Lattice Hadron Physics
Newport News, July 31 - August 3 2006

Lattice QCD and Nuclear Physics

Kostas Orginos
The College of William and Mary
JLab

NPLQCD Collaborators

- S. Beane (UNH)
- P. Bedaque (LBNL)
- E. Pallante (Gronigen)
- A. Parreno (Barcelona)
- M. Savage (UW)

LHPC COLLABORATORS

- R. Edwards (Jlab)
- G. Fleming (Yale)
- P. Hagler (Vrije Universiteit)
- J. Negele (MIT)
- A. Pochinsky (MIT)
- D. Renner (UofA)
- D. Richards (Jlab)
- W. Schroers (DESY)

Nuclear physics

- Connect Nuclear physics to QCD
- Two scale problem
  - QCD scale 1GeV
  - Nuclear binding energy ~ MeV
- Does it look hopeless?
- Not really!

Nuclear physics: what can we do?

- EFT description of nuclear forces
- Need low energy constants
- Use experiment
- Why not use lattice instead?

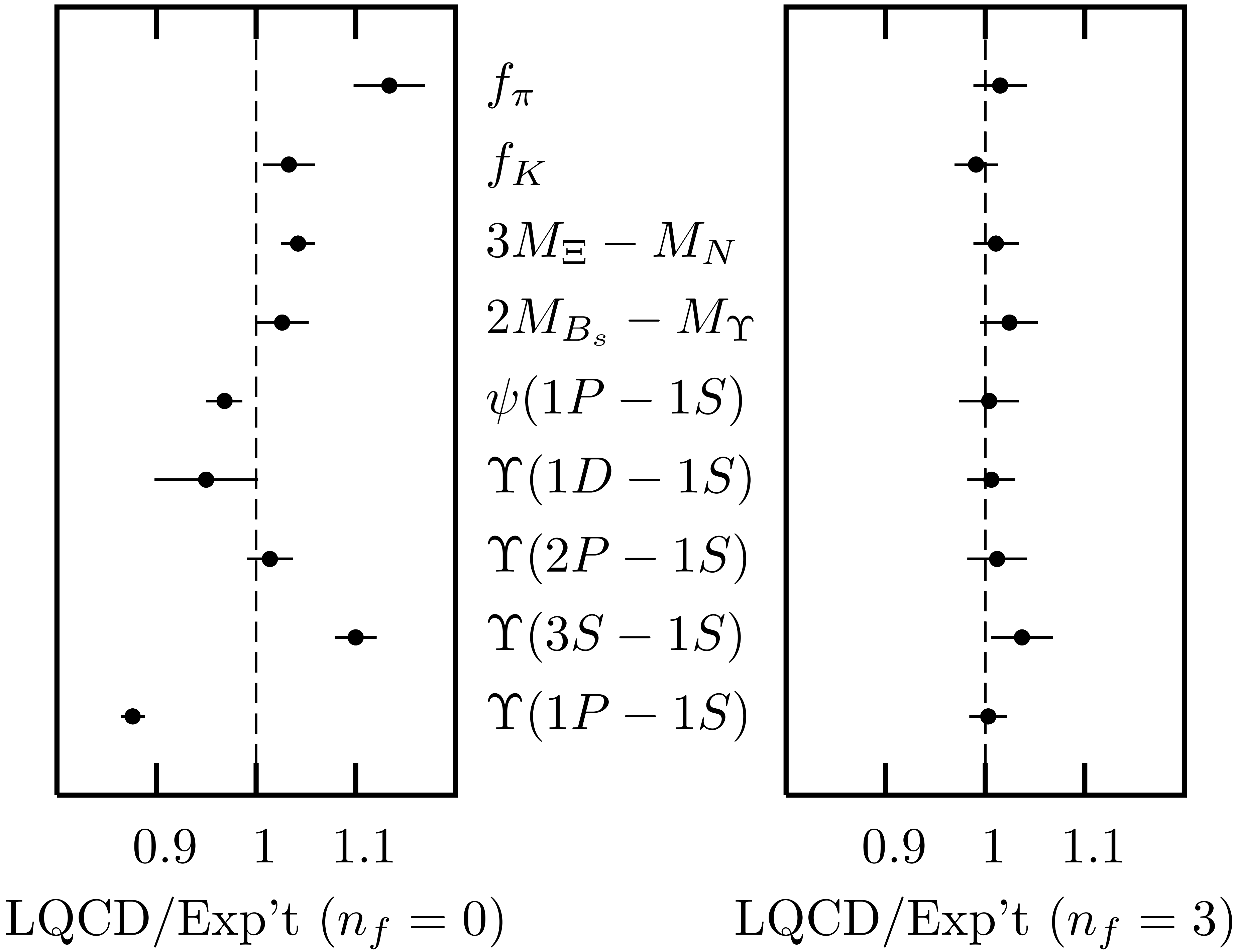
Nuclear physics: what can we do?

- Nucleon mass
  - Isospin breaking
- Decay constants and couplings
  - f_π, g_A, $g_{N\Delta}$, $g_{\Sigma\Sigma}$, $g_{\Xi\Xi}$, $g_{\Sigma\Lambda}$, ...
  - Gasser-Leutwyler coefficients
- Scattering lengths [NPLQCD]
  - Lattice Nuclear physics [Lee et al., Borasoy et al.]
- Lattice offers flexibility!
- Ask questions not accessible to experiment

Realistic Calculations

- 2+1 Dynamical flavors
  - 2 light (up down) 1 heavy (strange)
  - charm bottom top (treated in HQET)
- Light quark masses $m_\pi < 400$MeV
  - Chiral extrapolations
  - Finite volume corrections
  - Numerical algorithm slows down (algorithm scaling $\sim \frac{1}{m_q^{2.5}}$)
- Continuum extrapolations
  - compute at several lattice spacings (algorithm scaling $\sim \frac{1}{a^7}$)

Quenched vs Dynamical



MILC, HPQCD, UKQCD

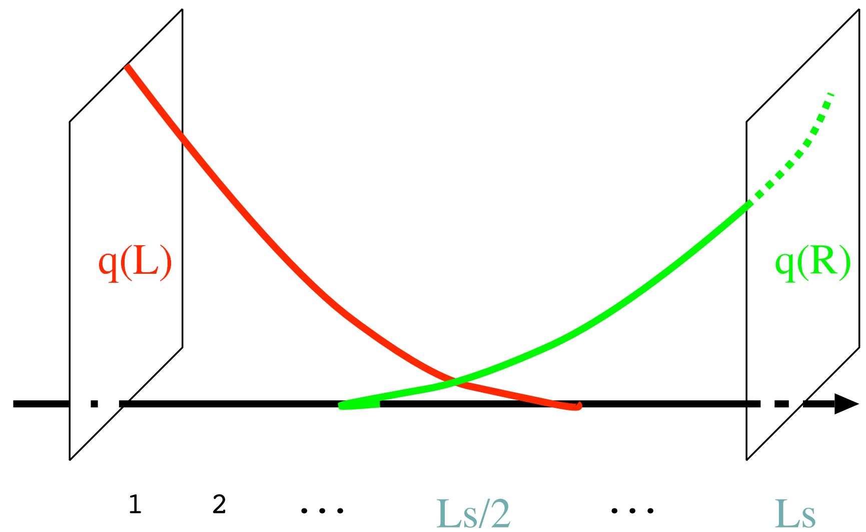
The Hybrid Action Program

- Domain wall fermions for valence (with hyp smeared links)
  - Chiral symmetry ($O(a^2)$ errors better scaling)
  - Ward Identities (renormalization, power divergent mixing)
- Kogut-Susskind 2+1 Dynamical flavors
  - Improved KS action (Asqtad: $O(a^4, g^2 a^2)$) [KO, Sugar, Toussaint '99]
  - MILC has generated lattices
- Light quark masses: Lightest pion $m_\pi \sim 250\text{MeV}$
- Volumes: 2.6 to 3.2 fm
- Future: Continuum extrapolation
  - MILC lattice spacings: a=0.125fm, 0.09fm
  - a=0.06fm in 1 - 2 years
- Problem: "Rooted" fermions? (Bernard, Shamir, Sharpe, Golderman, Durr, Creutz, Hassenfratz....)

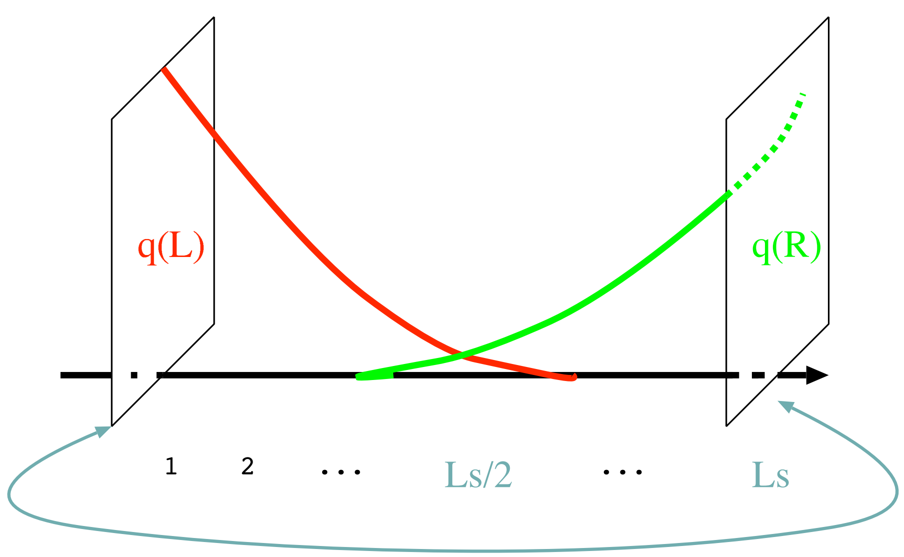
Ugly ⟶ Results are pretty ??

Domain Wall Fermions for QCD

Formulate the 5D Wilson fermions with mass $M \neq 0$ in $s \,\epsilon\, [1, L_s]$



For $-2 < M < 0$, light chiral modes are bound on the walls.
Only one Dirac fermion without doublers remains.



Fermion mass is introduced by explicitly coupling m_f of the walls. [Shamir, Furman & Shamir]

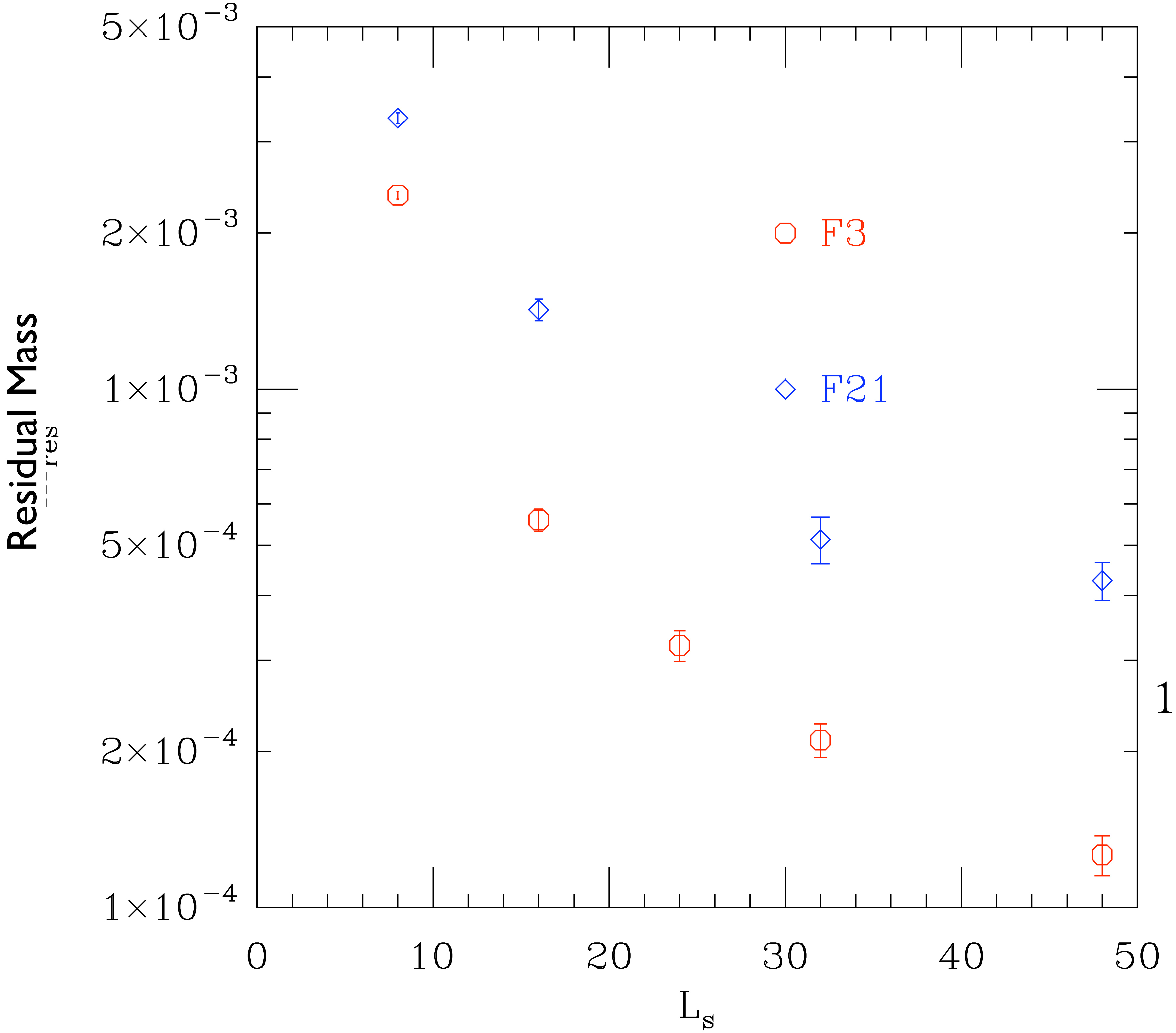
Chiral symmetry breaking

$$\Delta_\mu \langle \mathcal{A}^a_\mu(x)\mathcal{O}\rangle = 2\, m_f\, \langle J^a_5(x)\mathcal{O}\rangle + 2\langle J^a_{5q}(x)\mathcal{O}\rangle + i\langle \delta^a_x \mathcal{O}\rangle$$

- The size of $\langle J^a_{5q}(x)\mathcal{O}\rangle$ measures chiral symmetry breaking
- Let's use for the operator $\mathcal{O} = J^a_5(0)$
- Assume at long distances $J^a_{5q} \sim J^a_5$
- The proportionality constant is the residual mass

$$M_{\mathrm{res}} = \left.\frac{\sum_{x,y}\langle J^a_{5q}(y,t)J^a_5(x,0)\rangle}{\sum_{x,y}\langle J^a_5(y,t)J^a_5(x,0)\rangle}\right|_{t\geq t_{min}}$$

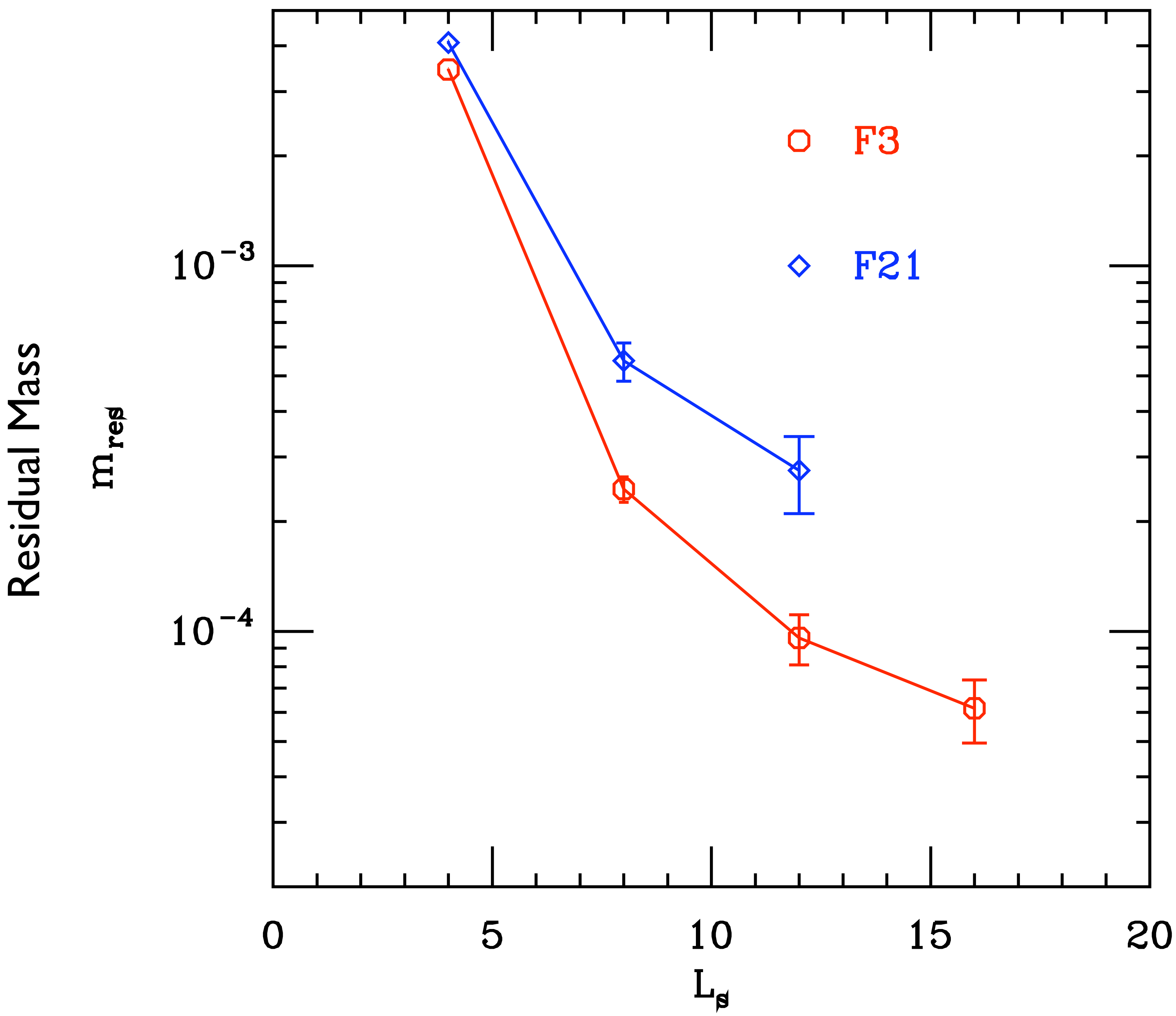
Residual Mass vs Ls



a=0.125fm

At Ls = 16:
1MeV < m_{res} <2.5MeV

Residual Mass vs Ls



a=0.09fm

At Ls = 12:
0.2MeV< m_{res} <0.7MeV

The 4D effective operator

With a little algebra we get

$$\mathcal{P}^{-1}\frac{1}{D_{dwf}(1)}D_{dwf}(m)\mathcal{P} = \begin{bmatrix} D_{ov}(m) & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ -(1-m)T^{-L_s/2+1}\frac{1}{T^{-L_s/2}+T^{L_s/2}} & 1 & 0 & 0 & \cdots & \cdots & 0 \\ -(1-m)T^{-L_s/2+2}\frac{1}{T^{-L_s/2}+T^{L_s/2}} & 0 & 1 & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \cdots & \vdots \\ -(1-m)\frac{1}{T^{-L_s/2}+T^{L_s/2}} & 0 & \cdots & \cdots & 1 & 0 & \cdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ -(1-m)T^{L_s/2-1}\frac{1}{T^{-L_s/2}+T^{L_s/2}} & 0 & \cdots & \cdots & \cdots & 0 & 1 \end{bmatrix}$$

$$\mathcal{P} = \begin{bmatrix} P_- & P_+ & \cdots & 0 \\ 0 & P_- & P_+\cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & P_+ \\ P_+ & 0 & \cdots & P_- \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -T^{-L_s+1}M_+ & 1 & 0 & 0 & \cdots \\ -T^{-L_s+2}M_+ & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -T^{-1}M_+ & 0 & \cdots & 0 & 1 \end{bmatrix}$$

$$M_- = P_- - m\,P_+ \qquad M_+ = P_+ - m\,P_-$$

$$T^{-1} = \frac{1+H_T}{1-H_T} \qquad H_T = \gamma_5 D$$

$$D_{ov}(m) = \frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \mathcal{E}_{L_s}[\gamma_5 D(M_5)]$$

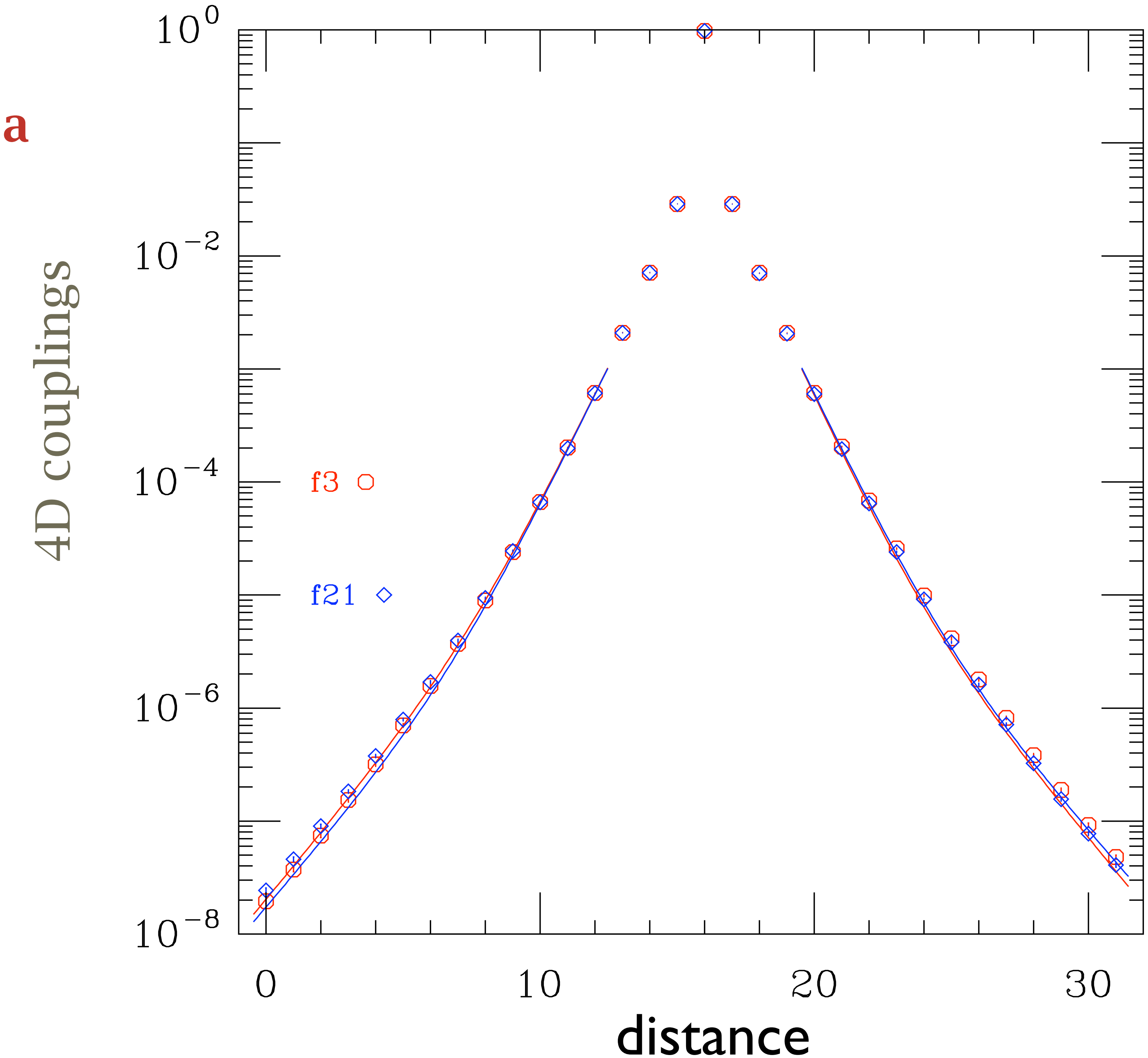
$$\varepsilon_{L_s} = \frac{T^{-L_s}-1}{T^{-L_s}+1} = \frac{(1+H_T)^{L_s}-(1-H_T)^{L_s}}{(1+H_T)^{L_s}+(1-H_T)^{L_s}}$$

$$D = (b_5+c_5)\frac{D_w}{2+(b_5-c_5)D_w} = \alpha\frac{D_w}{2+a_5D_w}$$

- Overlap: $\alpha=2$, $a_5=0$ (Borici)
- DWF: $\alpha=1$, $a_5=1$ (Shamir)

Locality of the 4D action

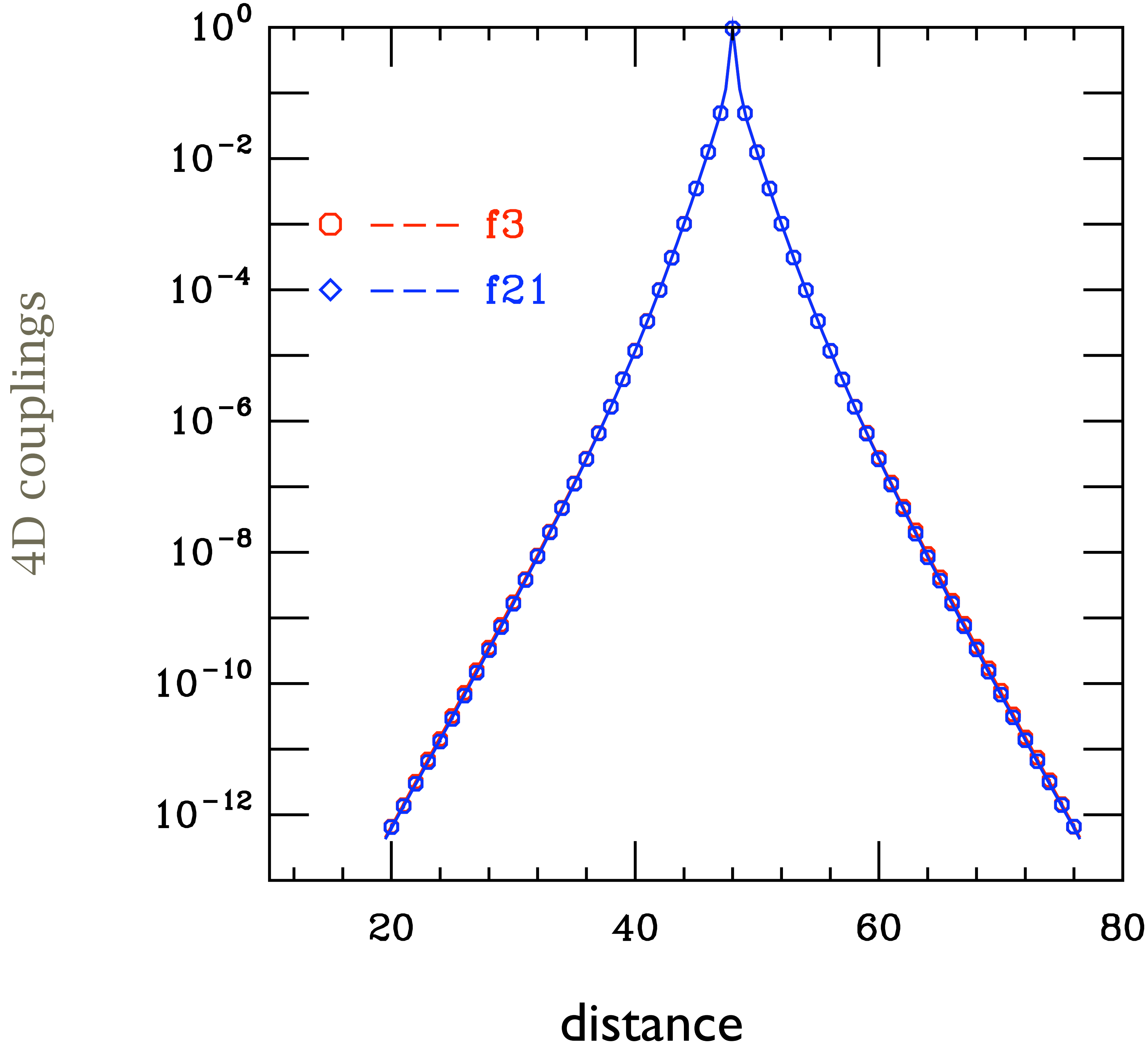
Localization: **~ 1.5a**



a=0.125fm

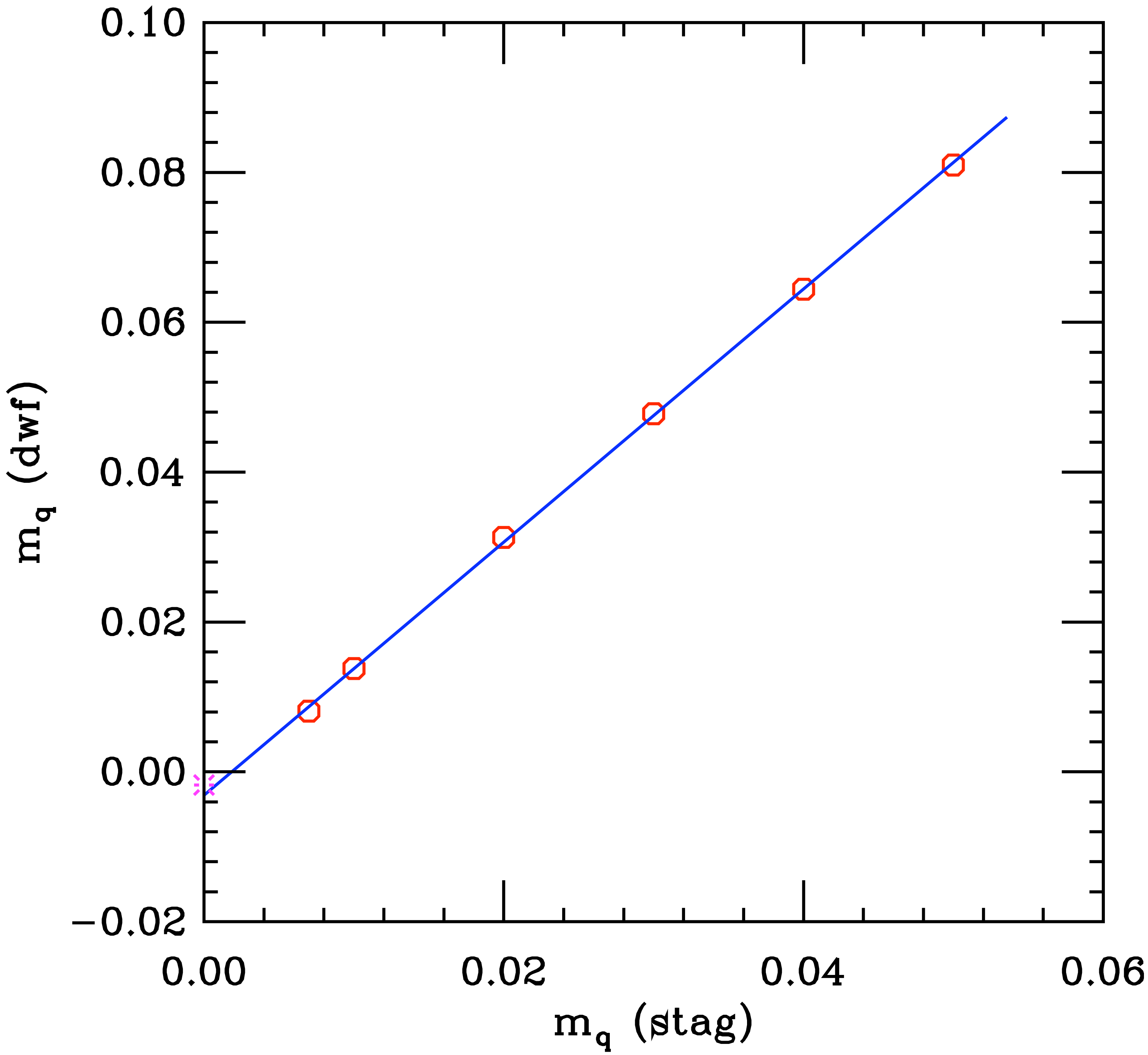
Locality of the 4D action

Localization: **~ 1.3a**



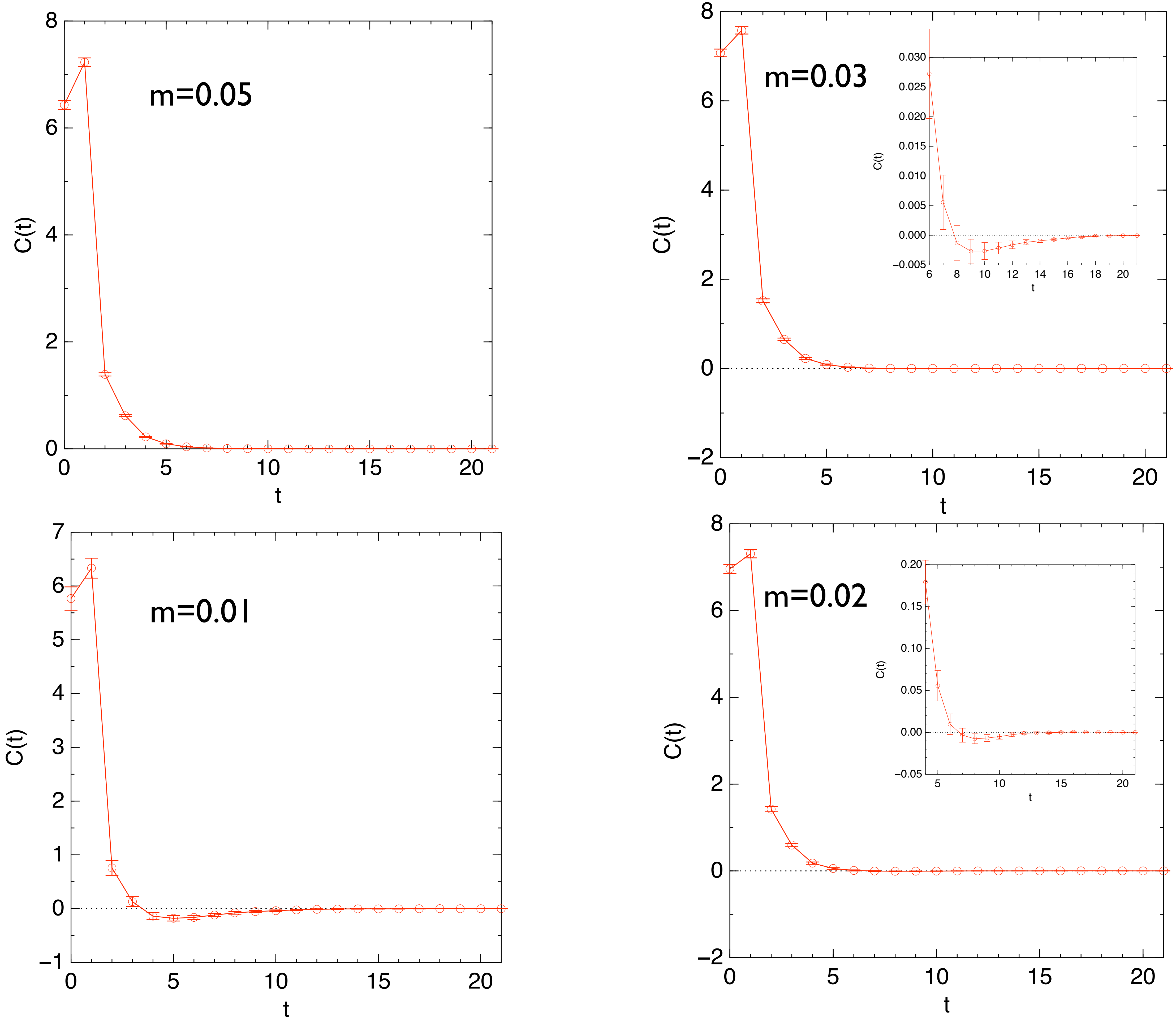
a=0.09fm

The DWF quark masses



a=0.125fm

IsoVector scalar correlator: Unitarity violation



SχPT calculation: Prelovsek '05

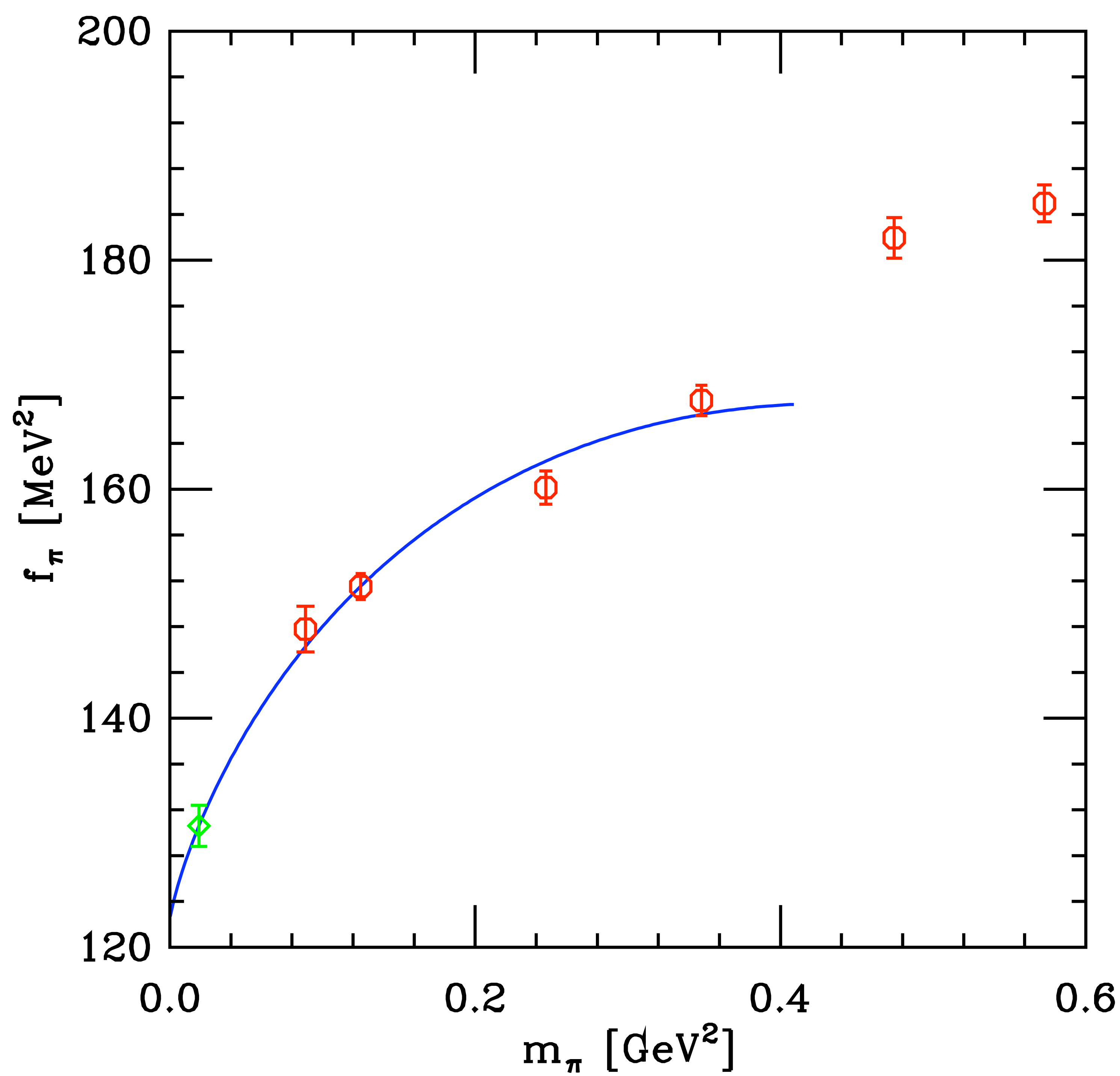
Pion Decay Constant

- Fit the lower 4 points
- Scale used a = 0.125 fm
- One loop χPT extrapolation:

  130.6(1.8)MeV

$$f_\pi = f\left[1 - \frac{m_\pi^2}{8\pi^2 f^2} log\left(\frac{m_\pi^2}{\mu^2}\right) + c(\mu^2) m_\pi^2\right]$$

- Systematic error:
  - chiral extr. 3 MeV
  - 2% from scale setting
- χ^2/d.o.f. ~ 2
- Need mixed χPT: Baer et.al.'05



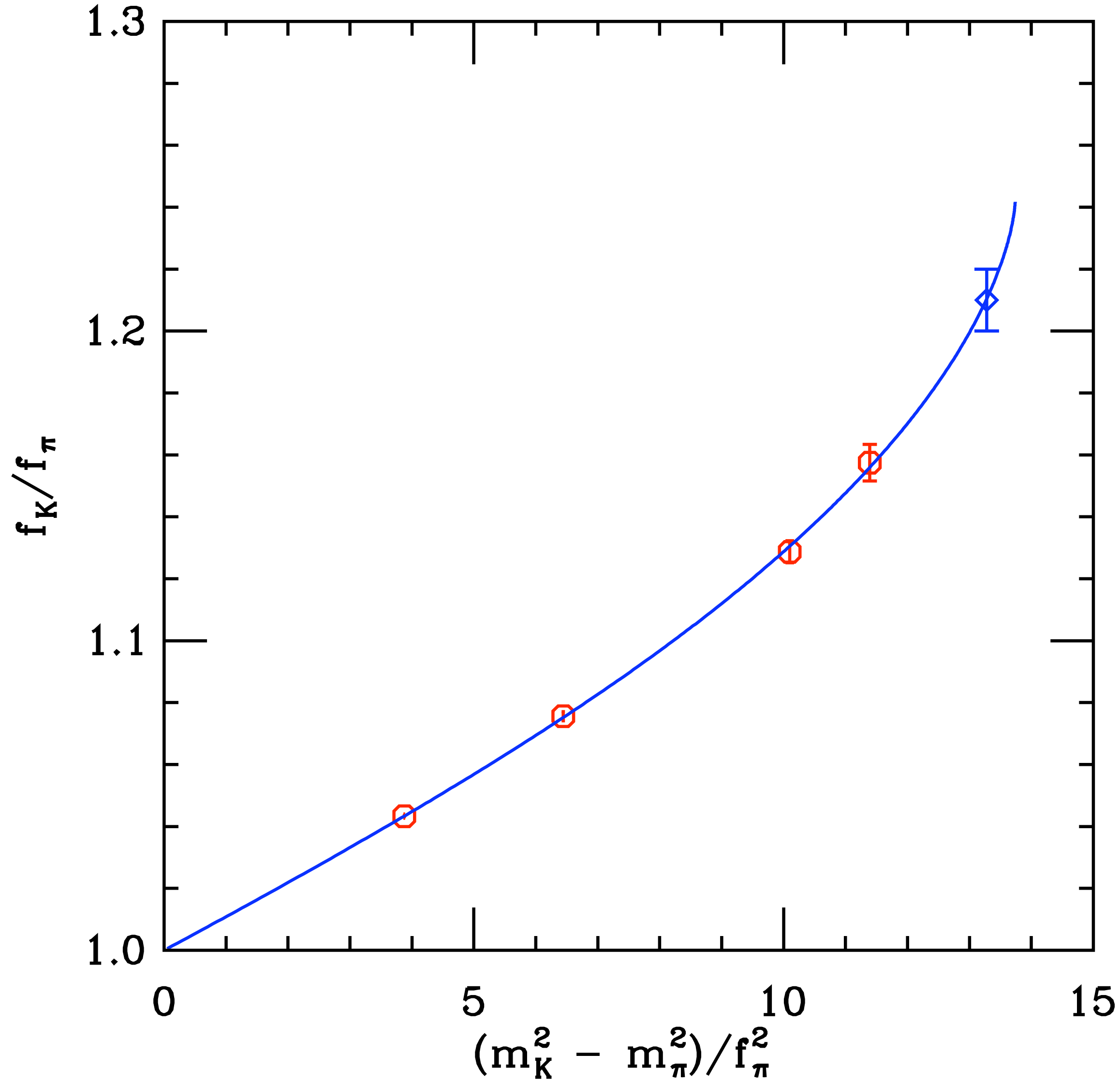
Fκ/Fπ

Beane, Bedaque, KO, Savage hep-lat/0606023

Gasser-Leutwyler:

$$\frac{f_K}{f_\pi} = 1 + \frac{5}{4} l_\pi(\mu) - \frac{1}{2} l_K(\mu) - \frac{3}{4} l_\eta(\mu) + \frac{8}{f^2}\left(m_K^2 - m_\pi^2\right) L_5(\mu)$$

$$l_i(\mu) = \frac{1}{16\pi^2}\frac{m_i^2}{f^2}\log\left(\frac{m_i^2}{\mu^2}\right)$$



$$\frac{f_K}{f_\pi} = 1.210(10)$$

$$\left.\frac{f_K}{f_\pi}\right|_{\text{exp.}} = 1.223(12)$$

Result comparable with MILC

$$\left.\frac{f_K}{f_\pi}\right|_{\text{MILC}} = 1.210(4)(13)$$

Need much higher precision to see effects of Mixed χPT Baer et.al.'05

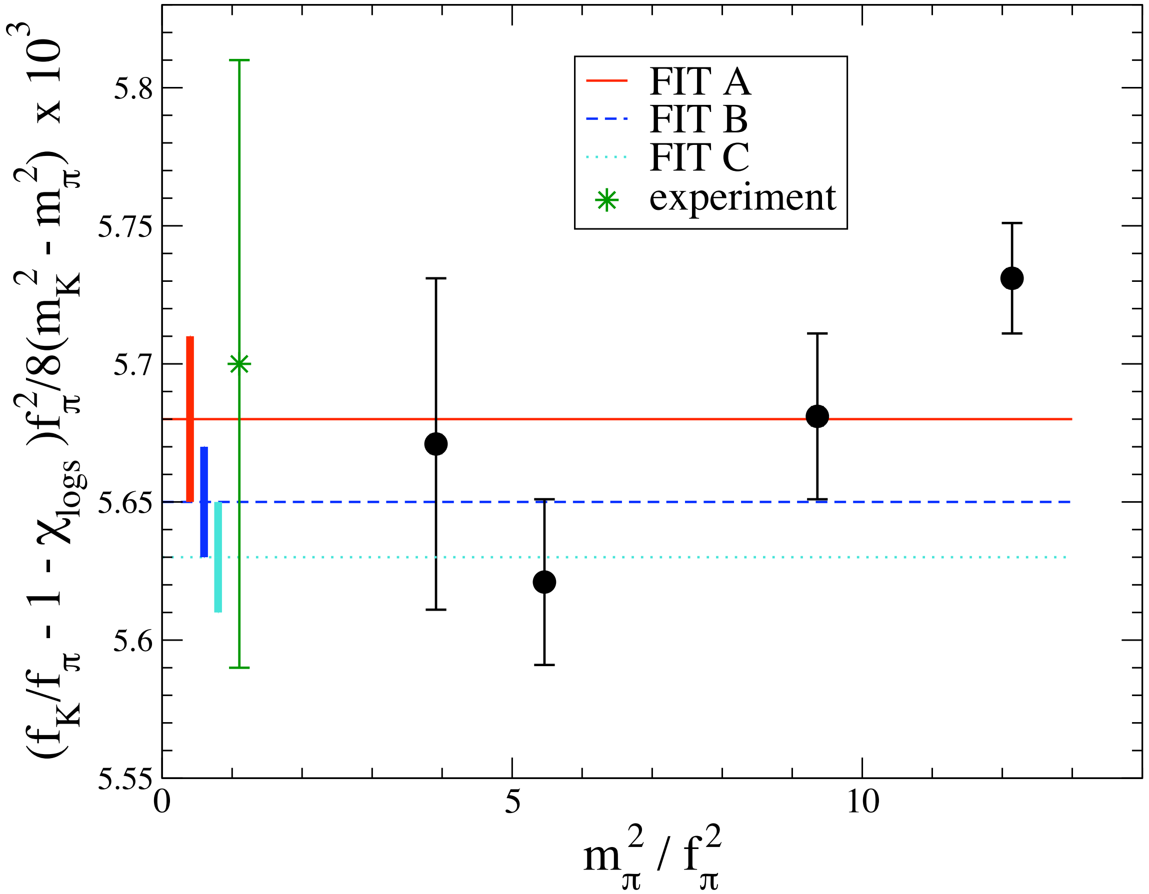
F_K/F_π

Beane, Bedaque, KO, Savage hep-lat/0606023

Gasser-Leutwyler:

$$\frac{f_K}{f_\pi} = 1 + \frac{5}{4} l_\pi(\mu) - \frac{1}{2} l_K(\mu) - \frac{3}{4} l_\eta(\mu) + \frac{8}{f^2}\left(m_K^2 - m_\pi^2\right) L_5(\mu)$$

$$l_i(\mu) = \frac{1}{16\pi^2} \frac{m_i^2}{f^2} \log\left(\frac{m_i^2}{\mu^2}\right)$$



Result comparable with MILC

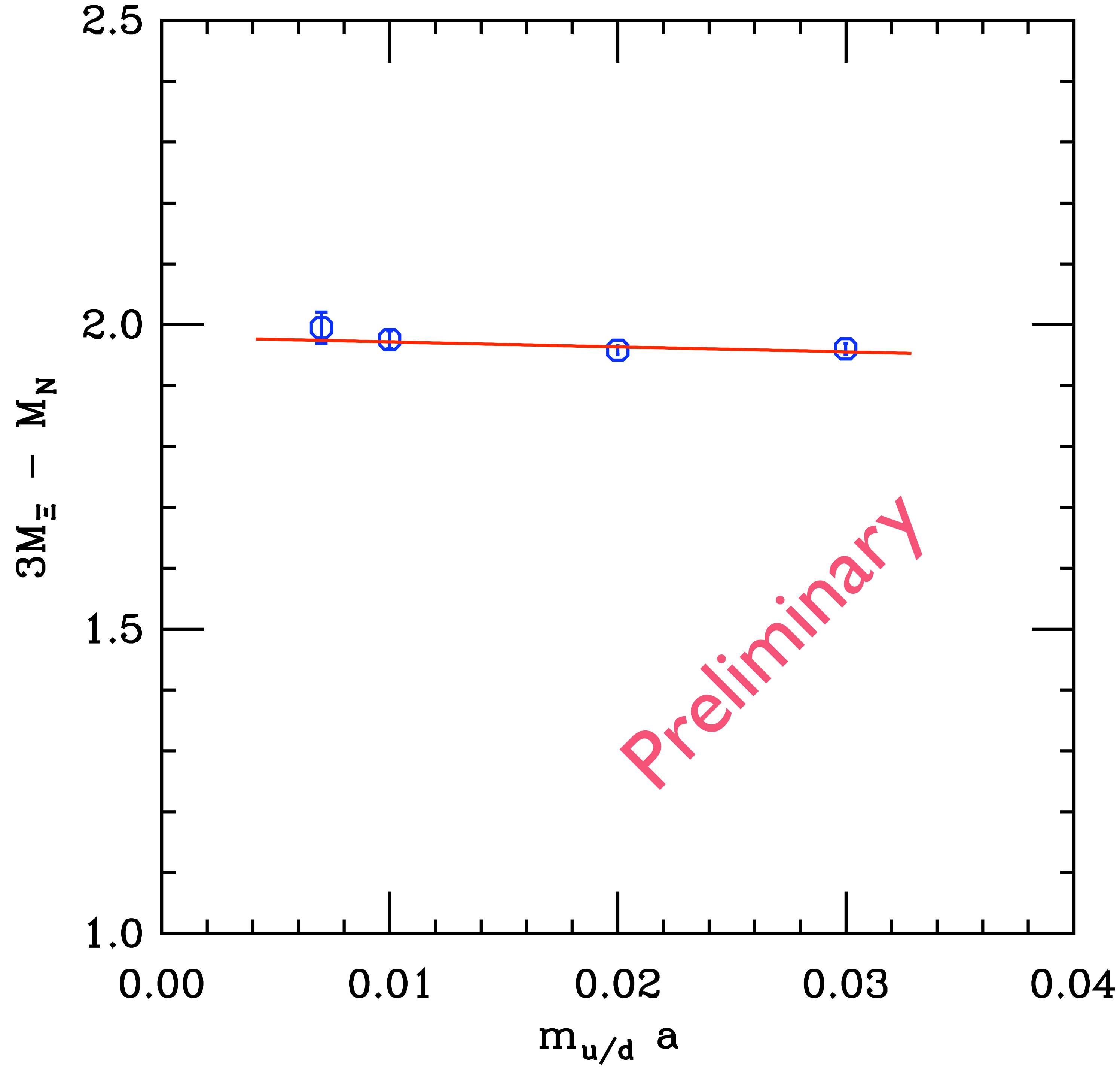
FIT	$L_5 \times 10^3$	f_K/f_π (extrapolated)	χ^2/dof
A	5.68(3)	1.221(3)	3.5
B	5.65(2)	1.218(2)	1.4
C	5.63(2)	1.215(2)	0.7

$$\left.\frac{f_K}{f_\pi}\right|_{\mathrm{MILC}} = 1.210(4)(13)$$

$$\left.\frac{f_K}{f_\pi}\right|_{\mathrm{exp.}} = 1.223(12)$$

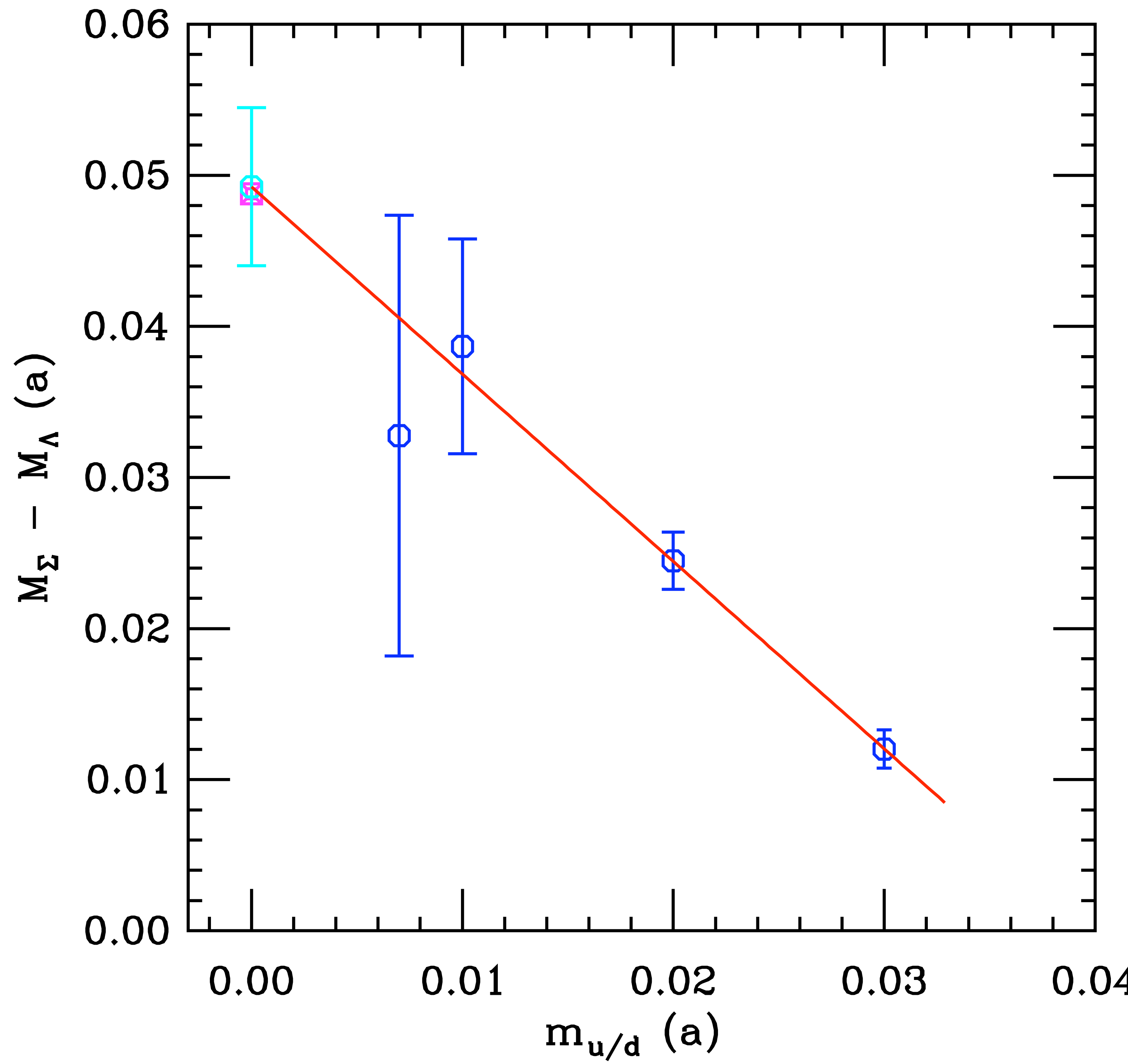
Cascade - Nucleon Mass Splitting

- Mild quark mass dependence
- Small systematic error due to chiral extrapolation
- Other systematic errors cancel
- Scale used a = 1588 MeV
- Latt./Exp. = 1.006(8)



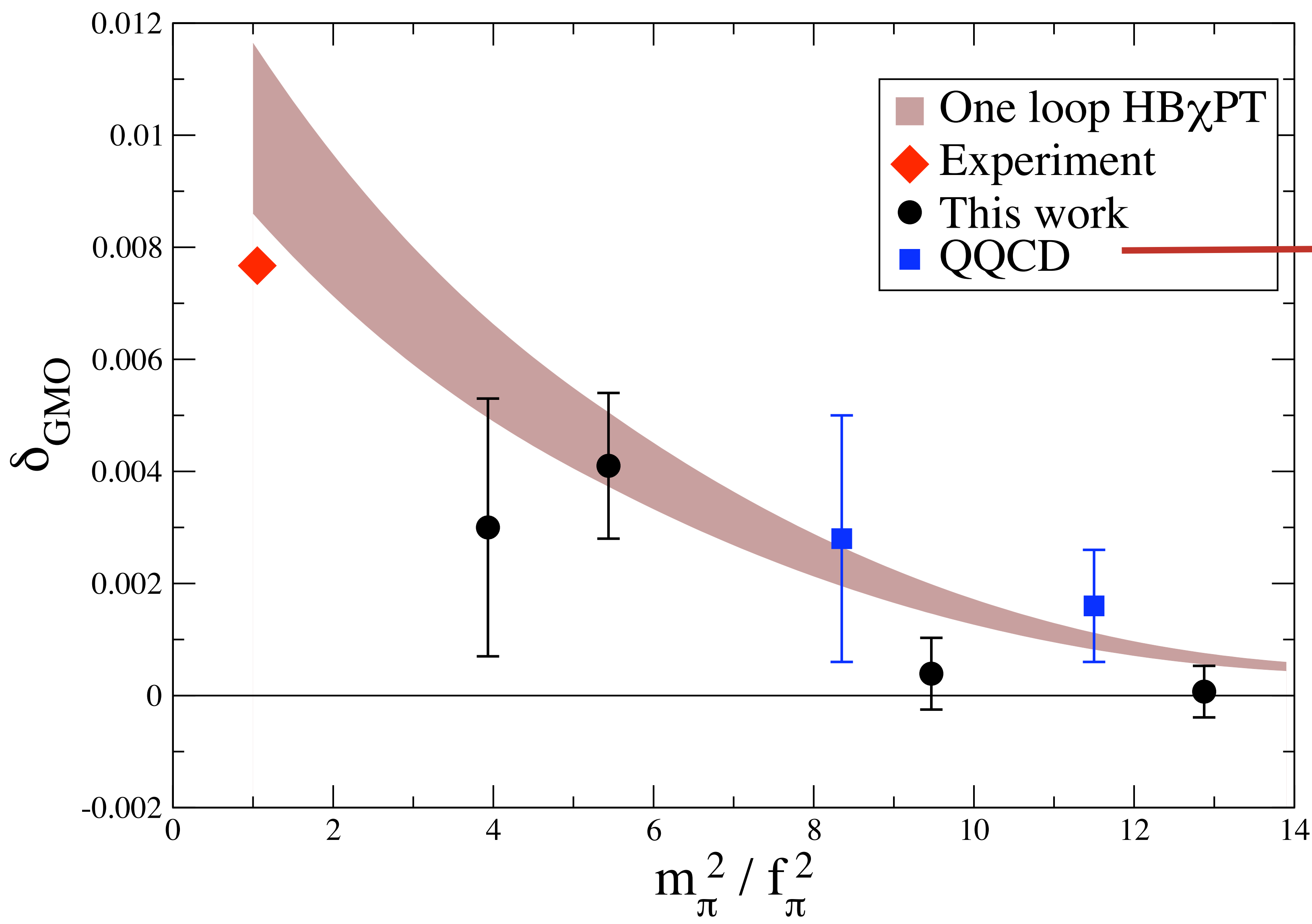
LAMBDA-SIGMA SPLITTING

- Data point towards experimental result
- Linear fit is good
- Need χPT
- Exper.: 77.47MeV
- Lat.: 78(8)MeV
- Scale used a = 1588 MeV



GMO Relation

Beane, KO, Savage hep-lat/0604013



Bhattacharya *et al*. hep-lat/9512021

$$\delta_{\rm GMO} = \frac{M_\Lambda + \frac{1}{3}M_\Sigma - \frac{2}{3}M_N - \frac{2}{3}M_\Xi}{\frac{1}{8}M_\Lambda + \frac{3}{8}M_\Sigma + \frac{1}{4}M_N + \frac{1}{4}M_\Xi}$$

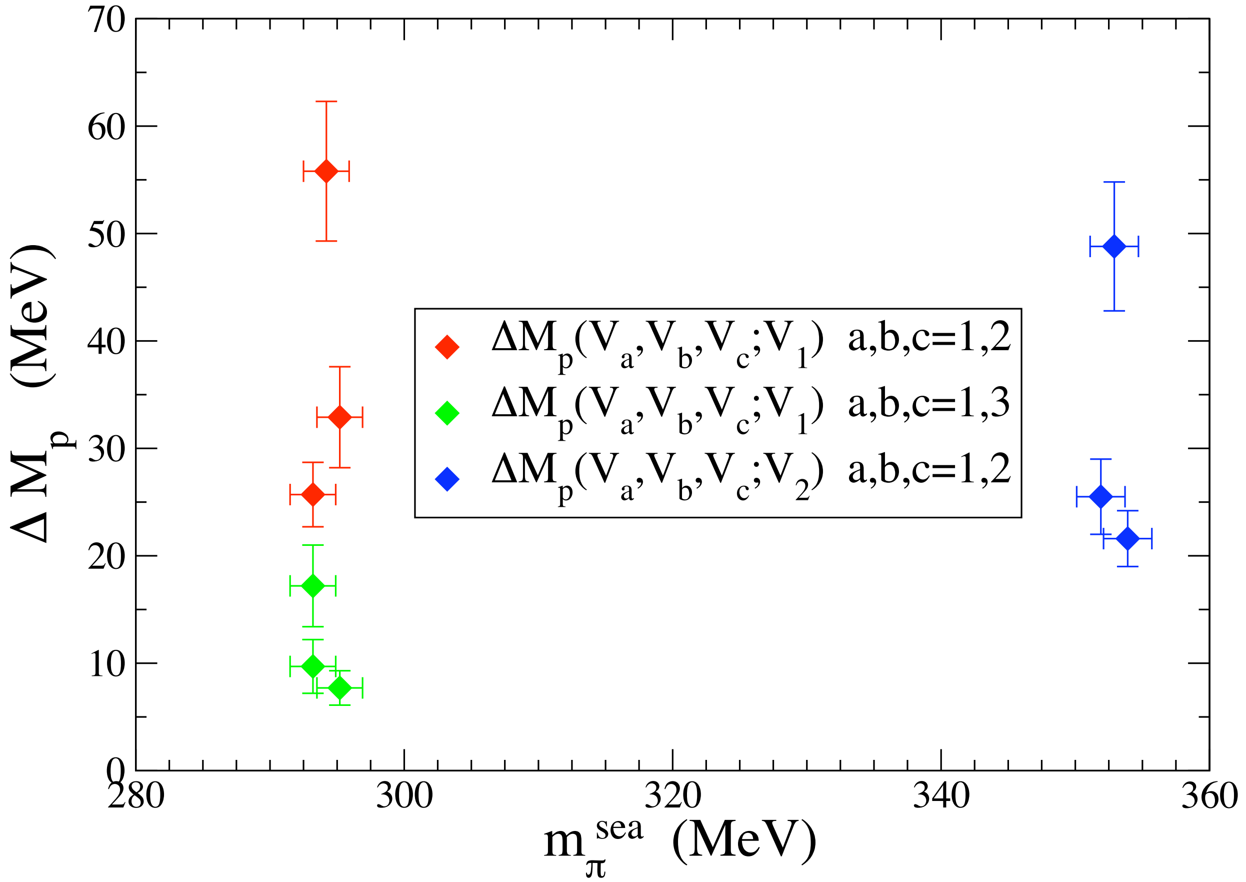
$$G^{\rm GMO}(t) = \frac{C_\Lambda(t) \quad C_\Sigma(t)^{1/3}}{C_N(t)^{2/3} \quad C_\Xi(t)^{2/3}} \quad \rightarrow \quad e^{-(M_\Lambda + M_\Sigma/3 - 2M_N/3 - 2M_\Xi/3)t}$$

ISOSPIN BREAKING

Beane, KO, Savage hep-lat/0605015

$$M_n - M_p|^{d-u} = \frac{2}{3}\left(2\overline{\alpha} - \overline{\beta}\right)\left(\frac{1-\eta}{1+\eta}\right) m_\pi^2$$

MILC: $\eta = m_u/m_d = 0.43(1)(8)$



Extraction	$M_n - M_p\|^{d-u}$ (MeV) at $m_\pi^{phys.}$
LO $\mathcal{O}(m_q)$	$1.96 \pm 0.92 \pm 0.37$
NLO $\mathcal{O}(m_q^{3/2})$	$2.26 \pm 0.57 \pm 0.42$

Extraction	$\frac{1}{3}\left(2\overline{\alpha} - \overline{\beta}\right)$ (l.u.)	$\overline{\alpha} + \overline{\beta}$ (l.u.)	g_1	$\|g_{\Delta N}\|$	χ^2/dof
LO $\mathcal{O}(m_q)$	0.198 ± 0.093	2.07 ± 0.08	$--$	$--$	0.56
NLO $\mathcal{O}(m_q^{3/2})$	0.229 ± 0.058	3.4 ± 1.1	-0.10 ± 0.35	0.60 ± 0.66	0.21

Exp. value: $M_n - M_p$ = 1.2933317(5) MeV —— minus EM part / Gasser Leutwyler '82 ——→ $M_n - M_p$ = 2.05(30) MeV

Scattering on the Lattice

- Miani-Testa no-go theorem ('90) [and C. Michael '89]

- Infinite Volume:

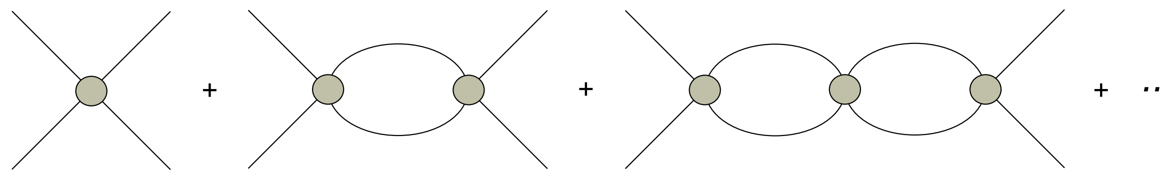
  Euclidean $\not\longrightarrow$ Minkowski

- Finite volume: discrete spectrum

Scattering on the lattice

Luscher

Scattering amplitude:

$A(p) =$ 

$$A(p) = \frac{4\pi}{m} \frac{1}{p\,cot\delta \; - \; i\,p}$$

At finite volume one can show: $\Delta E_n \equiv E_n - 2m = 2\sqrt{p_n^2 + m^2} - 2m$

p_n solutions of:

$$p\cot\delta(p) = \frac{1}{\pi L}\,\mathbf{S}\left(\frac{p^2L^2}{4\pi^2}\right)$$

$$\mathbf{S}(\eta) \equiv \sum_{\mathbf{j}}^{|\mathbf{j}|<\Lambda} \frac{1}{|\mathbf{j}|^2 - \eta} - 4\pi\Lambda$$

Effective range expansion:

$$p\cot\delta(p) = \frac{1}{a} + \frac{1}{2}rp^2 + \ldots.$$

a is the scattering length

LUSCHER FORMULA

Energy level shift in finite volume:

$$\Delta E_n \;\equiv\; E_n \;-\; 2m \;=\; 2\sqrt{p_n^2 \;+\; m^2} \;-\; 2m$$

p_n solutions of:

$$p \cot \delta(p) \;=\; \frac{1}{\pi L}\, \mathbf{S}\left(\frac{p^2 L^2}{4\pi^2}\right)$$

$$\mathbf{S}\,(\,\eta\,) \;\equiv\; \sum_{\mathbf{j}}^{|\mathbf{j}|<\Lambda} \frac{1}{|\mathbf{j}|^2 - \eta} \;-\; 4\pi\Lambda$$

$$p_n \cot \delta(p_n) = \frac{1}{a} + \cdots$$

$$\frac{1}{a} = \frac{1}{\pi L} S\left(\frac{p_0^2 L^2}{4\pi^2}\right) + \cdots$$

Expansion at p~0 :

$$\Delta E_0 \;=\; -\frac{4\pi a}{m L^3}\left[\,1 \;+\; c_1 \frac{a}{L} \;+\; c_2 \left(\frac{a}{L}\right)^2\,\right] \;+\; \mathcal{O}\left(\frac{1}{L^6}\right)$$

a is the scattering length

c_1 and c_2 are universal constants

Pion I=2 Scattering Length

S. Bean P. Bedaque KO and M. Savage hep-lat/0506013

$$C_{\pi^+}(t) = \sum_{\mathbf{x}} \langle \pi^-(t,\mathbf{x})\ \pi^+(0,\mathbf{0}) \rangle$$

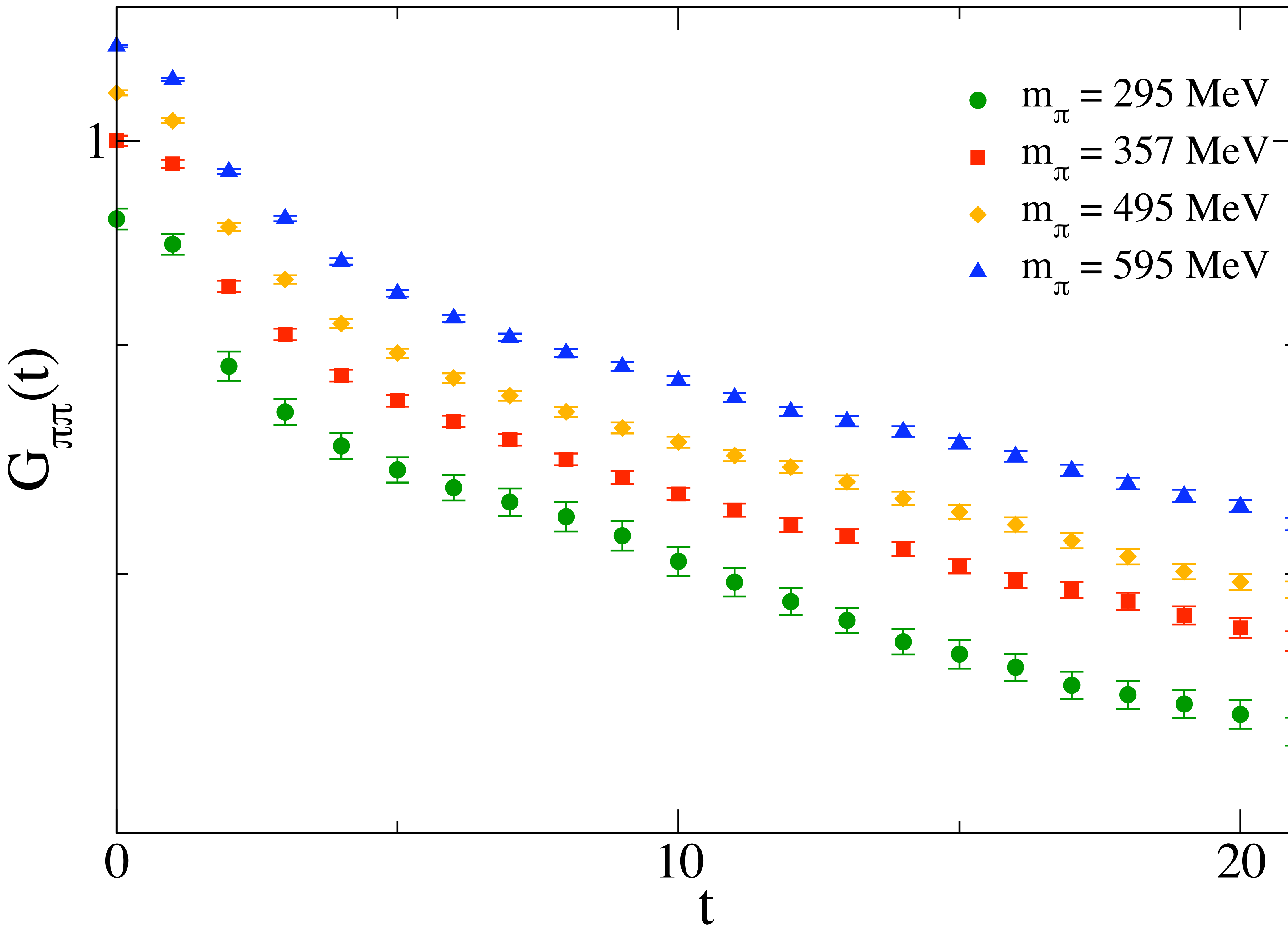
$$C_{\pi^+\pi^+}(p,t) = \sum_{|\mathbf{p}|=p} \sum_{\mathbf{x},\mathbf{y}} e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \langle \pi^-(t,\mathbf{x})\ \pi^-(t,\mathbf{y})\ \pi^+(0,\mathbf{0})\ \pi^+(0,\mathbf{0}) \rangle$$

$$G_{\pi\pi}(p,t) \equiv \frac{C_{\pi\pi}(p,t)}{C_\pi(t)^2} \rightarrow \sum_{n=0}^{\infty} \mathcal{A}_n\ e^{-\Delta E_n\ t}$$

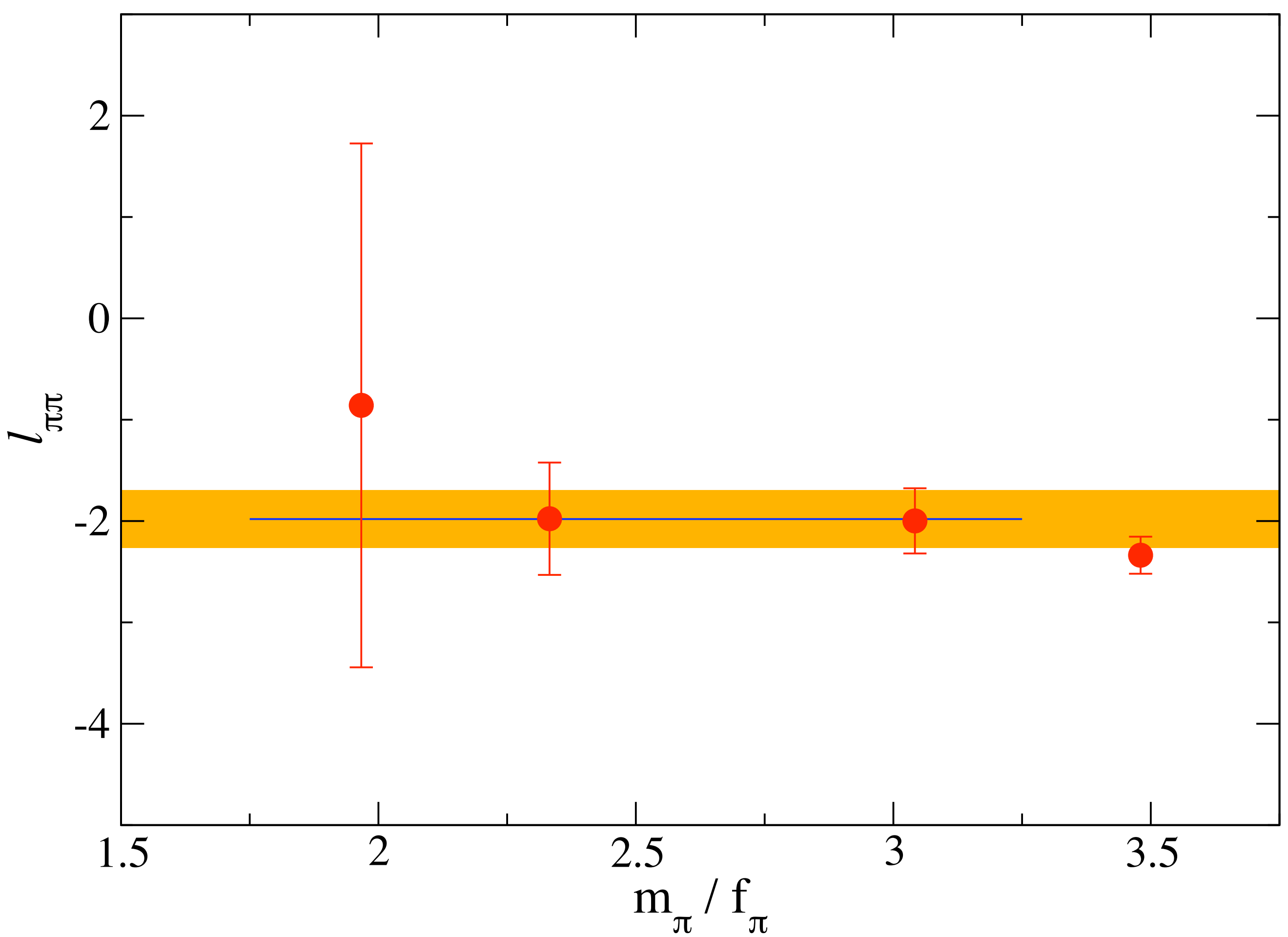
Quenched
Sharpe etal '92
Gupta etal '93
Kuramashi etal '93
Fugugita etal '94
C. Liu etal '02
J. Junk RBG '03
CP-PACS

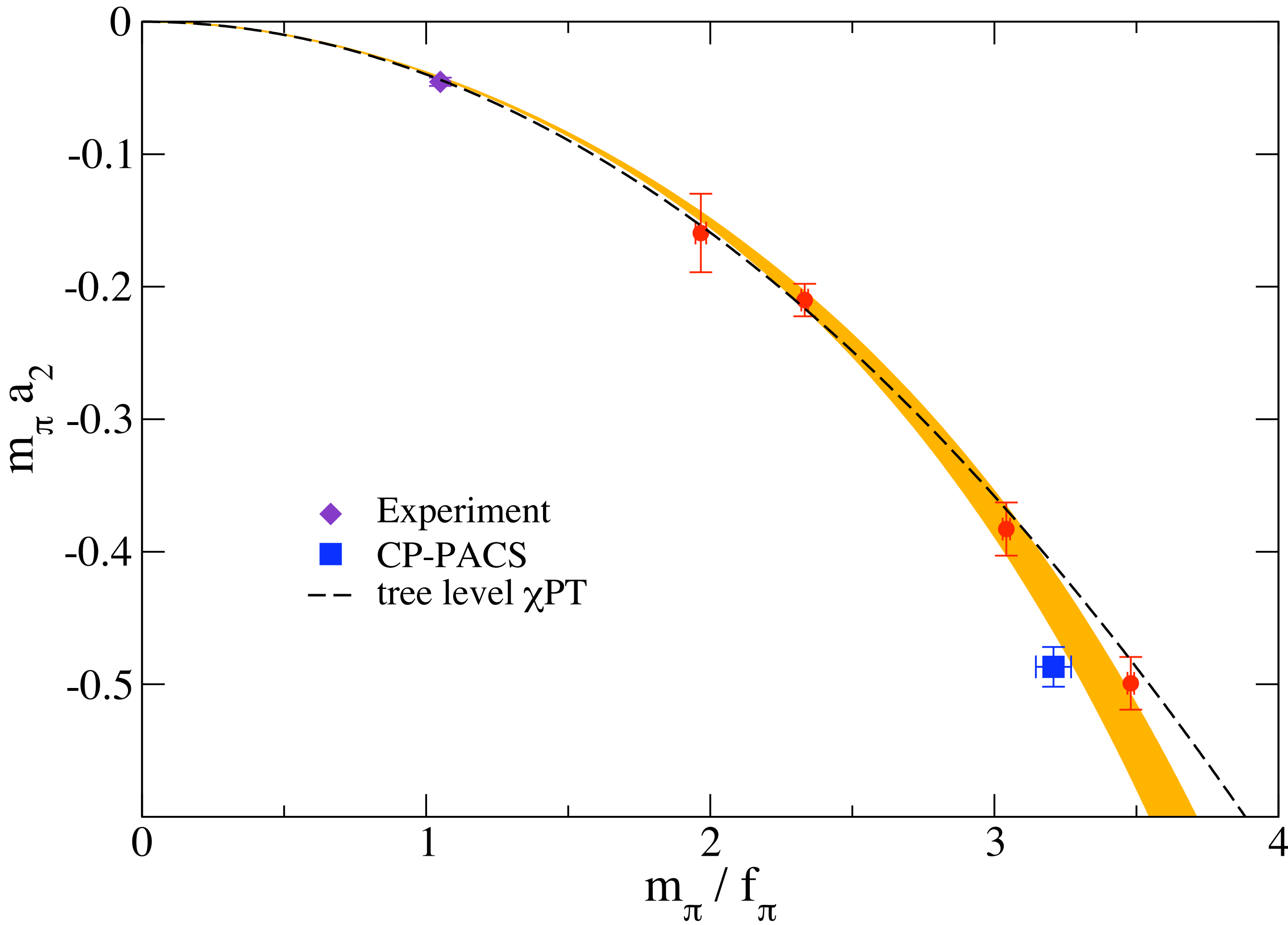
Dynamical
CP-PACS '04 (Wilson)
NPLQCD '05 (Hybrid)

CORRELATOR RATIO



I=2 Pion Scattering



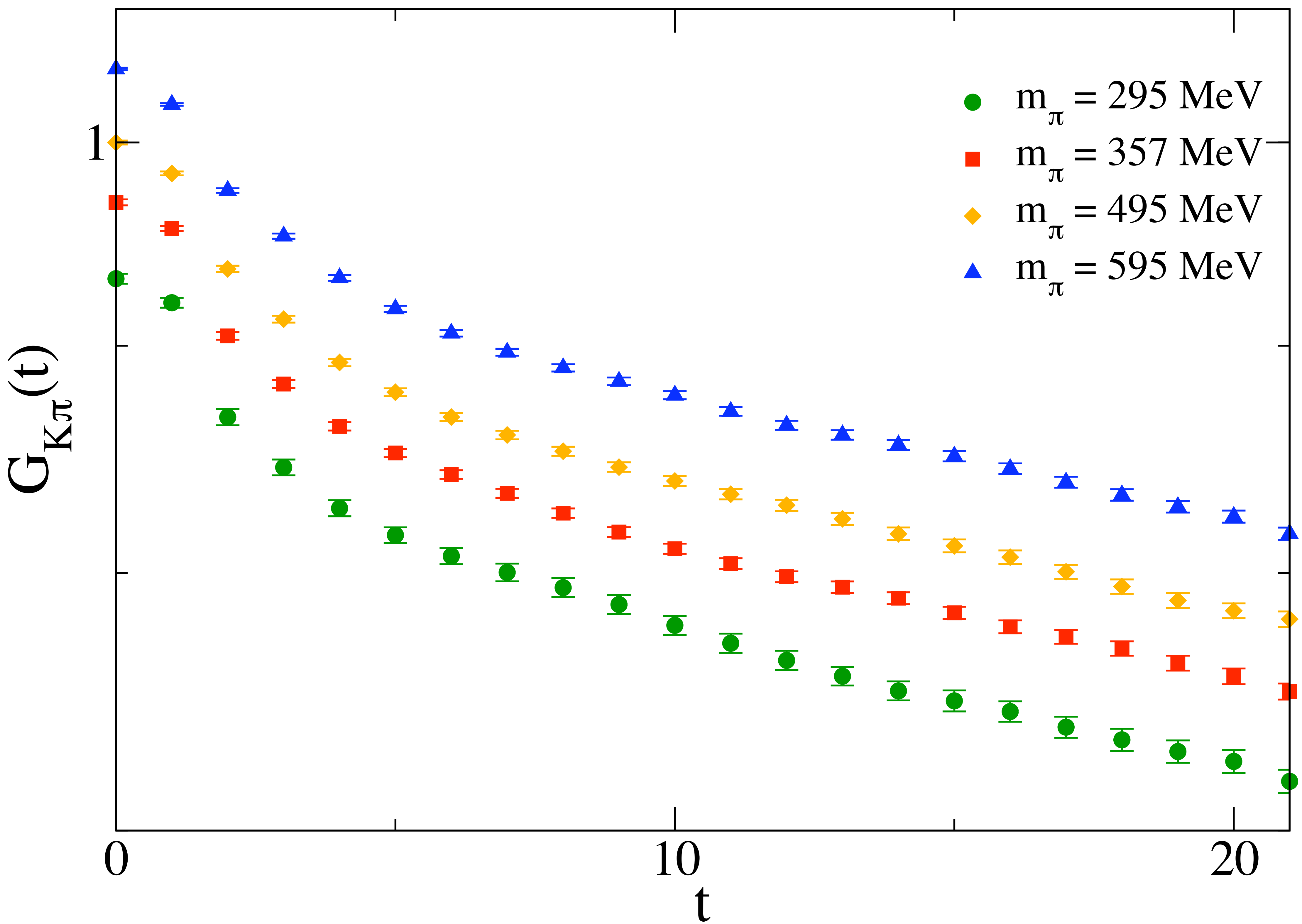


$$m_\pi a_2 = -\frac{m_\pi^2}{8\pi f_\pi^2}\left[1 + \frac{3m_\pi^2}{16\pi^2 f_\pi^2}\left(\log\frac{m_\pi^2}{\mu^2} + l_{\pi\pi}(\mu)\right)\right]$$

[Gasser-Leutwyler '84]
[Colangelo et al. '01]

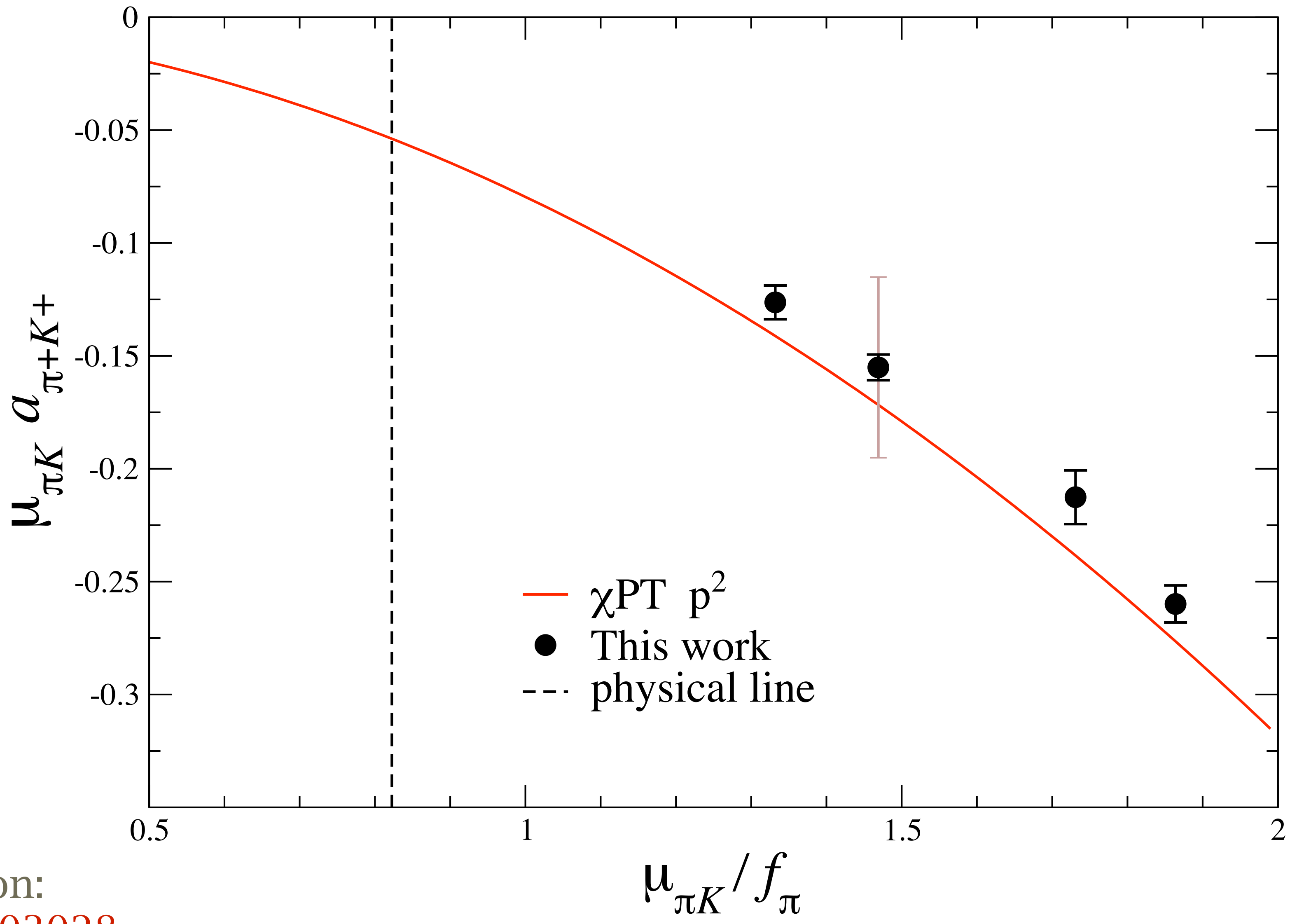
- $m_\pi\ a_2 = -0.0422(3)(18)$
- Experiment: $m_\pi\ a_2 = -0.0454(31)$
- SχPT has insignificant effect to the result [Chen et al. '05]

CORRELATOR RATIO



I=3/2 K-π Scattering

S. Bean P. Bedaque, T. Luu, KO, E. Pallante, A. Parreno and M. Savage hep-lat/0607036



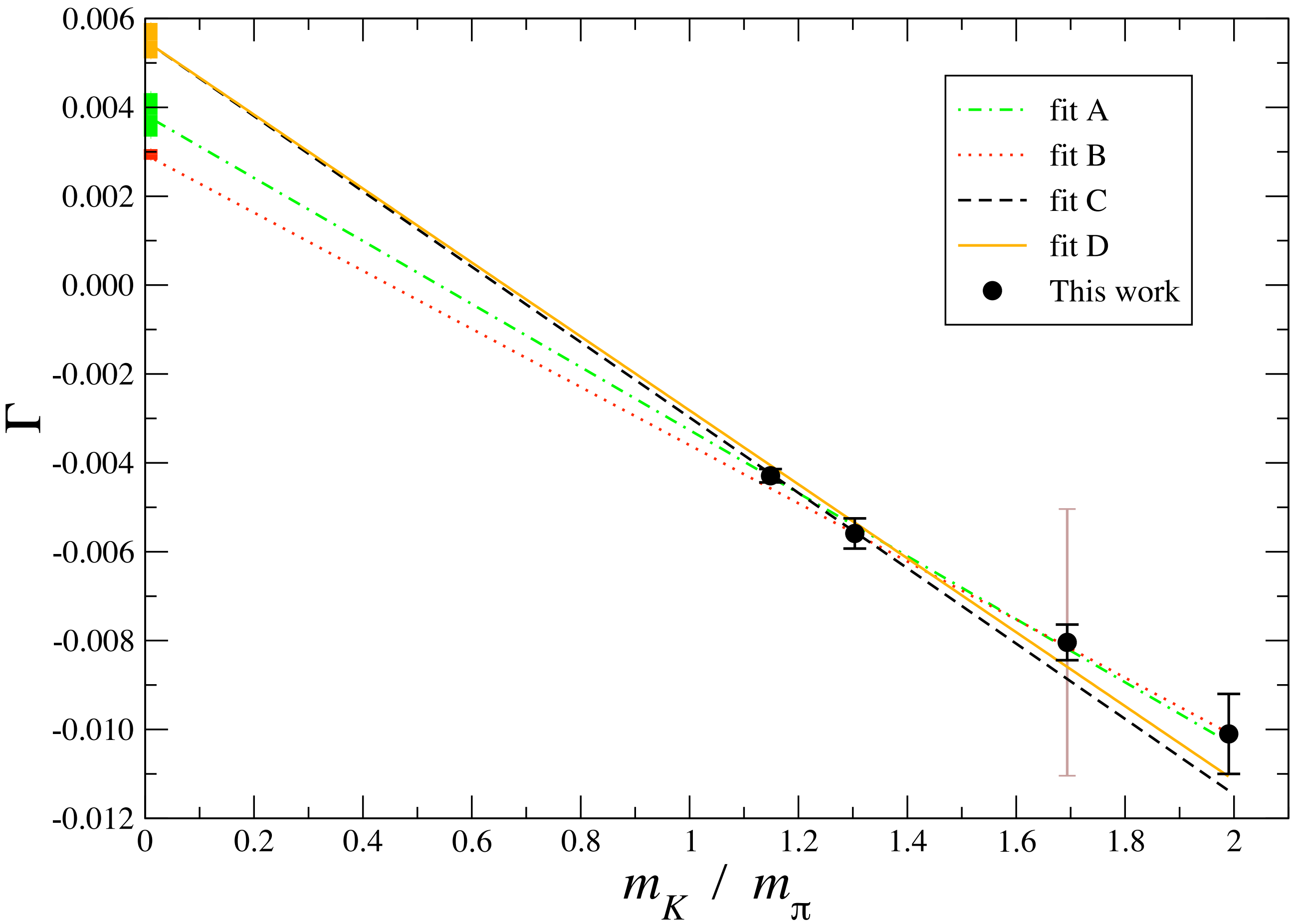
Quenched Calculation:
C. Miao et.al. hep-lat/0403028

Fitting to NLO ChiPT allows the extraction of both I=1/2 and I=3/2 scattering lengths

I=3/2 K-π SCATTERING

$$\Gamma\left(\frac{m_\pi}{f_\pi},\frac{m_K}{f_\pi}\right) \equiv -\frac{f_\pi^2}{16 m_\pi^2}\left(\frac{4\pi f_\pi^2}{\mu_{\pi K}^2}\left[\mu_{\pi K}\ a_{\pi^+K^+}\right] + 1 + \chi^{(NLO,-)} - 2\frac{m_K m_\pi}{f_\pi^2}\chi^{(NLO,+)}\right)$$

$$\Gamma \ = \ L_5(f_\pi^{\rm phys}) \ - \ 2\ \frac{m_K}{m_\pi}\ L_{\pi K}(f_\pi^{\rm phys})\ .$$

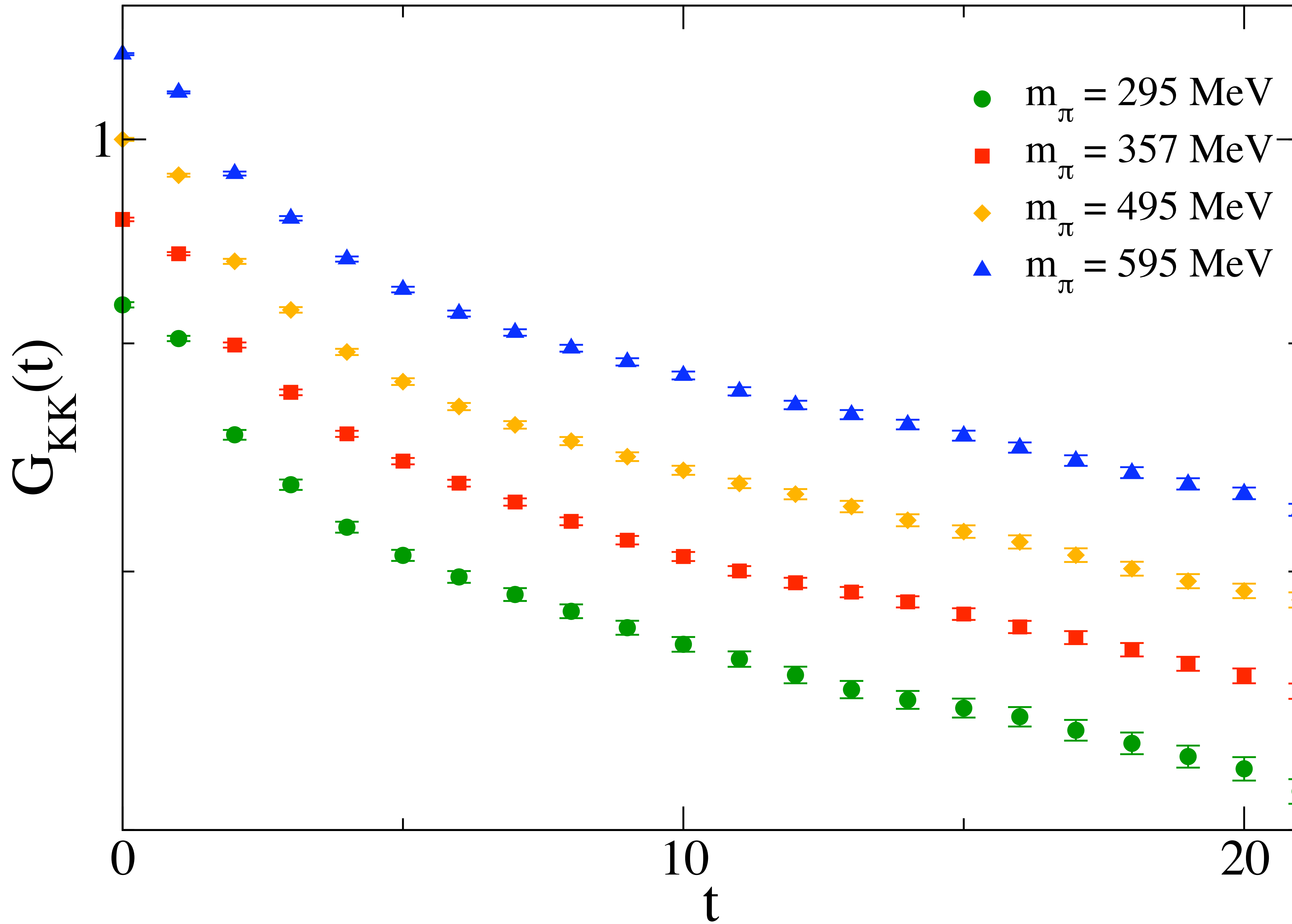


FIT	$L_5 \times 10^3$	$L_{\pi K} \times 10^3$	$m_\pi a_{3/2}$	$m_\pi a_{1/2}$	χ^2/dof
A	3.83 ± 0.49	3.55 ± 0.20	-0.0607 ± 0.0025	0.1631 ± 0.0062	0.17
B	2.94 ± 0.07	3.27 ± 0.02	-0.0620 ± 0.0004	0.1585 ± 0.0011	0.001
C	$5.65 \pm 0.02^{+0.18}_{-0.54}$ [a]	4.24 ± 0.17	-0.0567 ± 0.0017	0.1731 ± 0.0017	0.84
D	$5.65 \pm 0.02^{+0.18}_{-0.54}$ [a]	4.16 ± 0.18	-0.0574 ± 0.0016	0.1725 ± 0.0017	0.90

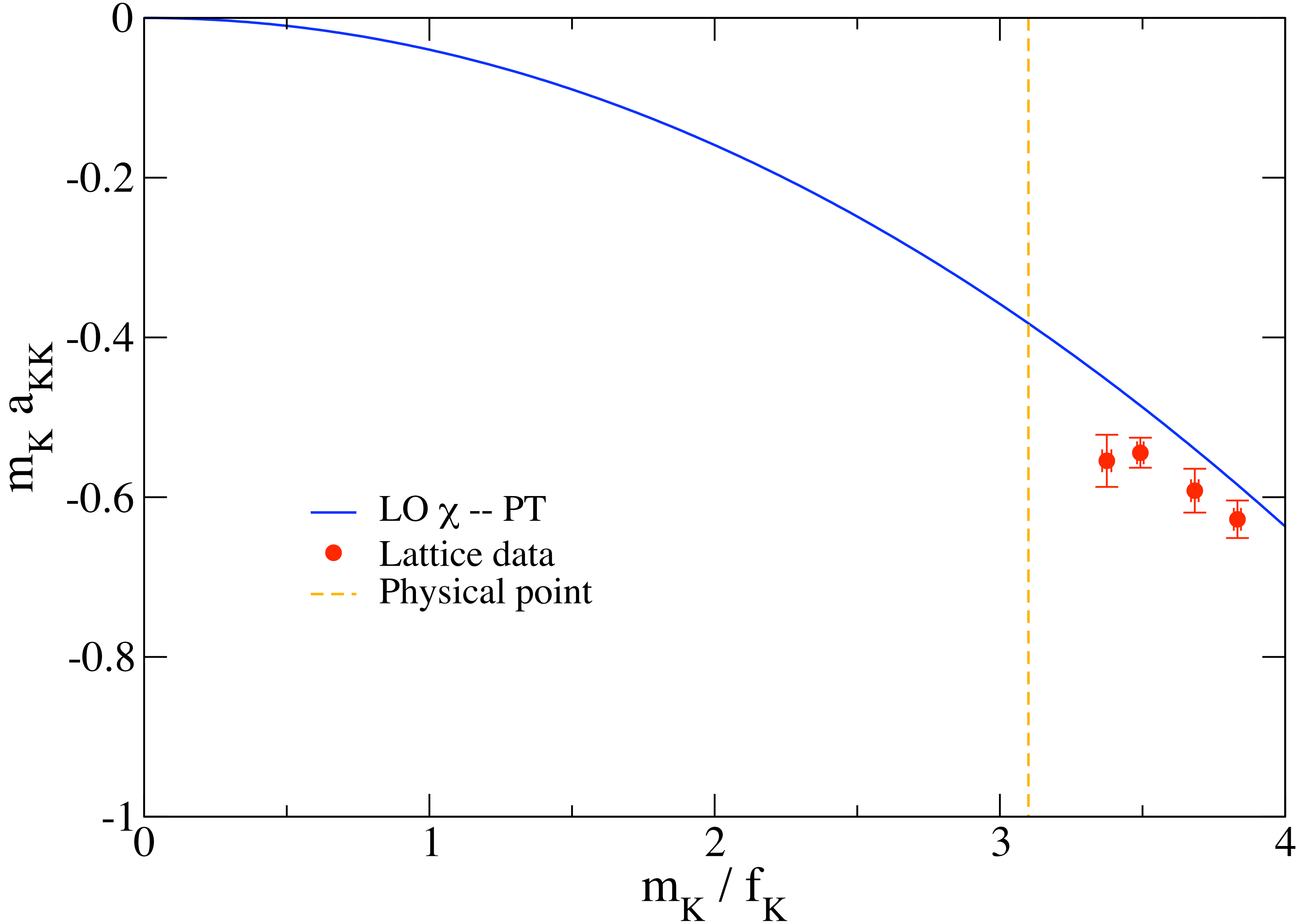
[a] Input from f_K/f_π [37].

$$m_\pi\ a_{3/2} \ = \ -0.0574 \pm 0.0016^{+0.0024}_{-0.0058}$$
$$m_\pi\ a_{1/2} \ = \ 0.1725 \pm 0.0017^{+0.0023}_{-0.0156}\ .$$

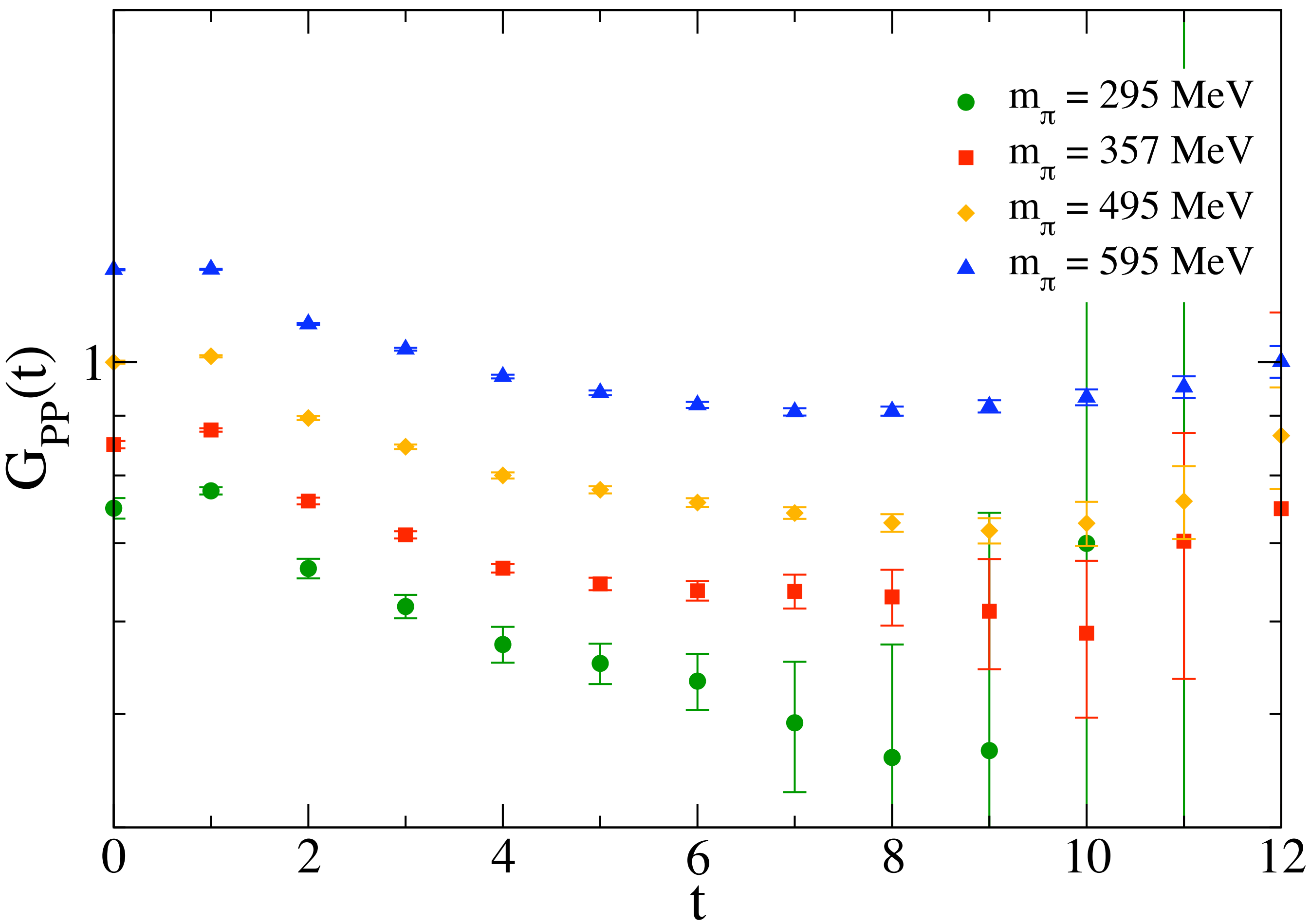
CORRELATOR RATIO

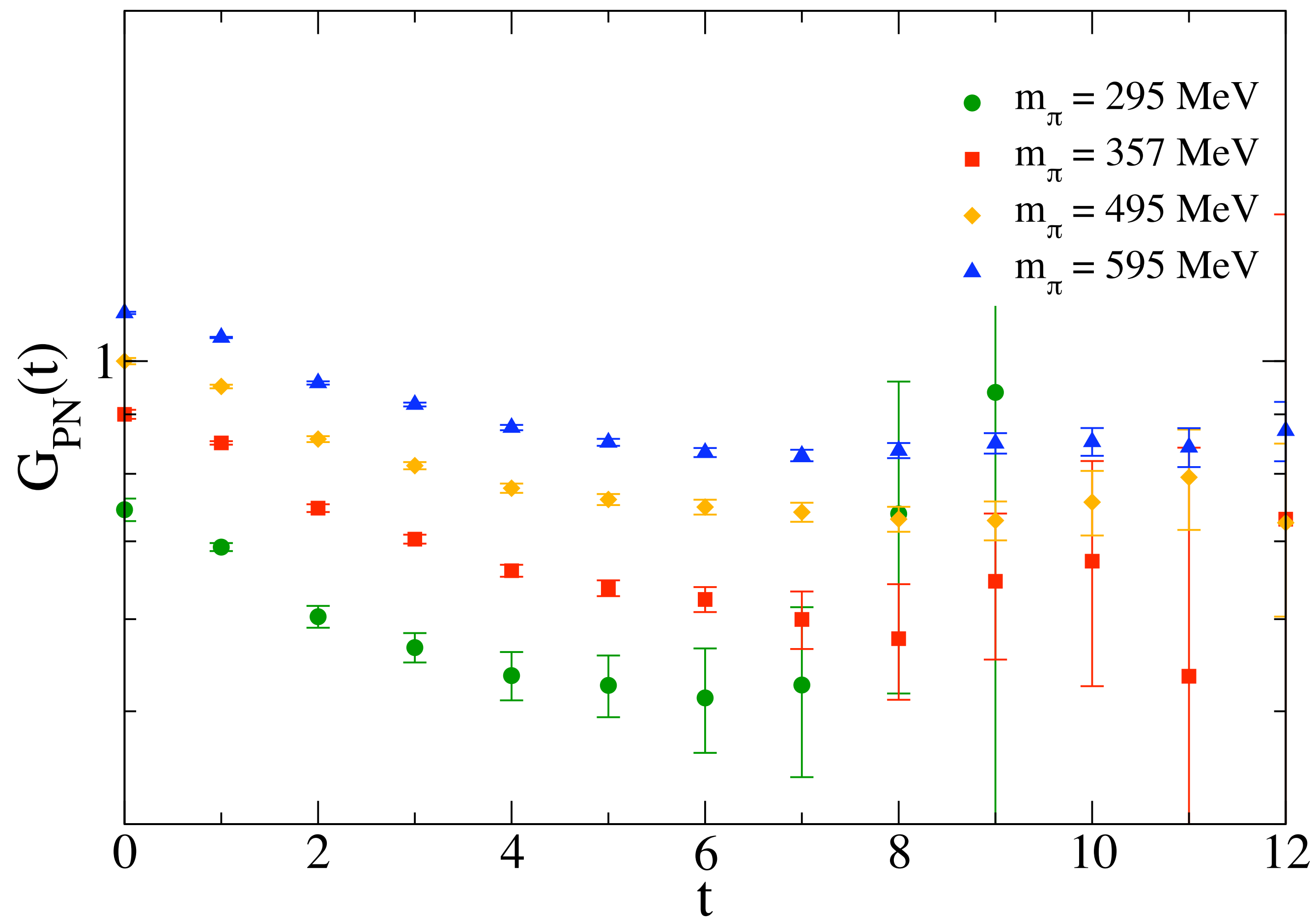


Kaon Scattering



NUCLEON-NUCLEON



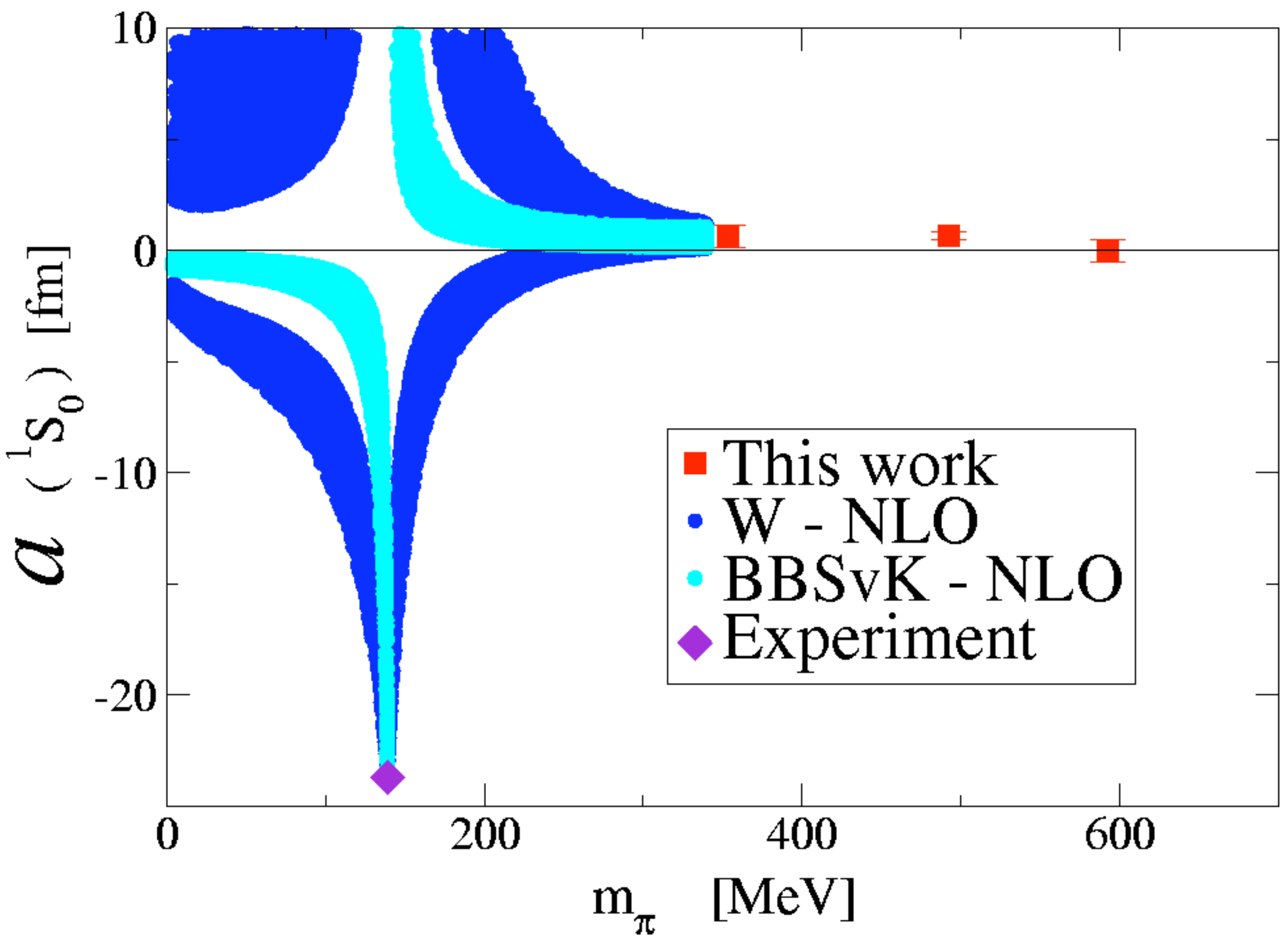


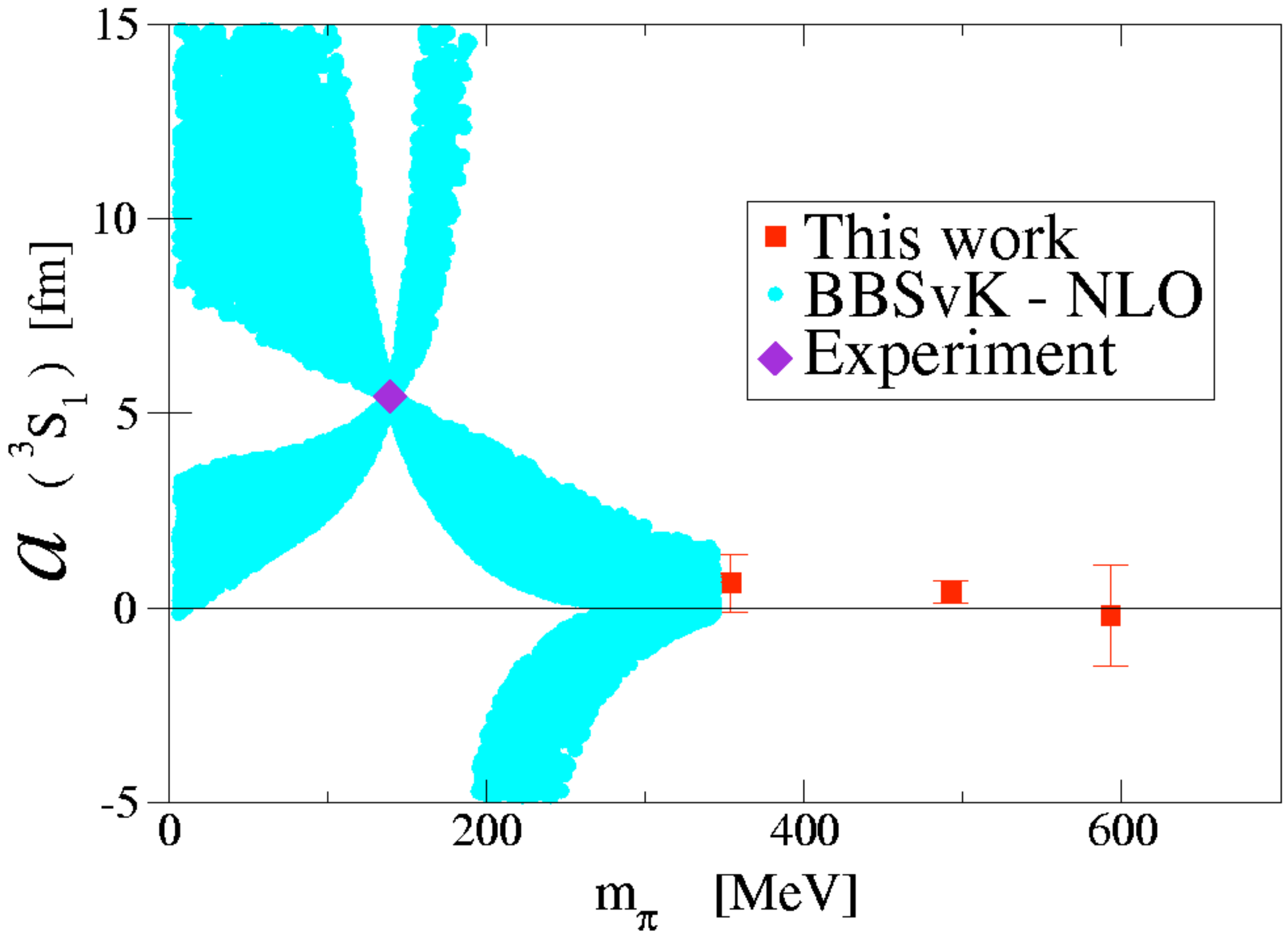
$^1S_0$ channel

$^3S_1$ channel

Nucleon-Nucleon

Beane, Bedaque, KO, Savage hep-lat/0602010





BBSvK: Beane Bedaque Savage van Kolck '02

W: Weinberg '90; Weingberg '91; Ordonez et.al '95

Fukugita et al. '95

FUTURE

- These calculations are the beginning of the beginning!
- Need lighter pion masses, multiple volume sizes, and lattice spacings
  - Determine we see scattering states
- K-π and K-K in the works
- Meson baryon channels: (K-n, K-Σ ...)
- Hyperon-Hyperon and Hyperon-Nucleon channels
- Higher statistics
- Need to make lattices designed for this project
- Turn to Wilson fermions (exact chiral symmetry not important) (JLAB program)
- Find a big computer!

Conclusions

- We have the means to perform high precision calculations relevant to hadronic physics
- Mixed action calculations with ChiPT can very accurately compute all Gasser-Leutwyler coefficients determining important parameters of the low energy effective field theories describing hadronic physics
- A careful study of systematic errors is still needed
- Opportunities for new calculations are now arising
- The computation of nucleon-nucleon scattering lengths is explored with encouraging results - new ideas are needed for obtaining phenomenologically interesting results.

INT Summer School on "Lattice QCD and its applications"

August 8 - 28, 2007

Seattle, WA
USA

http://www.int.washington.edu/PROGRAMS/07-2b.html