$I = 2$ $\pi\pi$ scattering length from two-pion wave function

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Reference: CP-PACS Collaboration PRD71 094504(2005)

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LHP 2006 @ Jefferson Lab.

Jul. 31 – Aug. 3

Contents

1. Introduction

Motivation

- Understanding of hadron dynamics based on lattice QCD

Strict test of Standard model requires comparison of theory and experiment. But one of main theoretical uncertainties is hadronic effect.

Especially $\pi\pi$ scattering case, many works employed effective theory such as chiral perturbation theory (ChPT) to estimate that effect.

In order to calculate the scattering based on QCD,
non-perturbative method is required.

$$\Downarrow$$

One of possible methods is lattice QCD.

- 'Dynamical' physical quantity

Most of lattice studies have focused on 'static' physical quantities, e.g. hadron spectrum.

$$\Downarrow$$

non-perturbative test of QCD

It is also important to study 'dynamical' physical quantities, e.g. scattering phase shift and decay width, beyond the static physical quantities.

- $I = 2$ $\pi\pi$ scattering is one of simple hadron scatterings.

Only scattering ($\pi\pi$) state exists in $I = 2$ system at low energy region.
In $I = 0$ or 1 system, not only scattering ($\pi\pi$) state but also unstable (σ or ρ) states exist.

- First step toward decays of hadrons

$I = 1$ Channel $\rho \to \pi\pi$

$I = 0$ Channel $\sigma \to \pi\pi$

$I = 0, 2$ Channel $K \to \pi\pi$ direct calculation

(Traditional calculation method is $K \to 0$ and $K \to \pi$ relate to $K \to \pi\pi$ with ChPT.)

Present status of $I = 2$ $\pi\pi$ scattering on lattice

Isospin $I = 2$ S-wave $\pi\pi$ Scattering amplitude

$$T(p) = \frac{16\pi E}{p}\frac{1}{2i}\left(e^{2i\delta(p)} - 1\right), \quad a_0 = \lim_{p\to 0} \delta(p)/p, \quad E = 2\sqrt{m_\pi^2 + p^2}$$

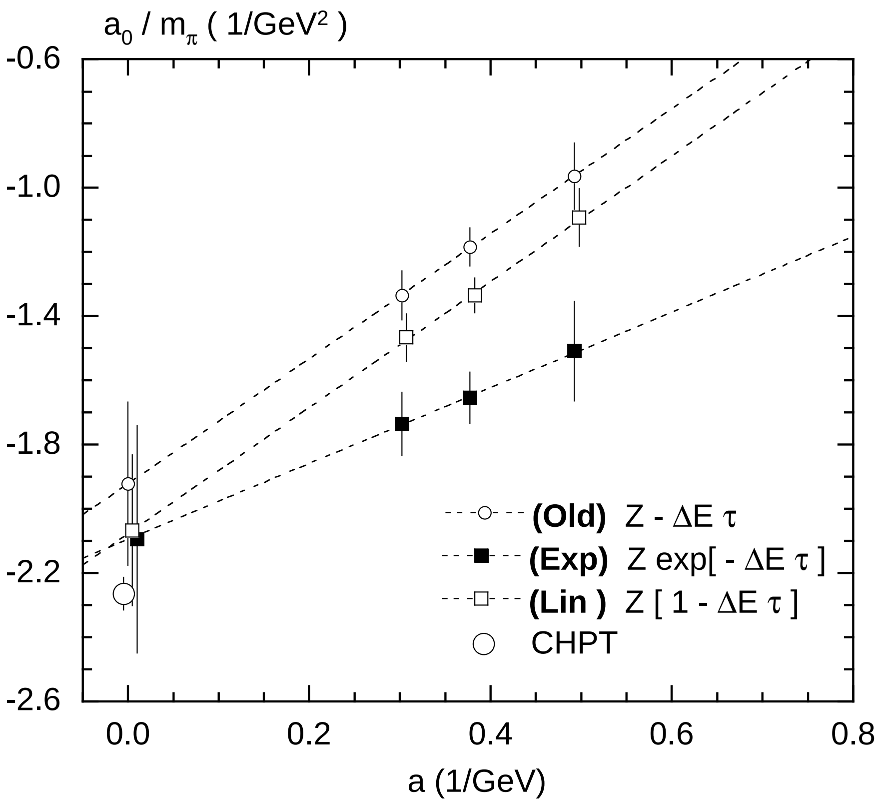
scattering length a_0 and scattering phase shift $\delta(p)$

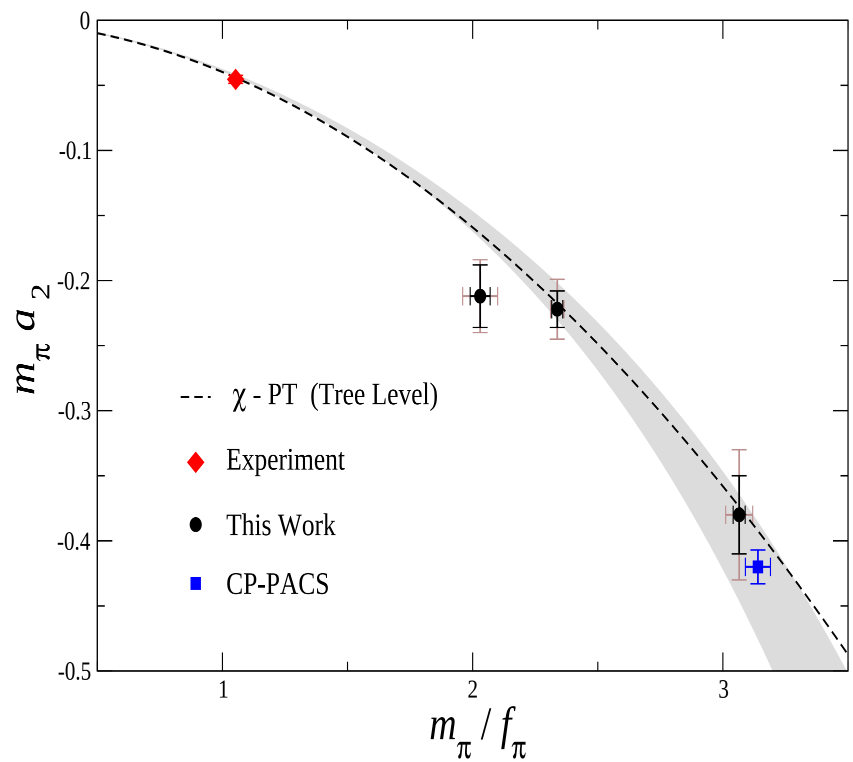
were calculated by many groups with finite volume method.

Lüscher, CMP105 153(1986)
NPB354 531(1991)

'92	Sharpe, Gupta and Kilcup
'93	Gupta, Patel and Sharpe
	Kuramashi *et al.*
'99	JLQCD Collaboration
'01	Liu, Zhang, Chen and Ma
'02	CP-PACS Collaboration
'03	BGR Collaboration
	Kim
'04	CP-PACS Collaboration
	Du, Meng, Miao and Liu
'05	CP-PACS Collaboration
	BGR Collaboration
	NPLQCD Collaboration

$I = 2$ $\pi\pi$ scattering length a_0

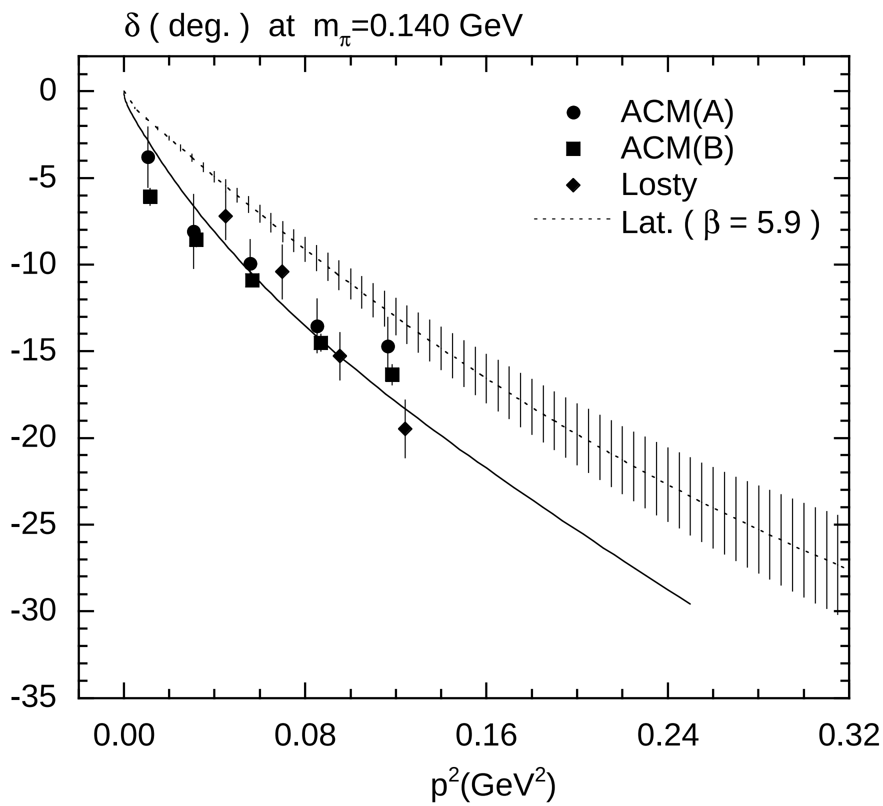




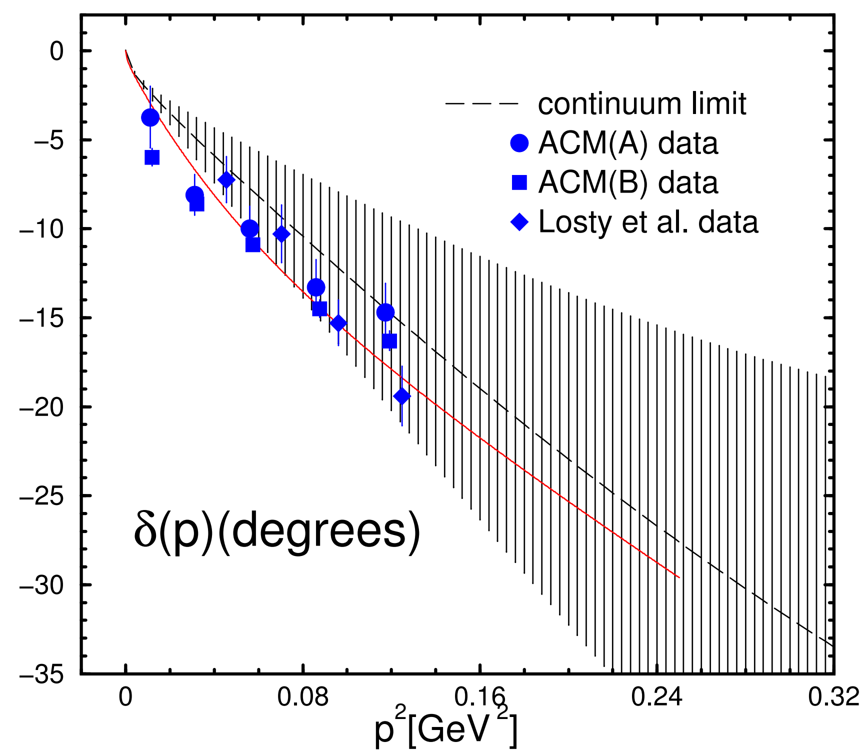
Quenched calculation
Wilson gauge + Wilson quark
$a \to 0$; $L^3 \approx (2\ \mathrm{fm})^3$
JLQCD PRD66 077501(2002)

$N_f = 2 + 1$ Staggered sea
Domain wall valence quark
$a \approx 0.125$ fm ; $L^3 \approx (2.5\ \mathrm{fm})^3$
NPLQCD PRD73 054503(2006)

$I = 2\ \pi\pi$ scattering phase shift $\delta(p)$

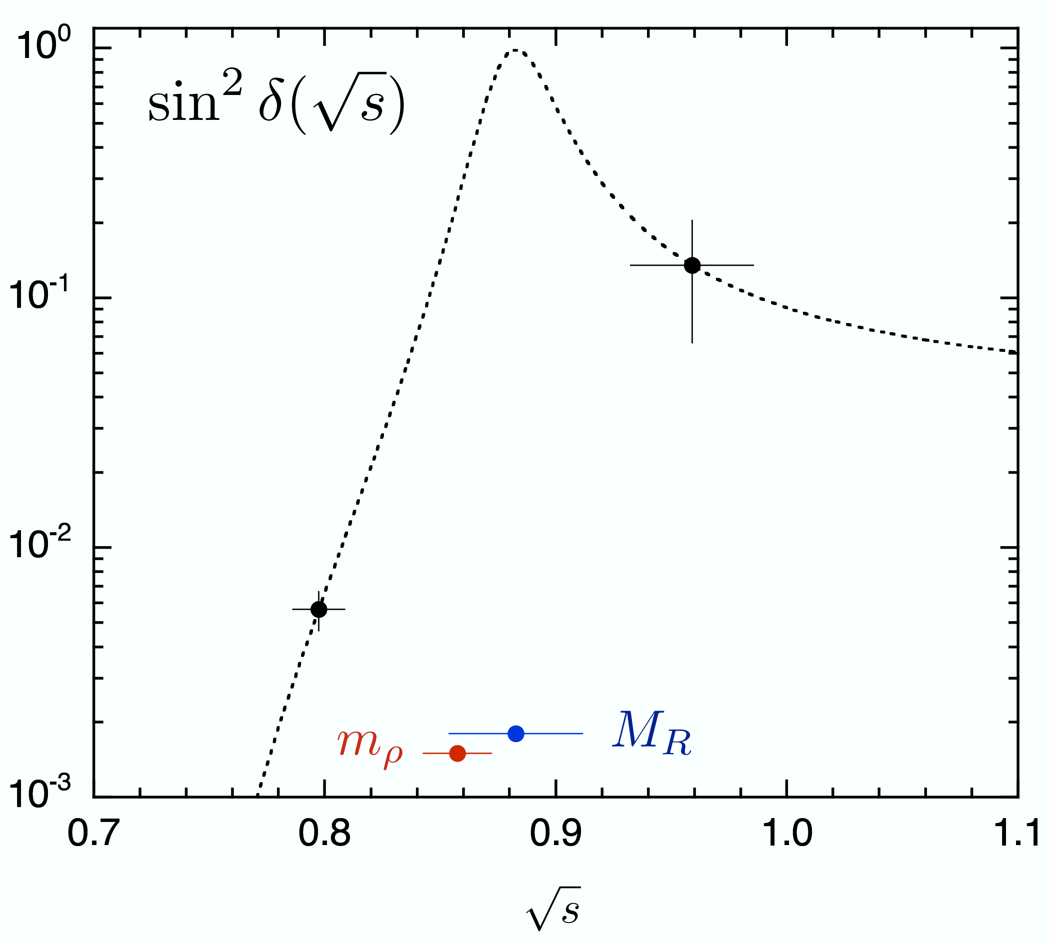


Quenched $a = 0.1$ fm
Wilson gauge
Wilson quark
$L^3 \approx (2.5 \sim 4.8\ \text{fm})^3$
CP-PACS PRD67 014502(2003)



$N_f = 2\ a \to 0$
Iwasaki gauge
Tad-pole imp. clover quark
$L^3 \approx (2.5\ \text{fm})^3$
CP-PACS PRD70 074513(2004)

$I = 1$ $\pi\pi$ scattering phase shift $\delta(p)$



$N_f = 2$ $a^{-1} = 0.91$ fm

Iwasaki gauge

Tad-pole imp. clover quark

$m_\pi / m_\rho = 0.42$ $L = 2.53$ fm

N. Ishizuka and K. Sasaki

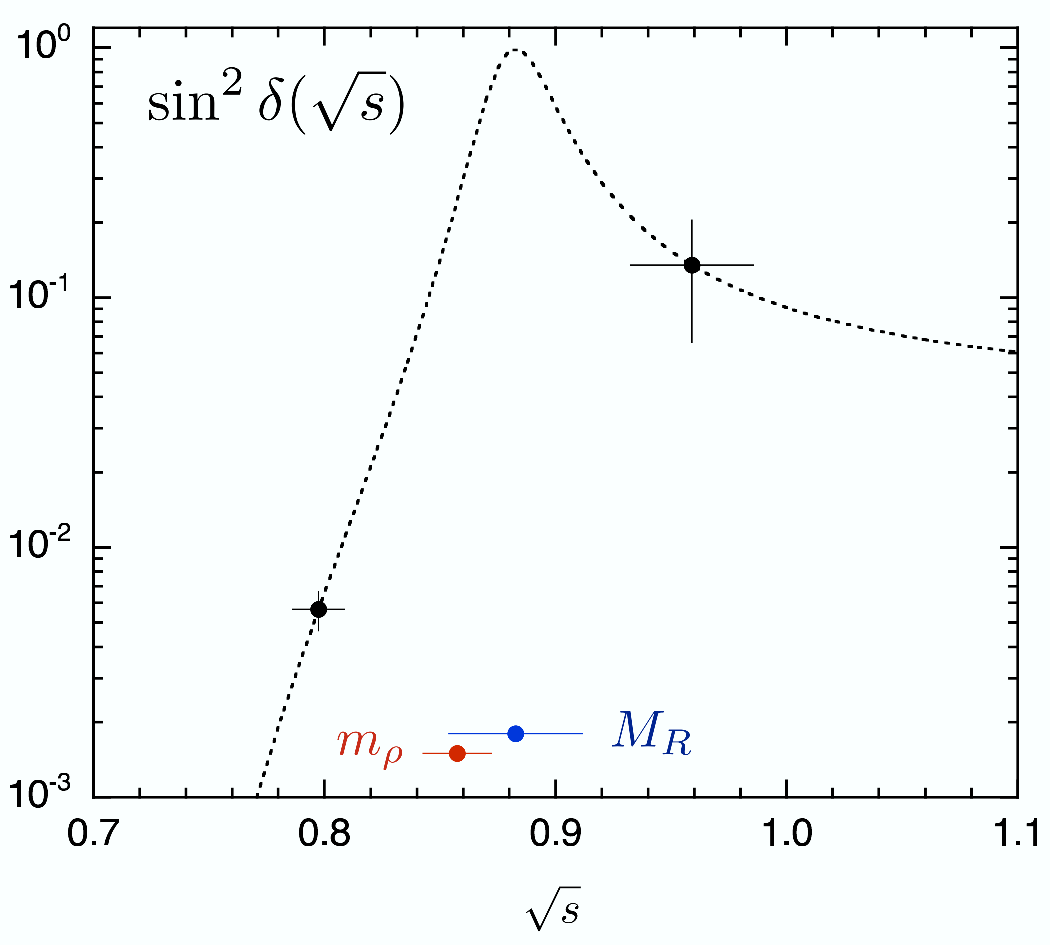
at Lattice2006

$$\tan \delta(\sqrt{s}) = \overline{g}^2 \frac{p^3}{\sqrt{s}(M_R^2 - \sqrt{s})}$$

Parameters: $\overline{g}$ and M_R

$$\Gamma_\rho = \overline{g}^2 \cdot p_\rho^3 / M_\rho^3$$

$I = 1$ $\pi\pi$ scattering phase shift $\delta(p)$



$N_f = 2$ $a^{-1} = 0.91$ fm
Iwasaki gauge
Tad-pole imp. clover quark
$m_\pi/m_\rho = 0.42$ $L = 2.53$ fm
Ishizuka and Sasaki
at Lattice2006

$$M_R = 0.883 \pm 0.031$$
$$m_\rho = 0.857 \pm 0.015$$
$$\Gamma_\rho = 130 \pm 31 \text{ MeV}$$
(exp. 150 MeV)

$\delta(p)$ was evaluated with

1. Diagonalization of correlation function matrix,

Lüscher and Wolff, NPB339 222(1990)

2. Finite volume method,

$$\tan\delta(p) = \frac{\pi^{3/2} q}{Z_{00}(1; q^2)} \text{ for S-wave } (l = 0),$$

$$\text{where } p^2 = \left(\frac{2\pi}{L}\right)^2 q^2, \quad Z_{00}(1; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\mathbf{n}\in\mathbf{Z}^3} \frac{1}{\mathbf{n}^2 - q^2}$$

In previous works for $I = 2$ $\pi\pi$ scattering, p^2 is determined from two-pion energy $E = 2\sqrt{m_\pi^2 + p^2}$ through time correlator.

In derivation of method, there is important assumption of size of interaction.

However, we did not confirm assumption is satisfied in previous works.

Our naive question;

Assumption is valid or not in present calculation.

2. Finite volume method

Lüscher, CMP105 153(1986)
NPB354 531(1991)

Conditions of finite volume method

1. Finite volume L^3 in center of mass system
   with periodic boundary condition in spatial directions

2. Two-pion wave function satisfies effective Schrödinger equation

$$\left(\nabla^2 + p^2\right)\phi(\mathbf{r}) = \int d\mathbf{r}' U_p(\mathbf{r},\mathbf{r}')\phi(\mathbf{r}')$$

$U_p(\mathbf{r},\mathbf{r}')$: Fourier transform of modified Bethe-Salpeter kernel
$\mathbf{r}$: relative coordinate of two pions, $\pi(\mathbf{x})\pi(\mathbf{y})$ $\mathbf{r} = \mathbf{x} - \mathbf{y}$
$p^2 = E_{\pi\pi}^2/4 - m_\pi^2 = (2\pi/L)^2 \cdot q^2, \quad q^2 \notin Z$
m_π is independent of L

2. Finite volume method

Lüscher, CMP105 153(1986)
NPB354 531(1991)

Conditions of finite volume method

1. Finite volume L^3 in center of mass system
   with periodic boundary condition in spatial directions

2. Two-pion wave function satisfies effective Schrödinger equation

$$\left(\nabla^2 + p^2\right)\phi(\mathbf{r}) = V_p(\mathbf{r})\phi(\mathbf{r})$$

$V_p(\mathbf{r})$: effective scattering potential

$\mathbf{r}$: relative coordinate of two pions, $\pi(\mathbf{x})\pi(\mathbf{y})$ $\mathbf{r} = \mathbf{x} - \mathbf{y}$

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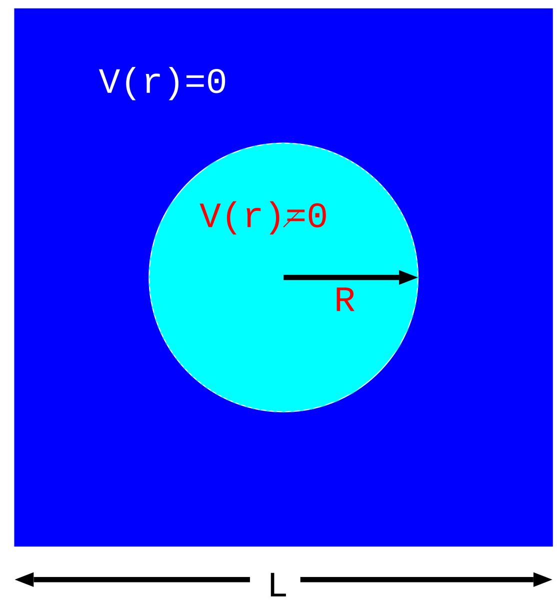


Important assumption

1. Two-pion interaction is small.
$\rightarrow$ Interaction range R exists.

$$V_p(r) \begin{cases} \neq 0 & (r \leq R) \\ = 0 \ (\sim e^{-cr}) & (r > R) \end{cases}$$

2. $V_p(r)$ is not affected by boundary. $\rightarrow R < L/2$

Helmholtz equation

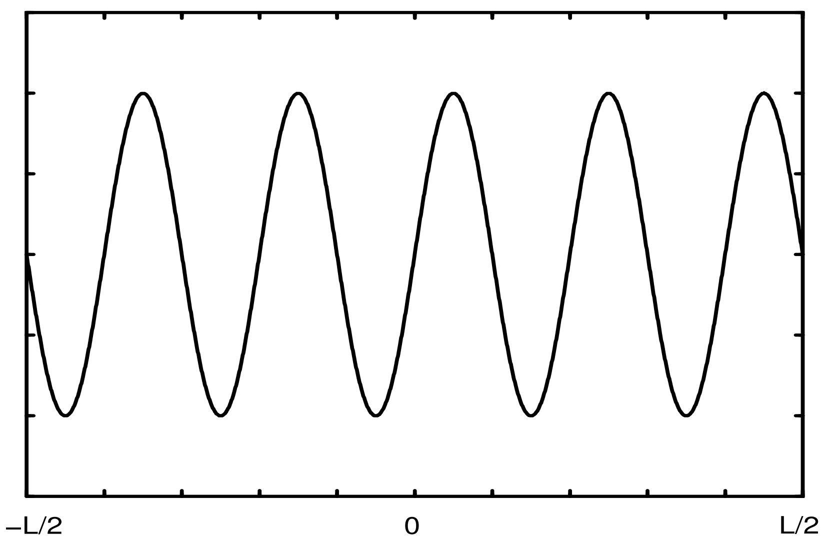
$$\left(\nabla^2 + p^2\right)\phi(\mathbf{r}) = 0 \text{ in } r > R \ (R < L/2)$$

One-dimension L case with periodic boundary condition

Two-pion wave function satisfies periodic boundary condition.

Free case

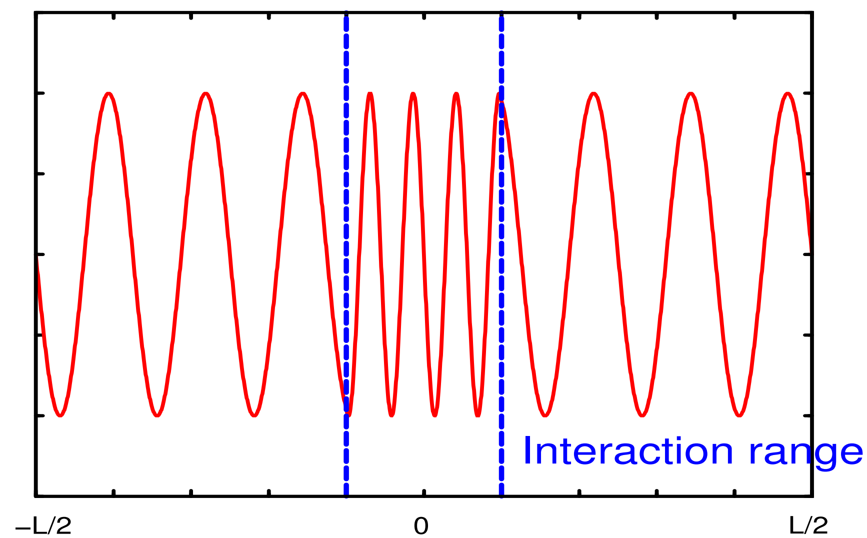
$$\left(\nabla^2 + p_0^2\right)\phi(r) = 0$$



$p_0^2 = (2\pi/L)^2 \cdot n$, n is integer.

Interacting case

$$\left(\nabla^2 + p^2\right)\phi(r) = V_p(r)\phi(r)$$



$L,\ \delta(p) \rightarrow p^2 = (2\pi/L)^2 \cdot q^2$,

q^2 is not integer.

$V_p(r) \neq 0$ in $r < R$

p^2 has information of $\delta(p)$.

Three-dimensional case with periodic boundary condition

In $r > R$ ($V_p(r) = 0$), $\phi(\mathbf{r})$ satisfies the Helmholtz equation

$$(\nabla^2 + p^2)\phi(\mathbf{r}) = 0.$$

1. Solution in $r > R$ (neglecting $l \geq 4$ scattering)

$$\begin{aligned}\phi(\mathbf{r}) &= C \cdot G(\mathbf{r}; p) \\ &= C \cdot \sum_{\mathbf{n} \in Z^3} \frac{e^{i\mathbf{r}\cdot\mathbf{n}(2\pi/L)}}{\mathbf{n}^2 - q^2}, \quad q^2 = \left(\frac{Lp}{2\pi}\right)^2\end{aligned}$$

2. Expansion by spherical Bessel $j_l(pr)$ and Noeman $n_l(pr)$ functions:

$$\phi(\mathbf{r}) = \beta_0(p) n_0(pr) + \sum_{lm} \sqrt{4\pi} Y_{lm}(\theta_r, \varphi_r) \alpha_l(p) j_l(pr)$$

3. Scattering phase shift $\delta_l(p)$ is defined by

$$\tan \delta_l(p) = \frac{\beta_l(p)}{\alpha_l(p)}$$

From relations 1.–3. Lüscher found

finite volume formula for S-wave ($l = 0$)

$$\tan\delta(p) = \frac{\pi^{3/2} q}{Z_{00}(1;q^2)}$$

where $p^2 = \left(\frac{2\pi}{L}\right)^2 q^2$, $Z_{00}(1;q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\mathbf{n}\in\mathbf{Z}^3} \frac{1}{\mathbf{n}^2 - q^2}$

Field theoretical derivation has already done.

C.-J.D. Lin *et al.* NPB619 465(2001)

CP-PACS PRD71 094504(2005)

Motivations of this work

1. Is it possible to calculate wave function?

   Two-pion wave function is useful to check validity of assumptions.

2. $\pi\pi$ interaction is small enough in present calculation?

   Check assumption of finite volume formula

   $$\frac{\nabla^2 \phi(\mathbf{r})}{\phi(\mathbf{r})} = V_p(\mathbf{r}) - p^2$$

   1. $V_p(\mathbf{r}) \approx 0$ in $|\mathbf{r}| > R$, R : interaction range
   2. R is included in finite volume ($R < L/2$)

3. Can we determine physical quantity from wave function?

   In derivation, $\delta(p)$ is determined by wave function in $r > R$.
   In principle we can obtain $\delta(p)$ from wave function.

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   Yes, at least in $L = 3.9$ fm box.
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   2. R is included in finite volume ($R < L/2$)

3. Can we determine physical quantity from wave function?
   Yes, if assumptions are satisfied.
   In derivation, $\delta(p)$ is determined by wave function in $r > R$.
   In principle we can obtain $\delta(p)$ from wave function.

4. Definition of wave function

C.-J.D. Lin *et al.* NPB619 465(2001)
J. Balog *et al.* NPB618[FS] 315(2001)

Definition of wave function $\phi(\mathbf{r})$

$$\phi(\mathbf{r}) = \sum_{\mathbf{R}} \sum_{\mathbf{X}} \langle 0|\pi(\mathbf{R}[\mathbf{r}] + \mathbf{X})\pi(\mathbf{X})|\pi\pi; p\rangle,$$

$\sum_{\mathbf{X}}$: projection to zero total momentum

$\sum_{\mathbf{R}}$: projection to A_1^+ sector $\sim$ S-wave up to $l \geq 4$

$|\pi\pi; p\rangle$: two-pion state with $E_{\pi\pi} = 2\sqrt{m_\pi^2 + p^2}$

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$$\phi(\mathbf{r}) = \sum_{\mathbf{R}} \sum_{\mathbf{X}} \langle 0 | \pi(\mathbf{R}[\mathbf{r}] + \mathbf{X}) \pi(\mathbf{X}) | \pi\pi; p \rangle,$$

Calculation of wave function

$$\begin{aligned} G_{\pi\pi}(\mathbf{r}, t) &= \sum_{\mathbf{R}} \sum_{\mathbf{X}} \langle 0 | \pi(\mathbf{R}[\mathbf{r}] + \mathbf{X}, t) \pi(\mathbf{X}, t) (W(t_0) W(t_0 + 1))^\dagger | 0 \rangle \\ &\to C \cdot \phi(\mathbf{r}) \cdot e^{-E_{\pi\pi} t}, \quad t \gg t_0 + 1, \quad E_{\pi\pi} = 2\sqrt{m_\pi^2 + p^2} \end{aligned}$$

Wall source : $W(t) = \sum_{\mathbf{x}} \pi(\mathbf{x}, t)$

$G_{\pi\pi}(t) = \sum_{\mathbf{r}} G_{\pi\pi}(\mathbf{r}, t)$ is usual wall-point two-pion four-point function.

$$\phi(\mathbf{r}) = \frac{G_{\pi\pi}(\mathbf{r}, t)}{G_{\pi\pi}(\mathbf{r_0}, t)} \quad \text{up to normalization}$$

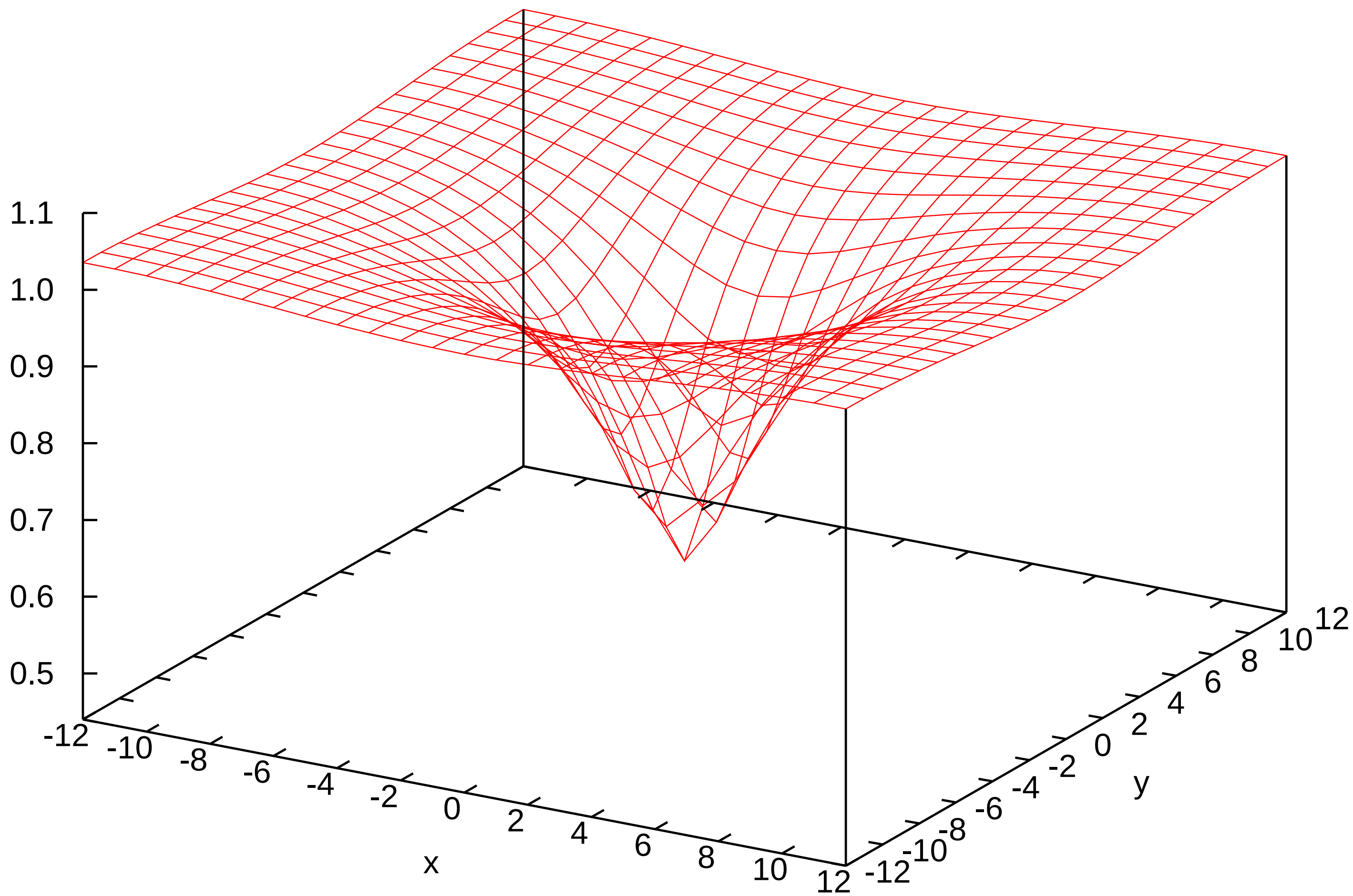
Parameters

- Only ground state ($p^2 \approx 0$) $\sim$ only scattering length a_0
- Iwasaki gauge action $\beta = 2.334$
  Clover quark action with tad-pole improved $c_{SW} = 1.398$
  $a^{-1} = 1.207$[GeV], $a = 0.1632$[fm]
- quenched approximation
- $L^3 \times T = 24^3(20^3, 16^3) \times 80$ $L = 3.92(3.26, 2.61)$ fm
- Source position $t_0 = 12$
- $m_\pi = 0.52, 0.58, 0.66, 0.76, 0.85$[GeV]
- # of Conf. $= 506$

5. Results

Wave function $\phi(\mathbf{r})$ at $(z, t) = (0, 52)$ slice

normalized at $r_0 \sim 9$ with $m_\pi = 0.52$ GeV



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Wave function $\phi(\mathbf{r})$ at $(z,t) = (0,52)$ slice

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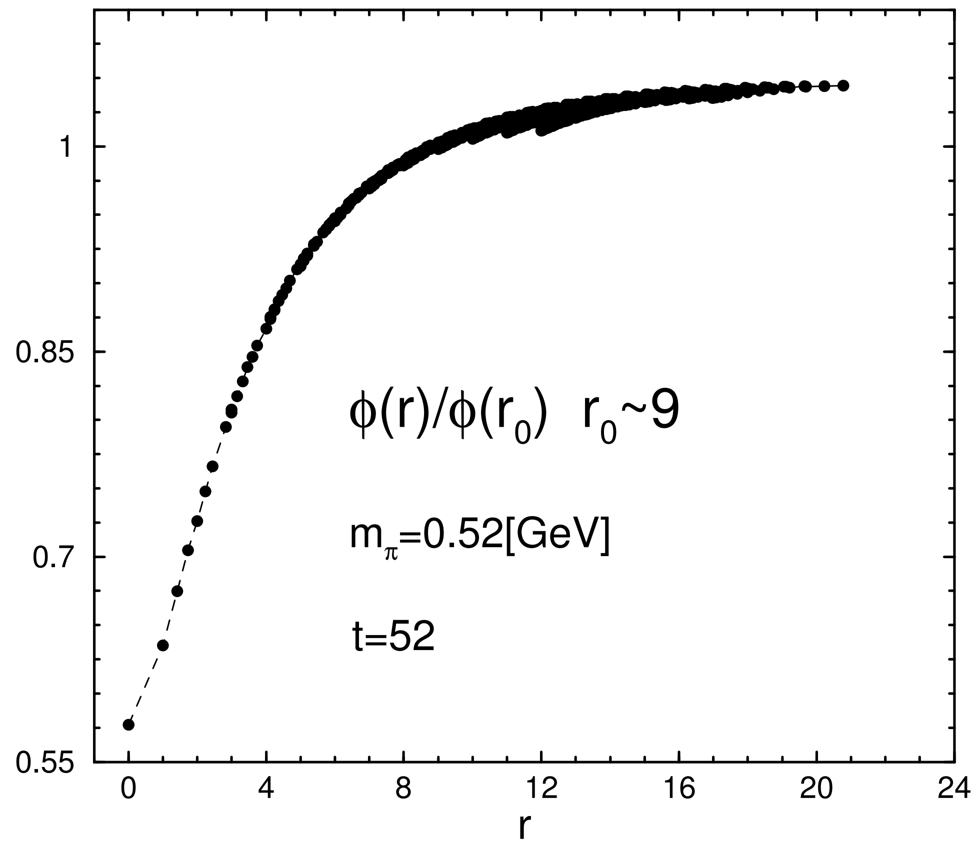
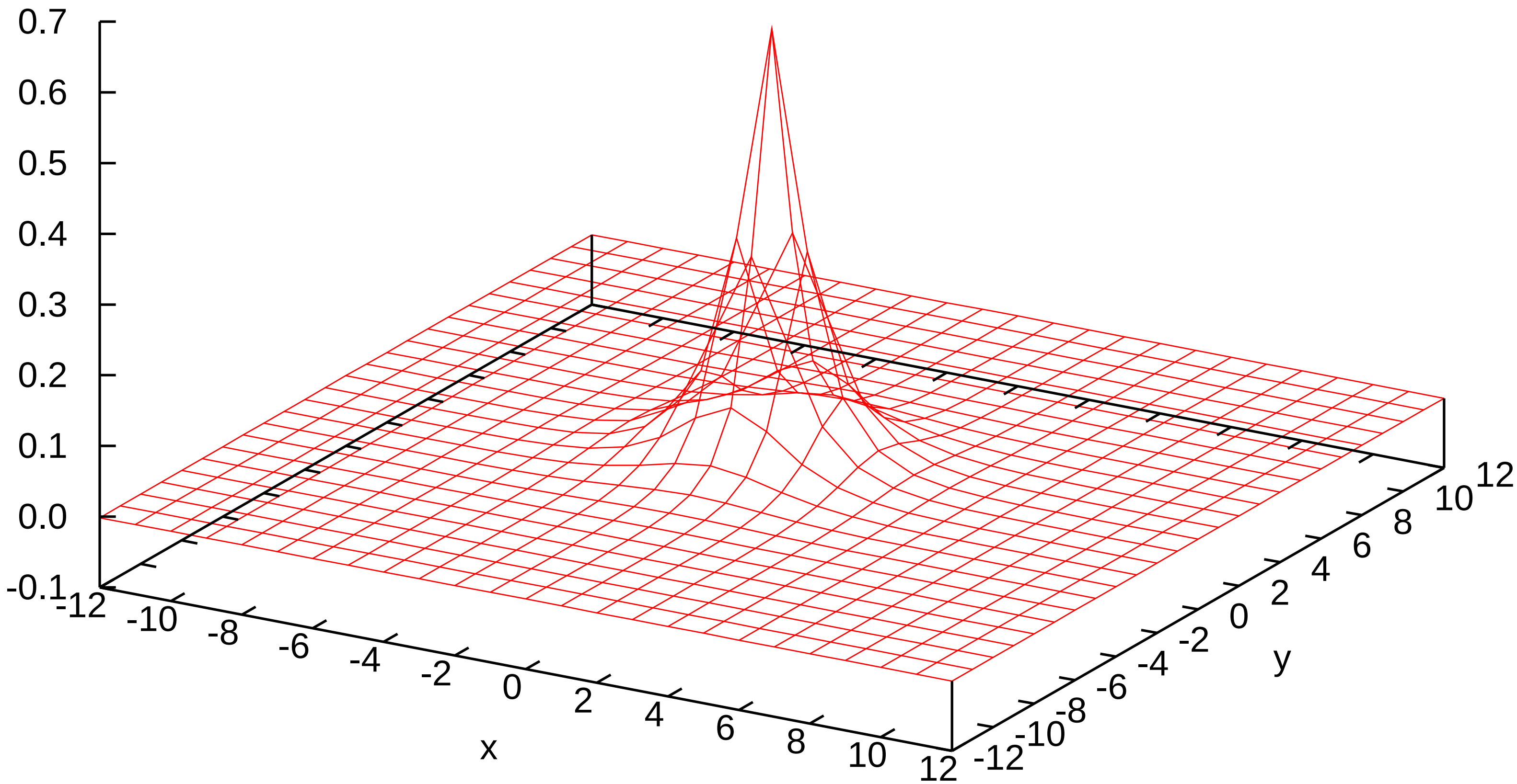


Signal is very clean.

$\phi(r)$ increases as r increases, which is consistent with repulsive interaction of $I = 2$ $\pi\pi$ channel.

Effective potential $V_p(r) - p^2 \equiv \frac{\nabla^2 \phi(\mathbf{r})}{\phi(\mathbf{r})}$ at $(z, t) = (0, 52)$ slice

with $m_\pi = 0.52$ GeV

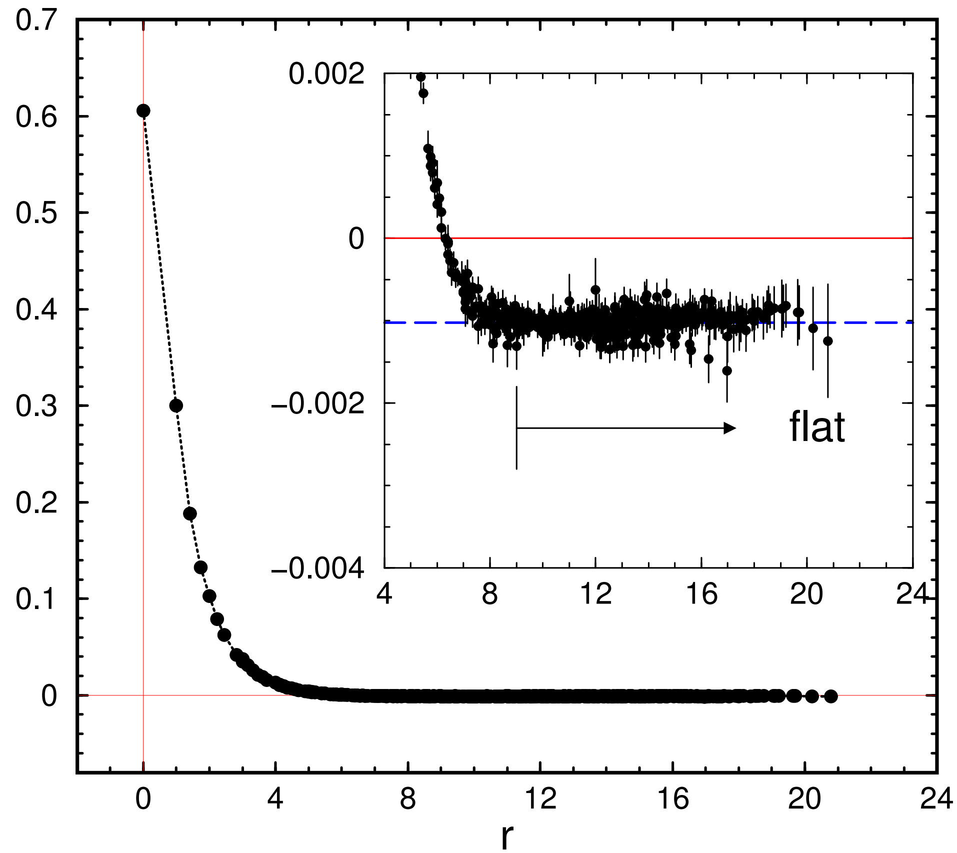


Repulsive, localized effective potential is seen.

We do not focus on form or structure of effective potential.

$(\nabla^2 \phi(\mathbf{r}))/\phi(\mathbf{r})$ in large r region seems to be flat.

Effective potential $V_p(r) - p^2 \equiv \dfrac{\nabla^2\phi(\mathbf{r})}{\phi(\mathbf{r})}$ at $t = 52$ with $m_\pi = 0.52$ GeV



Flat region starts from smaller than $L/2 = 12$.

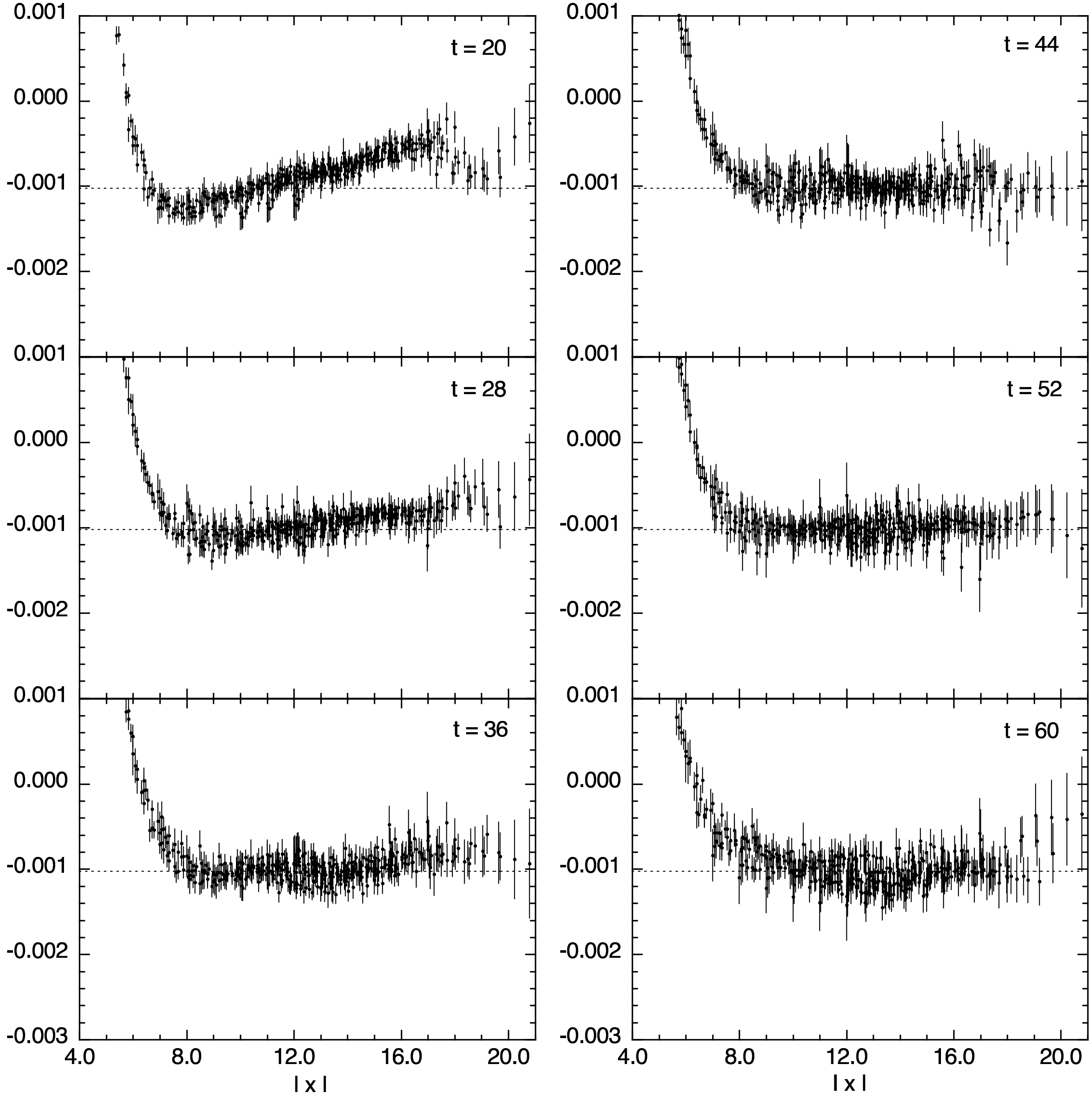
Interaction range $R \leq L/2$. $\rightarrow$ Assumption is satisfied.

Value in flat region is consistent with $-p^2$ obtained from $E_{\pi\pi} = 2\sqrt{m_\pi^2 + p^2}$.

R does not have direct relation to effective range r_0

$$p \cot\delta(p) = 1/a + r_0 p^2/2 + O(p^4)$$

Time dependence of $(\nabla^2\phi(\mathbf{r}))/\phi(\mathbf{r})$ with $m_\pi = 0.52$ GeV



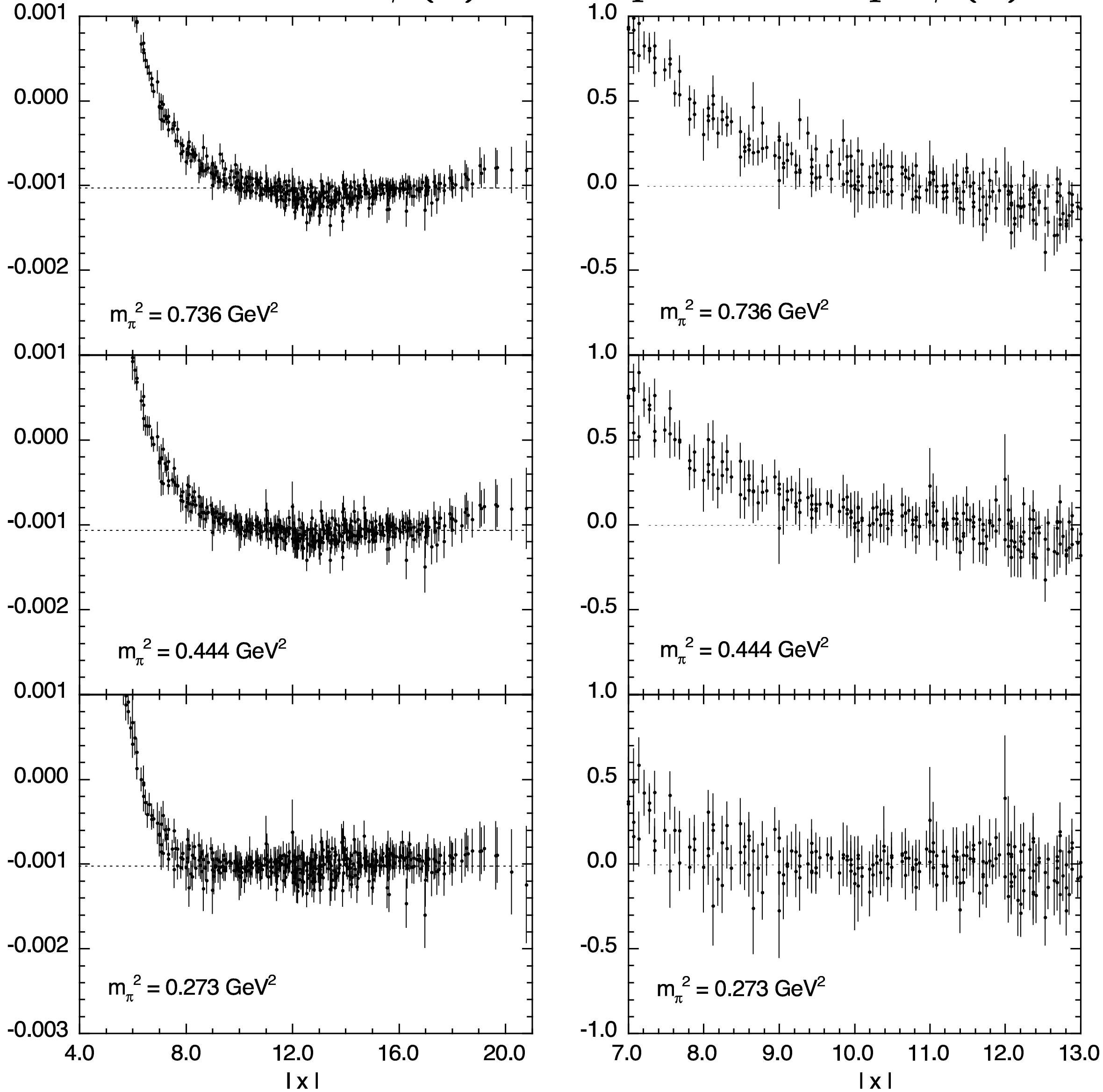
Dashed line : $-p^2$ from $E_{\pi\pi} = 2\sqrt{m_\pi^2 + p^2}$

$(\nabla^2\phi(\mathbf{r}))/\phi(\mathbf{r})$ is stable in $t \geq 44$.

In larger t and r, $(\nabla^2\phi(\mathbf{r}))/\phi(\mathbf{r})$ agrees with $-p^2$.

m_π dependence of effective potential $V_p(r) - p^2$ and $V_p(r)/p^2$ at $t = 52$

$$V_p(r) - p^2 \equiv \frac{\nabla^2 \phi(r)}{\phi(r)} \qquad \frac{V_p(r)}{p^2} \equiv \frac{\nabla^2 \phi(r) + p^2}{p^2 \phi(r)}$$



$V_p(r) - p^2$ at all m_π in large r are flat and agree with p^2 from time correlator.

$V_p(r)/p^2$ agrees with zero at $r \leq L/2 = 12$.

Assumptions are satisfied in all m_π region.

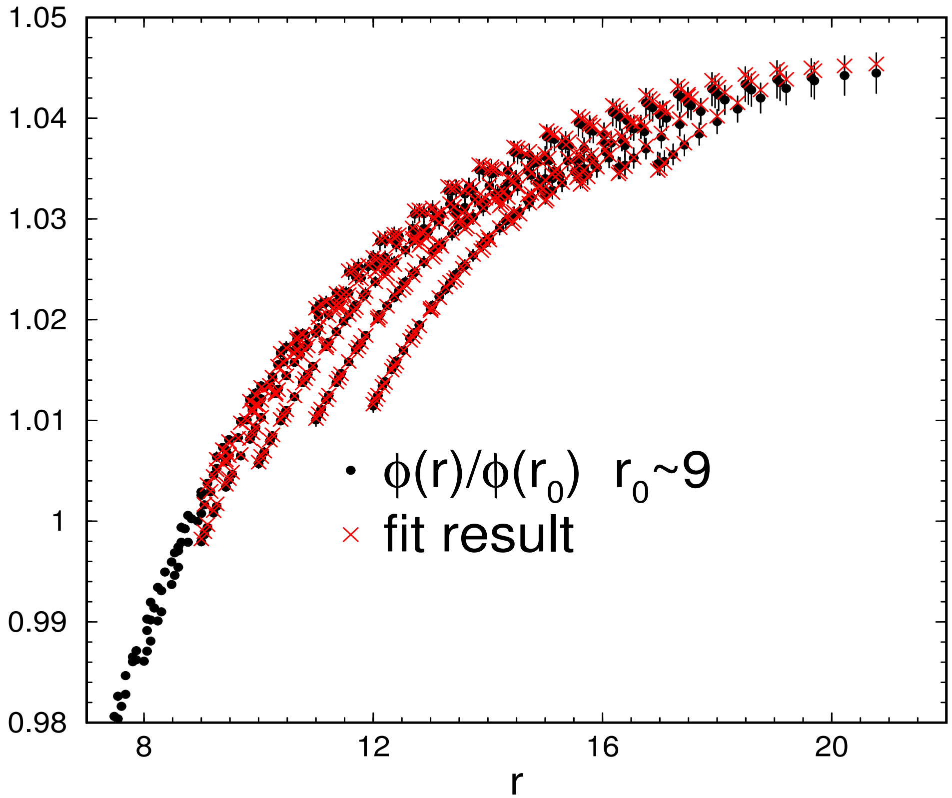
Interaction range R seems to increase as m_π increases.

Physical quantities from $\phi(\mathbf{r})$

Two parameters fit C, q^2 of wave function $\phi(r)$ in $r > R$ ($R \sim 9$)

Solution of Helmholtz equation on L^3

$$G(\mathbf{r}) = C \sum_{\mathbf{n} \in Z^3} \frac{e^{i\mathbf{r}\cdot\mathbf{n}(2\pi/L)}}{\mathbf{n}^2 - q^2}$$



We can fit $\phi(r)$ in $r > R$ very well.

O(100) data is fitted by two parameters.

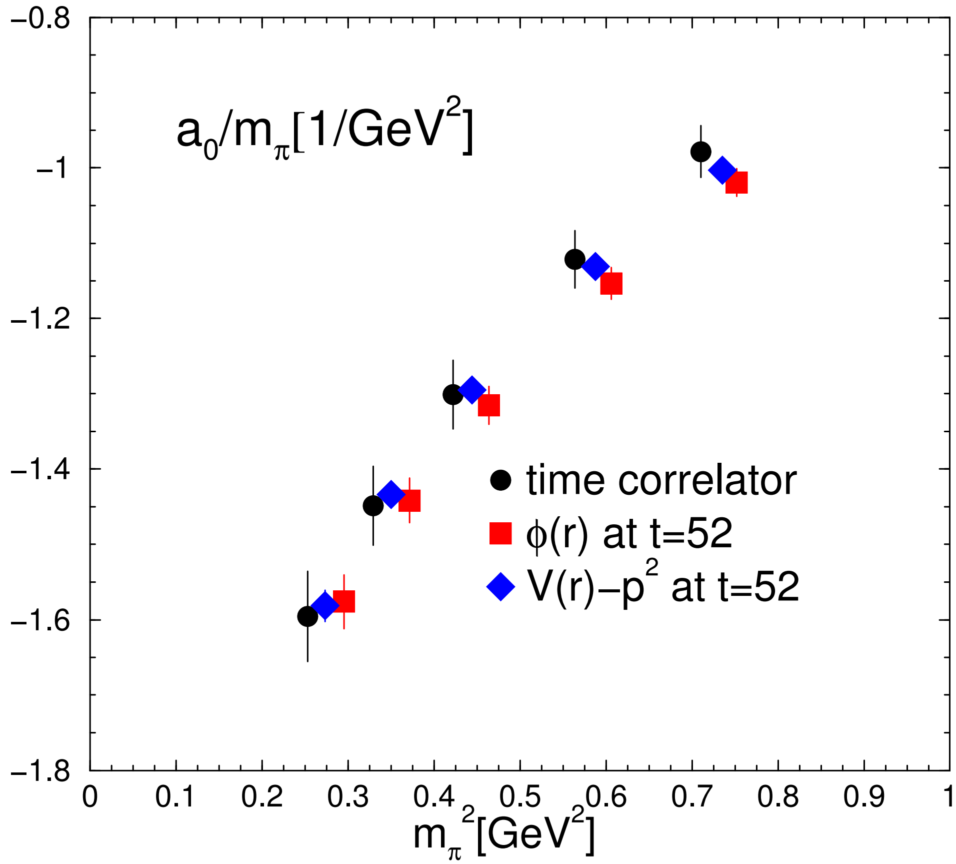
$p^2 = (2\pi/L)^2 \cdot q^2$ is extracted from $\phi(\mathbf{r})$.

$\rightarrow$ We can obtain $\delta(p)$ from p^2 through finite volume formula.

Comparisons of a_0/m_π

1. p^2 from $G_{\pi\pi}(t) \equiv \sum_{\mathbf{r}} G_{\pi\pi}(\mathbf{r}, t) \rightarrow A \cdot e^{-E_{\pi\pi} t}$, $E_{\pi\pi} = 2\sqrt{m_\pi^2 + p^2}$
2. p^2 from two parameter fit of $\phi(\mathbf{r})$ at $t = 52$
3. p^2 from $V_p(r) - p^2$ at $t = 52$ with constant fit in $r > R$

obtain a_0 using finite volume method



Consistency is very well.

Errors of $\phi(\mathbf{r})$ and $V_p(r) - p^2$ are smaller than $G_{\pi\pi}(t)$ in $L = 24$.

- $E_{\pi\pi} - 2m_\pi \propto 1/L^3$
  disadvantage of $G_{\pi\pi}(t)$
- large # of $\phi(r)$ in $r > R$
  advantage of $\phi(\mathbf{r})$ and $V_p(r) - p^2$

Comparisons of a_0/m_π

1. p^2 from $G_{\pi\pi}(t) \equiv \sum_{\mathbf{r}} G_{\pi\pi}(\mathbf{r},t) \to A \cdot e^{-E_{\pi\pi}t}$, $E_{\pi\pi} = 2\sqrt{m_\pi^2 + p^2}$
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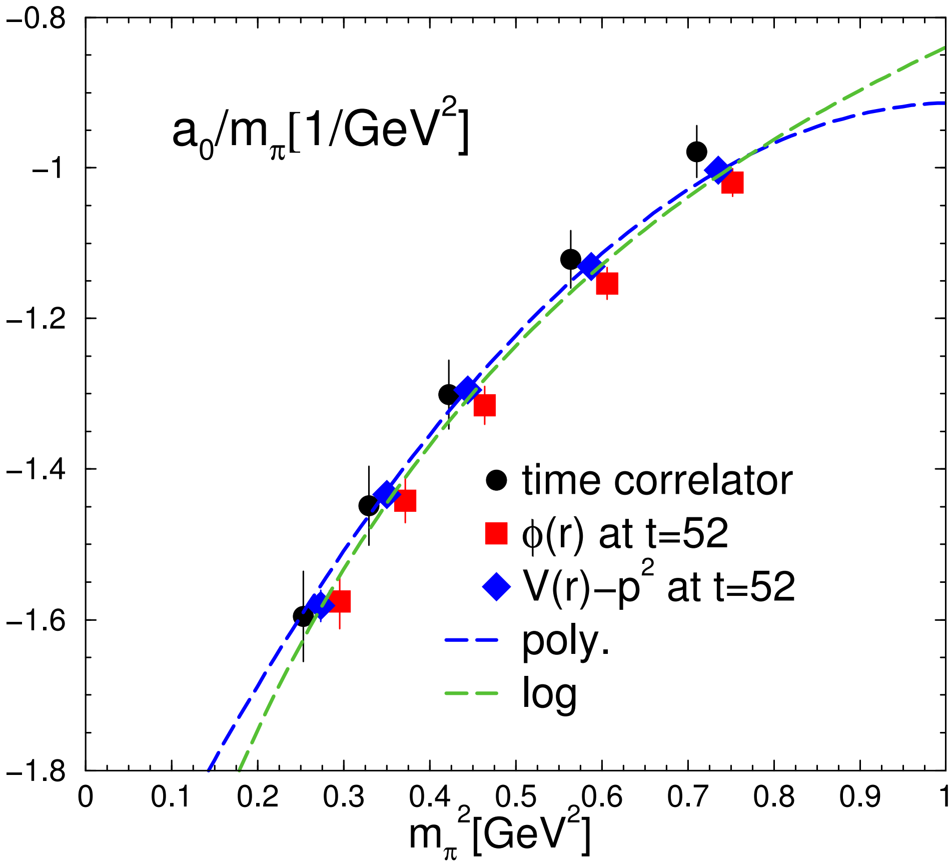


Fit with $V_p(r) - p^2$

Fitting forms

- $A_1 + B_1 m_\pi^2 + C_1 m_\pi^4$
- $A_2/(1 + B_2 m_\pi^2 \log(m_\pi^2/C_2))$

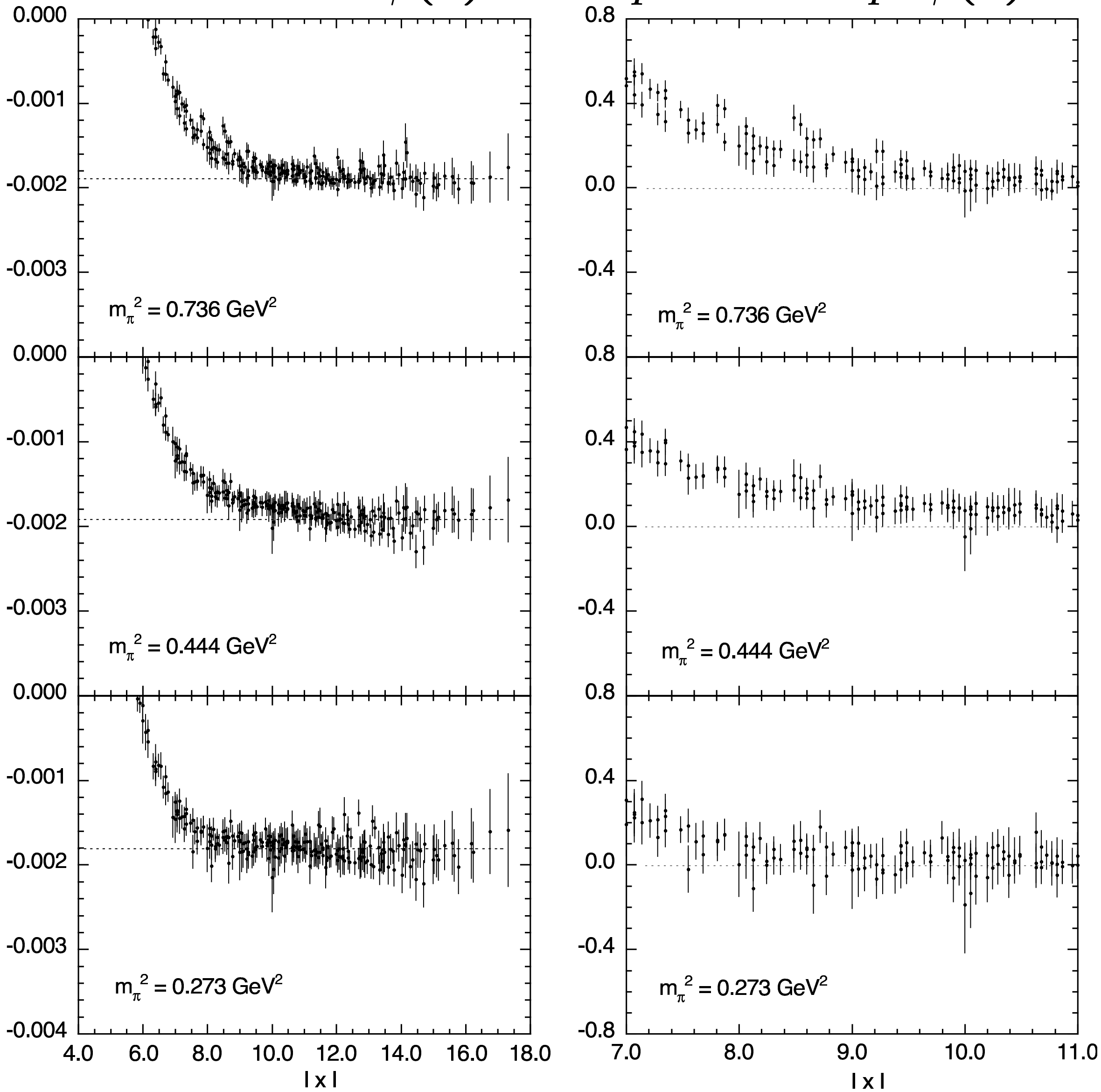
Both fits are reasonable.

a_0/m_π [1/GeV^2] at chiral limit

$-2.117(83)$ (A_1)
$-2.39(16)$ (A_2)

Smaller volume $L = 20$ (3.27 fm) at $t = 52$

$$V_p(r) - p^2 \equiv \frac{\nabla^2\phi(r)}{\phi(r)} \qquad \frac{V_p(r)}{p^2} \equiv \frac{\nabla^2\phi(r) + p^2}{p^2\phi(r)}$$

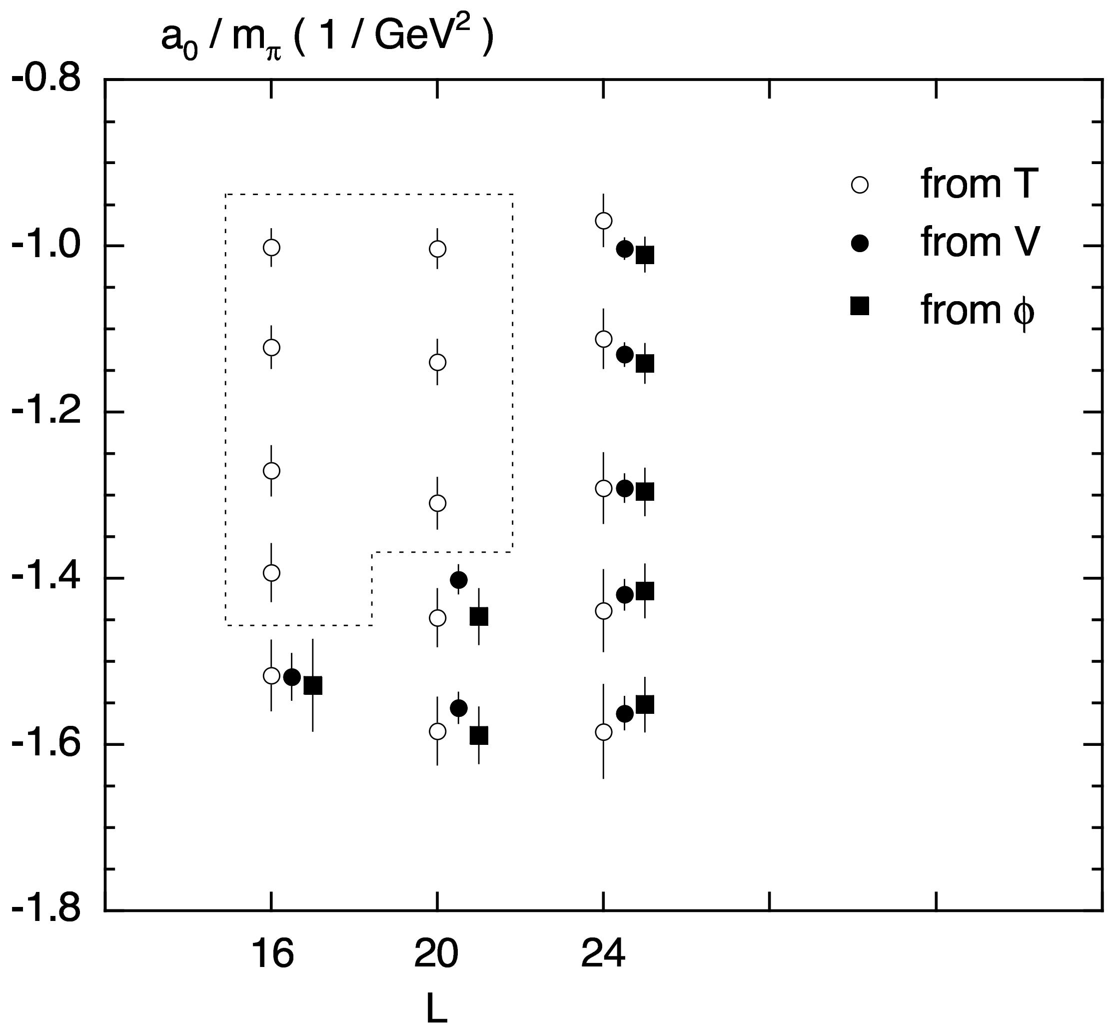


$V_p(r) - p^2$ at all m_π are flat and agree with p^2 from time correlator in large r.

But $V_p(r)/p^2$ seems to disagree with zero at $r \leq L/2 = 10$ at heavier m_π.

Because interaction range R seems to increase as m_π increases.

Volume dependence of a_0/m_π



$V_p(r)/p^2 \neq 0$ at $r = L/2$ in heavier m_π.
$\phi(r)$ and $V_p(r) - p^2$ analyses are not applied.

However,
significant volume dependence of a_0/m_π obtained from $G_{\pi\pi}(t)$ (T) is not seen in $L = 16, 20, 24$ (2.61, 3.26, 3.92 fm).

Effects of deformation of two-pion interaction due to finite size effects is smaller than statistical error in $L \geq 2.6$ fm.

6. Conclusions

- We have investigated validity of finite volume formula.
  Assumptions are satisfied in $L = 3.92$ fm.
  R increases as m_π increases.
  Effects of deformation of two-pion interaction is smaller than statistical error in $L \geq 2.6$ fm.

- Physical quantity can be extracted from two-pion wave function
  ◦ solution of Helmholtz equation
  ◦ flat region of effective potential
  Consistency between two methods and traditional method is very well.
  Statistical error of two methods are smaller than traditional method at larger volume.

Future works

- Higher energy state
  Scattering phase shift from two-pion wave function
- Non-zero total momentum system
  Similar wave function discussion has been done
  in Rummukainen and Gottlieb NPB450, 397(1995).
- Other scattering system
  wave function of $^1S_0$ NN scattering system, Ishii *et al.* at Lattice2006
- Decay system

$\Delta I = 3/2 K \rightarrow \pi\pi$ decay

Calculation of Re(ε'/ε) with reduction method

$(K \to 0, K \to \pi) \to (K \to \pi\pi)$

'03 CP-PACS, RBC Collaboration

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \begin{cases} (-7.7 \pm 2.0) \times 10^{-4} & \text{(CP−PACS)} \\ (-4.0 \pm 2.3) \times 10^{-4} & \text{(RBC)} \end{cases}$$

Sign is opposite to experiment. ($\mathrm{Re}\left(\varepsilon'/\varepsilon\right) = 16.6 \pm 1.6$)

Main systematic errors

- Reduction method
- Leading order Chiral perturbation theory(ChPT)
- Quenched approximation
- Finite lattice spacing

Reduction method may cause large systematic error, because final state interaction effect is expected to play an important role in the decay process.

To get rid of the systematic errors,

it is important to calculate with direct method.

(without reduction method and ChPT)

Problems of direct calculation

1. $K \to \pi(p)\pi(-p)$ with $p = 206$ MeV

   We cannot directly treat $K \to \pi(p)\pi(-p)$ by traditional analysis method, because $|\pi(0)\pi(0)\rangle$ is ground state. ‘90 Maiani and Testa

   solutions: Diagonalization ('02 Ishizuka), H-Parity boundary condition ('04 Kim)

   Non-zero total momentum (Lab) system ('05 Boucaud et al.)

   $|\pi(P)\pi(0)\rangle$ is ground state, which relates to $|\pi(p)\pi(-p)\rangle$ with $p \neq 0$ in center-of-mass(CM) system.

2. Calculation on finite volume (2-3 fm)

   Finite volume effect of two-particle state is large, we need LL formula to connect decay amplitude in infinite volume to finite volume.

   $$|A_\infty^{\mathrm{CM}}| = F(E_{\pi\pi}, \delta)|M_V^{\mathrm{CM}}|$$ (‘01 Lellouch and Lüscher)

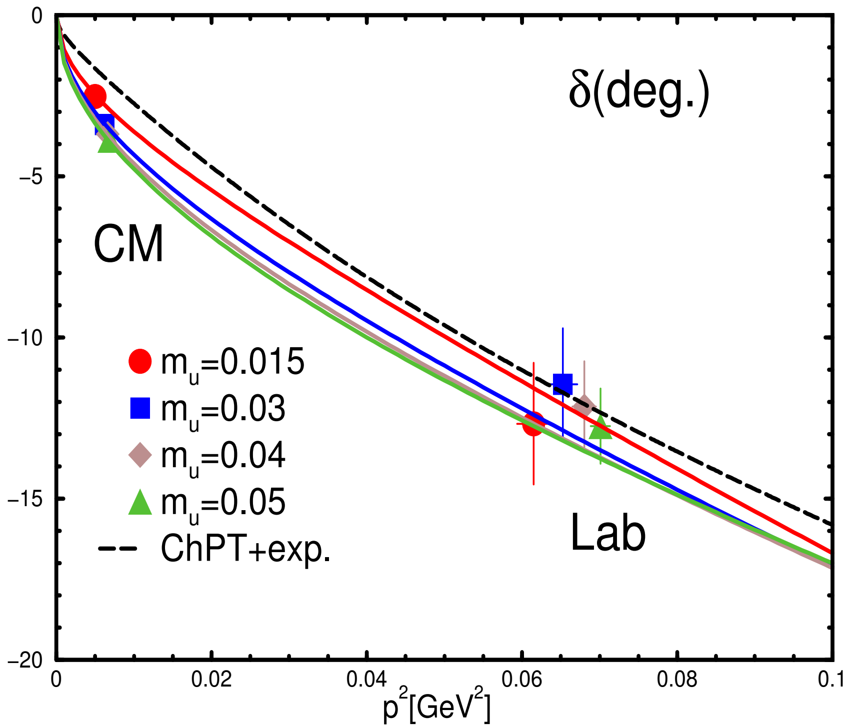
   However, LL formula is in CM system.

   Extension of LL formula to Lab system '05 Kim et al. and Christ et al.

   $$|A_\infty^{\mathrm{CM}}| = F(E_{\pi\pi}, \delta, \gamma)|M_V^{\mathrm{Lab}}|$$

To apply two methods to $\Delta I = 3/2$ $K \to \pi\pi$ decay

$I = 2$ $\pi\pi$ Scattering phase shift



To obtain $\dfrac{\partial\delta}{\partial p}$, we employ global fitting for m_π^2 and p^2.

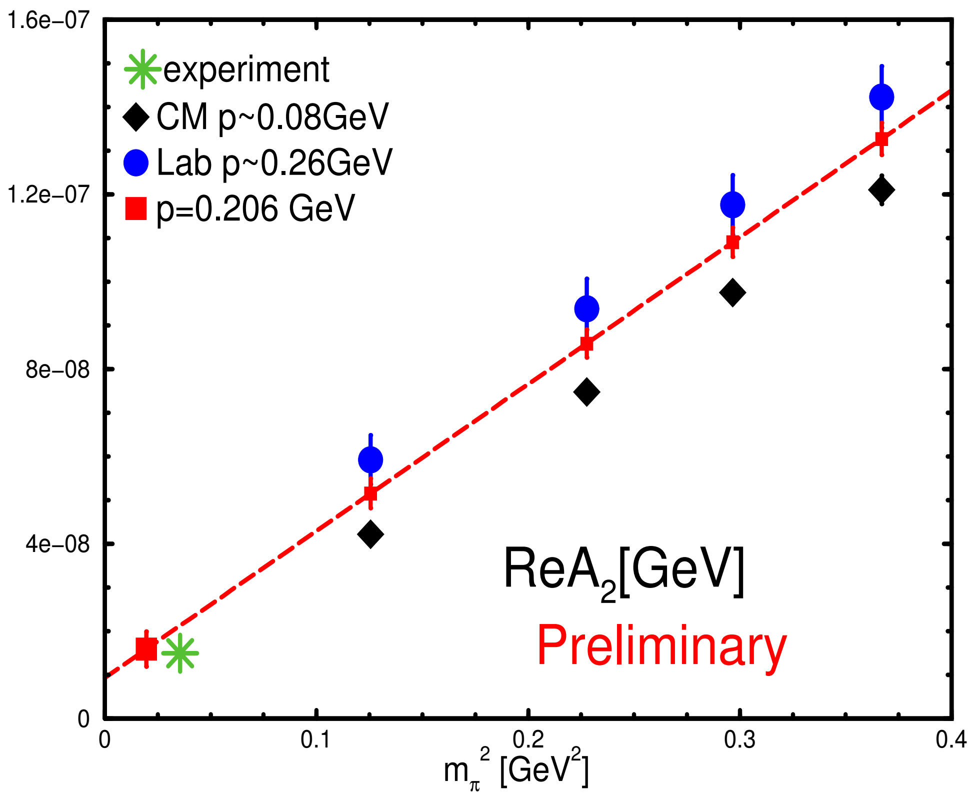
$$\frac{\tan\delta(p)}{p}\frac{E_{\pi\pi}}{2} = A_{10}m_\pi^2 + A_{20}m_\pi^4 + A_{01}p^2 + A_{11}m_\pi^2 p^2$$

where $E_{\pi\pi} = 2\sqrt{m_\pi^2 + p^2}$

A_{10}[GeV^{-2}]	A_{20}[GeV^{-4}]	A_{01}[GeV^{-2}]	A_{11}[GeV^{-4}]	χ^2/d.o.f.
-1.813(99)	1.26(21)	-2.8(1.0)	3.2(2.2)	0.98

$\partial\delta/\partial p$ is extracted from fit result.(Solid lines)

Preliminary result of ReA_2[GeV]



Non-perturbative renormalization at $\mu = 1.44$[GeV]

'04 Kim for RBC

Physical point $m_K^2 = 4(m_\pi^2 + p^2)$

$$m_\pi = 0.140\text{[GeV]}$$
$$m_K = 0.498\text{[GeV]}$$
$$p = 0.206\text{[GeV]}$$

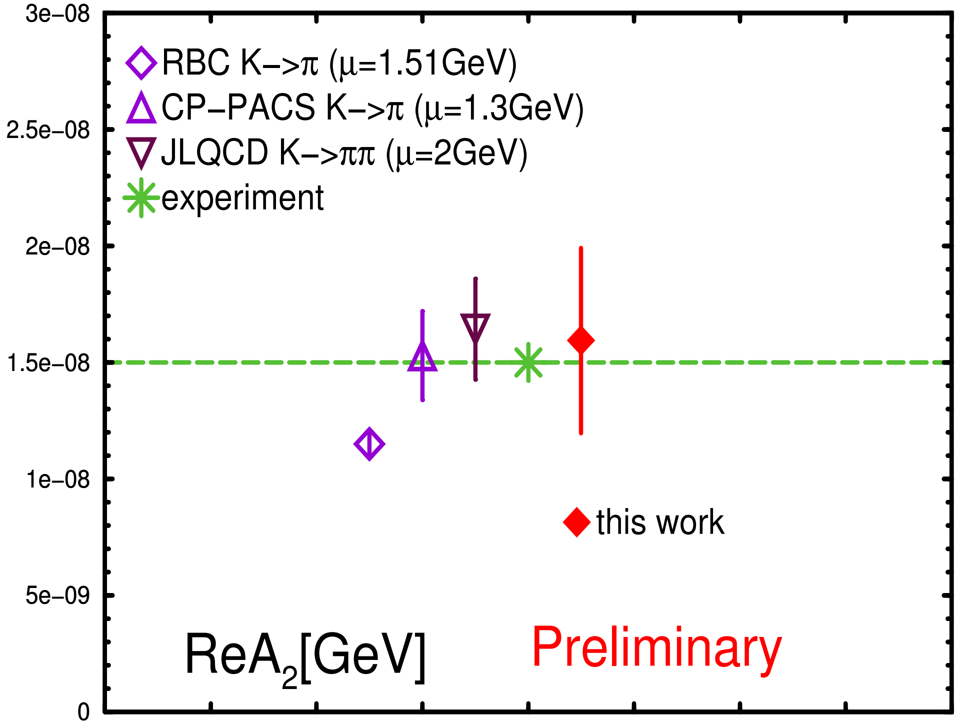
To extract ReA_2 at physical point, we employ global fitting of ReA_2 for m_π^2 and p^2.

$(p^2 = m_K^2/4 - m_\pi^2)$

Fitting form $C_{10}m_\pi^2 + C_{01}p^2 + C_{11}m_\pi^2 p^2$

Calculation with non-zero total momentum is possible.

Preliminary result of ReA_2[GeV] (cont'd)



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Fitting form $C_{10}m_\pi^2 + C_{01}p^2 + C_{11}m_\pi^2 p^2$

ReA_2	experiment	this work	CP-PACS	RBC	JLQCD
[$\times 10^{-8}$GeV]	1.50	1.59(40)	1.53(19)	1.151(52)	1.64(22)

Error is large, but result is consistent with experiment.