D-Branes in the QCD Vacuum

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LHP06, Jefferson Lab, July 31-Aug. 3, 2006

References:
QCD Results:

CPN-1 Results:
Y. Lian and HT, hep-lat/0607026

SUSY Relics: P. Keith-Hynes and HT, Lattice 2006 Proceedings
A “chirally smooth” definition of topological charge in QCD:

\[ q(x) = \frac{1}{2} \text{tr} \gamma^5 D \]

\[ D = \text{Overlap Dirac operator} \]

Results of first study of overlap q(x) distribution in 4D QCD (Horvath, et al, Phys. Rev. D (2003)):

Extended coherent 3-dimensional sheets in 4-D space!!

Results:
-- Only small 4D coherent structures found with sizes of O(a) and integrated \( q(x) \ll 1 \). (No instantons.)

-- Large coherent structures are observed which are locally 3-D sheets in 4-D space (surfaces of codimension 1), typically only ~1 or 2 lattice spacings thick in transverse direction.

-- In each configuration \(~(1.5 \text{ fm})^4\), two sheets of opposite charge are found, which are everywhere close to each other. Possibly a single membrane with a dipole layer of topological charge

⇒ Short range, negative TCh correlator (required by spectral arguments).

(Note: \( <q(x)q(0)> \) correlator must be \( \leq 0 \) for all \( |x| \neq 0 \))
2D slice of $Q(x)$ distribution for 4D QCD

Note: Topological charge distributed more-or-less uniformly throughout membrane, not concentrated in localized lumps. (Horvath, et al, Phys.Lett. B(2005))
**CP(N-1) models on the lattice** (Lenaghan, Ahmad, and HT, PRD 2005)

\[ S = \beta N \sum_{x, \mu} z^*(x) U(x, x + \hat{\mu}) z(x + \hat{\mu}) + h.c \]

Here \( z = N \)-component scalar, and \( U = U(1) \) gauge field

As in QCD, we study the topological charge distribution using \( q(x) \) constructed from overlap Dirac operator.

To exhibit coherent structure, look for nearest-neighbor-connected structures. Plot largest structure.

For best visualization plot 1 for sites on structure, 0 otherwise.

To normalize expectations, first look for connected structures on random \( q(x) \) distributions, then compare with \( q(x) \) distributions in CP\(^{N-1} \) Monte Carlo configurations:
Largest coherent structure from a random TC distribution:
Another random TC distribution:
Coherent structure: $CP^3 \quad 50 \times 50 \quad \beta = 1.2 \quad (\xi \approx 20)$
Another CP(3) configuration:
Still another CP(3) configuration:
Plot sign(q(x)) for CP3 config:

"Backbone" of coherent 1D regions is only 1 to 2 sites thick (~range of nonultralocality). Positive and negative regions everywhere close.
QCD Topological charge density correlator

\[ \langle q(r) q(r') \rangle \text{[lattice units]} \]

- Blue line: \( a = 0.1650 \text{ fm} \)
- Green line: \( a = 0.1100 \text{ fm} \)
- Red line: \( a = 0.0825 \text{ fm} \)

r[\text{lattice units}]
Summary of Monte Carlo results: In both 4D QCD and 2D CP^{N-1} (for N>4), topological charge comes in the form of extended membranes of codimension 1, with opposite sign sheets (or lines in CP^{N-1}) juxtaposed in dipole layers.

An interesting exception -- small instantons in CP1 and CP2 (Y. Lian and HT, hep-lat/0607026):

- CP1 and CP2 are dominated by small instantons (Luscher, 1982) with radii of order a (so correlator remains negative for nonzero separation in continuum limit). Small instantons are easily seen with overlap q(x).
- CP3 is on the edge of the instanton melting point – has some instantons but mostly coherent line excitations.
- CP4 and higher have no instantons – only line excitations.

Crude estimate (lower bound) for instanton melting point = “tipping point” of integral over instanton size in semiclassical calculation (Luscher, 82):

$$\Rightarrow \text{for CP}^{N-1}, \quad N_{\text{crit}}=2 \quad \text{for QCD,} \quad N_{\text{crit}}=12/11$$
CP1, beta=1.6, Q = 1
CP1, $\beta=1.6$, $Q = -2$
CP2, beta=1.8, Q = 1
CP9, beta=0.9, Q = -1
Plot integrated $q(x)$ in highest structure (within 2 sites of highest peak) for all configs with $Q = \pm 1$

(End of digression on melting instantons.)
Fundamental new development in QCD: ADS/CFT duality ("holography")

- 4D QCD ≈ IIA String Theory in a 5D black hole metric.

\[ R_4 \times S_1 \times R_5 \rightarrow R_4 \times D \times S_4 \]

5D Chern-Simons term → 4D \( \theta \) term

Among other things, ADS/CFT confirms Witten’s (1979) large-\( N_c \) view of topological charge--Instantons “melt” and are replaced by (Witten, PRL 98):

- Multiple vacuum states ("k-vacua") with \( \theta_{\text{eff}} = \theta + 2\pi k \)
- Local k-vacua separated by domain wall = membrane
- Domain wall = fundamental D6-brane of IIA string wrapped around \( S_4 \)
- \( \theta = \) Wilson line around D=disk with BH singularity at center ~ Aharonov-Bohm phase around “Dirac string”
- \( k = \) integer is a Dirac-type quantization of 6-brane charge
- TC is dual to Ramond-Ramond charge in string theory.
Holographic view of domain wall in CP(N-1):

In Witten’s brane construction, 4D Yang-Mills is viewed from 6 dimensions = $R_4 \times D$, where $D = S_1 \times$ radial coordinate of black-hole metric. Radius of $S_1$ is an ultraviolet cutoff, analogous to lattice spacing.

Analog in (1+1)-D is (3+1)-dimensional solid cylinder.

(1+1)-D case is equivalent to Laughlin’s gedankenexperiment for topological understanding of integer quantum Hall effect. (Corbino disk)

Longitudinal component of monopole field ($B$) is dual to topological charge (= longitudinal E field) in CP(N-1) model.

$\theta = \oint A \cdot dy = 0$

$\theta = \oint A \cdot dy = 2\pi$

$\Rightarrow$ Domain wall = dipole layer of topological charge!
What are coherent sheets of TC in QCD? Are they D-branes?

The ADS/CFT holographic view of topological charge in the QCD vacuum has an analog in 2D U(1) theories:

--Multiple discrete k-vacua characterized by an effective value of $\theta$ which differs from the $\theta$ in the action by integer multiples of $2\pi$.

-- Interpretation of effective $\theta$ similar to Coleman’s discussion of 2D massive Schwinger model (Luscher (1978), Witten (1979,1998)), where $\theta = \text{background E field}$.

In 2D U(1) models (CP(N-1) or Schwinger model): Domain walls between k-vacua are world lines of charged particles:
Precise analogy between U(1) in 2D and SU(N) in 4D (Luscher, 1978):

◆ Identify Chern-Simons currents for the two theories.

\[ A_\mu \rightarrow A_{\mu\nu\sigma} \equiv -Tr \varepsilon_{\mu\nu} A_\nu A_\sigma + \frac{3}{2} A_\mu \partial_\nu A_\sigma \]

\[ j_\mu^{\text{CS}} = \varepsilon_{\mu\nu} A_\nu \rightarrow j_\mu^{\text{CS}} = \varepsilon_{\mu\nu\sigma\tau} A_{\nu\sigma\tau} \]

\[ Q = \partial_\mu j_\mu^{\text{CS}} \rightarrow Q = \partial_\mu j_\mu^{\text{CS}} \]

Wilson line \rightarrow integral over 3-surface ("Wilson bag")
charged particle \rightarrow charged membrane
(= domain wall) (= domain wall)

In both cases, CS current correlator has massless pole \( \sim 1/q^2 \)

This analogy suggests that the coherent 1D structures in \( \mathbb{CP}^{N-1} \) are charged particle world lines, and the 3D coherent structures in QCD are Wilson bags=excitation of Chern-Simons tensor on a 3-surface.
The emerging picture -- A “laminated” vacuum:

- Alternating sign sheets (or lines) of topological charge:

**Possible dynamics of vacuum lamination in \( \text{CP}^{N-1} \):**

- Spectrum consists of nonsinglet and singlet z-zbar pairs.
- \( M_{\text{singlet}} > M_{\text{nonsinglet}} \) due to annihilation diagrams:
- Singlet pairs pop out of vacuum, but they can propagate farther by forming nonsinglet pairs with members of neighboring singlet pairs:

Two degenerate vacua with **topological order** (ala Wen and Zee in quantum hall eff.)
Conformal field theory between the branes:

\( \text{CP}^{N-1} \text{ in Lorentz gauge} \quad \partial_\mu A_\mu = 0 \)

\[ \Rightarrow A_\mu = \varepsilon_{\mu\nu} \partial_\nu \Phi \]

\[ J^\text{CS}_\mu = \partial_\mu \Phi \]

\[ \nabla^2 \Phi = 2\pi q(x) \]

Thickness of coherent lines of \( q(x) \) is of order \( a \to 0 \) in continuum.

Consider an idealized brane vacuum configuration where \( q(x) \) is confined to one-dimensional subspaces (or zero-dimensional for small instantons).

In voids between branes, \( \nabla^2 \Phi = 0 \) so \( \Phi(x_1, x_2) \) can be written as the real part of a holomorphic fcn of \( z=x_1+ix_2 \)

\[ \Phi(x_1, x_2) = \varphi(z) + \varphi(\bar{z}) \]

--- \( \varphi(z) \) has branch cuts at branes (and/or poles at small instantons)

\( \varphi(z) \sim \) string coordinate for Coulomb flux tube between charged particles (= Dbranes = string endpoints)!
- Note that singlet pairs must all polarize in the same direction to form nonsinglet mesons with nearest neighbors.

A "dimerization" (lamination) of the vacuum. Similar to Peierls transition in one-dimensional chain of atoms.

Like antiferromagnetic order, but not tied to even-odd sublattice (hence topological)

This mechanism is also compatible with long range behavior of large N solution:

\[
\Delta^{-1}_{\mu \nu} = \Delta_{\mu \nu} (q) \quad \Delta_{\mu \nu} (q) \quad \Rightarrow \quad \frac{2 \pi}{N} \delta_{\mu \nu} - \frac{q_\mu q_\nu}{q^2} \frac{6 M^2}{q^2} + \frac{3}{5} + O \left( \frac{q^2}{M^2} \right)
\]

In large N, singlet pairs are treated individually, but with massive z-propagators.

\[
\text{dimerization} \quad \leftrightarrow \quad \text{Spontaneous generation of constituent z-mass}
\]
Topological Charge Correlators from CFT

Static dilute brane approximation $M^2 \gg q^2$ (in QCD, large ps glueball mass)

- Effective theory with z’s integrated out:
  $$S \rightarrow -\frac{1}{4} F_{\mu\nu}^2$$

- OPE for Chern-Simons current correlator:
  $$\left\langle J_{\mu}^{CS}(x)J_{\mu}^{CS}(0) \right\rangle \sim C_{1\mu\nu}(x) + C_{2\mu\nu}(x) \left\langle F^2(0) \right\rangle + \ldots$$

Form of OPE coefficients completely determined up to 2 overall constants by CFT arguments:

$$C_{1\mu\nu}(x) = c_1 \frac{x^2 \delta_{\mu\nu} - 2x_\mu x_\nu}{(x^2)^2}$$
$$C_{2\mu\nu}(x) = c_2 \left( \delta_{\mu\nu} \ln x^2 + \frac{2x_\mu x_\nu}{x^2} \right)$$

Result for TC correlator is sum of contact terms:

$$G(t) = \int dx \left\langle q(x,t)q(0,0) \right\rangle = -c_1 \delta''(t) + c_2 \delta(t) , \quad c_2 = \chi_t$$

Gives a good fit to all lattice correlators using

$$\delta(t) \rightarrow \frac{1}{d \sqrt{\pi}} \exp \left[ -\frac{t^2}{d^2} \right]$$

With $d \approx 1.5$ lattice spacings, essentially independent of beta.
Fit correlator to
SUSY Relics in QCD (Armony, Shifman, and Veneziano (2002)):

- Holographic projection of orientifold compactification of string theory predicts “planar (large N) equivalence” between $\mathcal{N} = 1$ SUSY YM and ordinary 1-flavor QCD.

- A “SUSY relic” prediction that can be tested by Monte Carlo: Degeneracy between the scalar and pseudoscalar mesons in 1-flavor QCD (they belong to the same WZ multiplet in SUSY YM chiral Lagrangian (Veneziano, Yankielowicz)).

- Prediction is tested using MC results for scalar and pseudoscalar valence and hairpin diagrams. (N. Isgur and HT, PRD (2001))

**pseudoscalar hairpin (quenched):**

$$\propto m_0^2 \propto \chi_t \quad m_{\eta'}^2 = m_\pi^2 + m_0^2$$

**scalar hairpin:**

$$- \text{const.} \propto m_{sc}^2 \quad m_\sigma^2 = m_{a0}^2 + m_{sc}^2$$
Test of scalar-pseudoscalar degeneracy in 1-flavor QCD:
(with Patrick Keith-Hynes)

\[(\text{valence mass})^2 + [315(6)]^2 + [1416(14)]^2 + [1350(90)]^2 + [427+249-756]^2\]

\[(\text{hairpin mass shift})^2 + [407(11)]^2\]

\[(\text{total mass})^2 + [515(13)]^2\]
Conclusions:

- Topological charge in QCD is ADS/CFT dual to Ramond-Ramond charge in string theory. As such it provides a central focus on the string theory/holography aspects of gauge theory.

- Interpretation of observed sheets of topological charge as D-branes (as suggested by Witten’s brane construction of QCD) seems plausible and fits into a very appealing theoretical framework.

- $\text{CP}^{N-1}$ sigma models capture many of the essential aspects of QCD in a computationally and theoretically simpler context. MC results in these models provide strong support for the Dbrane vacuum scenario.

- Dbrane vacuum may suggest connections between XSB & confinement, chiral anomaly & conformal anomaly, ...

- SUSY relics are another interesting (and presumably related) implication of string/gauge holography.
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