Flavor Twisting for Isovector Form Factors

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Flavor Twisted Boundary Conditions
and
Isovector Form Factors

- Quantized momentum and form factors
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- Twisted boundary conditions
Outline

Flavor Twisted Boundary Conditions and Isovector Form Factors

- Quantized momentum and form factors
- Twisted boundary conditions
- Kinematical effects
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- Quantized momentum and form factors
- Twisted boundary conditions
- Kinematical effects
- Dynamical effects
Limitations near $q = 0$

- Operator insertion method

$$\langle H'(p')|O|H(p)\rangle = \sum_j O_j f_j(q^2)$$
Limitations near $q = 0$

Operator insertion method

$$\langle H'(p')|\mathcal{O}|H(p)\rangle = \sum_j O_j f_j(q^2)$$

![Graph showing $f_j(q^2)$ vs $q^2$]
Limitations near $q = 0$

- Operator insertion method: 
  \[ \langle H'(p')|\mathcal{O}|H(p)\rangle = \sum_j O_j f_j(q^2) \]

![Graph showing $f_j(q^2)$ vs. $q^2$](image)

- $O_j \propto q$

- $L = 24a, a = 2\text{ GeV}^{-1}$: $q_{\text{min}} = 2\pi/L \sim 500\text{ MeV}$
Nucleon isovector form factor

\begin{itemize}
  \item Definition & Chiral expansion
  \[ F_2(q^2) = G_M^p(q^2) - G_M^n(q^2) = \mu^{\text{iso}} + f(q^2/4m_\pi^2) \]
\end{itemize}
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- Deviation
  \[ \Delta F_2(q^2) = \frac{F_2(q^2) - F_2(0)}{q^2 F'_2(0)} \]
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- **Deviation**
  \[ \Delta F_2(q^2) = \frac{F_2(q^2) - F_2(0)}{q^2 F_2'(0)} \]

- **Chiral corrections, recoil corrections, lattice point**
Twisted boundary conditions

\[ U^\dagger U = 1 \]  

global symmetry of action, e.g.  

\[ U = \exp i \theta^a T^a \]

TwBCs  

\[ \psi(x_i + L) = U\psi(x_i) \]
Twisted boundary conditions

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\text{global symmetry of action, e.g.} \quad U = \exp i \theta_i^a T_C^a

- TwBCs \quad \psi(x_i + L) = U \psi(x_i)
- \sum_x \overline{\psi}(\not{D} + m_Q)\psi \quad \text{single valued}
Twisted boundary conditions

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- TwBCs \( \psi(x_i + L) = U \psi(x_i) \)
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- Momentum modes \( p_i = 2\pi n_i/L + \theta_i/L \)
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- **TwBCs** \( \psi(x_i + L) = U \psi(x_i) \)
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\[ \tilde{\psi}(x) = \exp(-i \theta^a \cdot x T^a_C / L) \psi(x) \] periodic
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- **Rewrite**  \( \tilde{\psi}(x) = \exp(-i \theta^a \cdot x T^a_c / L)\psi(x) \) periodic
- \( \sum_x \overline{\tilde{\psi}}(\tilde{D} + m_Q)\tilde{\psi} \)
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\text{Rewrite} & \quad \tilde{\psi}(x) = \exp(-i \theta^a \cdot x T_C^a / L) \psi(x) \quad \text{periodic} \\
\sum_x \overline{\tilde{\psi}}(\tilde{\slashed{D}} + m_Q)\tilde{\psi} \\
\tilde{D}_\mu &= D_\mu + i B_\mu \quad \text{uniform gauge field}
\end{align*} \]
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- \[ B_\mu = (\theta^a_i T^a / L, 0) \] flavor charges
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Partial twisting \[ T_C^a \in SU(N|N)_{val} \in SU(N + M|N) \]
Hadrons pointlike in chiral EFTs
Field momenta of hadrons

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- Mesons $\tilde{D}_\mu \phi = D_\mu \phi + i [B_\mu, \phi]$

\[ E_{\pi^+} = \sqrt{m_\pi^2 + B_{\pi^+}^2}, \quad B_{\pi^+} = B_u - B_d \]

Sachrajda & Villadoro PLB 609
Field momenta of hadrons

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Sachrajda & Villadoro PLB 609

- Baryons $\tilde{D}_\mu B^{ijk} = D_\mu B^{ijk} + i (B^i_\mu + B^j_\mu + B^k_\mu) B^{ijk}$

$$E_p = M_p + \frac{B^2_p}{2M_p}, \quad B_p = 2B_u + B_d$$

Tiburzi PLB 617
Numerical investigations

Meson dispersion relations
Quenched de Divitiis, Petronzio & Tantalo PLB595

Dynamical partially twisted Flynn, Jüttner & Sachrajda PLB632
Flavor changing operators

- Twist flavors differently get momentum transfer!
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Lattice correlator

$$C(t, t') = \sum_{x, x'} \langle 0 | \hat{P}(x, t) \tilde{J}_5^+ (x', t') \tilde{N}(0, 0) | 0 \rangle$$
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Lattice correlator

$$C(t, t') = \sum_{x, x'} \langle 0 | \tilde{\mathcal{P}}(x, t) \tilde{J}_{5\mu}^+(x', t') \tilde{\mathcal{N}}(0, 0) | 0 \rangle$$

$$C(t, t') = \langle \mathcal{P}_{B_p}(t) J_{5\mu}^+(t') \mathcal{N}_{B_n}(0) \rangle$$
Flavor changing operators

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Lattice correlator

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C(t, t') = \sum_{x, x'} \langle 0 | \tilde{P}(x, t) \tilde{J}_{5\mu}^+ (x', t') \tilde{N}(0, 0) | 0 \rangle
\]

\[
C(t, t') = \langle P_{Bp}(t) J_{5\mu}^+ (t') N_{Bn}(0) \rangle
\]

- Momentum transfer \( q = B_p - B_n = B_{\pi^+} \)

Tiburzi PLB 617
Further numerical investigations

**Meson decay constants** Flynn, Jüttner & Sachrajda PLB632

\[ \langle \tilde{\pi}^+(0) | \tilde{J}_5^+ | 0 \rangle = i f_\pi B_{\pi^+} \]

\( f_\pi \) reliably extracted

\( |\Delta S| = 1 \) currents Guadagnoli, Mescia and Simula PRD73

\[ \langle \pi^-(p') | \bar{s}_\gamma_\mu u | K^0(p) \rangle \]

\( K \to \pi \) form factors

Systematics?
Flavor non-changing currents?
Isospin relations

- Flavor non-changing currents?
- Isospin rotation $\rightarrow$ isovector currents

\[
\langle p | \bar{u} \Gamma d | n \rangle = \langle p | \bar{u} \Gamma u | p \rangle - \langle n | \bar{u} \Gamma u | n \rangle
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Isospin relations

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- Arbitrary bilinears, e.g. $\Gamma = \gamma_5\gamma_\mu D_{\mu_1} \ldots D_{\mu_n}$
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- Calculate isospin change with TwBCs relate to isovector observables
- Arbitrary bilinears, e.g. $\Gamma = \gamma_5 \gamma \{ \mu D_{\mu_1} \ldots D_{\mu_n} \}$
- Vector current $\rightarrow$ electromagnetic current

$$\langle p \mid \bar{u} \gamma_{\mu} d \mid n \rangle = \langle p \mid J_{\mu}^{em} \mid p \rangle - \langle n \mid J_{\mu}^{em} \mid n \rangle$$
Dynamical effects due to twisting

- Exploited isospin breaking $B^u \neq B^d$ for kinematical gain
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- Isospin splittings with $m_u = m_d$

$$m_{\pi \pm}^2 - m_{\pi 0}^2 = \frac{m_\pi^2}{f^2_{\pi}} \left[ \mathcal{I}(0, m_{\pi}^2) - \mathcal{I}(B_{\pi^+}, m_{\pi}^2) \right]$$
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- Shifts $\sim 1\%$ for $m_{\pi} \sim 300$ MeV, 2.5 fm
Axial form factors in finite volume

• Special: besides tree-level, $q^2$ dependence arises from recoil terms that are competitive with two-loops
Axial form factors in finite volume

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- Axial radius and pseudoscalar form factor have negligible volume dependence.

\[
\langle \tilde{p}(0)|\tilde{J}_5^+|\tilde{n}(0)\rangle = \sigma \left[ G_A(0) - \frac{<r_A^2>}{6} B_{\pi^+}^2 \right] + B_{\pi^+} (B_{\pi^+} \cdot \sigma) \left[ \frac{g_A}{B_{\pi^+}^2 + m_\pi^2} + \frac{<r_A^2>}{3} \right]
\]
Axial form factors in finite volume

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- Ideal testing ground • TwBCs • Chiral physics
Take $B_2^u = B \neq 0$ and $J_{\mu=3}^+$

$$\langle \tilde{p}(0) | \tilde{J}_3^+ | \tilde{n}(0) \rangle = -\frac{iB}{2M} F_2(B^2) + L_{FV} + K_{FV}$$
Isovector magnetic moment

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- $L_{FV}$ ordinary FV modified with TwBCs Beane PRD70
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![Diagram with graphs and equations]
Isovector magnetic moment

Extracting $F_2(B^2)$: Volume Effects, $B^2$ resolving power

$q \sim 500\,\text{MeV} \rightarrow 25\,\text{MeV}$ in fixed volume
Comparison with background fields

- Study dependence of observables with continuous parameter: TwBCs $\theta$, BFs $A_\mu$
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  TwBCs different operator insertion, some propagators can be recycled
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- BFs isoscalar can be done time consumingly TwBCs ???
TwBCs produce continuous hadron momentum
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Flavor changing currents
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Further studies needed: EFT & Lattice
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