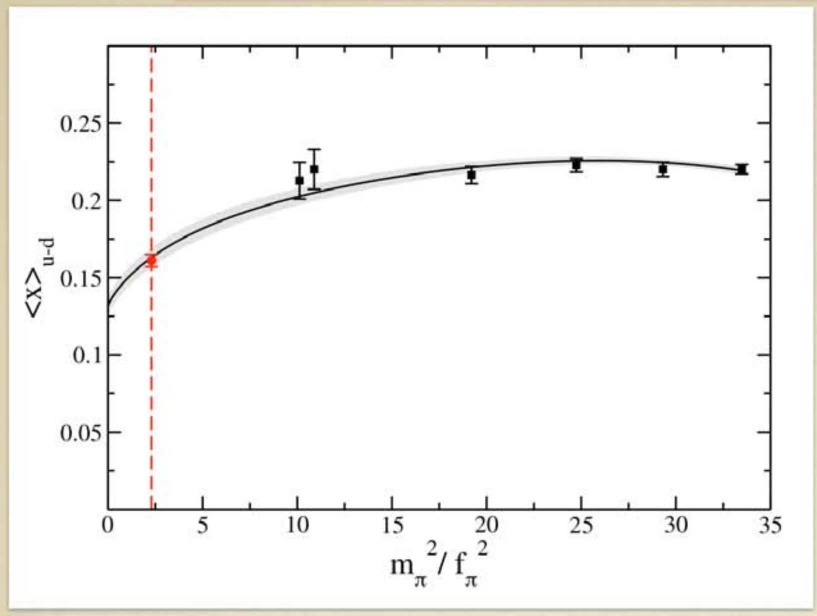
$$\langle x^n \rangle_{u-d} = a_n \left(1 - \frac{(3g_{A,\text{lat}}^2 + 1)}{(4\pi)^2} \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \ln\left(\frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}\right)\right) + b_n \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}$$

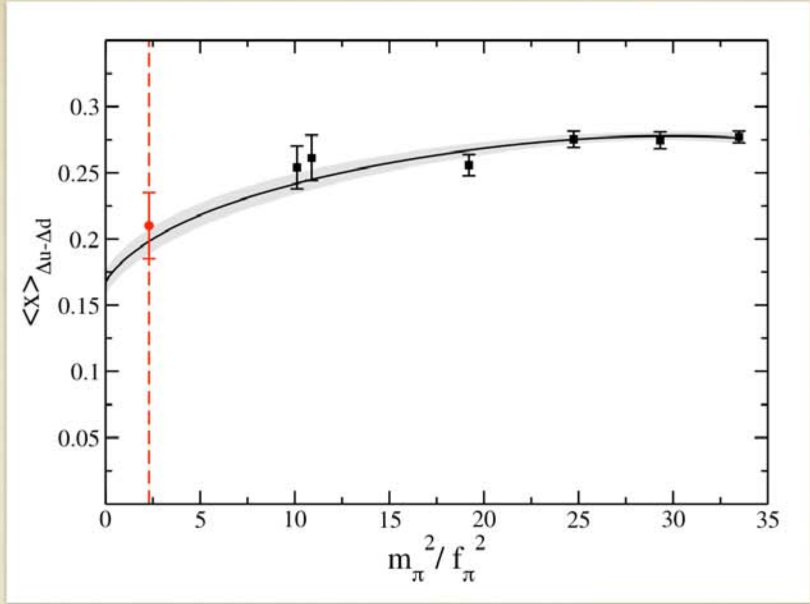
- similarly for the helicity and transversity moments

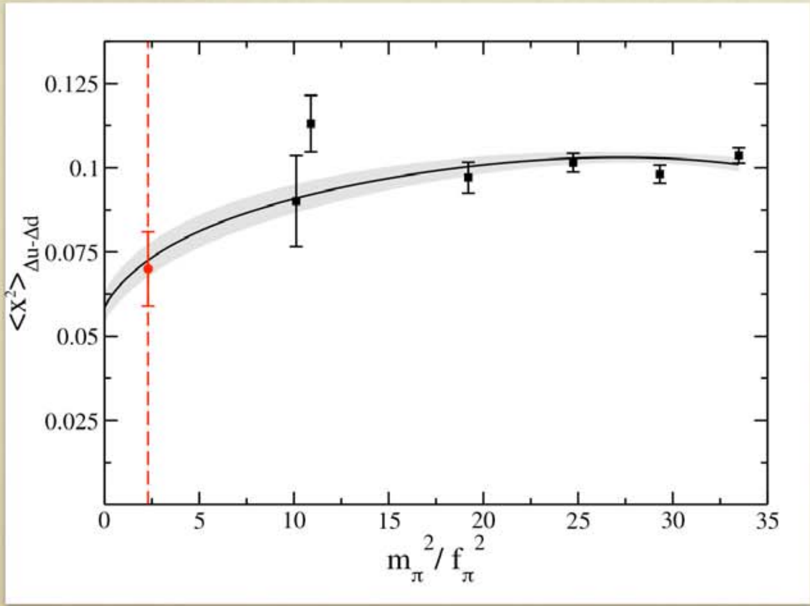
$$\langle x^n \rangle_{\Delta u-\Delta d} = \Delta a_n \left(1 - \frac{(2g_{A,\text{lat}}^2 + 1)}{(4\pi)^2} \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \ln\left(\frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}\right)\right) + \Delta b_n \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}$$

$$\langle x^n \rangle_{\delta u-\delta d} = \delta a_n \left(1 - \frac{(4g_{A,\text{lat}}^2 + 1)}{2(4\pi)^2} \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \ln\left(\frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}\right)\right) + \delta b_n \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}$$

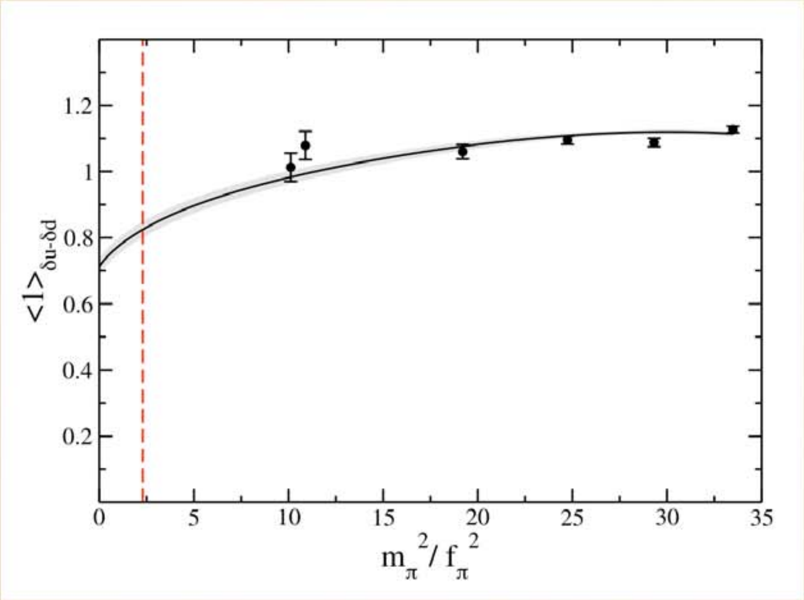
# Chiral Extrapolation of Moments

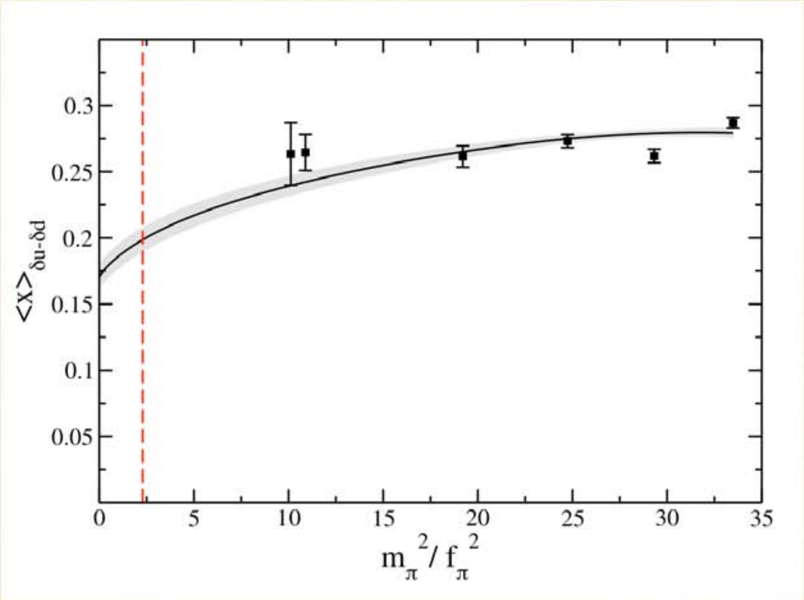


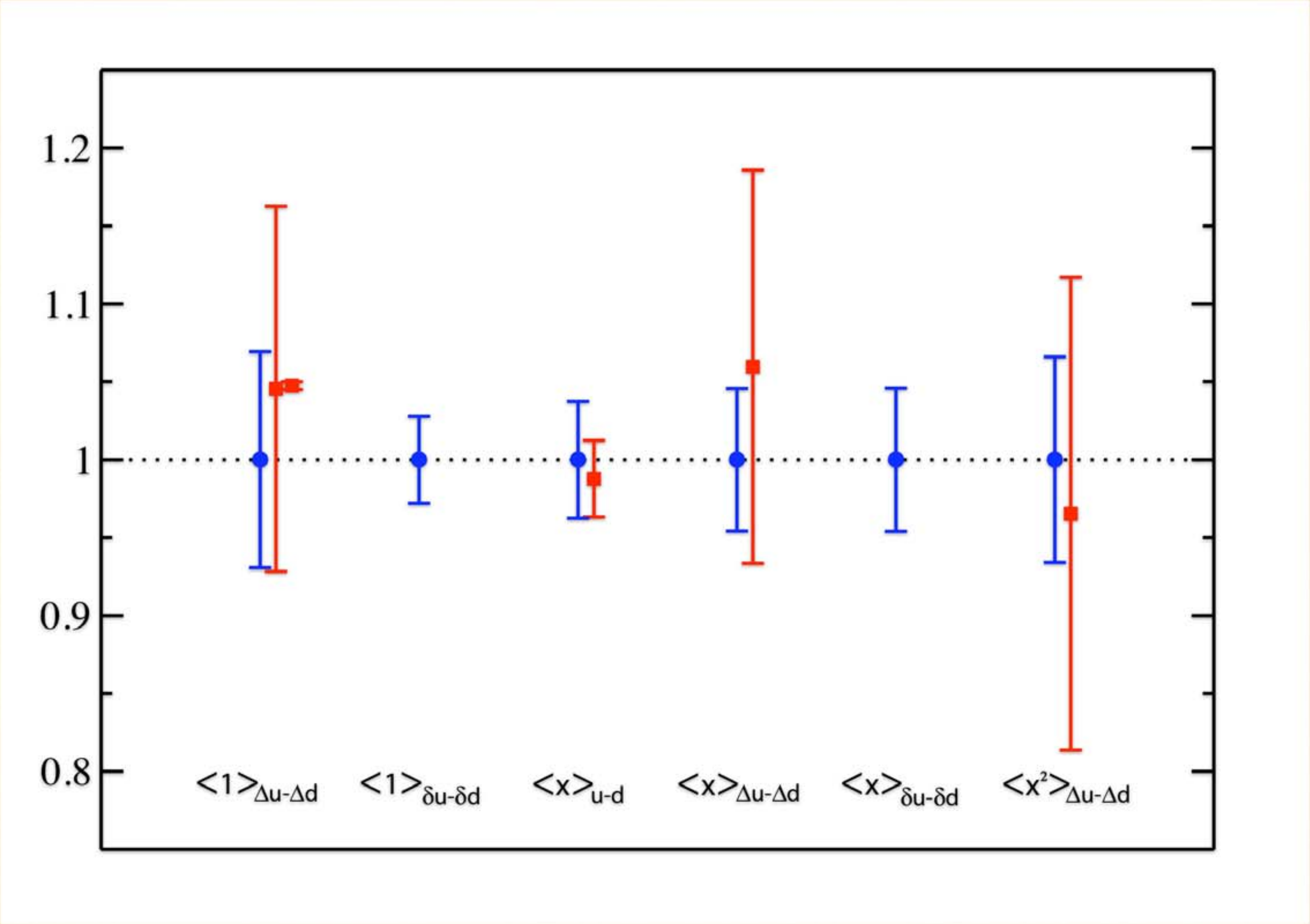



# Chiral Extrapolation of Moments



# Chiral Extrapolation of Moments



# Electromagnetic form factors

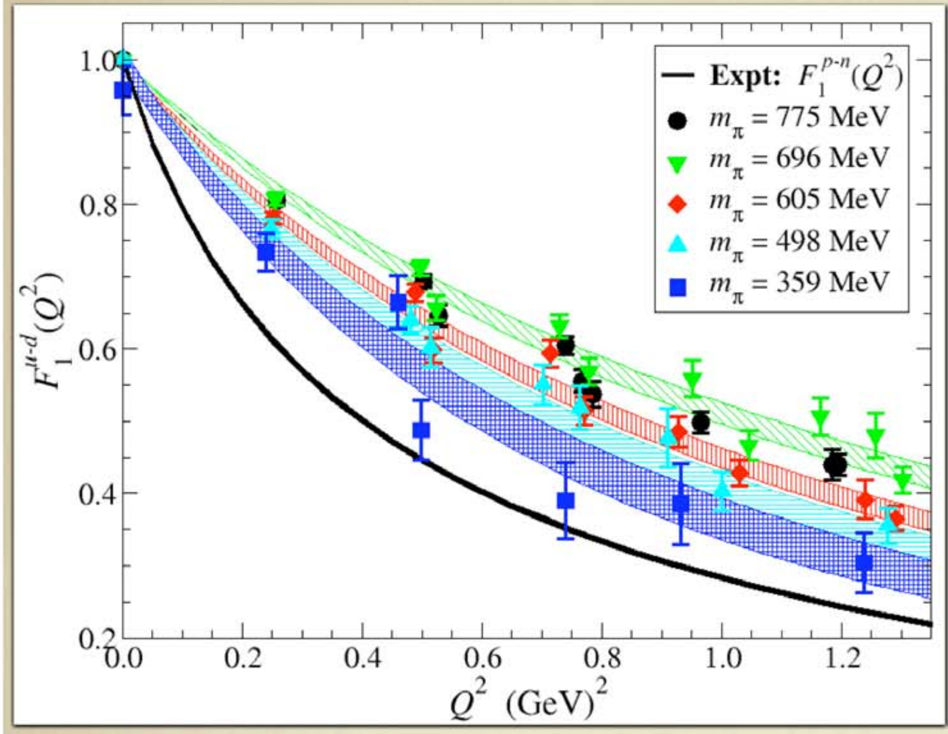
- Simplest off-diagonal matrix element

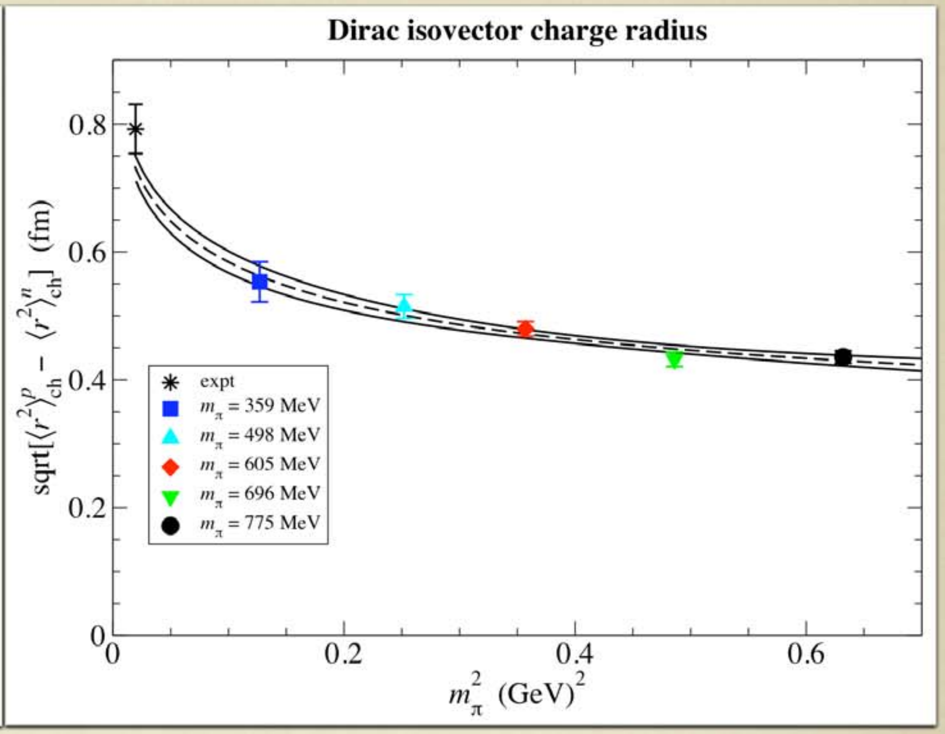
$$\langle p|\bar{\psi}\gamma^\mu\psi|p'\rangle = \bar{u}(p)[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m}]u(p')$$

$$G_E(q^2) = F_1(q^2) - \frac{q^2}{4M^2}F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

- Fourier transform of charge density if $\mathrm{L_{system}} \gg \mathrm{L_{wavepacket}} \gg \frac{1}{m}$
  - Pb: 5 fm >> $10^{-5}$ fm, Proton: 0.8 fm ~ 0.2 fm: marginal
  - For transverse Fourier transform of light cone w. f., m → $p_+$ ~ ∞
- Large $q^2$: ability of one quark to share $q^2$ with other constituents to remain in ground state - $q^2$ counting rules

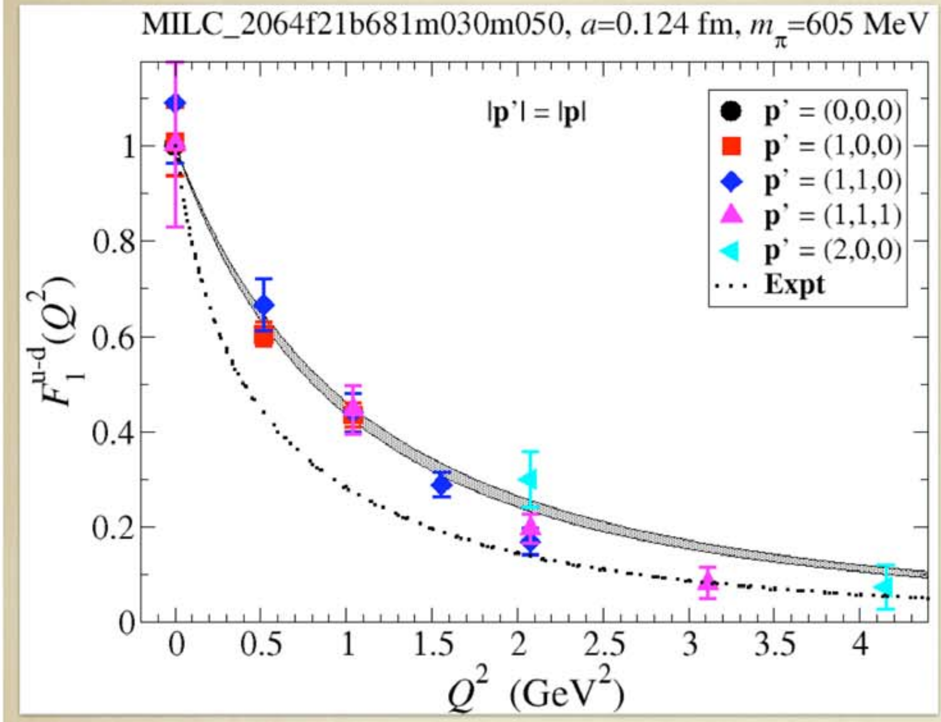
# $F_1$ Isovector Form Factor

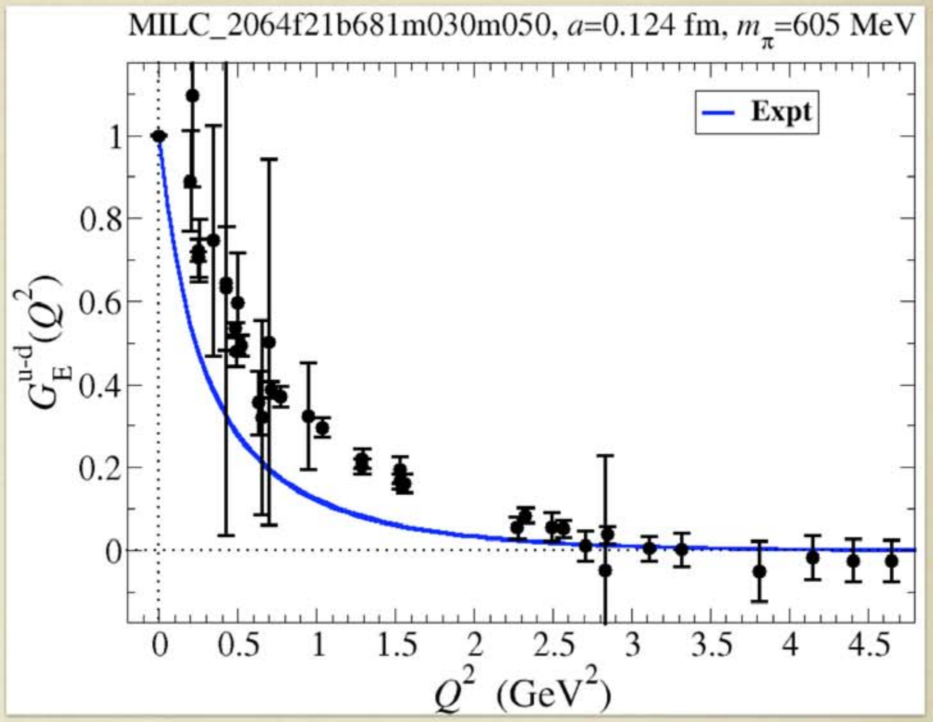


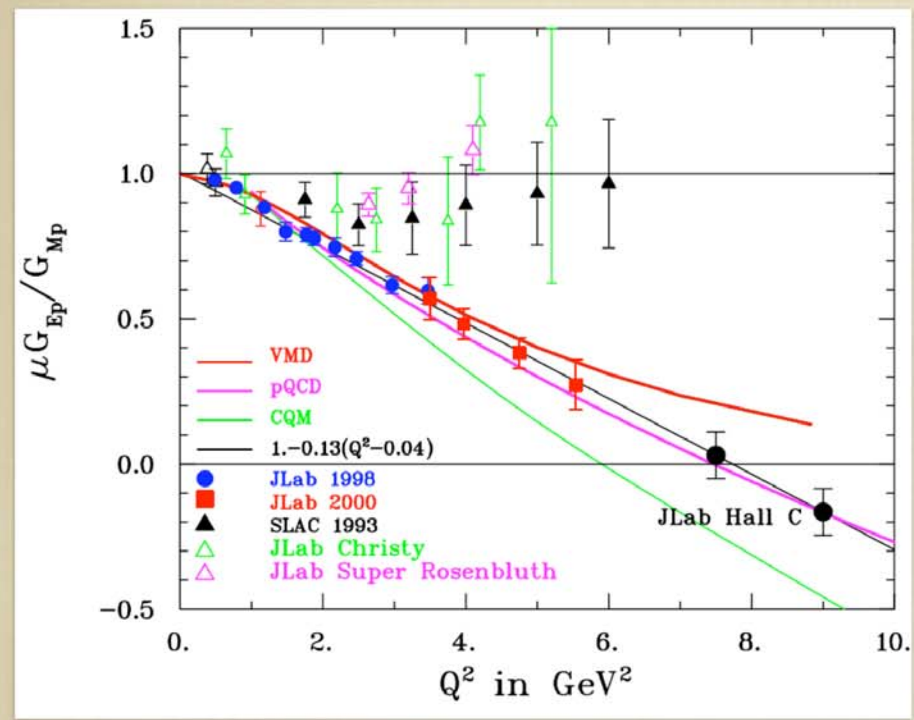

$$\langle r^2\rangle^{u-d} = a_0 - \frac{(1+5g_A^2)}{(4\pi f_\pi)^2}\log\left(\frac{m_\pi^2}{m_\pi^2+\Lambda^2}\right)$$

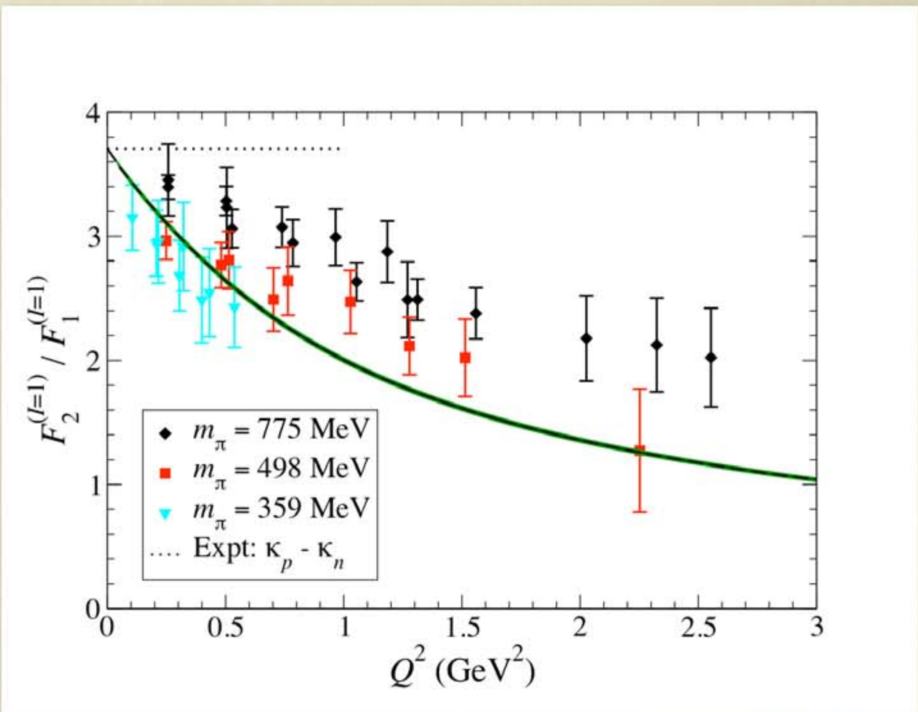
# Isovector Form Factors at higher $Q^2$



# Form factor ratio: $F_2 / F_1$



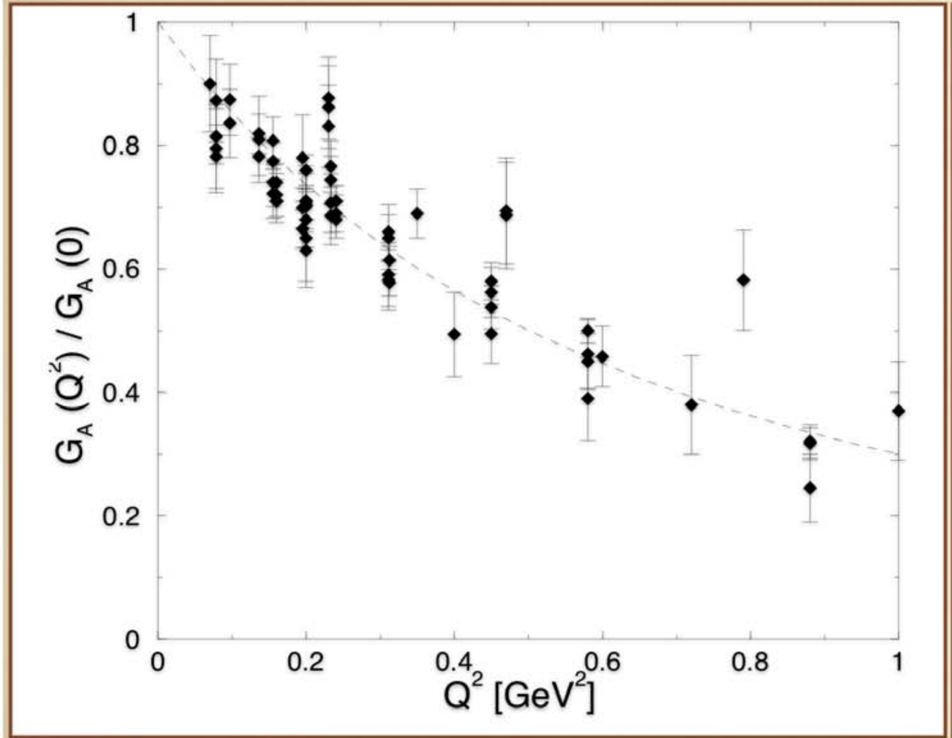



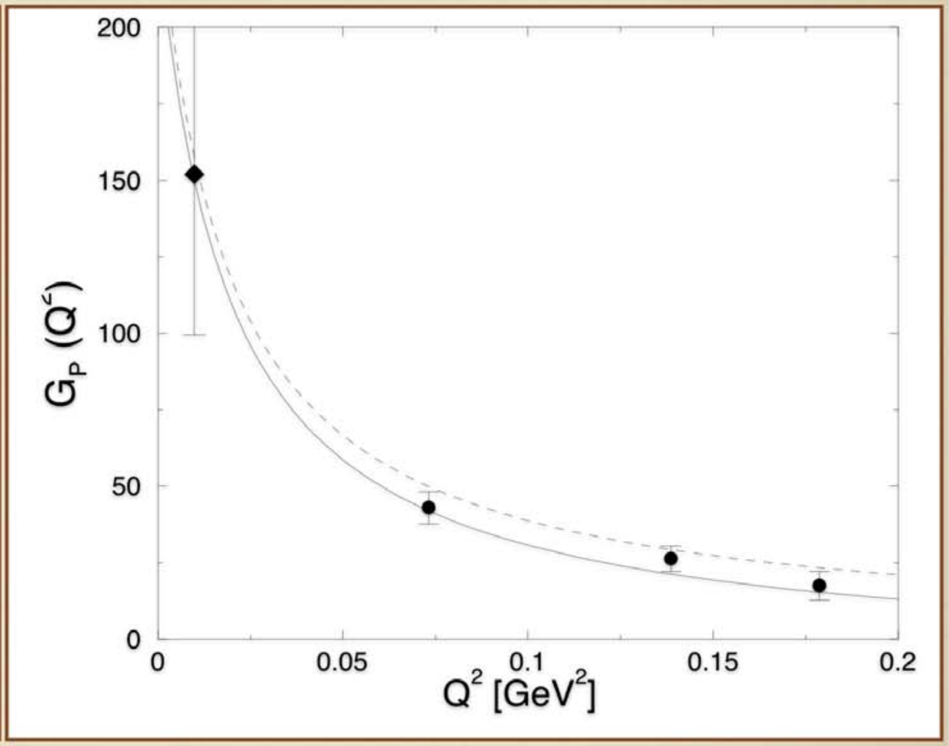

Polarization transfer at JLab

Lattice results

# Polarized Nucleon Form Factors $G_A$ and $G_P$

$$\langle p|\bar{\psi}\gamma^\mu\gamma_5\psi|p'\rangle = \bar{u}(p)[G_A(q^2)\gamma^\mu\gamma_5 + q^\mu\gamma_5 G_P(q^2) + \sigma^{\mu\nu}\gamma_5 q_\nu G_M(q^2)]u(p')$$




Bernard, Elouadrhiri, Meissner, J. Phys. G Nucl. Part. Phys. 2002, R1

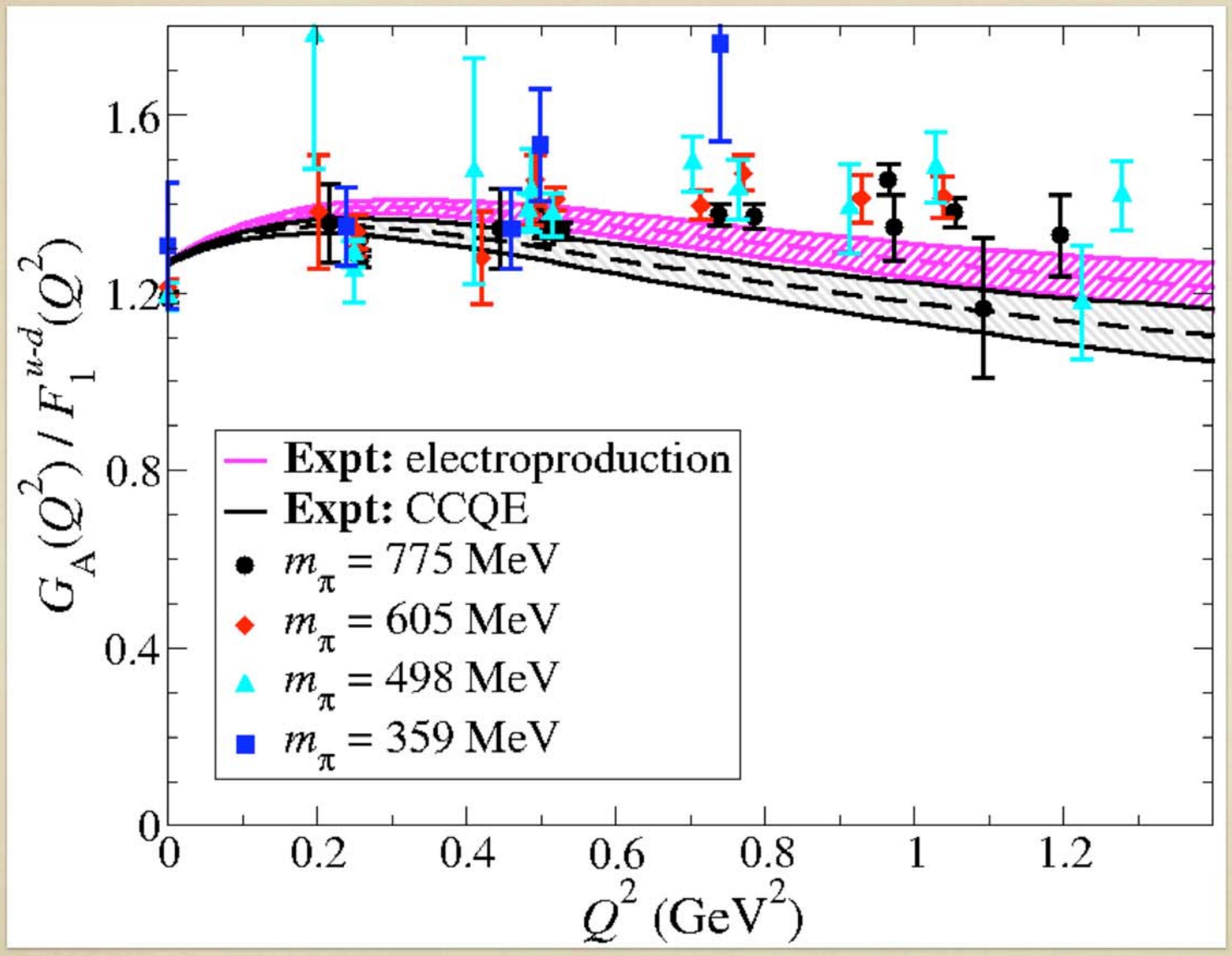
pion electroproduction ◆

$\nu_\mu n \rightarrow \mu^- p$

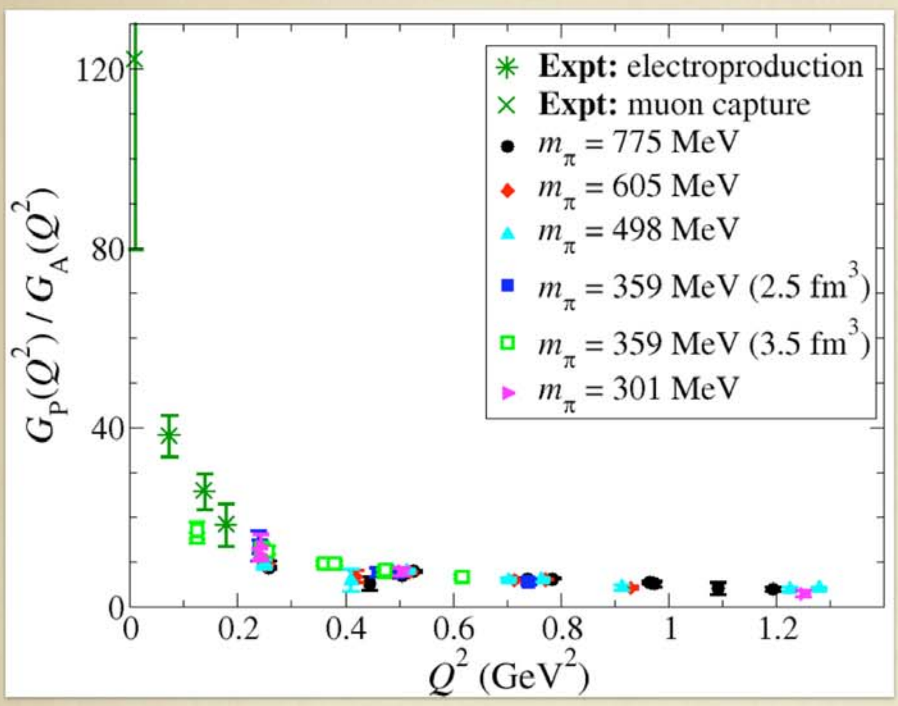
pion electroproduction ●

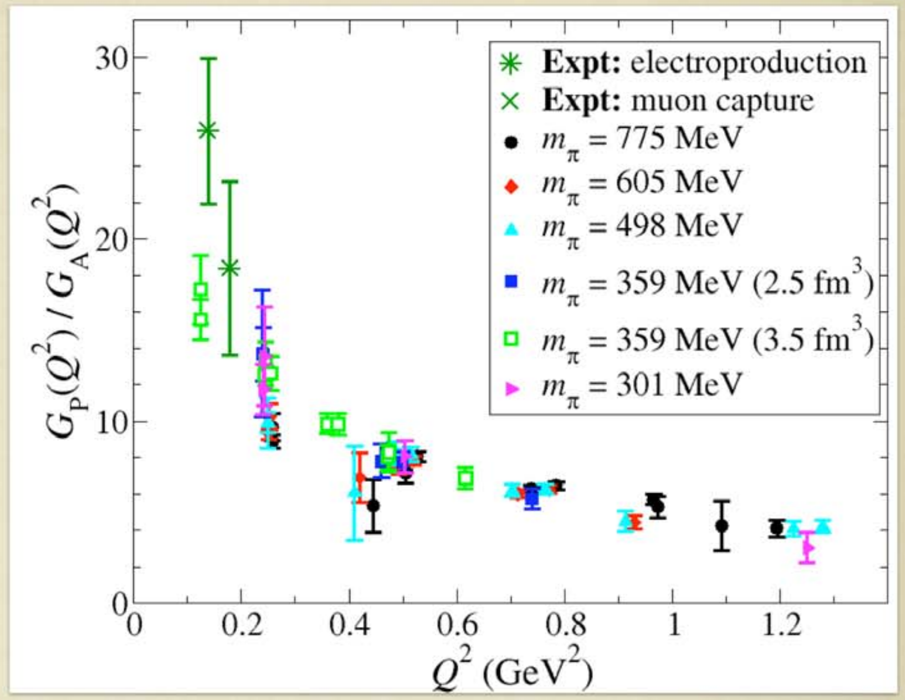
$\mu^- p \rightarrow \nu_\mu n$ ◆

# Form factor ratio: $G_A/F_1$



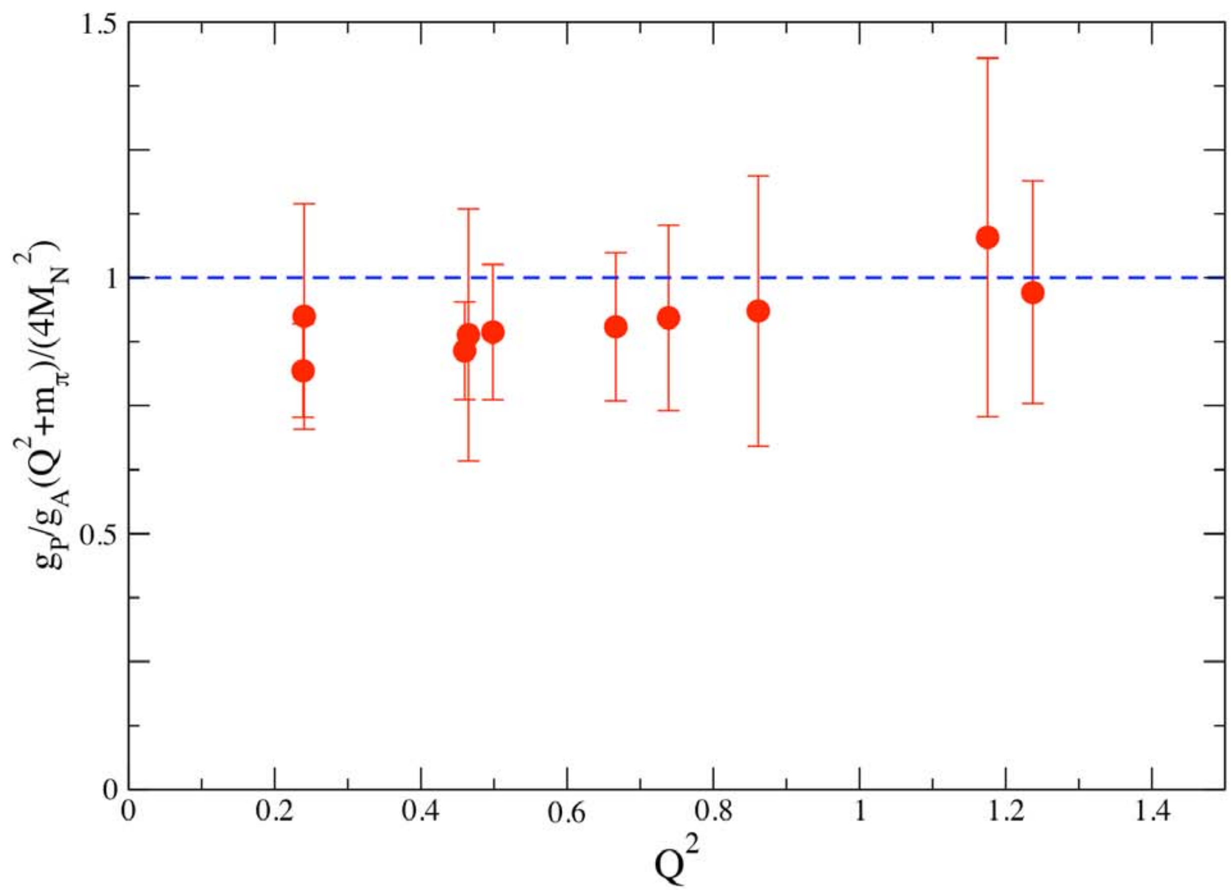
# Form factor ratio: $G_P/G_A$




soft pion pole: $$G_P(q^2) \sim \frac{4M^2 G_A(q^2)}{q^2 - m_\pi^2}$$

# Form factor ratio: $G_P/G_A$



# Generalized form factors

$$\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}} = \overline{\psi}_q\gamma^{\{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi_q$$

$$\bar{P} = \tfrac{1}{2}(P' + P)$$

$$\Delta = P' - P$$

$$t = \Delta^2$$

$$\begin{aligned}
\langle P'|\mathcal{O}^{\mu_1}|P\rangle &= \langle\!\langle\gamma^{\mu_1}\rangle\!\rangle A_{10}(t) \\
&+ \frac{i}{2m}\langle\!\langle\sigma^{\mu_1\alpha}\rangle\!\rangle\Delta_\alpha B_{10}(t)\,,
\end{aligned}$$

$$\begin{aligned}
\langle P'|\mathcal{O}^{\{\mu_1\mu_2\}}|P\rangle &= \bar{P}^{\{\mu_1}\langle\!\langle\gamma^{\mu_2\}}\rangle\!\rangle A_{20}(t) \\
&+ \frac{i}{2m}\bar{P}^{\{\mu_1}\langle\!\langle\sigma^{\mu_2\}\alpha}\rangle\!\rangle\Delta_\alpha B_{20}(t) \\
&+ \frac{1}{m}\Delta^{\{\mu_1}\Delta^{\mu_2\}}C_2(t)\,,
\end{aligned}$$

$$\begin{aligned}
\langle P'|\mathcal{O}^{\{\mu_1\mu_2\mu_3\}}|P\rangle &= \bar{P}^{\{\mu_1}\bar{P}^{\mu_2}\langle\!\langle\gamma^{\mu_3\}}\rangle\!\rangle A_{30}(t) \\
&+ \frac{i}{2m}\bar{P}^{\{\mu_1}\bar{P}^{\mu_2}\langle\!\langle\sigma^{\mu_3\}\alpha}\rangle\!\rangle\Delta_\alpha B_{30}(t) \\
&+ \Delta^{\{\mu_1}\Delta^{\mu_2}\langle\!\langle\gamma^{\mu_3\}}\rangle\!\rangle A_{32}(t) \\
&+ \frac{i}{2m}\Delta^{\{\mu_1}\Delta^{\mu_2}\langle\!\langle\sigma^{\mu_3\}\alpha}\rangle\!\rangle\Delta_\alpha B_{32}(t),
\end{aligned}$$

# Limits of generalized form factors

- Moments of parton distributions $t \to 0$

$$A_{n0} = \int dx x^{n-1} q(x)$$

- Electromagnetic form factors

$$A_{10} = F_1(t), \quad B_{10} = F_2(t)$$

- Total quark angular momentum

$$J_q = \frac{1}{2}[A(0)_{20} + B(0)_{20}]$$

# Transverse structure of nucleon

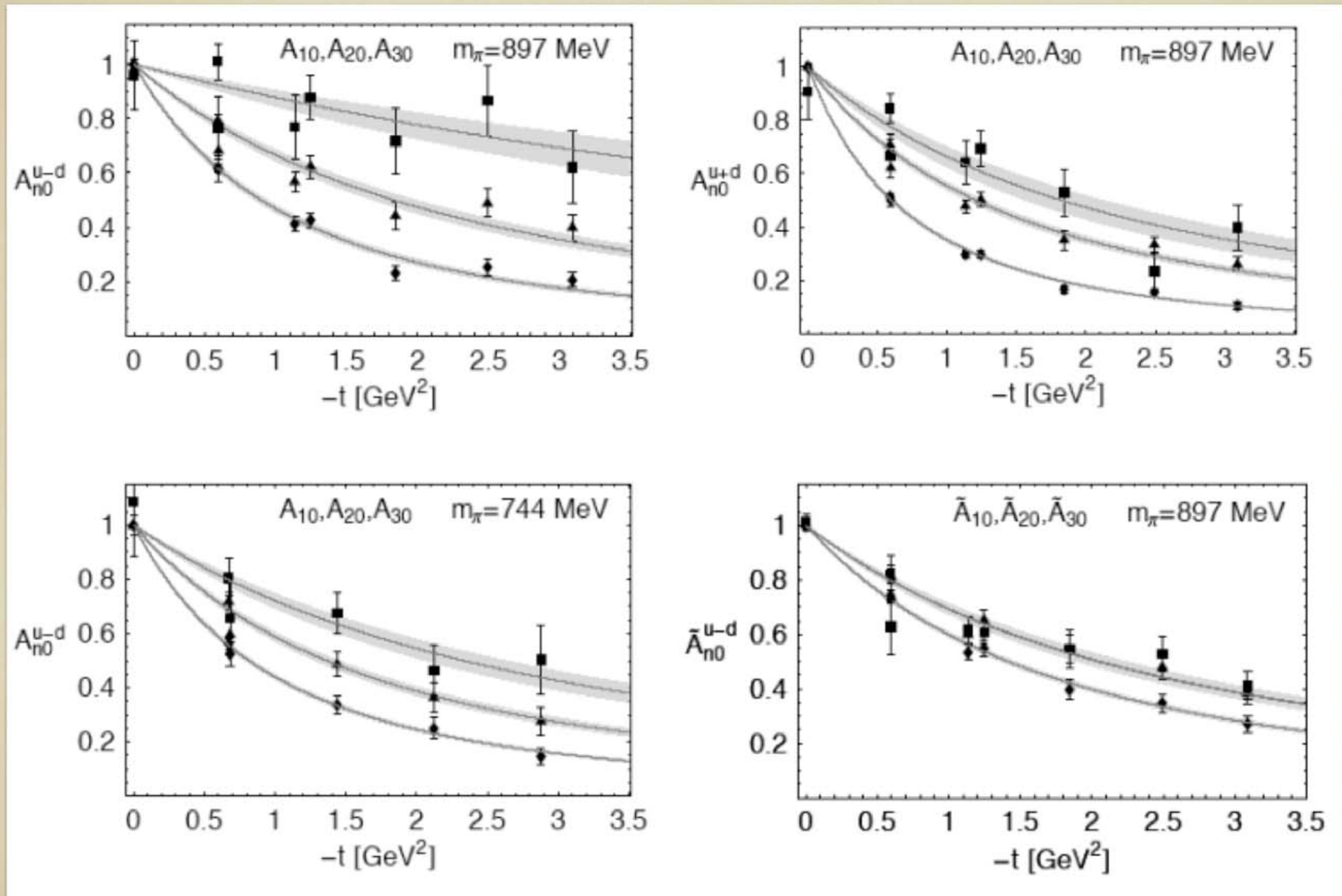
H(x, 0, - $\Delta^2_\perp$ ) is transverse Fourier transform of light cone quark distribution q(x,$r_\perp$ ) at momentum fraction x

$$q(x, r_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\Delta_\perp^2)\, e^{-ir_\perp \Delta_\perp}$$

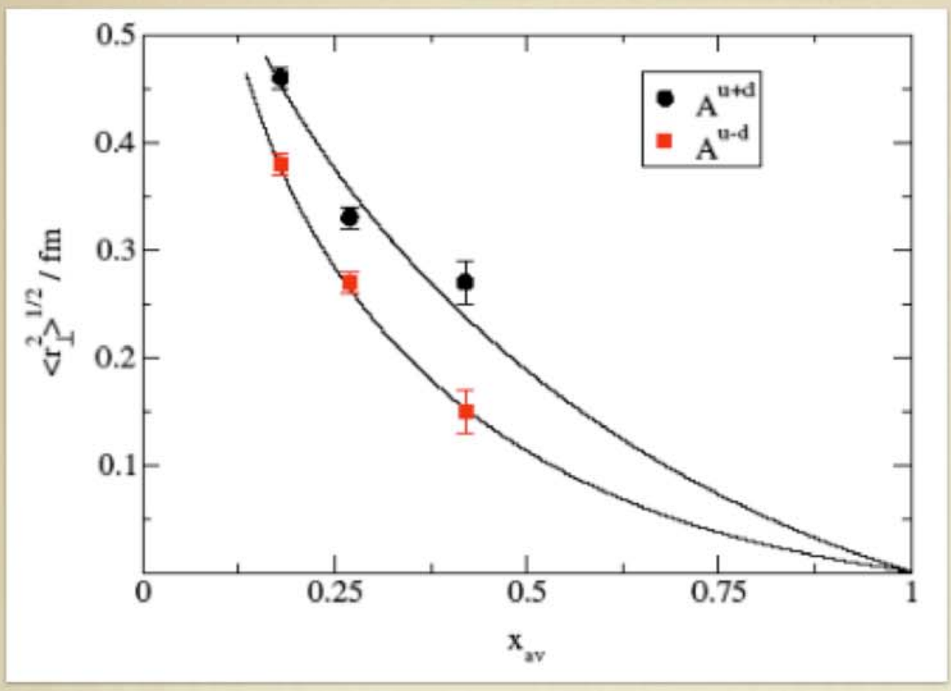
$$\int dx x^{n-1} q(x, r_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} A(-\Delta_\perp^2)\, e^{-ir_\perp \Delta_\perp}$$

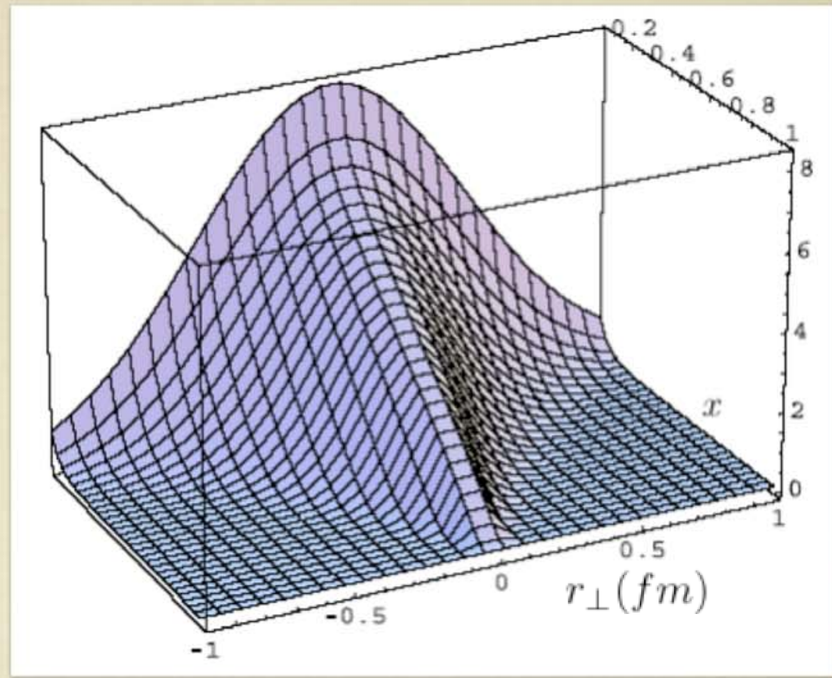
- x → 1: Single Fock space component ⇒ slope → 0
- x ≠ 1: Transverse structure ⇒ slope steeper

# Generalized form factors from lattice



# Transverse size of light-cone wave function

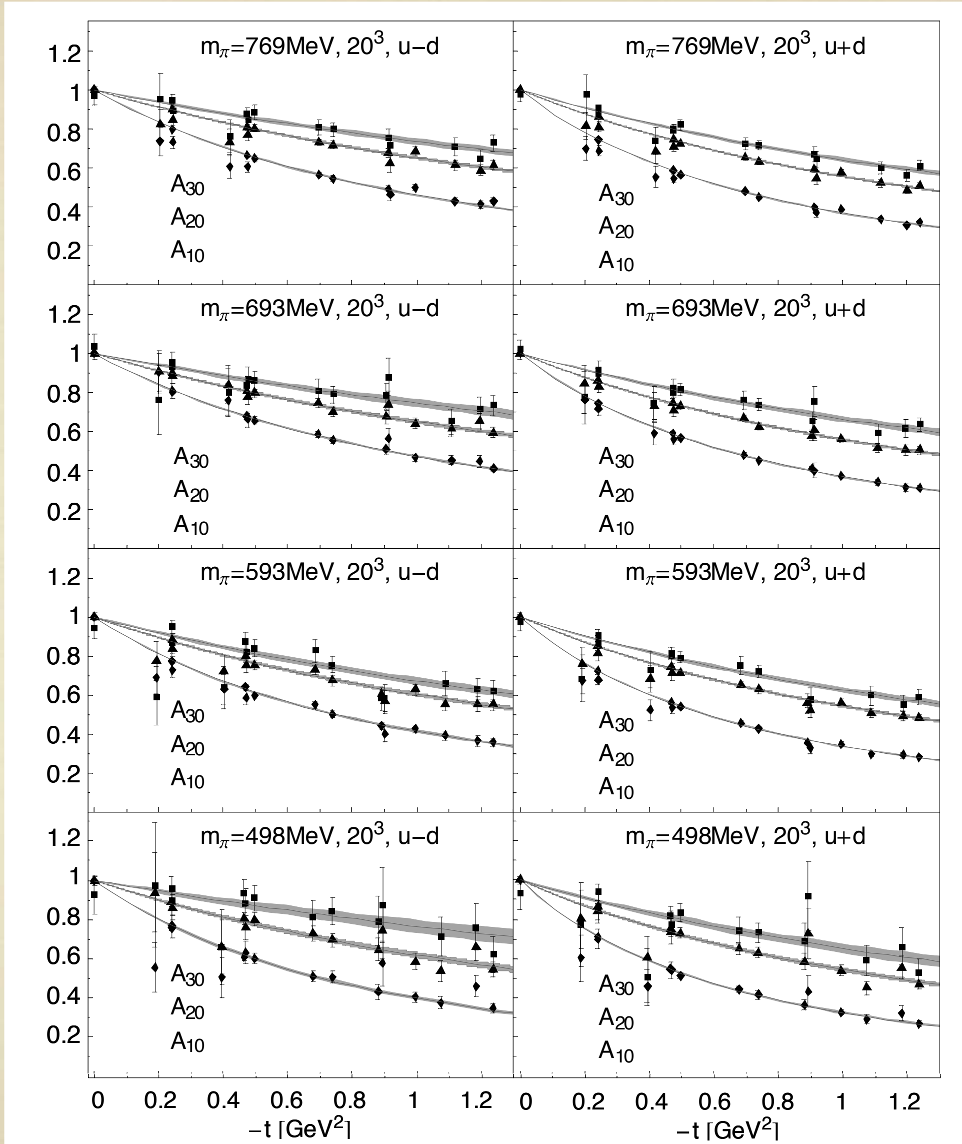



$$x_{\rm av}^n = \frac{\int d^2 r_\perp \int dx\, x \cdot x^{n-1} q(x,\vec{r}_\perp)}{\int d^2 r_\perp \int dx x^{n-1} q(x,\vec{r}_\perp)}$$

$q(x,\vec{r}_\perp)$ model (Burkardt hep-ph/0207047)

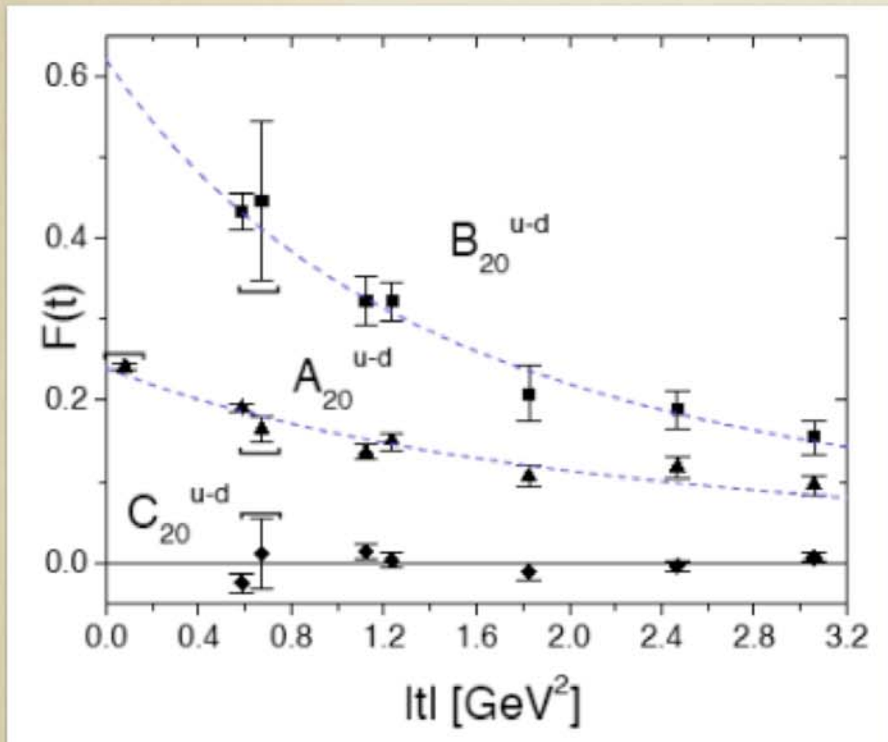
# Generalized form factors $A_{10}$, $A_{20}$, $A_{30}$

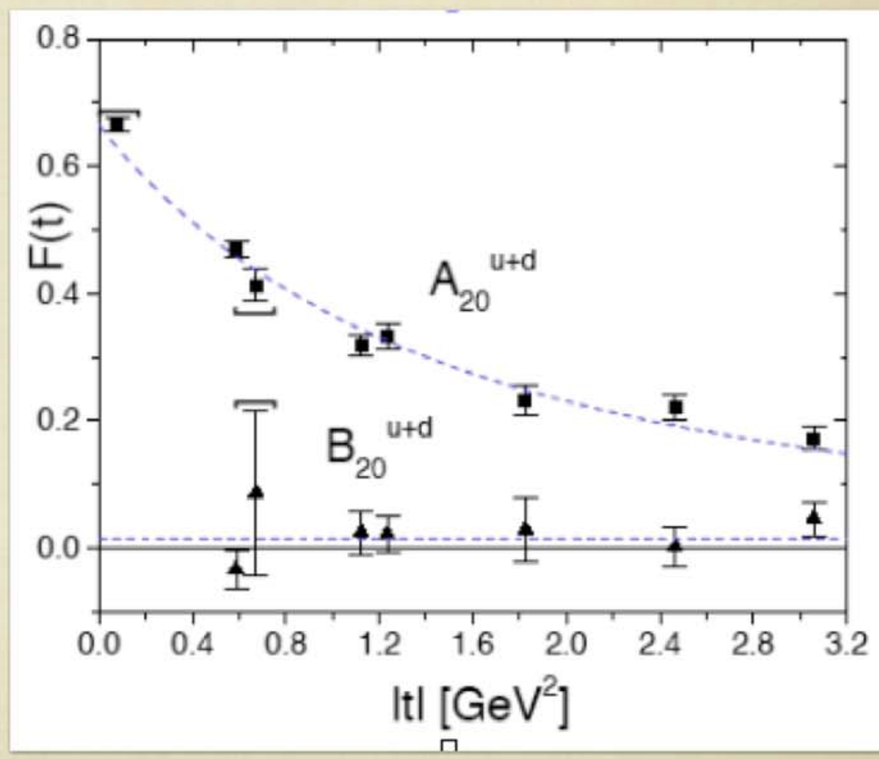


# First x moments: $A_{20}, B_{20}, C_{20}$

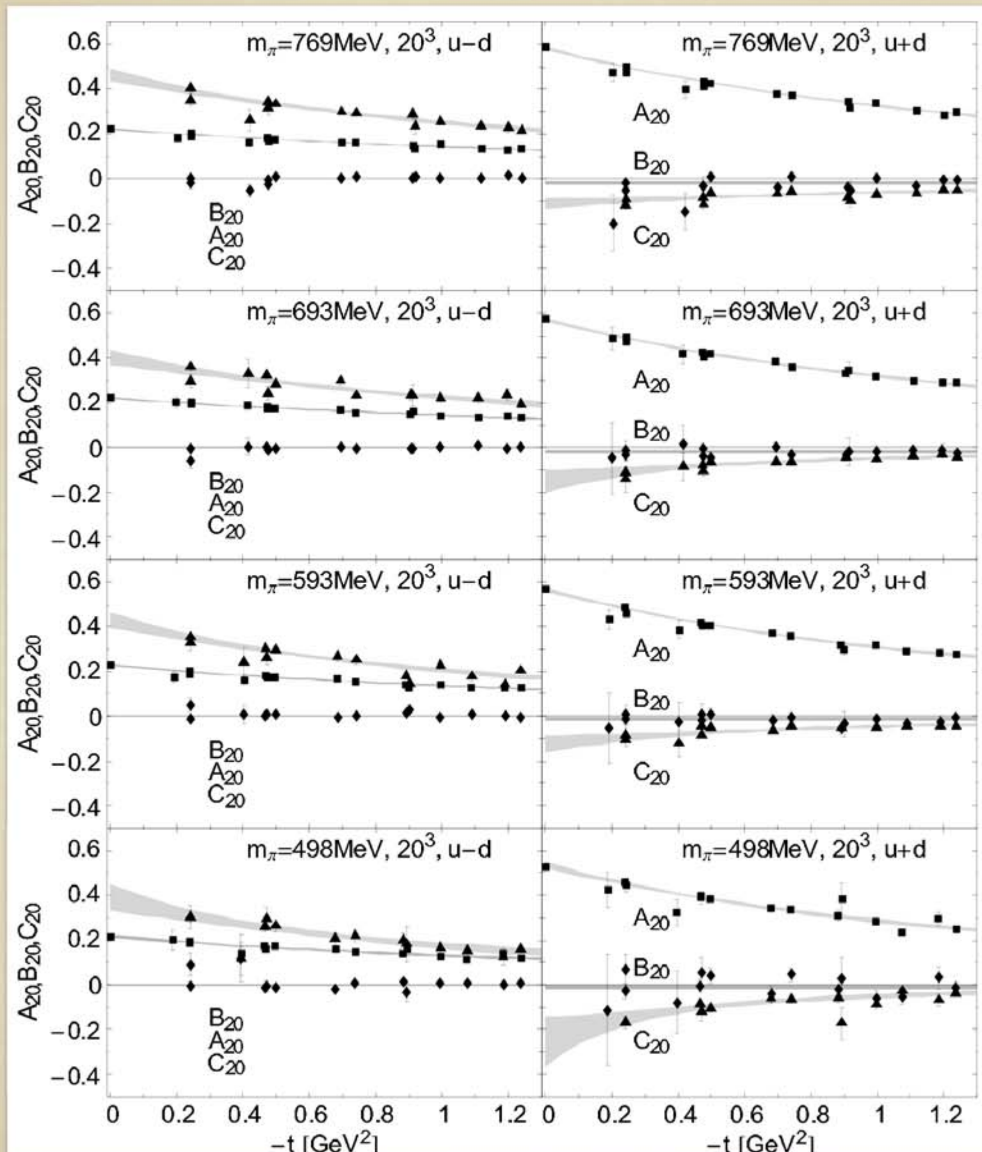
$m_\pi$ = 897 MeV

LHPC hep-lat/0304018



# First x moments: $A_{20}, B_{20}, C_{20}$




$B_{20}^{u-d} > A_{20}^{u-d}$

$A_{20}^{u+d} > B_{20}^{u+d} \sim 0$

$C_{20}^{u-d} \sim 0$

$C_{20}^{u+d} < 0$

Large $N_c$ behavior

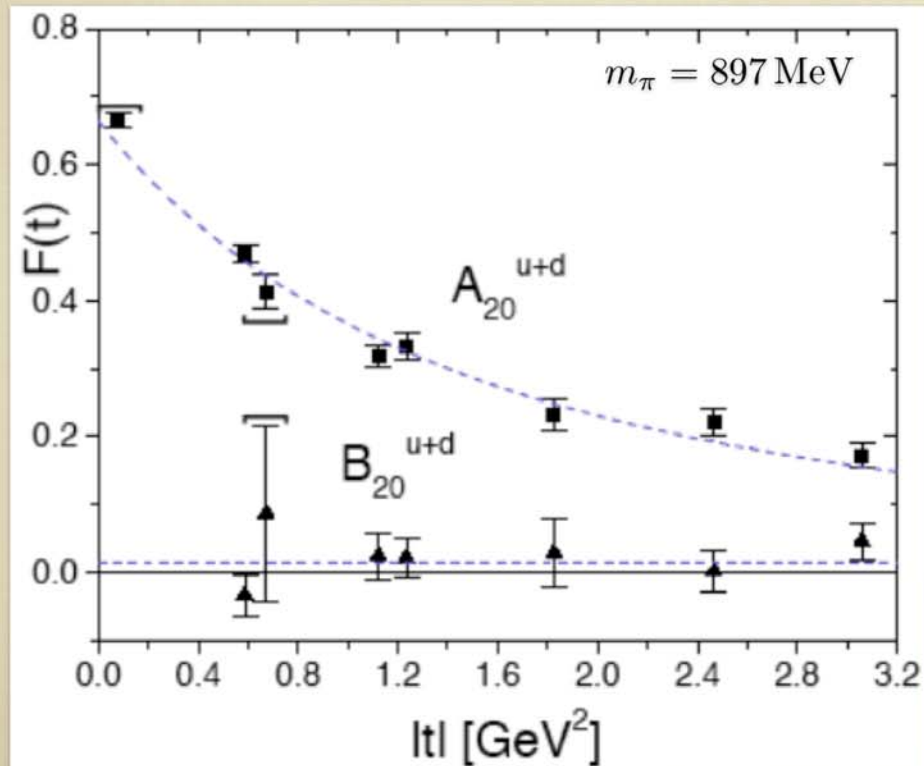
# Origin of nucleon spin

"Spin crisis" - only ~ 30% arises from quark spins

quark spin contribution $\frac{1}{2}\Delta\Sigma = \frac{1}{2}\langle 1\rangle_{\Delta u+\Delta d} \sim \frac{1}{2}0.682(18)$

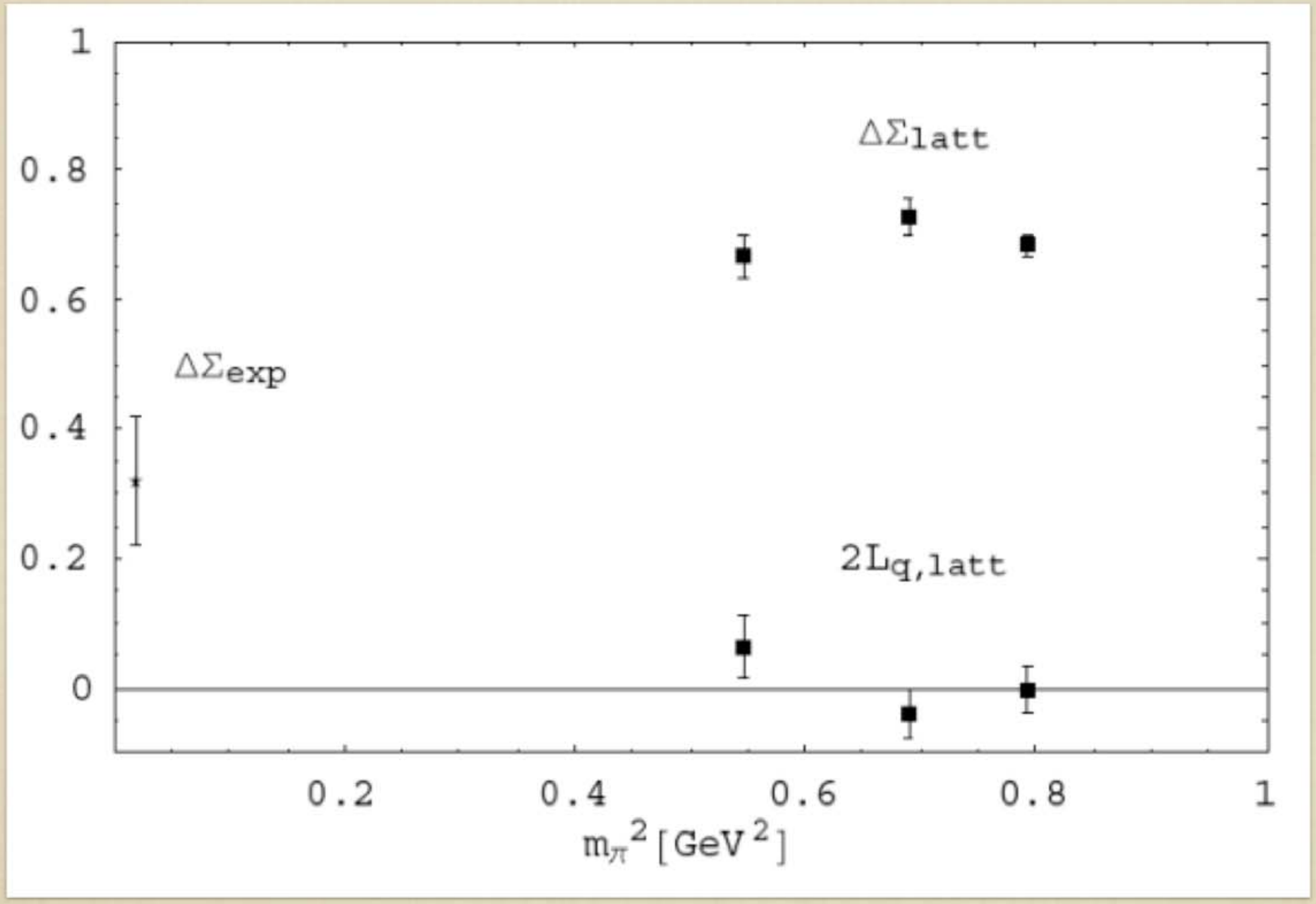
total quark contribution (spin plus orbital)

$$J_q = \frac{1}{2}[A_{20}^{u+d}(0) + B_{20}^{u+d}(0)] = \frac{1}{2}[\langle x\rangle_{u+d} + B_{20}^{u+d}(0)] \sim \frac{1}{2}0.675(7)$$

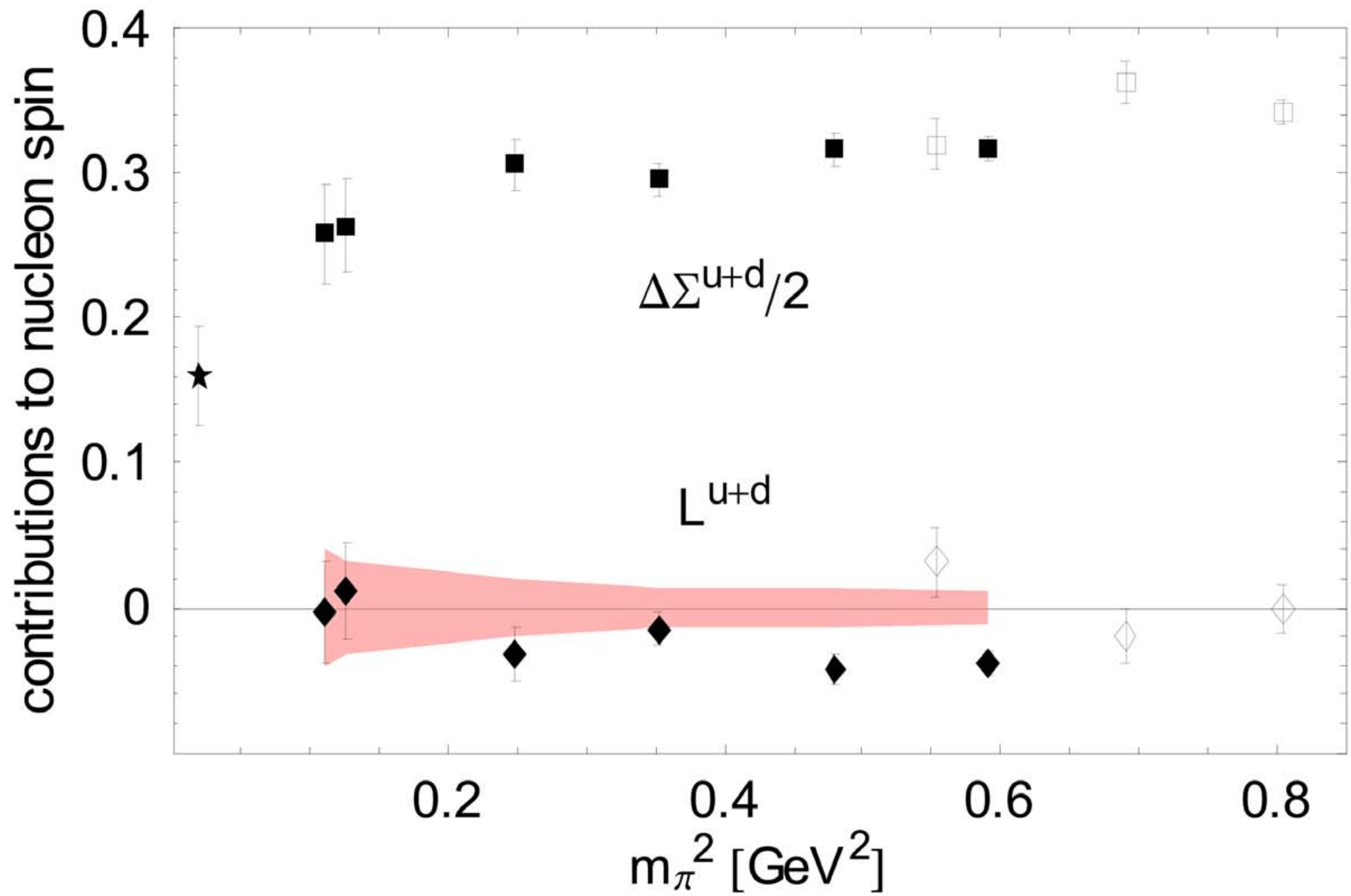



Spin Inventory
68% quark spin
0% quark orbital
32% gluons

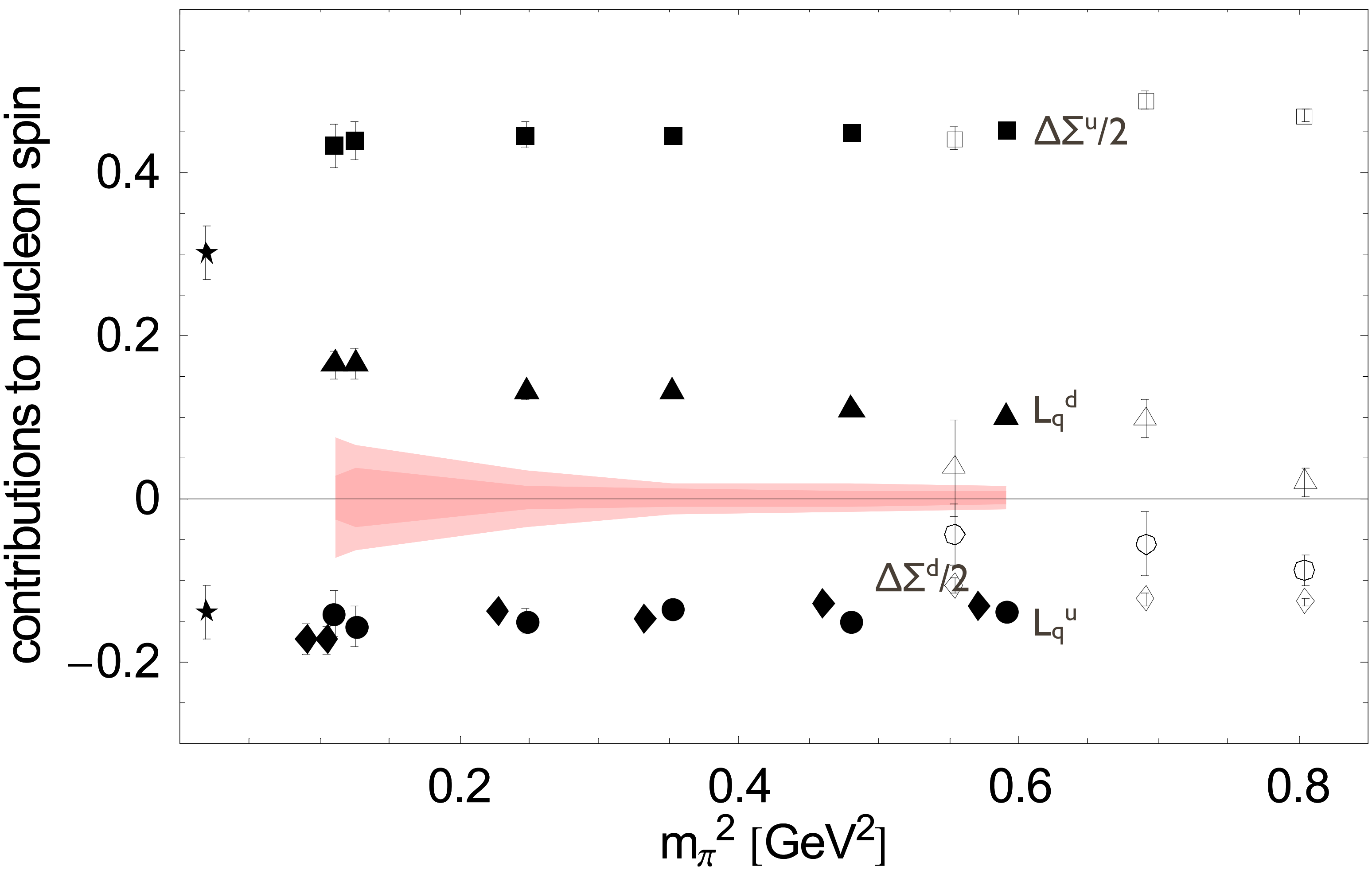
# Nucleon spin decomposition



# Nucleon spin decomposition



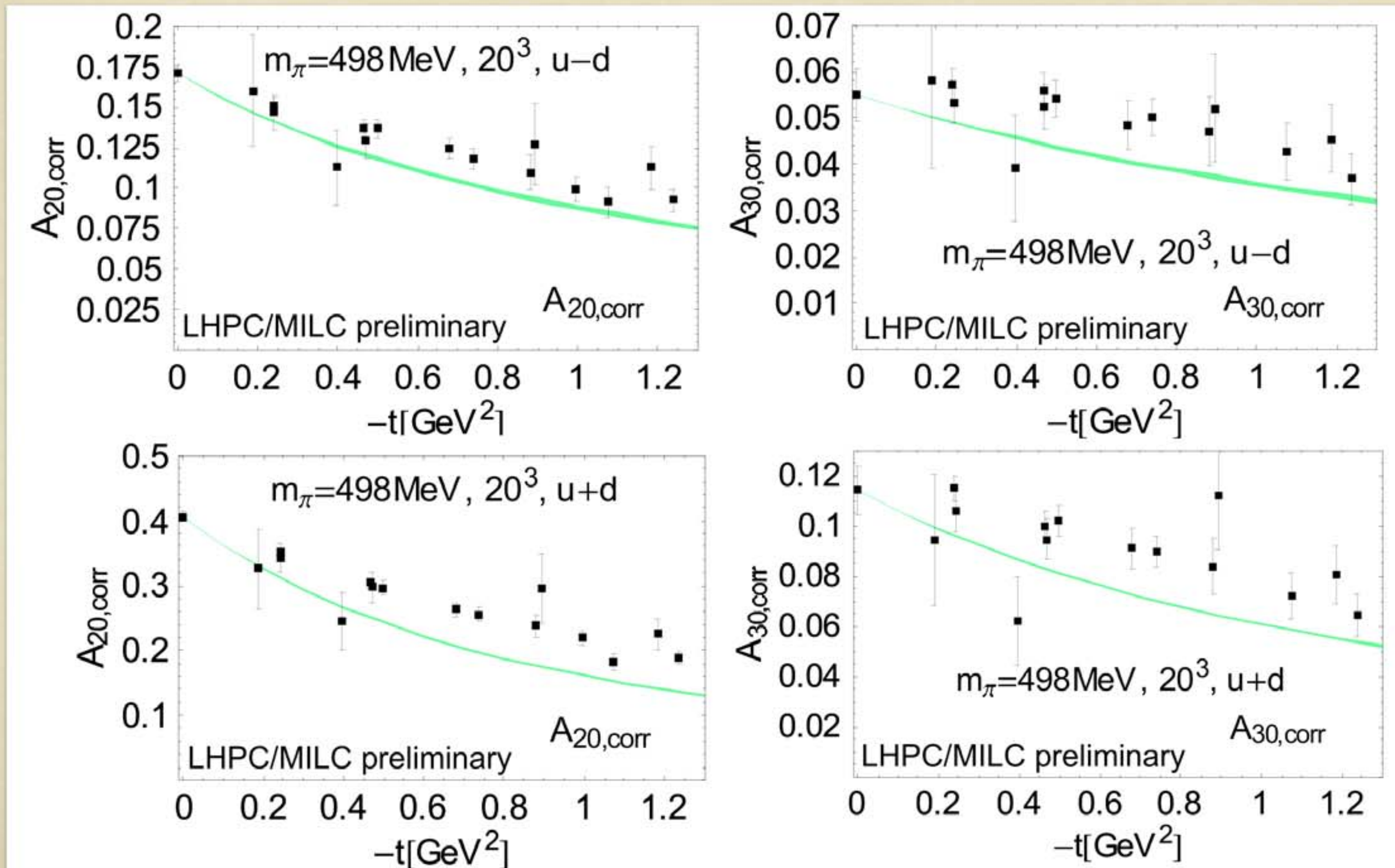
# Nucleon spin decomposition



# Comparison with Phenomenology

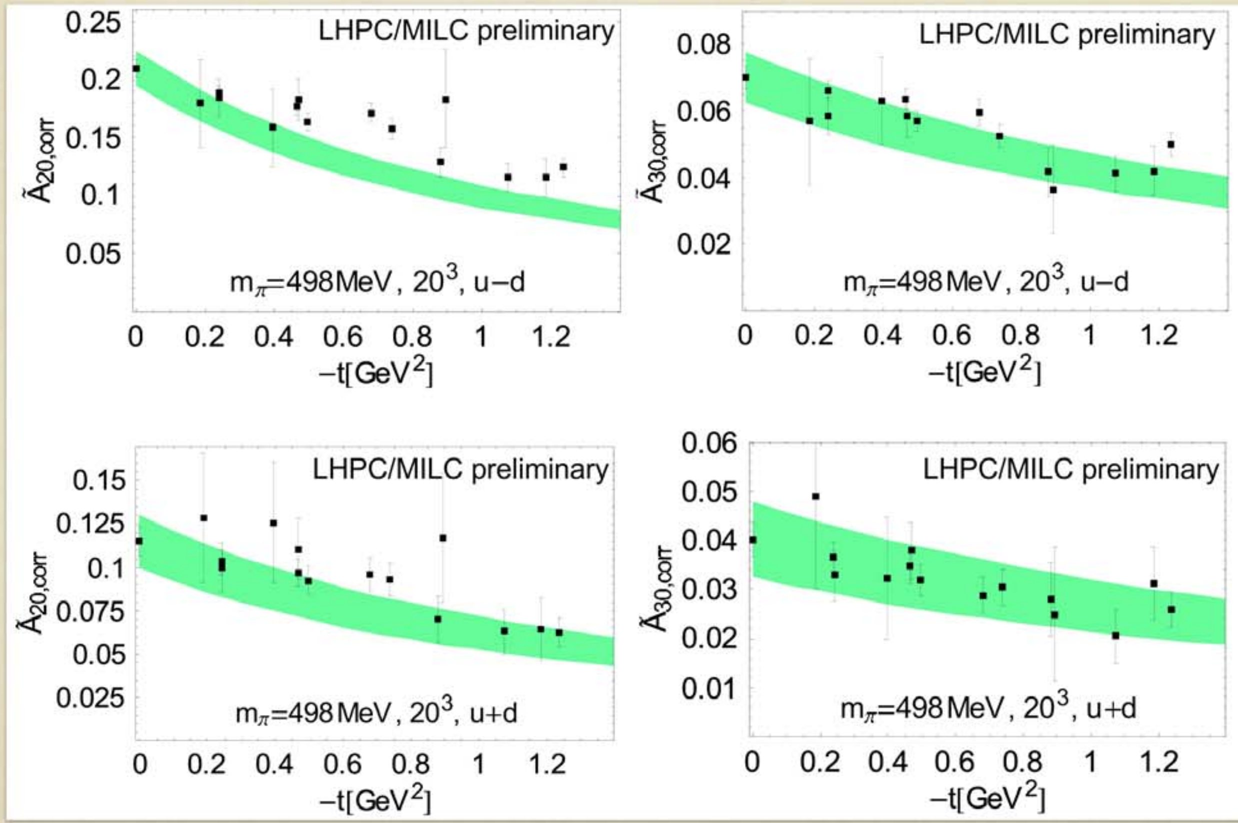
**GPD parameterization:** Diehl, Feldmann, Jakob, Kroll EPJC 2005
nucleon form factors, CTEQ parton distributions, Regge, Ansatz

$$A_{20} = \int dx\, x\, H(x,0,t) \qquad A_{30} = \int dx\, x^2\, H(x,0,t)$$

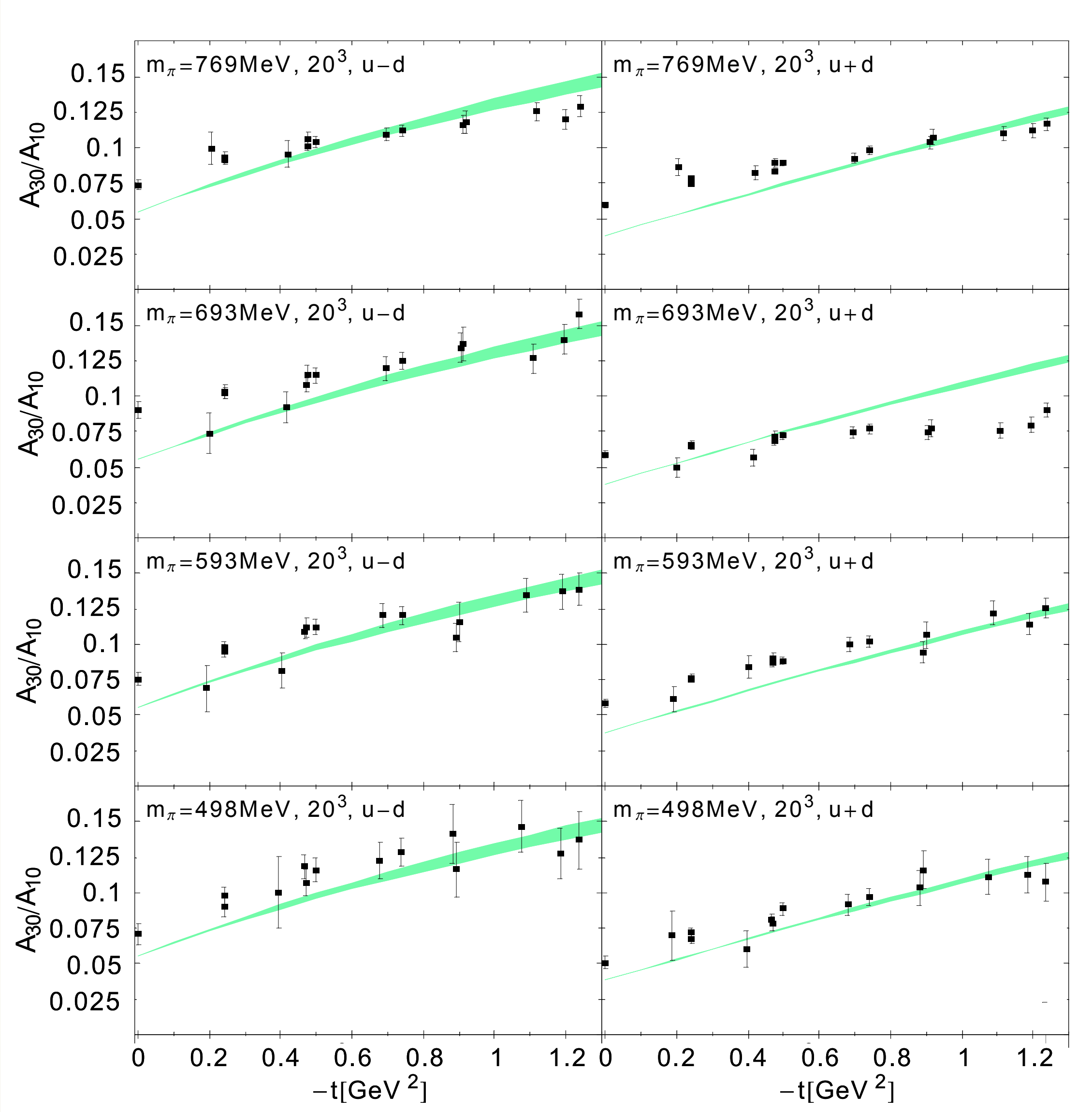


# Comparison with Phenomenology

$$\tilde{A}_{20} = \int dx\, x\, \tilde{H}(x,0,t) \qquad \tilde{A}_{30} = \int dx\, x^2\, \tilde{H}(x,0,t)$$



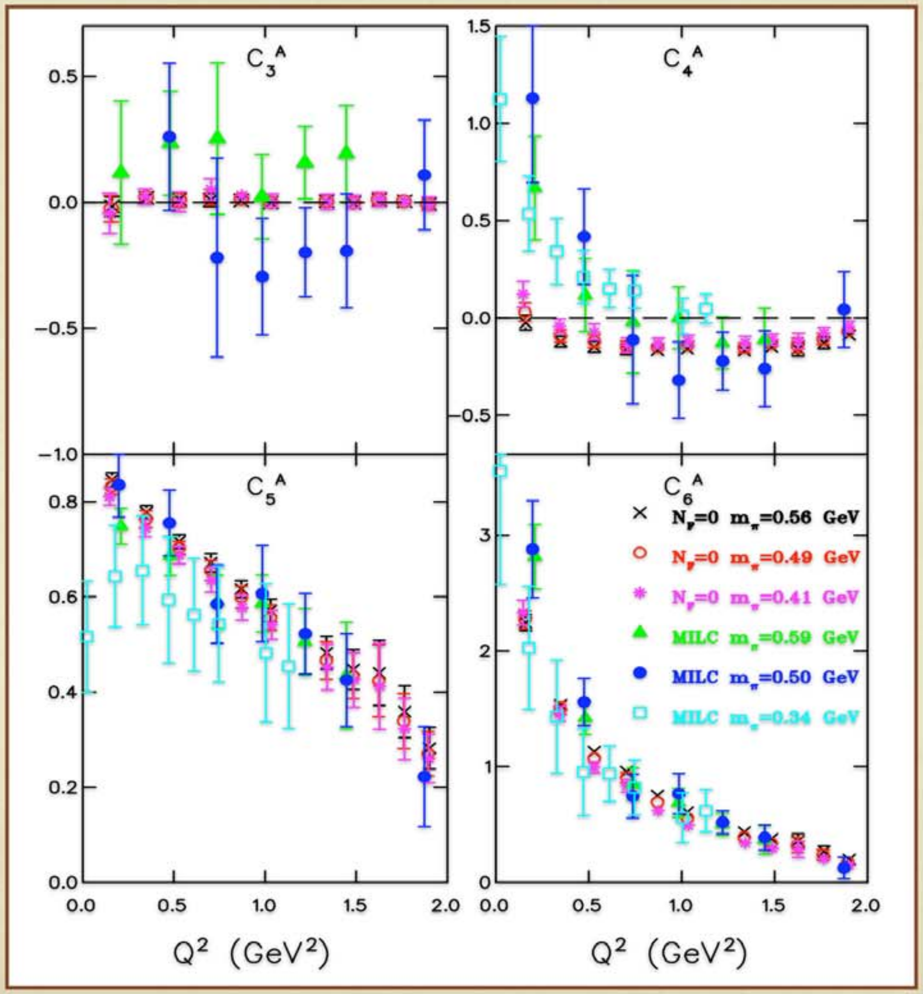
# Comparison with Phenomenology




Slope improves with decreasing masss

# Axial N-Delta transition form factors

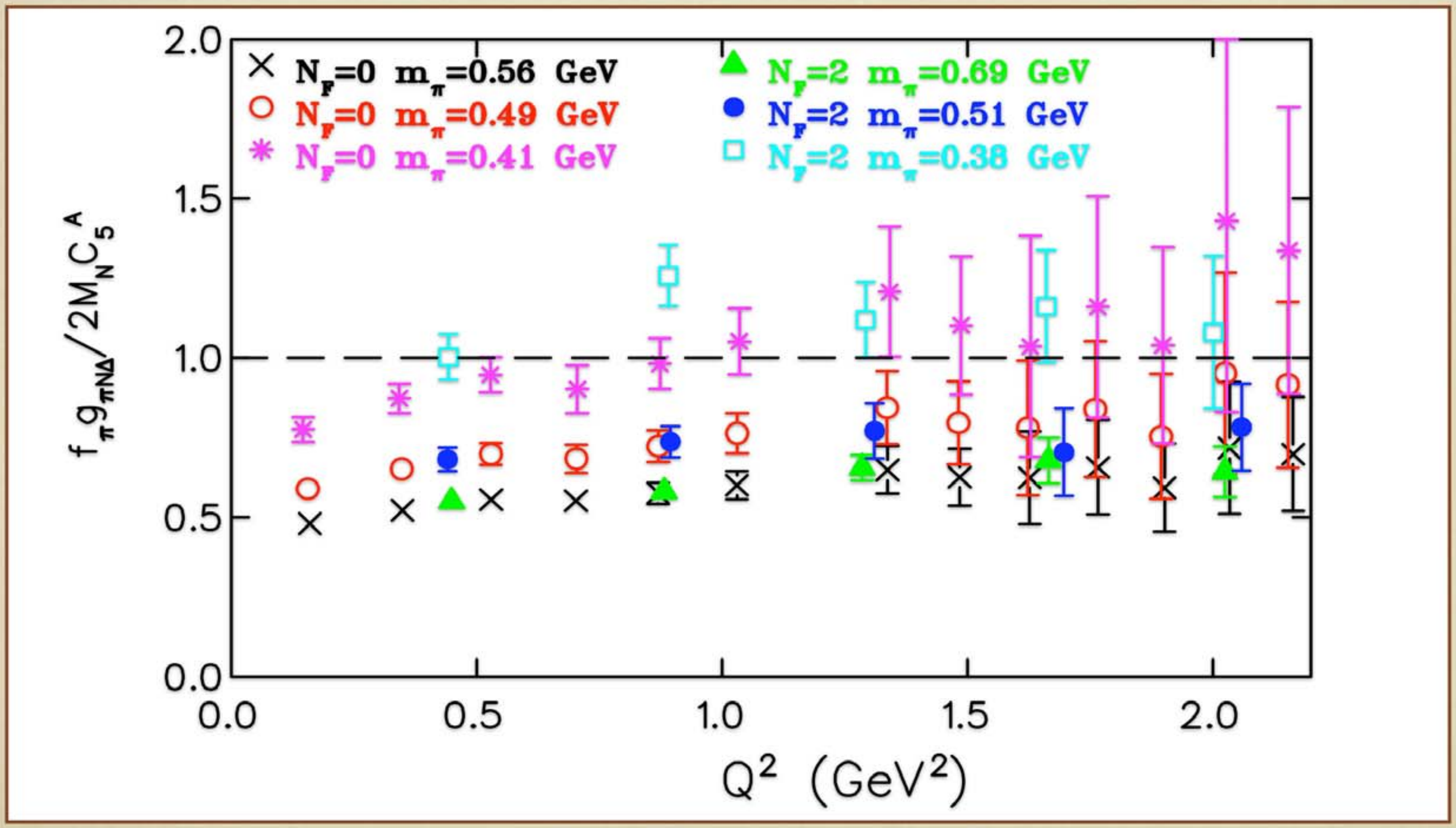
$$\langle\Delta(p',s')|A_\mu|N(p,s)\rangle \propto \bar{u}^\lambda(p',s')\left[\left(\frac{C_3^A(q^2)}{M}\gamma^\nu+\frac{C_4^A(q^2)}{M^2}p'^\nu\right)(g_{\lambda\mu}g_{\rho\nu}-g_{\lambda\rho}g_{\mu\nu})q^\rho+C_5^A(q^2)g_{\lambda\mu}+\frac{C_6^A(q^2)}{M^2}q_\lambda q_\mu\right]u(p,s)$$




Alexandrou, Leontiou, Tsapalis, JN

# Axial N-Delta transition form factors

Off-diagonal Goldberger-Treiman relation $$C_5^A(q^2) = \frac{F_\pi g_{\pi N\Delta}(q^2)}{2M}$$



# Summary

- Entering era of quantitative solution in chiral regime
  - Form factors: $F_1$ , $F_2$ , $G_A$ , $G_P$
  - Generalized form factors A B C
  - Transverse structure
  - Origin of nucleon spin
  - Transition form factors
- Opportunity for theory and experiment to work in consort
  - Validate by agreement with key experiments
  - GPD's: Expt. convolution, Theory moments, combine
  - Resolve experimental discrepancies
    - $F_2$ : 2-$\gamma$ contributions to Rosenbluth, pol. transfer
    - $G_A$ : $\nu$ vs $\pi$-electroproduction

# Future Challenges

- Lower pion masses and finer lattices
- Partially quenched hybrid chiral perturbation theory
- Form factors at high momentum transfer
- Disconnected diagrams
- Nonperturbative renormalization
- Full QCD with chiral fermions
- Gluon observables
- Transition form factors for unstable states