D-Branes in the QCD Vacuum

Hank Thacker
University of Virginia

LHP06, Jefferson Lab, July 31-Aug. 3, 2006

References:

QCD Results:

I. Horvath, et al. Phys.Rev. D68:114505(2003);Phys.Lett. B612: 21(2005);B617:49(2005).

CPN-1 Results:

J. Lenaghan, S. Ahmad, and HT Phys.Rev. D72:114511 (2005)

Y. Lian and HT, hep-lat/0607026

SUSY Relics: P. Keith-Hynes and HT, Lattice 2006 Proceedings

A "chirally smooth" definition of topological charge in QCD:

$$q(x) = \frac{1}{2} tr \gamma^5 D \qquad D = \text{ Overlap Dirac operator}$$

Results of first study of overlap q(x) distribution in 4D QCD (Horvath, et al, Phys. Rev. D (2003)):

Extended coherent 3-dimensional sheets in 4-D space !!

Results:

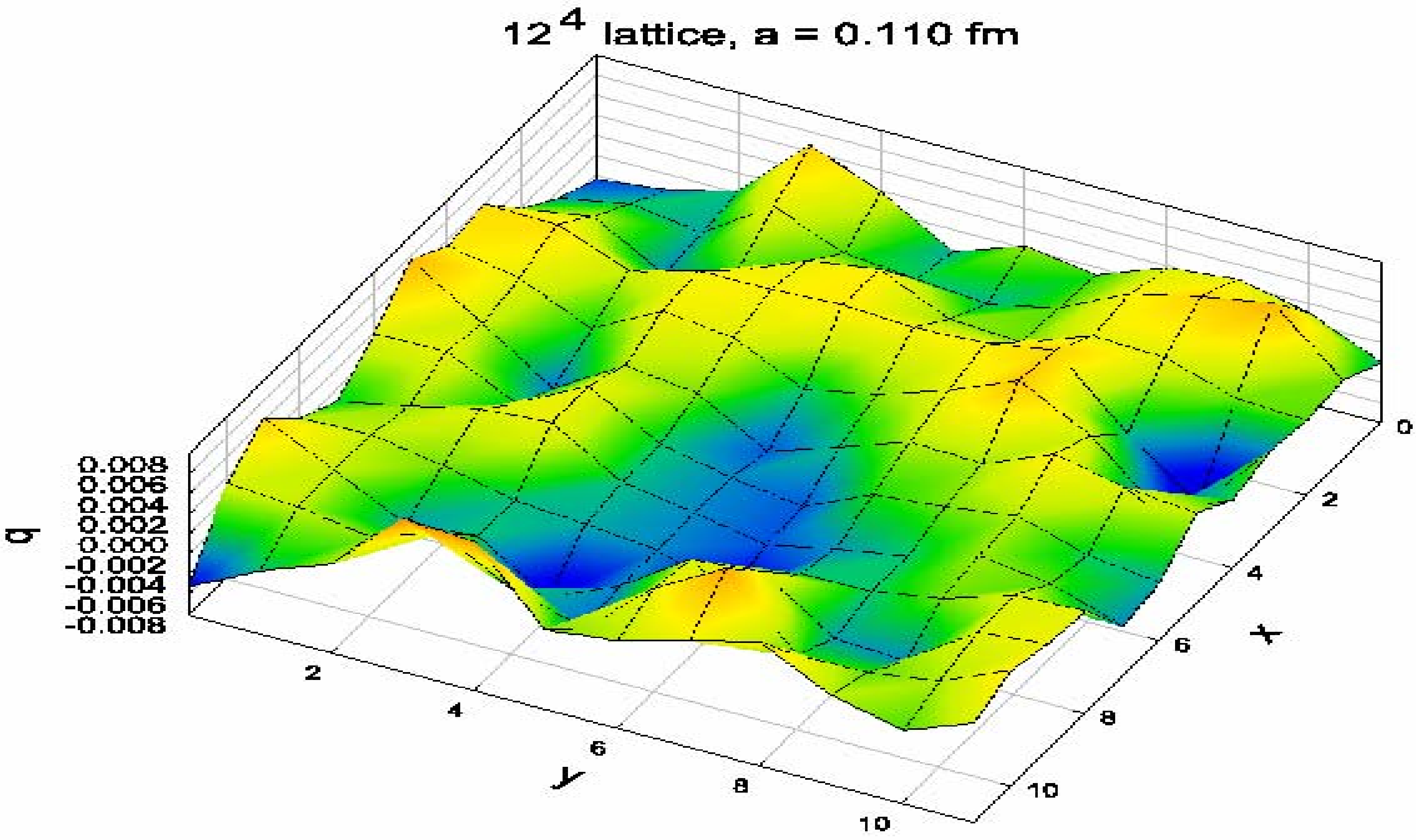
-- Only small 4D coherent structures found with sizes of O(a) and integrated q(x)<<1. (No instantons.)

-- Large coherent structures are observed which are locally 3-D sheets in 4-D space (surfaces of codimension 1), typically only ~1 or 2 lattice spacings thick in transverse direction.

-- In each configuration ~(1.5 fm)^4, two sheets of opposite charge are found, which are everywhere close to each other - - Possibly a single membrane with a dipole layer of topological charge

$\Rightarrow$ Short range, negative TCh correlator (required by spectral arguments).

(Note: <q(x)q(0)> correlator <u>must</u> be ≤ 0 for all $|x| \neq 0$)



2D slice of Q(x) distribution for 4D QCD

Note: Topological charge distributed more-or-less uniformly throughout membrane, not concentrated in localized lumps. (Horvath, et al, Phys.Lett. B(2005))

CP(N-1) models on the lattice (Lenaghan, Ahmad, and HT, PRD 2005)

$$S = \beta N \sum_{x,\hat{\mu}} z^*(x)U(x, x+\hat{\mu})z(x+\hat{\mu}) + h.c$$

Here z = N-component scalar, and U = U(1) gauge field

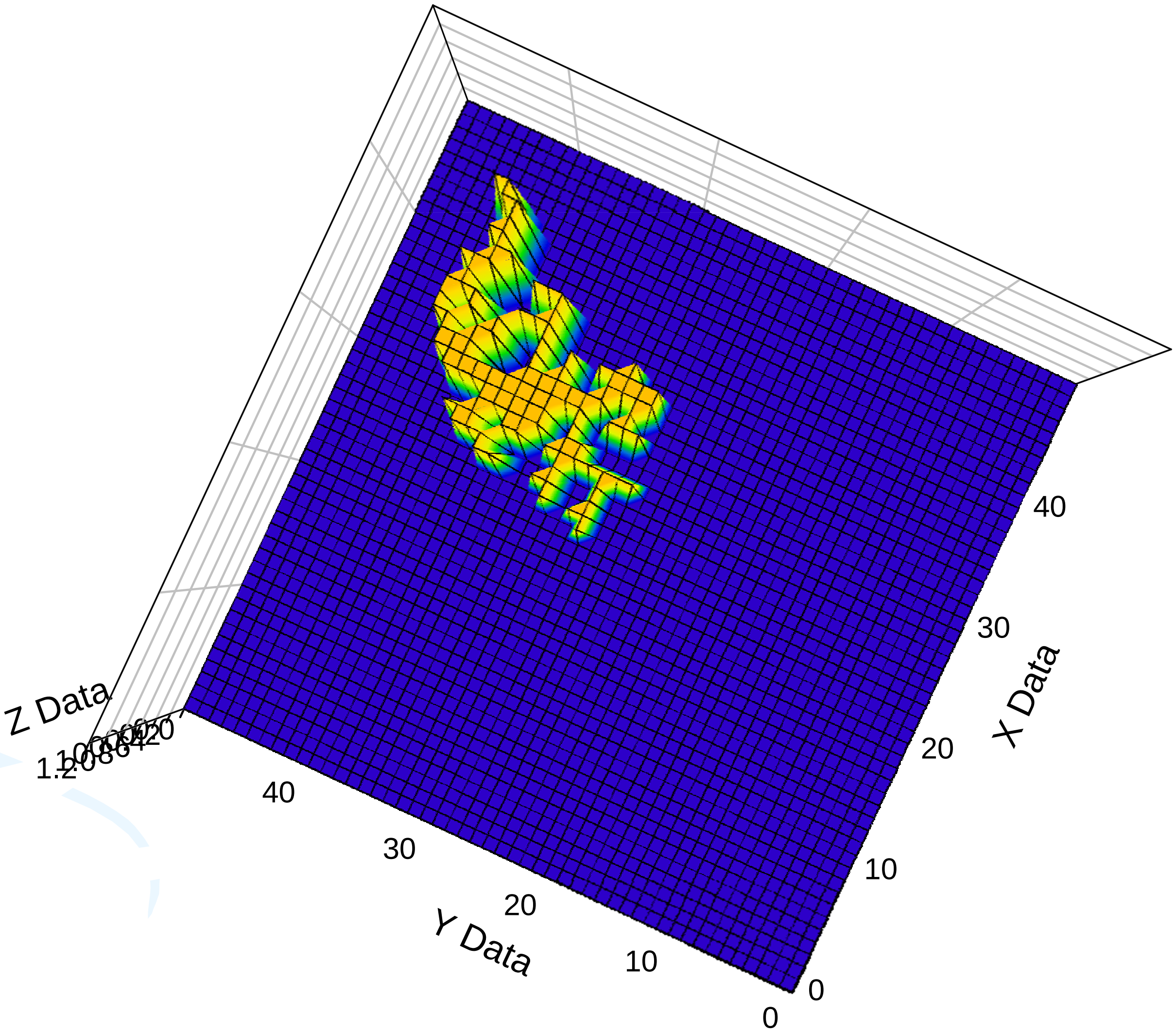
As in QCD, we study the topological charge distribution using q(x) constructed from overlap Dirac operator.

To exhibit coherent structure, look for nearest-neighbor-connected structures. Plot largest structure.

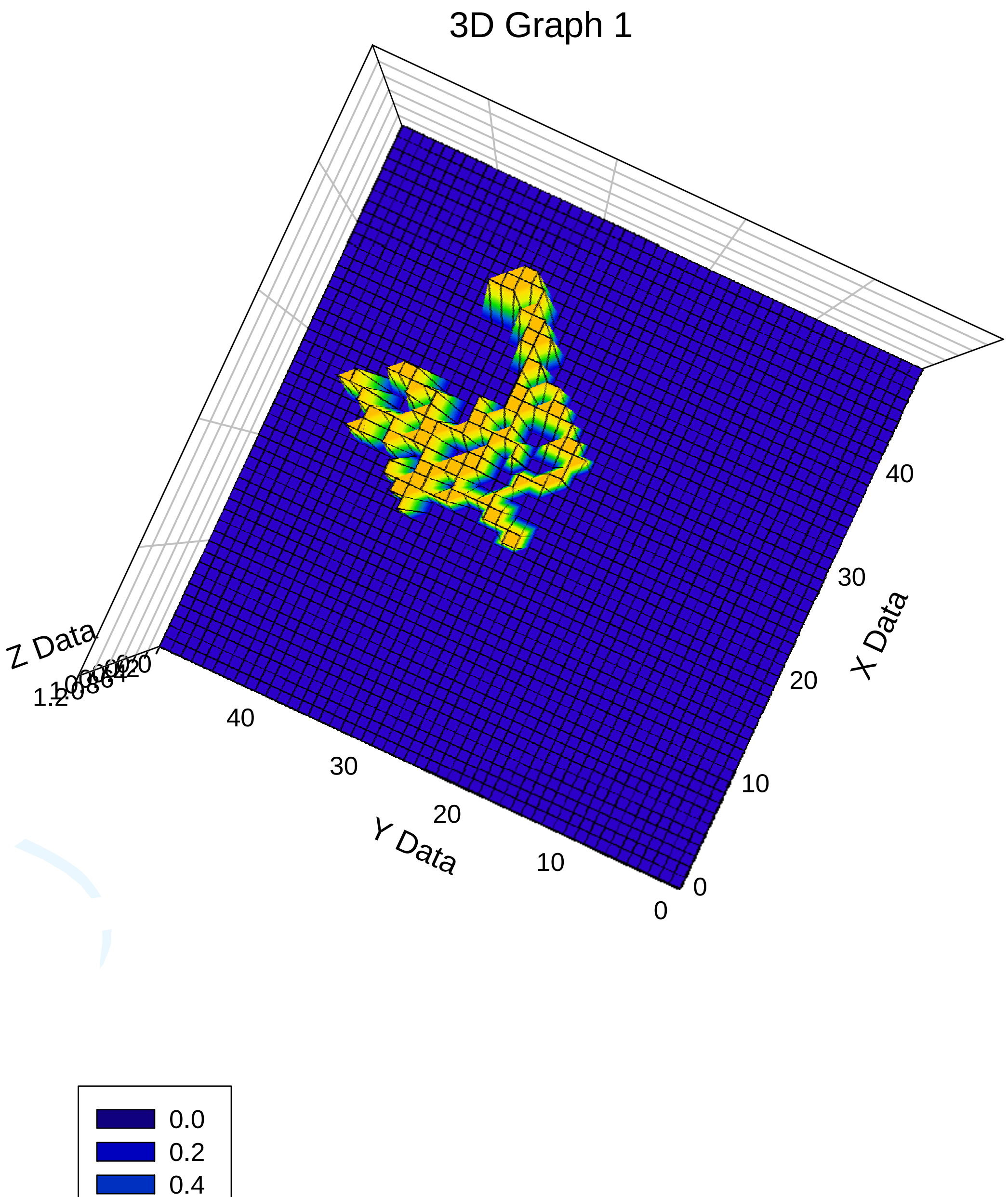
For best visualization plot 1 for sites on structure, 0 otherwise.

To normalize expectations, first look for connected structures on random q(x) distributions, then compare with q(x) distributions in CP^{N-1} Monte Carlo configurations:

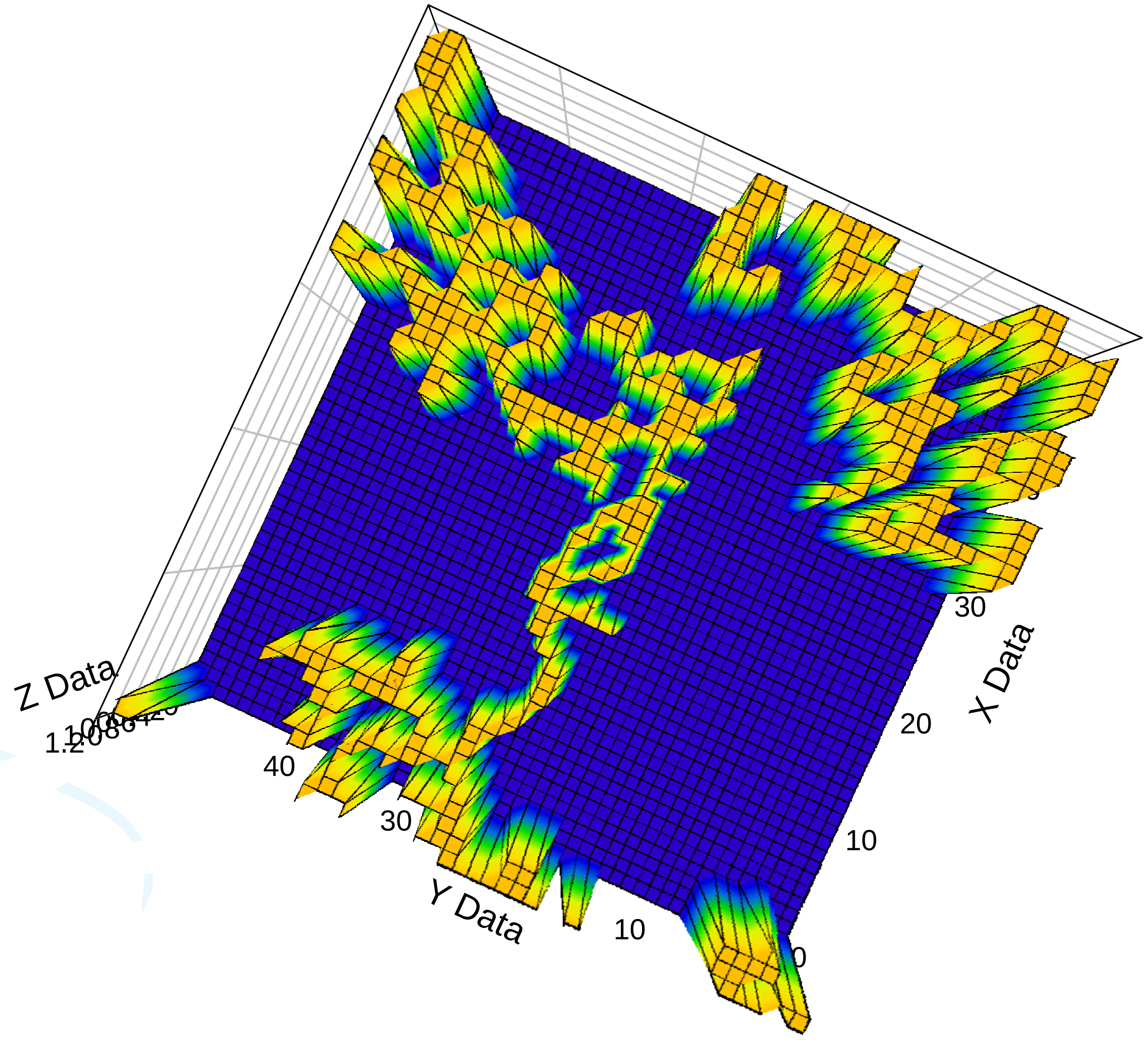
Largest coherent structure from a random TC distribution:



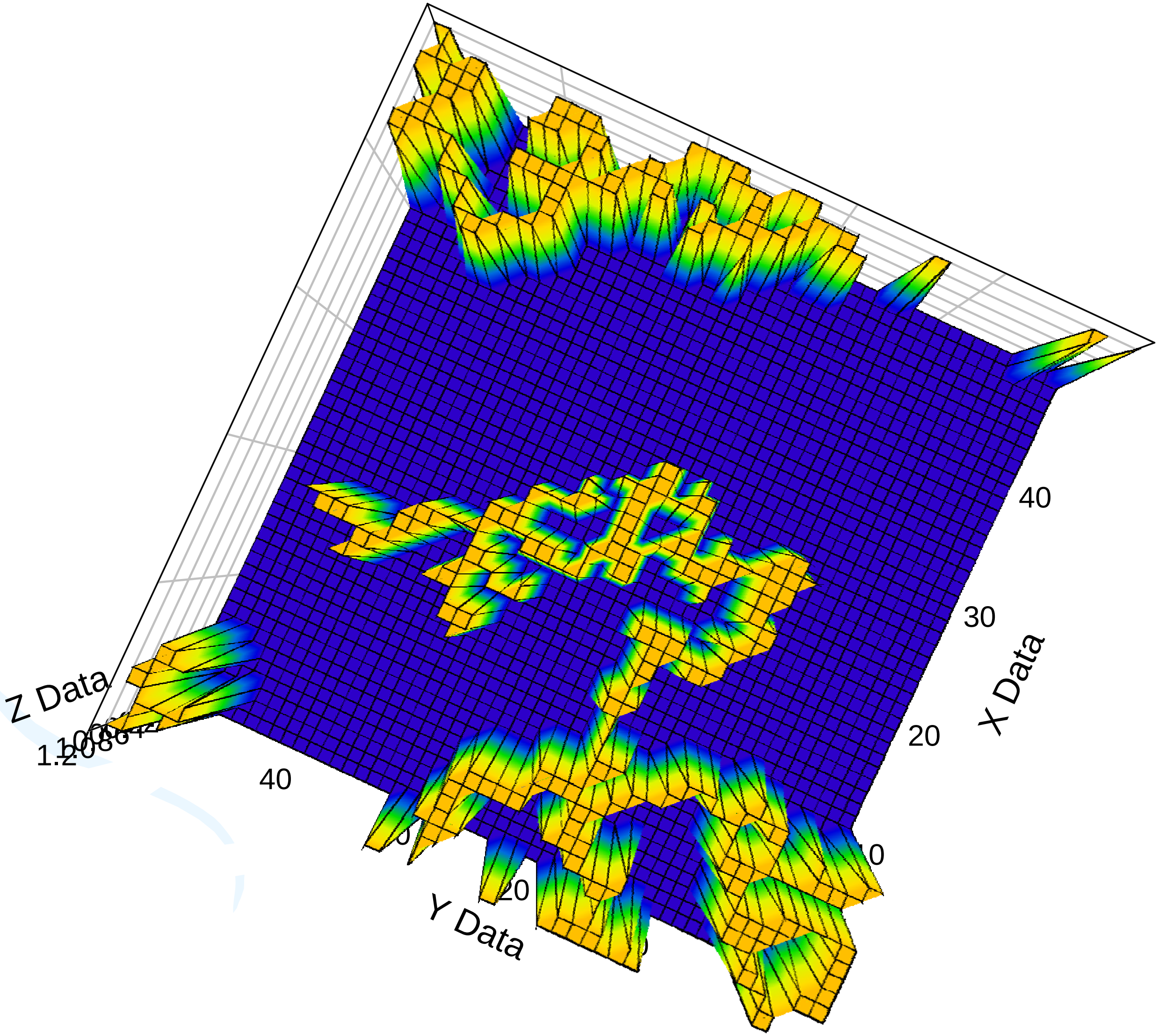
Another random TC distribution:



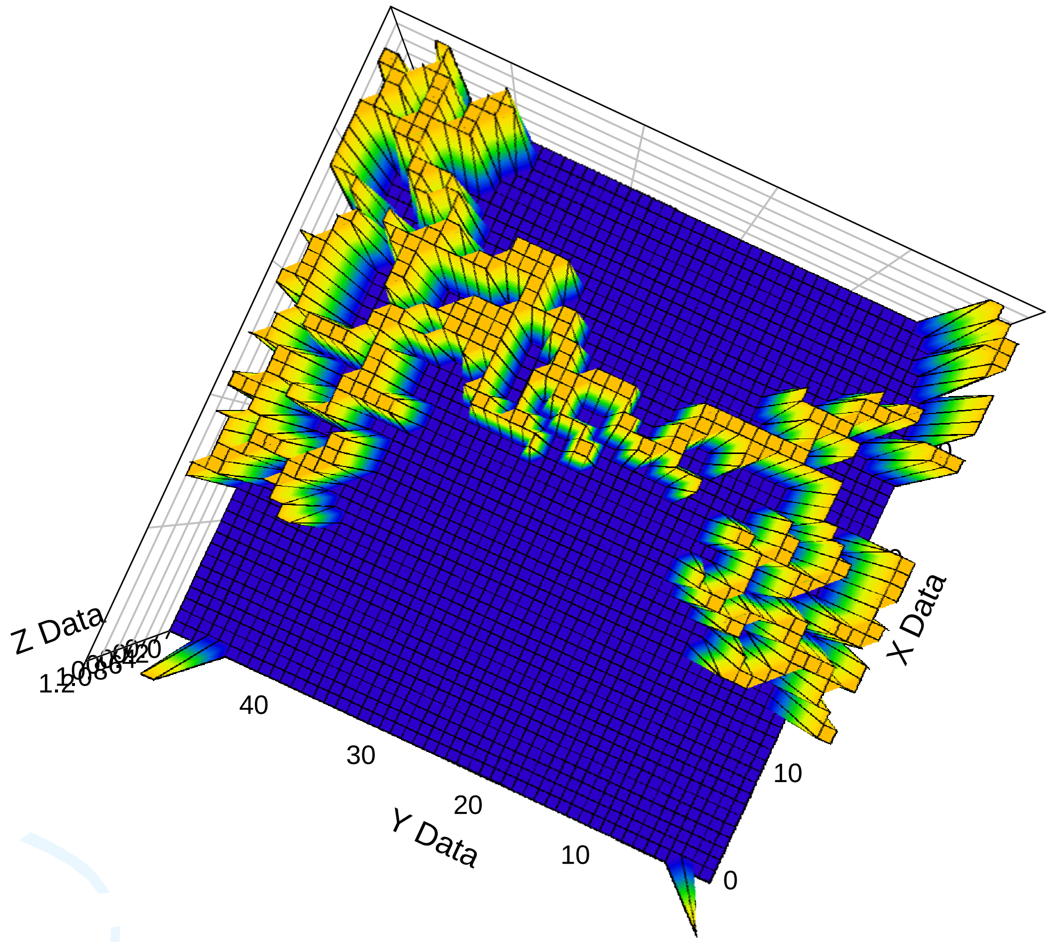
Coherent structure: CP^3 50×50 $\beta = 1.2$ $(\xi \approx 20)$



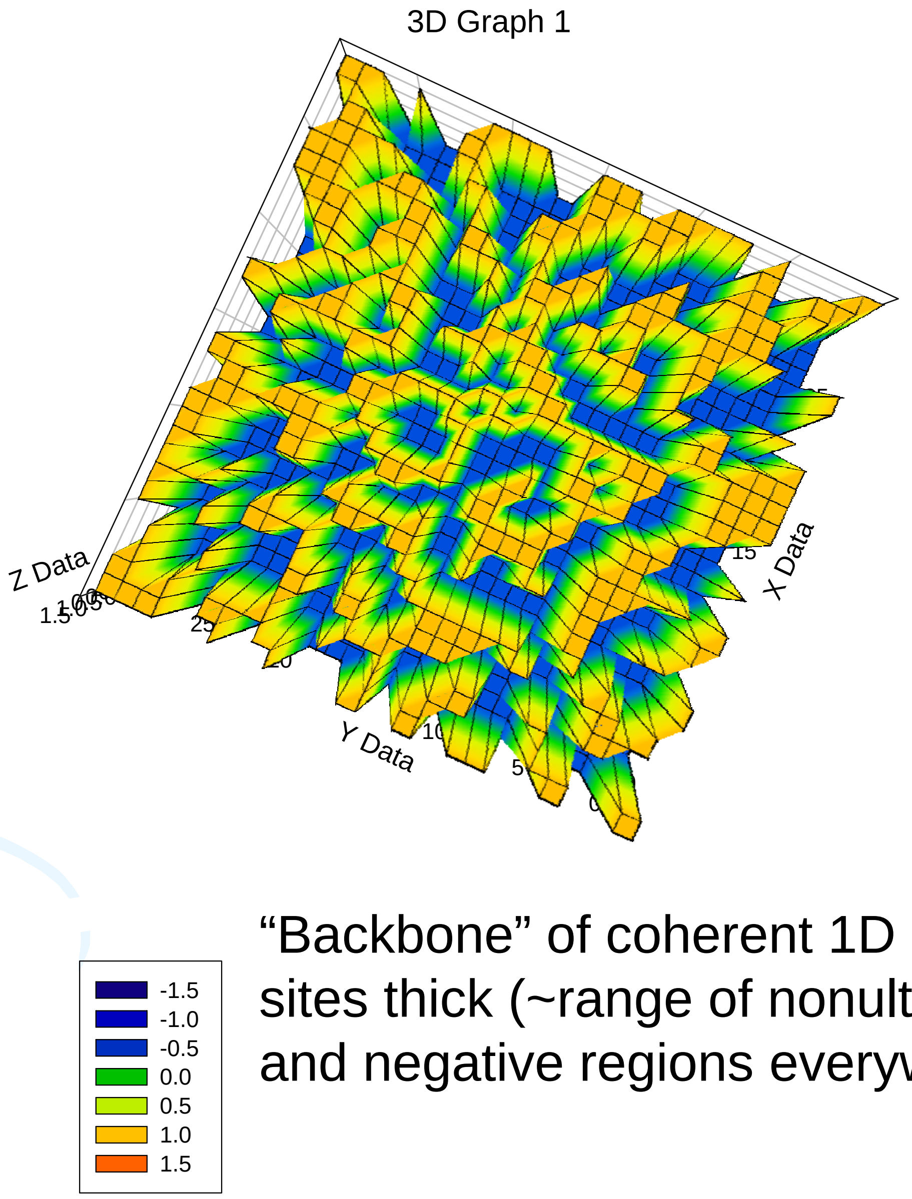
Another CP(3) configuration:



Still another CP(3) configuration:

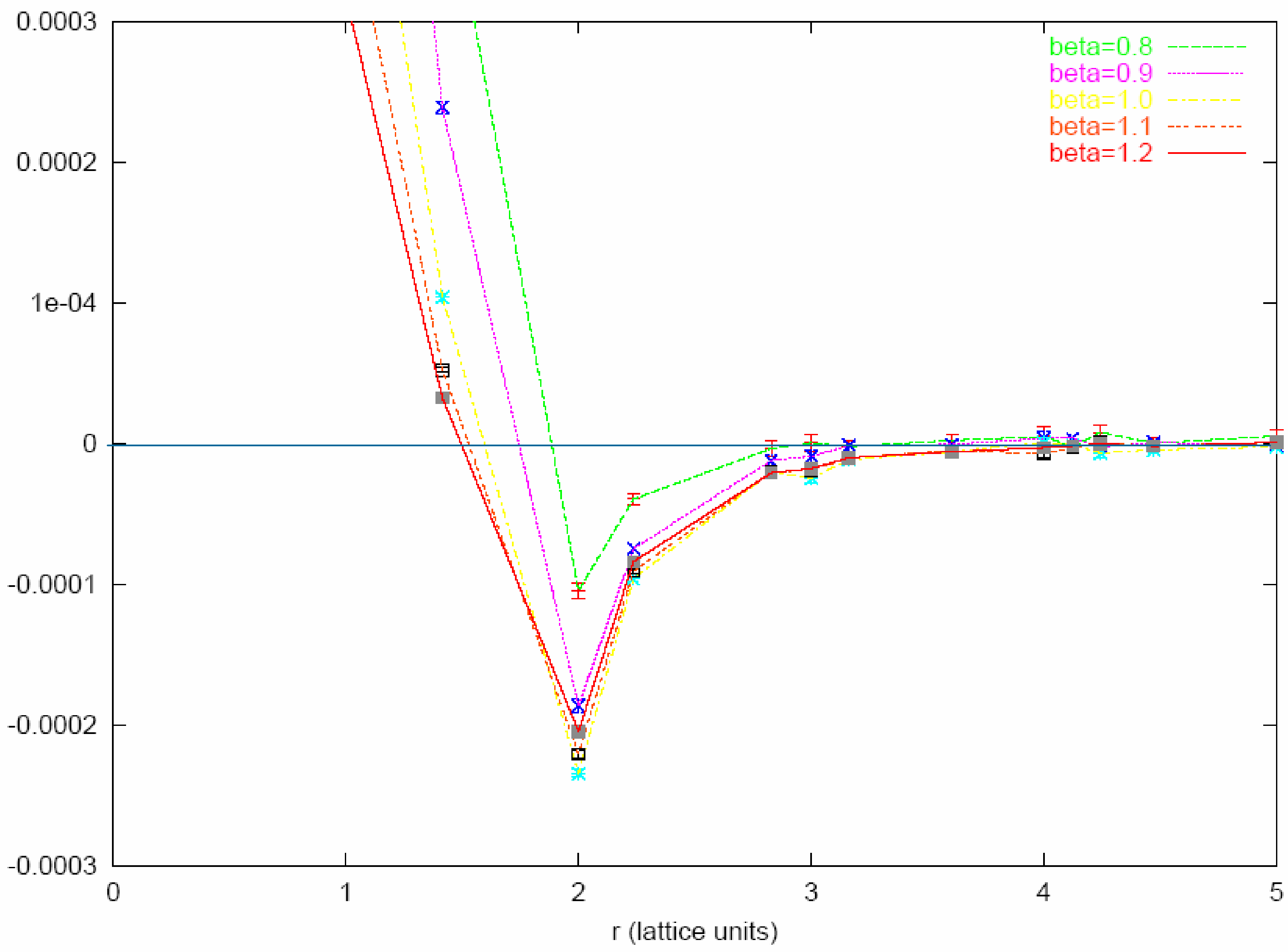


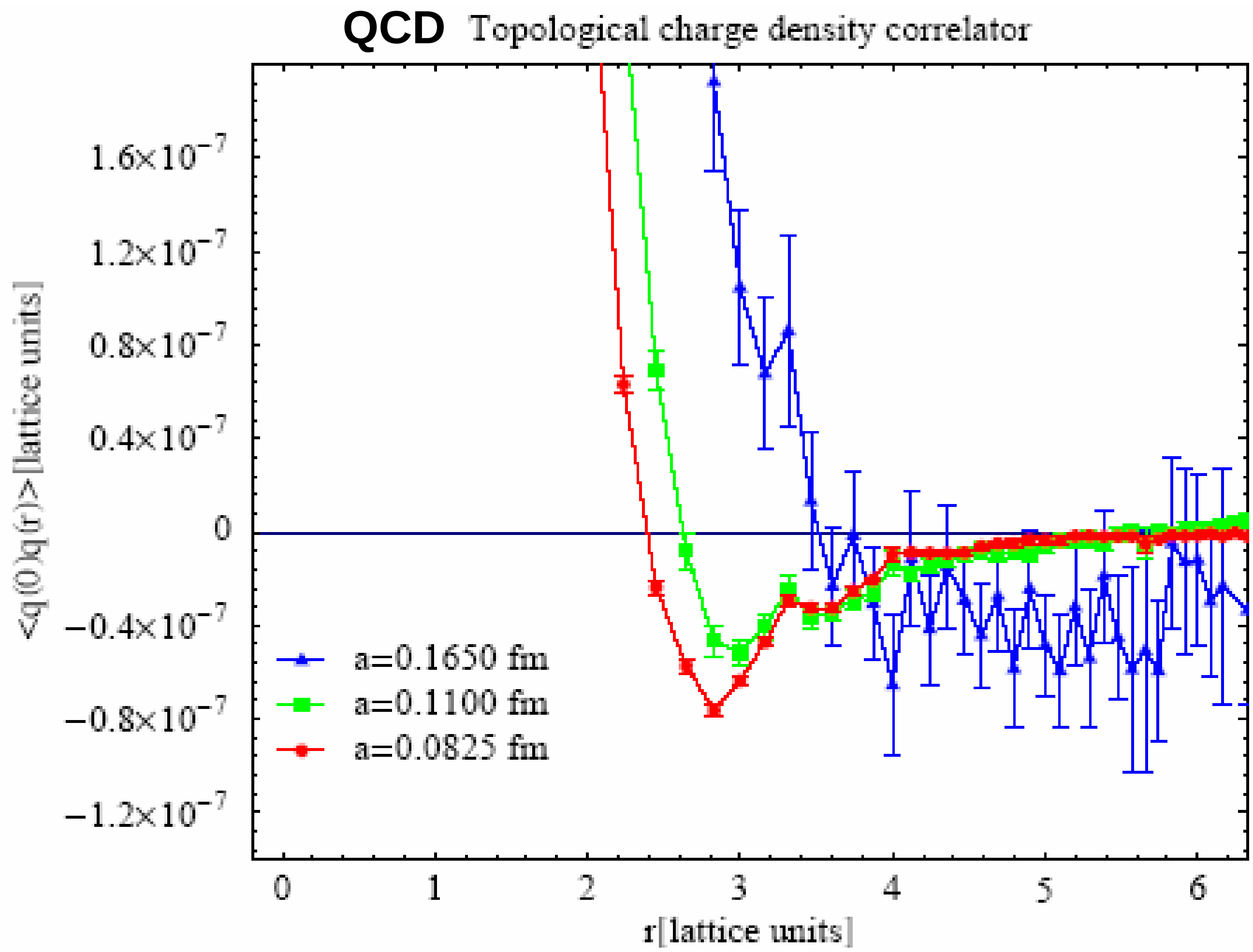
Plot sign(q(x)) for CP3 config:



"Backbone" of coherent 1D regions is only 1 to 2 sites thick (~range of nonultralocality). Positive and negative regions everywhere close.

Topological charge correlator for CP(3)




QCD Topological charge density correlator
1.6×10^-7
1.2×10^-7
0.8×10^-7
0.4×10^-7
0
−0.4×10^-7
−0.8×10^-7
−1.2×10^-7
<q(0)q(r)>[lattice units]
0
1
2
3
4
5
6
r[lattice units]
a=0.1650 fm
a=0.1100 fm
a=0.0825 fm

Summary of Monte Carlo results: In both 4D QCD and 2D CP^{N-1} (for N>4), topological charge comes in the form of extended membranes of codimension 1, with opposite sign sheets (or lines in CP^{N-1}) juxtaposed in dipole layers.

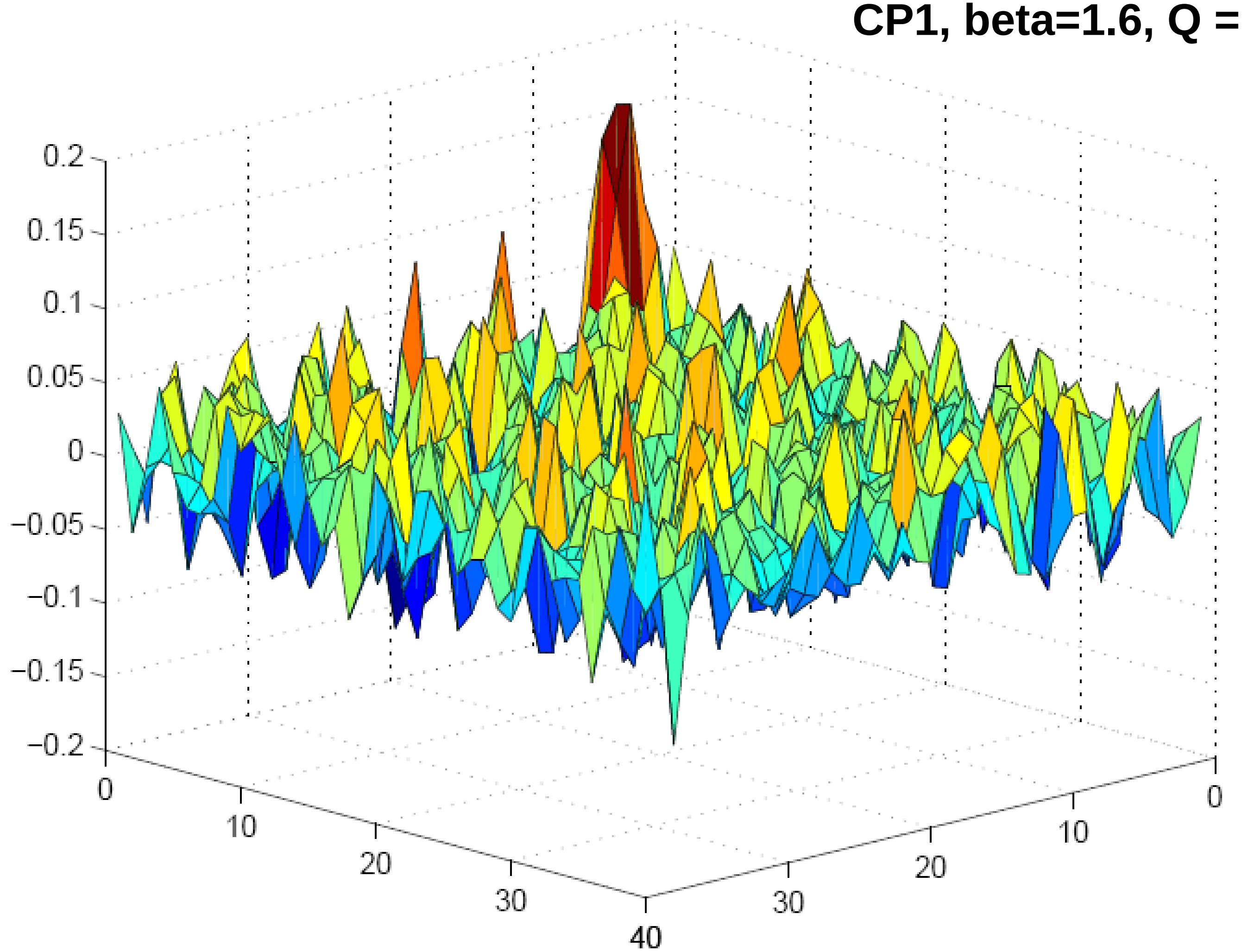
An interesting exception -- *small instantons in CP1 and CP2* (Y. Lian and HT, hep-lat/0607026):

- CP1 and CP2 are dominated by small instantons (Luscher, 1982) with radii of order a (so correlator remains negative for nonzero separation in continuum limit). Small instantons are easily seen with overlap q(x).
- CP3 is on the edge of the instanton melting point – has some instantons but mostly coherent line excitations.
- CP4 and higher have no instantons – only line excitations.

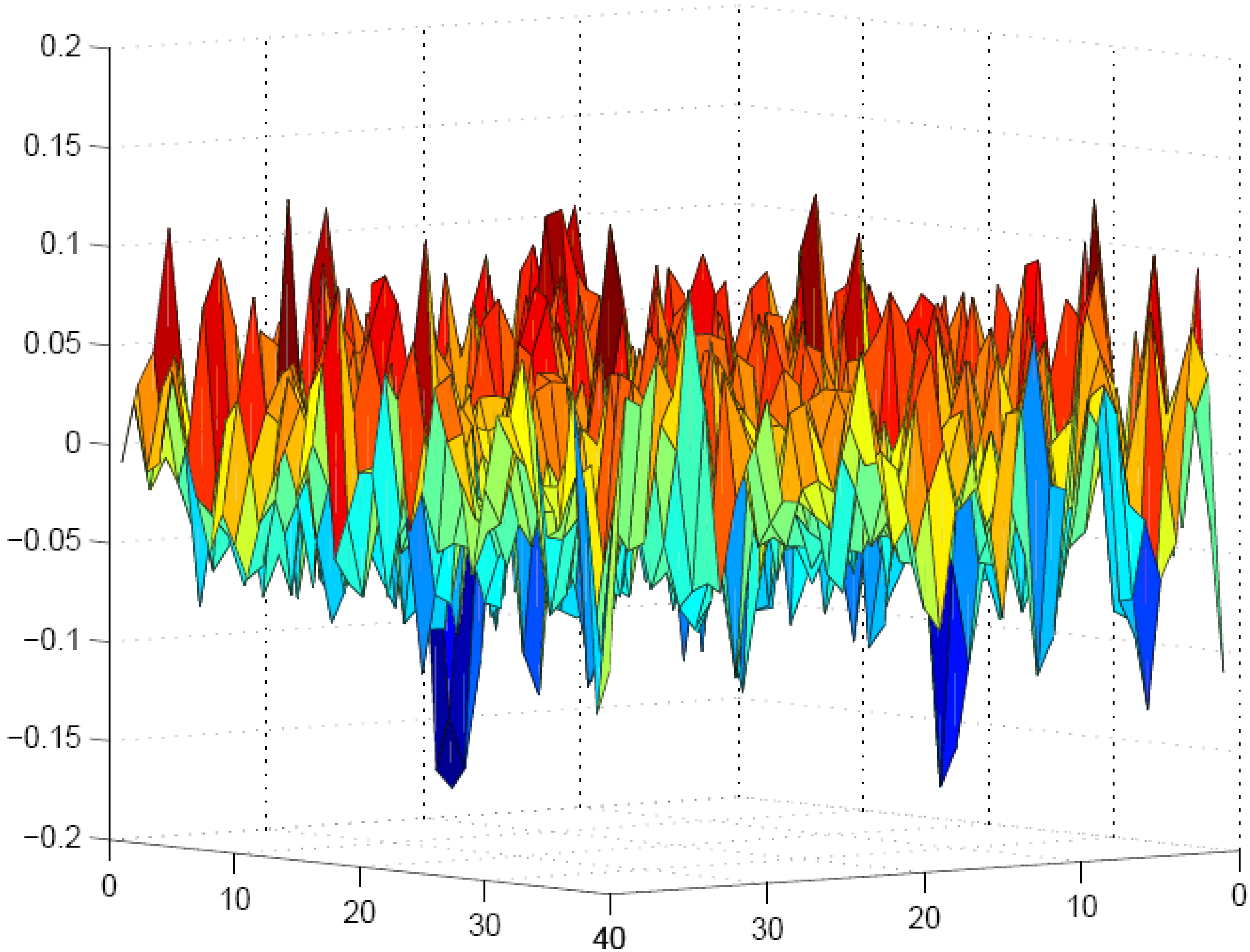
Crude estimate (lower bound) for instanton melting point = "tipping point" of integral over instanton size in semiclassical calculation (Luscher, 82):

$\Longrightarrow$ for CP^{N-1}, $N_{crit}=2$ for QCD, $N_{crit}=12/11$

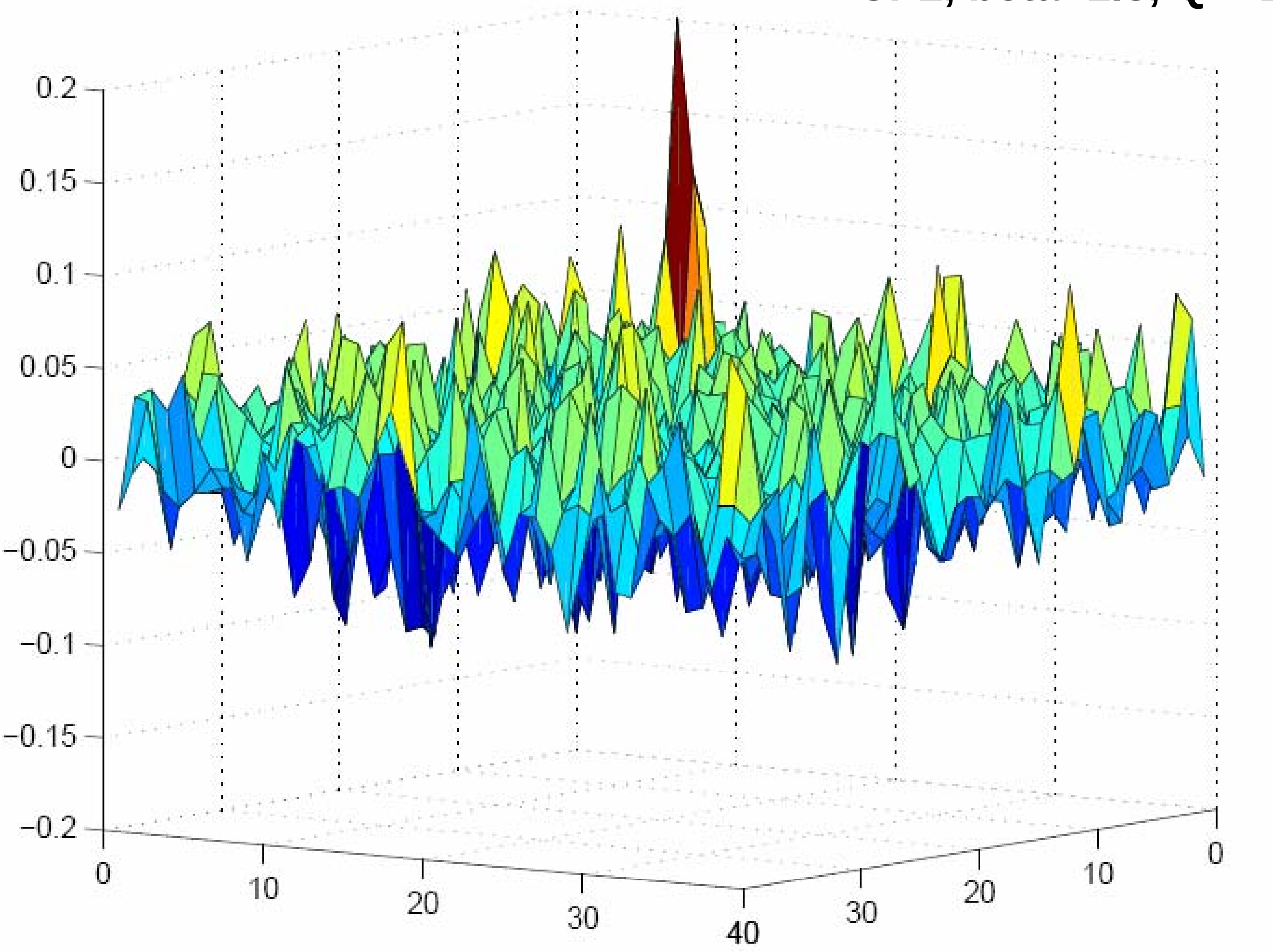
CP1, beta=1.6, Q = 1



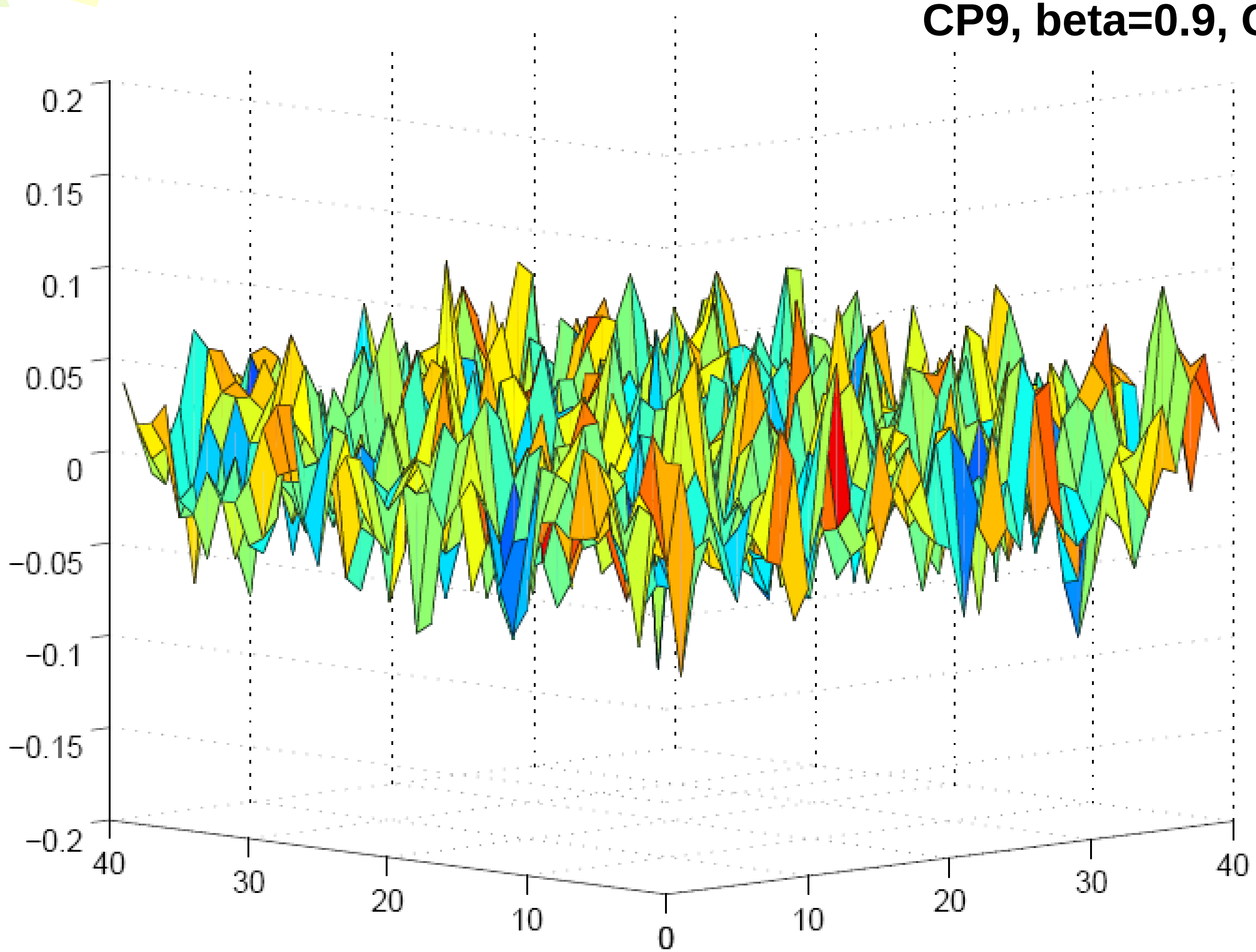
CP1, beta=1.6, Q = -2



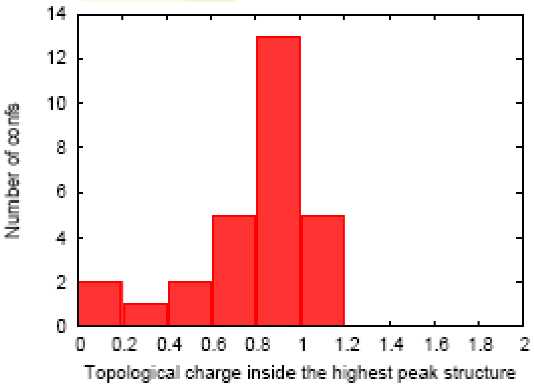
CP2, beta=1.8, Q = 1



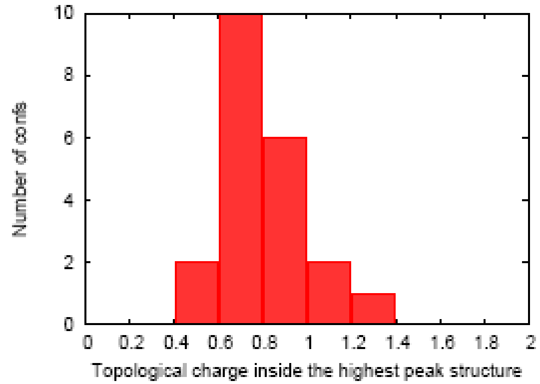
CP9, beta=0.9, Q = -1



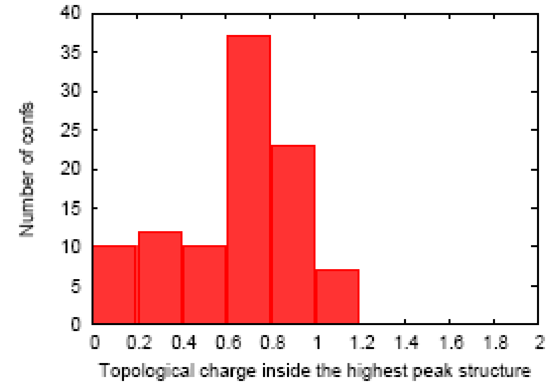
Plot integrated q(x) in highest structure (within 2 sites of highest peak) for all configs with Q = +-1



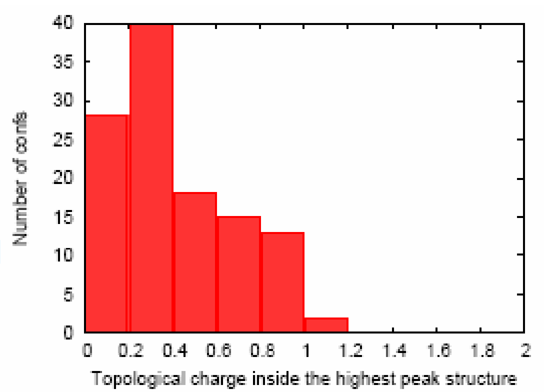
cp1



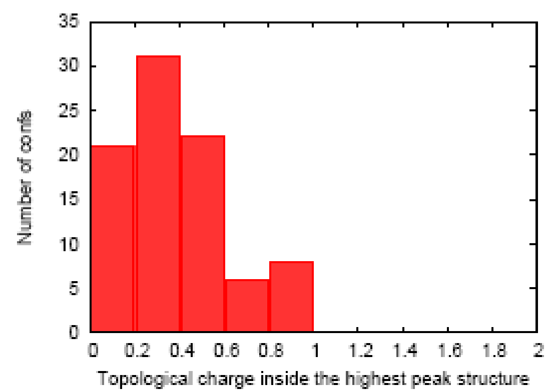
cp2



cp3

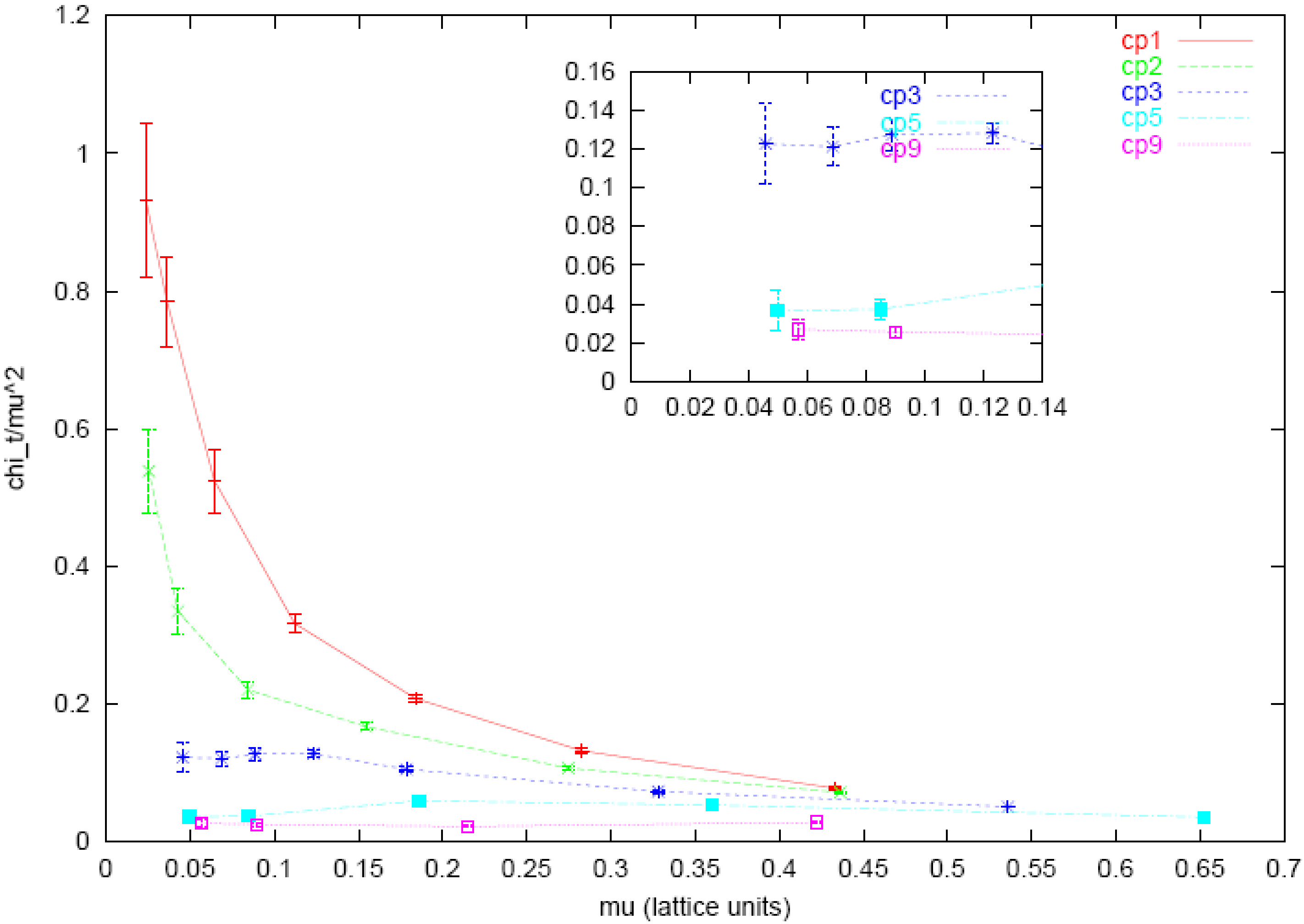


cp5



cp9

(End of digression on melting instantons.)


chi_t/mu^2
mu (lattice units)
cp1
cp2
cp3
cp5
cp9

Fundamental new development in QCD: ADS/CFT duality ("holography")

- 4D QCD ≈IIA String Theory in a 5D black hole metric.

$$R_4 \times S_1 \times R_5 \to R_4 \times D \times S_4$$

5D Chern-Simons term → 4D θ term

Among other things, ADS/CFT confirms Witten's (1979) large-N_c view of topological charge--Instantons "melt" and are replaced by (Witten, PRL 98):

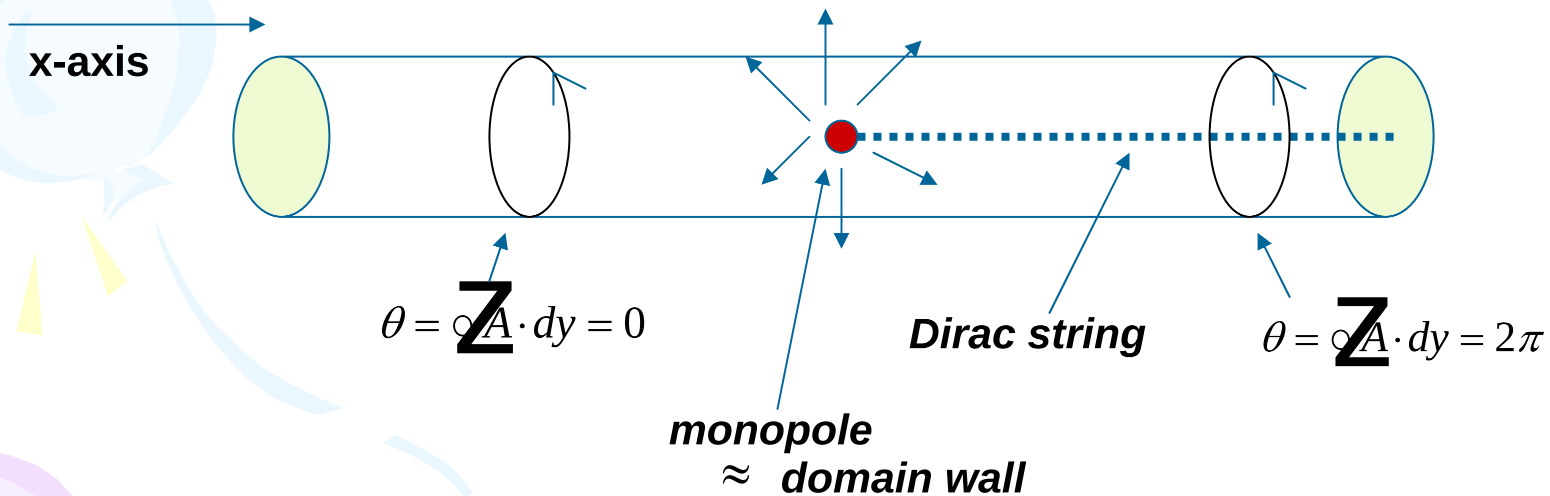
- Multiple vacuum states ("k-vacua") with $\theta_{\text{eff}} = \theta + 2\pi k$
- Local k-vacua separated by domain wall = membrane
- Domain wall = fundamental D6-brane of IIA string wrapped around S_4
- θ = Wilson line around D=disk with BH singularity at center ~ Aharonov-Bohm phase around "Dirac string"
- k=integer is a Dirac-type quantization of 6-brane charge
- TC is dual to Ramond-Ramond charge in string theory.

Holographic view of domain wall in CP(N-1):

In Witten's brane construction, 4D Yang-Mills is viewed from 6 dimensions = R_4 x D, where D = S_1 x radial coordinate of black-hole metric. Radius of S_1 is an ultraviolet cutoff, analogous to lattice spacing.

Analog in (1+1)-D is (3+1)-dimensional solid cylinder.

(1+1)-D case is equivalent to Laughlin's gedankenexperiment for topological understanding of integer quantum Hall effect. **(Corbino disk)**



Longitudinal component of monopole field (B) is dual to topological charge (= longitudinal E field) in CP(N-1) model.

$\Rightarrow$ **Domain wall = dipole layer of topological charge!**

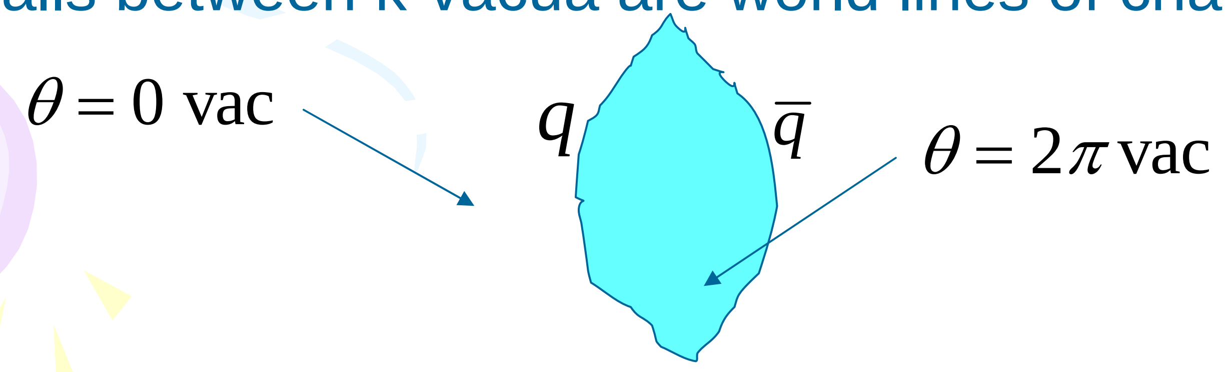
What are coherent sheets of TC in QCD? Are they D-branes?

The ADS/CFT holographic view of topological charge in the QCD vacuum has an analog in 2D U(1) theories:

--Multiple discrete k-vacua characterized by an effective value of θ which differs from the θ in the action by integer multiples of 2π.

-- Interpretation of effective θ similar to Coleman's discussion of 2D massive Schwinger model (Luscher (1978), Witten (1979,1998)), where $\theta =$ background E field.

In 2D U(1) models (CP(N-1) or Schwinger model): Domain walls between k-vacua are world lines of charged particles:



Precise analogy between U(1) in 2D and SU(N) in 4D (Luscher, 1978):

◆ **Identify Chern-Simons currents for the two theories.**

$$A_\mu \quad\rightarrow\quad A_{\mu\nu\sigma} \equiv -Tr\left[A_\mu A_\nu A_\sigma + \frac{3}{2} A_{[\mu}\partial_\nu A_{\sigma]}\right]$$

$$j_\mu^{CS} = \varepsilon_{\mu\nu} A_\nu \quad\rightarrow\quad j_\mu^{CS} = \varepsilon_{\mu\nu\sigma\tau} A_{\nu\sigma\tau}$$

$$Q = \partial_\mu j_\mu^{CS} \quad\rightarrow\quad Q = \partial_\mu j_\mu^{CS}$$

Wilson line → integral over 3-surface ("Wilson bag")

charged particle → charged membrane

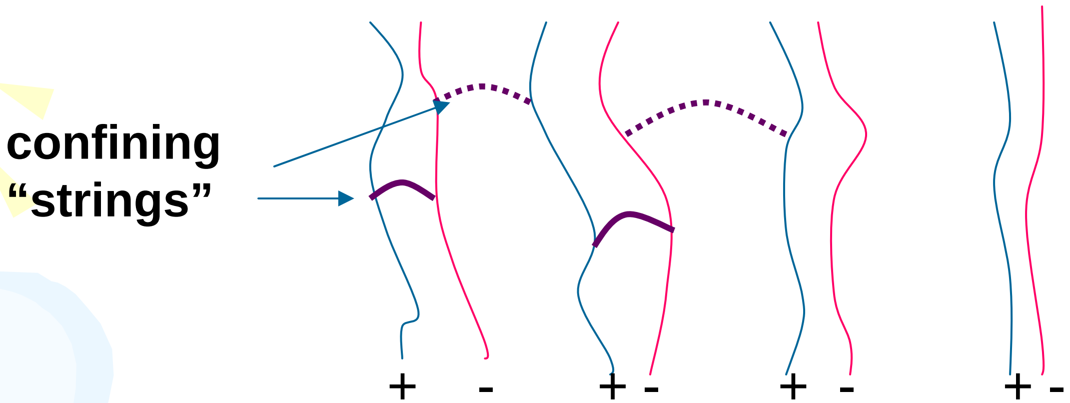
(= domain wall) (= domain wall)

In both cases, CS current correlator has massless pole $\sim 1/q^2$

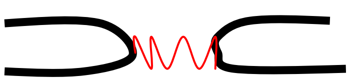
This analogy suggests that the coherent 1D structures in CP^{N-1} are charged particle world lines, and the 3D coherent structures in QCD are Wilson bags=excitation of Chern-Simons tensor on a 3-surface.

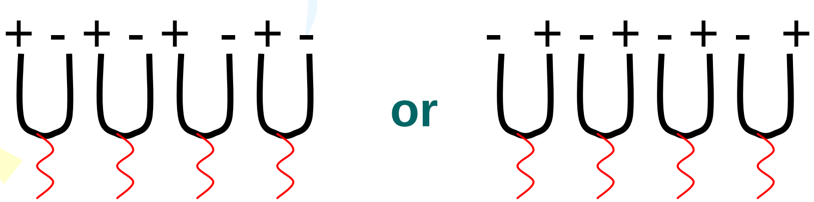
The emerging picture -- A “laminated” vacuum:

❖ **Alternating sign sheets (or lines) of topological charge:**



Possible dynamics of vacuum lamination in CP^{N-1}:

- **Spectrum consists of nonsinglet and singlet z-zbar pairs.**
- **$M_{singlet} > M_{nonsinglet}$ due to annihilation diagrams:** 
- **singlet pairs pop out of vacuum, but they can propagate farther by forming nonsinglet pairs with members of neighboring singlet pairs:**



Two degenerate vacua with *<u>topological order</u> (ala Wen and Zee in quantum hall eff.)*

Conformal field theory between the branes:

CP^{N-1} in Lorentz gauge $\partial_\mu A_\mu = 0$

$$\Rightarrow \quad A_\mu = \varepsilon_{\mu\nu}\partial_\nu \Phi$$

$$J_\mu^{CS} = \partial_\mu \Phi$$

$$\nabla^2 \Phi = 2\pi q(x)$$

Thickness of coherent lines of q(x) is of order a $\rightarrow 0$ in continuum.

Consider an idealized brane vacuum configuration where q(x) is confined to one-dimensional subspaces (or zero-dimensional for small instantons).

In voids between branes, $\nabla^2 \Phi = 0$ so $\Phi(x_1, x_2)$ can be written as the real part of a holomorphic fcn of $z=x_1+ix_2$

$$\Phi(x_1, x_2) = \varphi(z) + \varphi(\bar{z})$$

-- $\varphi(z)$ has branch cuts at branes (and/or poles at small instantons)

$\varphi(z) \overset{?}{\sim}$ string coordinate for Coulomb flux tube between charged particles (= Dbranes = string endpoints) !

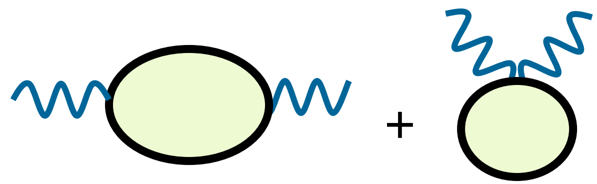
- **Note that singlet pairs must all polarize in the same direction to form nonsinglet mesons with nearest neighbors.**

$\Rightarrow$ **A "dimerization" (lamination) of the vacuum. Similar to Peierls transition in one-dimensional chain of atoms .**

Like antiferromagnetic order, but not tied to even-odd sublattice (hence topological)

This mechanism is also compatible with long range behavior of large N solution :

Vacuum polarization tensor $\Delta^{-1}_{\mu\nu} =$



$$\Delta_{\mu\nu}(q) \underset{q^2 << M^2}{\longrightarrow} \frac{2\pi}{N}\left[\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\left(\frac{6M^2}{q^2} + \frac{3}{5} + O\left(\frac{q^2}{M^2}\right)\right)\right]$$

In large N, singlet pairs are treated individually, but with <u>massive</u> z-propagators.

dimerization $\leftrightarrow$ **Spontaneous generation of constituent z-mass**

Topological Charge Correlators from CFT

Static dilute brane approximation $M^2 >> q^2$ (in QCD, large ps glueball mass)

- **Effective theory with z's integrated out:** $S \to -\frac{1}{4} F_{\mu\nu}^2$
- **OPE for Chern-Simons current correlator:**

$$\left\langle J_\mu^{CS}(x) J_\mu^{CS}(0) \right\rangle \sim C_{1\mu\nu}(x) + C_{2\mu\nu}(x) \left\langle F^2(0) \right\rangle + \ldots$$

Form of OPE coefficients completely determined up to 2 overall constants by CFT arguments:

$$C_{1\mu\nu}(x) = c_1 \frac{x^2 \delta_{\mu\nu} - 2x_\mu x_\nu}{(x^2)^2} \qquad C_{2\mu\nu}(x) = c_2 \left(\delta_{\mu\nu} \ln x^2 + \frac{2x_\mu x_\nu}{x^2} \right)$$

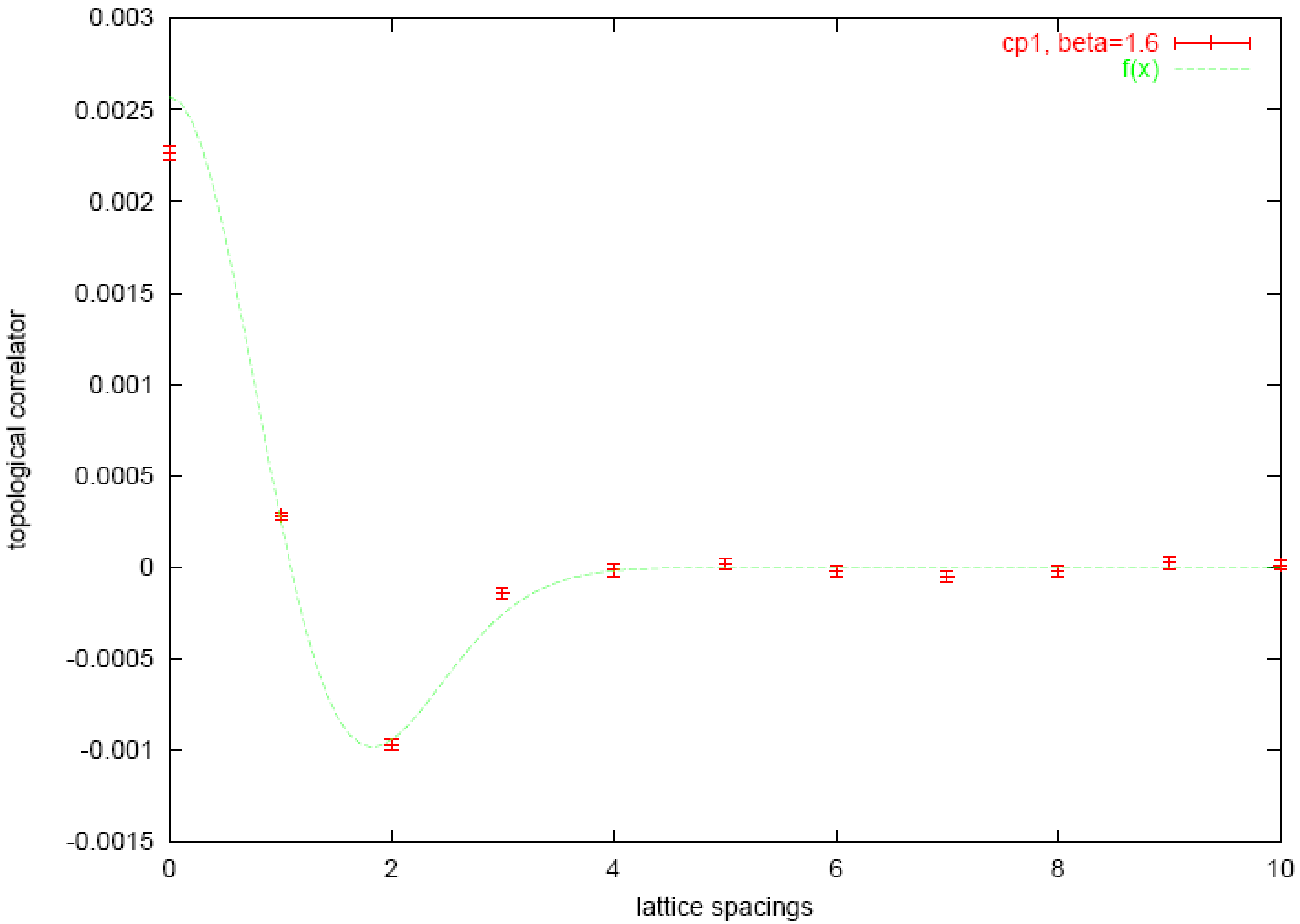
Result for TC correlator is sum of contact terms:

$$G(t) = \int dx \left\langle q(x,t) q(0,0) \right\rangle = -c_1 \delta''(t) + c_2 \delta(t) \ , \qquad c_2 = \chi_t$$

Gives a good fit to all lattice correlators using $\delta(t) \to \frac{1}{d\sqrt{\pi}} \exp\left(-\frac{t^2}{d^2} \right)$

With $d \approx 1.5$ **lattice spacings, essentially independent of beta.**

Fit correlator to



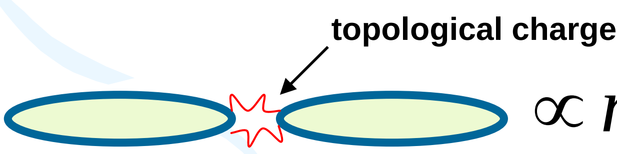
SUSY Relics in QCD (Armony, Shifman, and Veneziano (2002)):

- Holographic projection of orientifold compactification of string theory predicts "planar (large N) equivalence" between $\mathcal{N}=1$ SUSY YM and ordinary 1-flavor QCD.

- A "SUSY relic" prediction that can be tested by Monte Carlo: Degeneracy between the scalar and pseudoscalar mesons in 1-flavor QCD (they belong to the same WZ multipltet in SUSY YM chiral Lagrangian (Veneziano, Yankielowicz)).

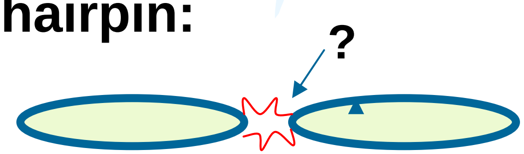
- Prediction is tested using MC results for scalar and pseudoscalar valence and hairpin diagrams. (N. Isgur and HT, PRD (2001))

pseudoscalar hairpin (quenched):



$$\propto m_0^2 \propto \chi_t \qquad m_{\eta'}^2 = m_\pi^2 + m_0^2$$

scalar hairpin:

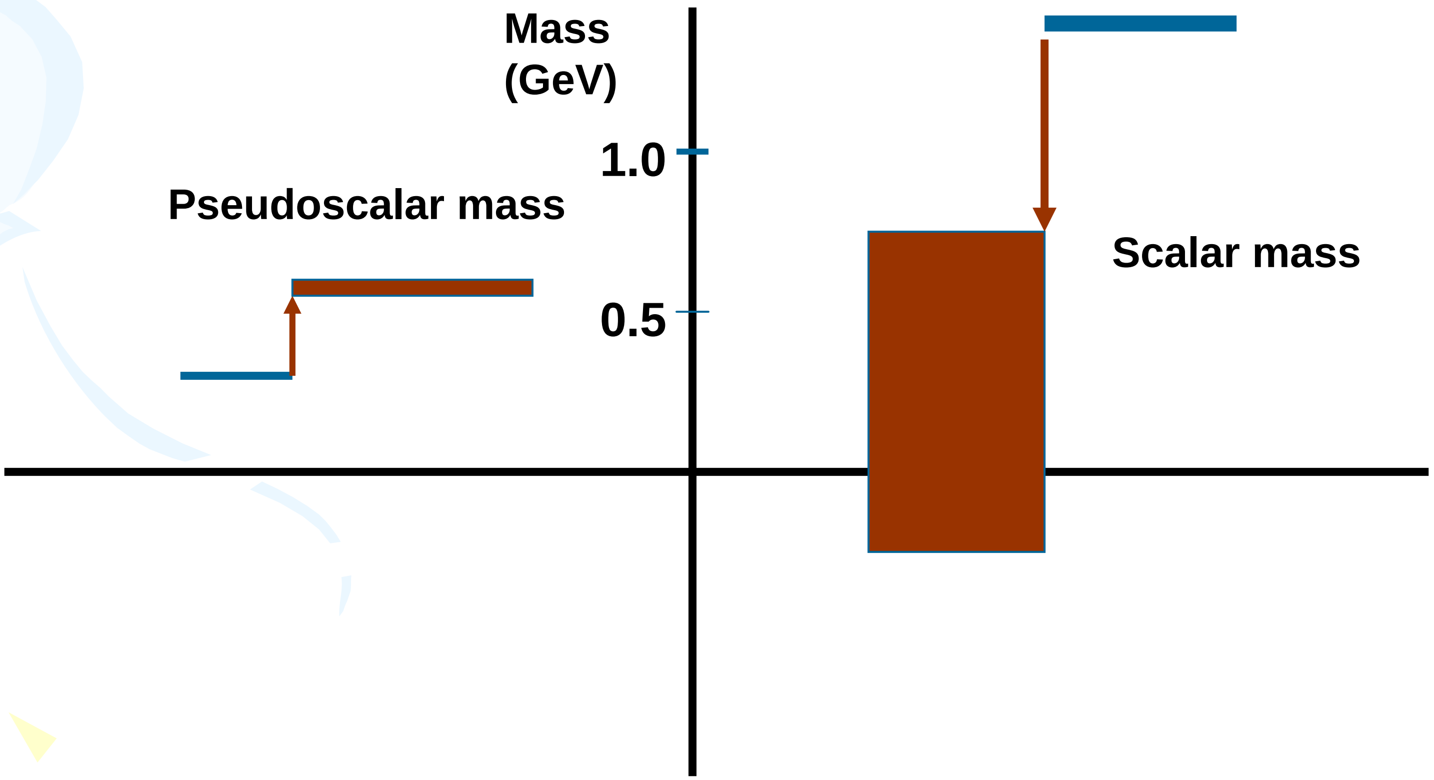


$$-\ \text{const.} \quad \propto m_{sc}^2 \qquad m_\sigma^2 = m_{a_0}^2 + m_{sc}^2$$

Test of scalar-pseudoscalar degeneracy in 1-flavor QCD:

(with Patrick Keith-Hynes)

(valence mass)^2	(hairpin mass shift)^2	(total mass)^2
$+[315(6)]^2$	$+[407(11)]^2$	$+[515(13)]^2$
$+[1416(14)]^2$	$-[1350(90)]^2$	$+[427+249-756]^2$



Conclusions:

- Topological charge in QCD is ADS/CFT dual to Ramond-Ramond charge in string theory. As such it provides a central focus on the string theory/holography aspects of gauge theory.

- Interpretation of observed sheets of topological charge as Dbranes (as suggested by Witten's brane construction of QCD) seems plausible and fits into a very appealing theoretical framework.

- CP^{N-1} sigma models capture many of the essential aspects of QCD in a computationally and theoretically simpler context. MC results in these models provide strong support for the Dbrane vacuum scenario.

- Dbrane vacuum may suggest connections between XSB & confinement, chiral anomaly & conformal anomaly, …

- SUSY relics are another interesting (and presumably related) implication of string/gauge holography.

Summary of Monte Carlo results: In both 4D QCD and 2D CP^{N-1} (for N>4), topological charge comes in the form of extended membranes of codimension 1, with opposite sign sheets (or lines in CP^{N-1}) juxtaposed in dipole layers.

An interesting exception -- *small instantons in CP1 and CP2* (Y. Lian and HT, hep-lat/0607026)

Conformal field theory between the branes:

CP^{N-1} in Lorentz gauge $\partial_\mu A_\mu = 0$

$$\Rightarrow \quad A_\mu = \varepsilon_{\mu\nu}\partial_\nu \Phi$$

$$J_\mu^{CS} = \partial_\mu \Phi$$

$$\nabla^2 \Phi = 2\pi q(x)$$

Thickness of coherent lines of q(x) is of order a $\to 0$ in continuum.

Consider an idealized brane vacuum configuration where q(x) is confined to one-dimensional subspaces (or zero-dimensional for small instantons).

In voids between branes, $\nabla^2\Phi = 0$ so $\Phi(x_1, x_2)$ can be written as the real part of a holomorphic fcn of $z=x_1+ix_2$

$$\Phi(x_1, x_2) = \varphi(z) + \varphi(\bar{z})$$

-- $\varphi(z)$ has branch cuts at branes (and/or poles at small instantons)

$\varphi(z) \overset{?}{\sim}$ string coordinate for Coulomb flux tube between charged particles (= Dbranes = string endpoints) !

The emerging picture -- A “laminated” vacuum:

- Alternating sign sheets (or lines) of topological charge: