# The chiral critical line of $N_f = 2+1$ QCD at zero and non-zero baryon density

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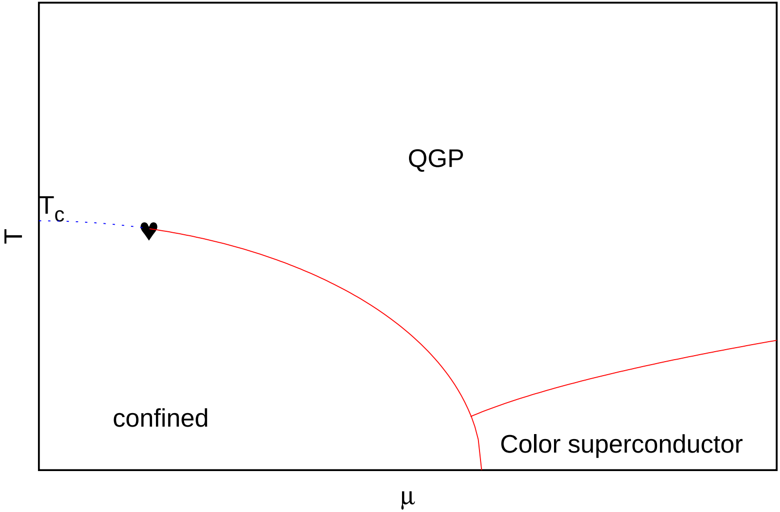
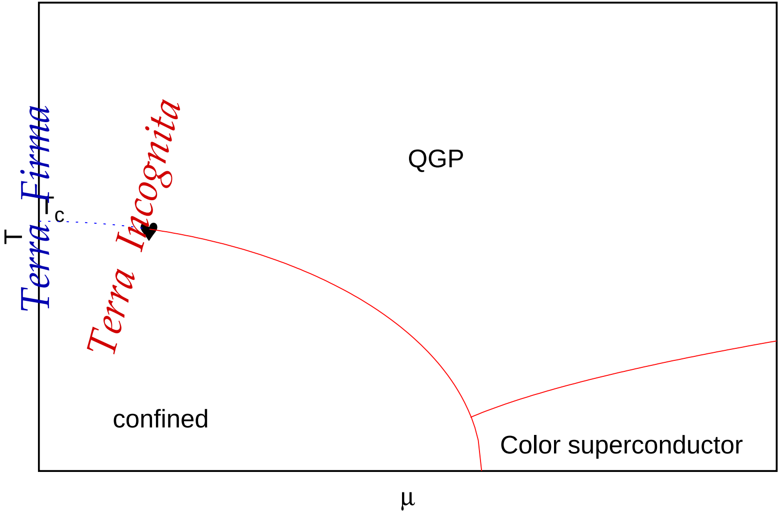
hep-lat/0607017

# Phase diagram of QCD




To be checked by lattice QCD simulations

# Phase diagram of QCD



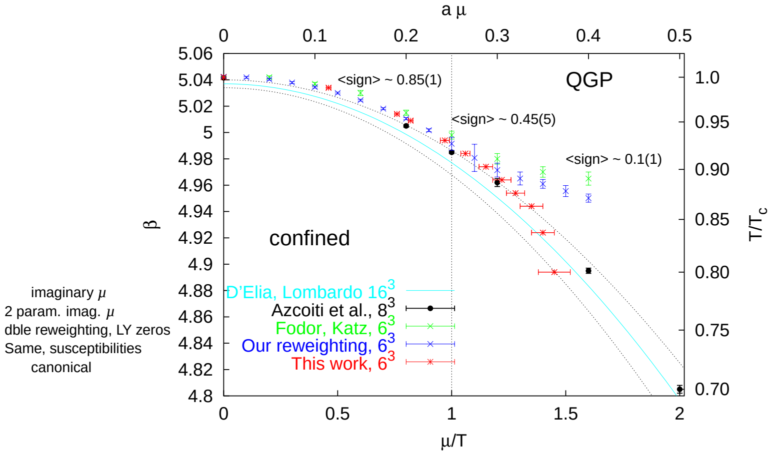

To be checked by lattice QCD simulations

- transition temperature $T_c(\mu)$
- order of transition/critical point ?

# Transition temperature under control

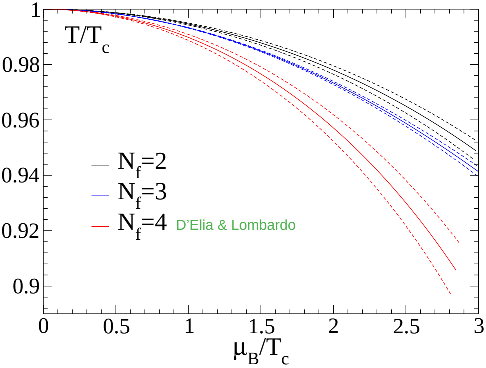
All with $N_f = 4$ staggered fermions, $am_q = 0.05, N_t = 4$ $(a \sim 0.3$ fm)

**Agreement for $\mu/T \lesssim 1$ ie. $\mu_B \lesssim 500$ MeV**




de Forcrand & Kratochvila

# Transition temperature: results



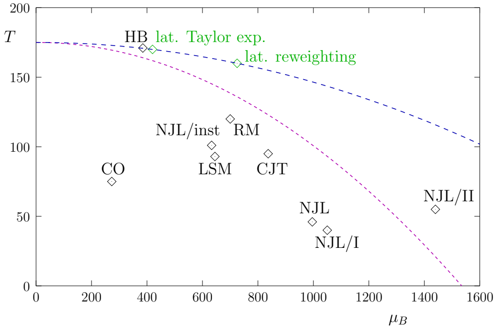

$$\frac{T_c(\mu)}{T_c(\mu=0)} = 1 - c(N_f, m_q)\left(\frac{\mu}{\pi T}\right)^2 + ..$$

$\frac{\mu}{\pi T}$ natural expansion parameter; expect $c \sim \mathcal{O}(1)$

$c \approx 0.500(34), 0.602(9), 0.93(10)$ for $N_f = 2, 3, 4,\ m_q/T \ll 1$

$c \propto N_f/N_c$, Toublan

# Critical point?



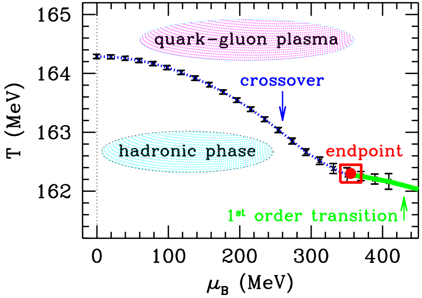

M. Stephanov, hep-ph/0502115

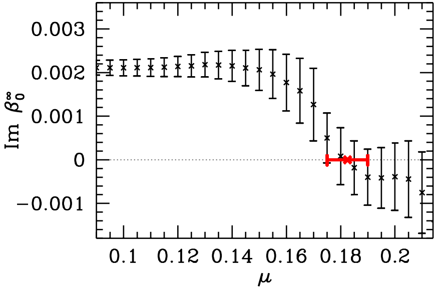
Lattice: much harder task than $T_c(\mu)$

detect divergence of correlation length on small lattice (??)

# Critical point: Fodor & Katz, hep-lat/0402006

Simulate at $(\mu = 0, \beta_c)$, reweight along $\beta_c(\mu)$



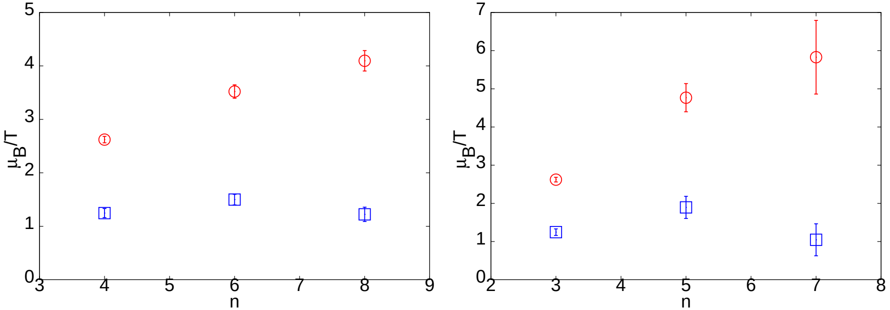

$N_t = 4 \Rightarrow a \sim 0.3$ fm (like everyone else)
$N_f = 2+1$, physical quark masses
$(T_E, \mu_E) = (162(2), 120(13))$ MeV

# Critical point: Gavai & Gupta, hep-lat/0412135

Simulate at $\mu = 0$, measure Taylor coeffs of susceptibility

Near pole $x_0$, susceptibility diverges as $\frac{1}{1-(x/x_0)^2} = 1 + \sum \frac{1}{x_0^{2k}} x^{2k}$

$$\longrightarrow \lim_{k\to\infty} \frac{c_{2k}}{c_{2k+2}} = x_0^2$$




$T = 0.95 T_c$, $8^3 \times 4$ and $24^3 \times 4$ $\Rightarrow$ need $m_\pi L \gtrsim 5$

*"convergence radius"* $\left|\frac{c_0}{c_{2n}}\right|^{1/2n}$ and $\left|\frac{c_{2n}}{c_{2n+2}}\right|^{1/2} \longrightarrow$

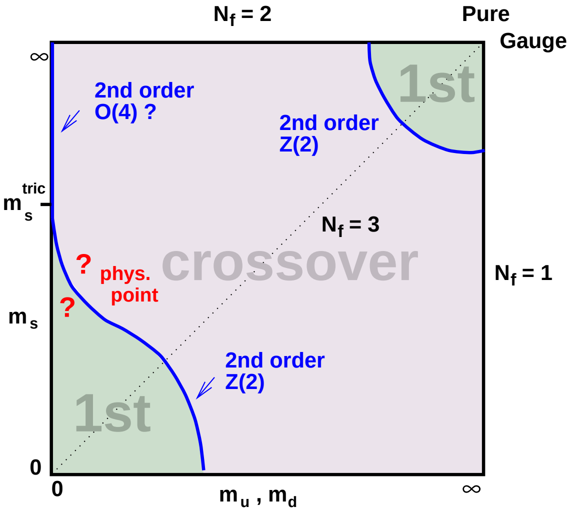
$(T_E, \mu_E) \sim (0.95 T_c, \sim T_c)$ N.B. $N_f = 2$

# Our strategy

- Learn about QCD by generalizing to arbitrary $(m_{u,d}, m_s)$ quark masses
- Simulate at imaginary $\mu$ (no sign pb.; cheap)
- Ask a simple question

# Phase diagram vs $(m_{u,d}, m_s)$, $T$ and $\mu$

$\mu = 0$



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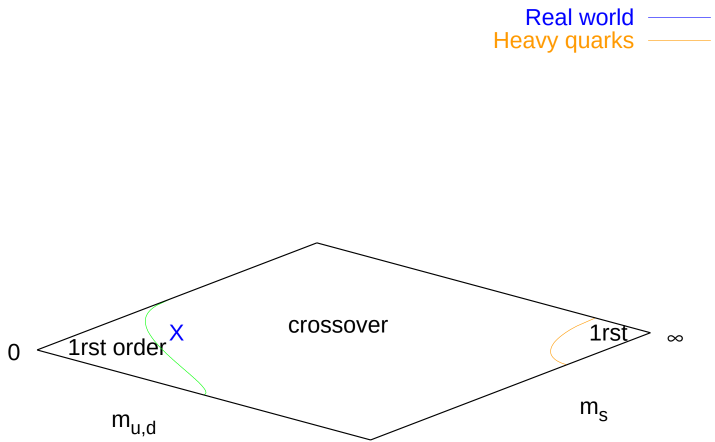
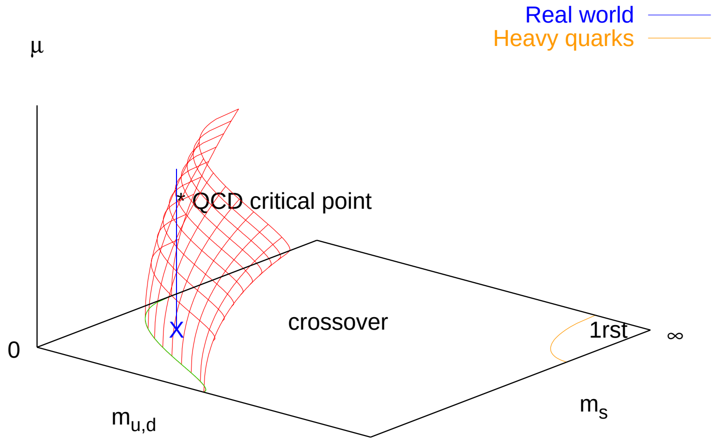



**Now turn on $\mu$**

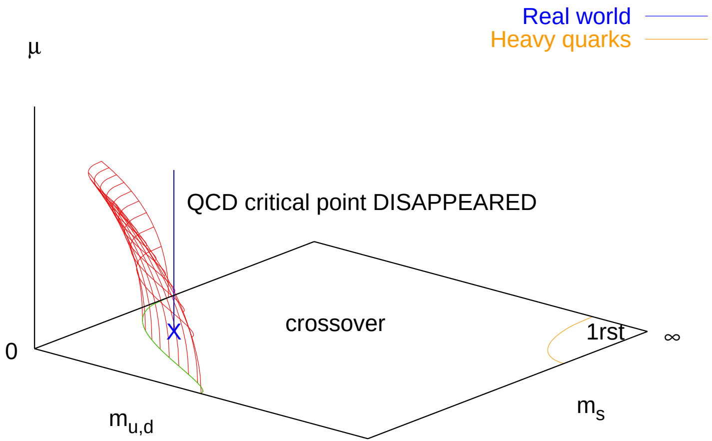
# Phase diagram vs $(m_{u,d}, m_s)$, $T$ and $\mu$

$\mu \neq 0$



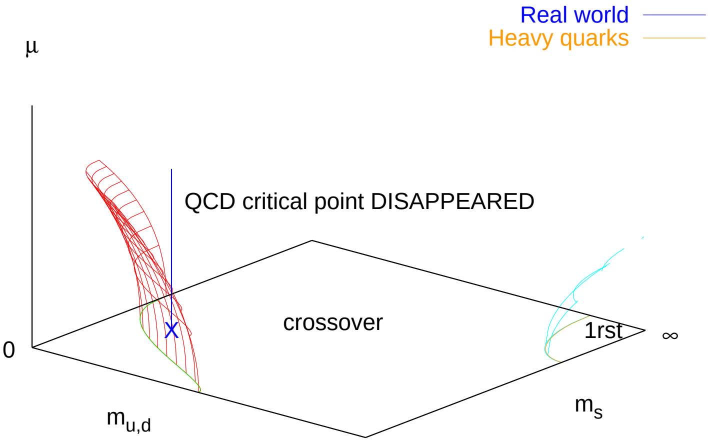

Conventional wisdom: first-order region expands with real $|\mu|$

# Phase diagram vs $(m_{u,d}, m_s)$, $T$ and $\mu$



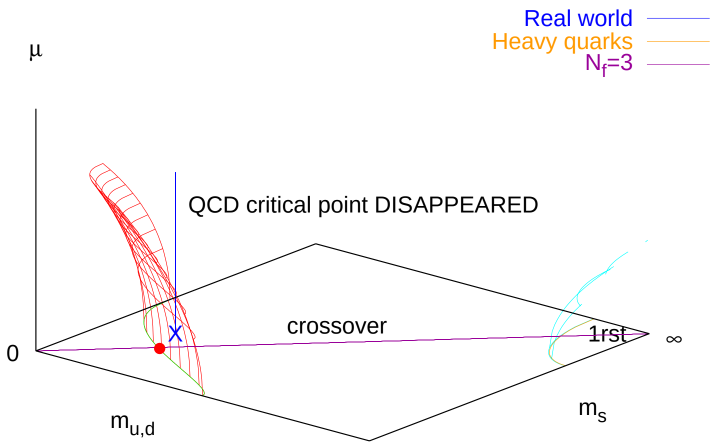

Exotic scenario: first-order region shrinks with real $|\mu|$ $\quad \frac{d\ m_c}{d\mu^2}\Big|_{\mu=0} < 0$

# Phase diagram vs $(m_{u,d}, m_s)$, $T$ and $\mu$




For heavy quarks, first-order region shrinks (PdF, Kim & Takaishi)

# Phase diagram vs $(m_{u,d}, m_s)$, $T$ and $\mu$

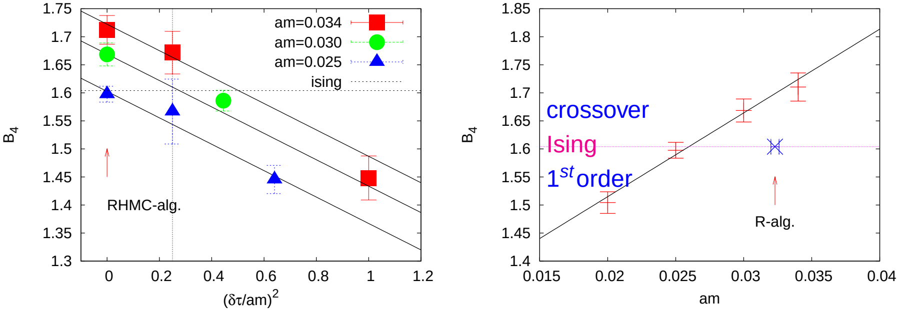



Detect 2nd order with Binder cumulant $B_4 \equiv \frac{\langle(\delta\bar{\psi}\psi)^4\rangle}{(\langle(\delta\bar{\psi}\psi)^2\rangle)^2}$. Check $N_f = 3$ first.

# Eliminating the stepsize error: R $\longrightarrow$ RHMC

- Old R-alg. requires $\delta\tau \to 0$ extrapolation

$\delta\tau \neq 0 \to$ bias transition towards 1rst-order small $B_4$ (Kogut Sinclair)



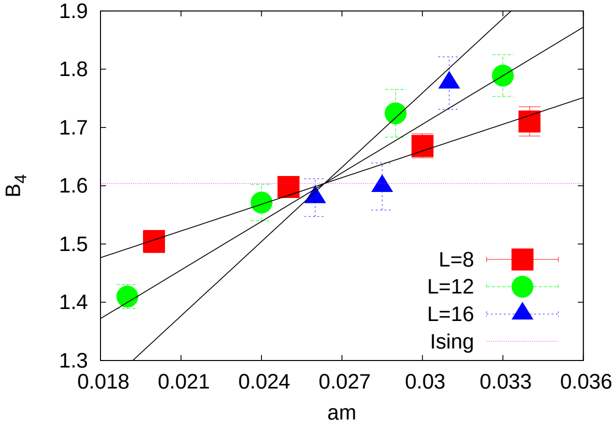

- New RHMC (Clark & Kennedy) exact and more efficient

$N_f = 3$: critical quark mass for 2nd order P.T. at $\mu = 0$ down by $\sim 25\%$ !

Lattice renorm.? No: corresponding $\frac{m_\pi}{T_c}$ down by $\sim 10\%$

**RHMC mandatory**

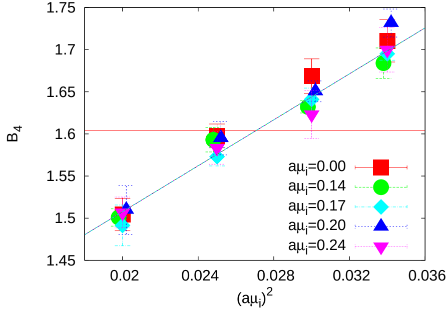
# $N_f = 3$, $\mu = 0$: finite size scaling

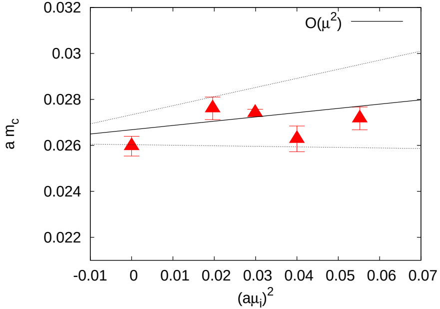



$$\xi \propto |m - m_c|^{-\nu} \quad \rightarrow \quad B_(m,L) = B_4^{Ising} + bL^{1/\nu}(m - m_c)$$

Fit: $\nu = 0.67(13)$ versus $\nu^{Ising} \approx 0.63$

# $N_f=3$, $\mu\neq 0$: critical mass versus $\mu$




No significant variation in $am_c \propto \frac{m_c}{T_c}$ versus $a\mu \propto \frac{\mu}{T}$. **Physical units?**

$$\frac{d(m_c/T_c)}{d(\mu/T)^2} \approx 0 \Rightarrow \frac{d\ \ m_c(\mu)/m_c(0)}{d(\mu/T)^2} \approx \frac{d\ \ T_c(\mu)/T_c(0)}{d(\mu/T)^2}$$

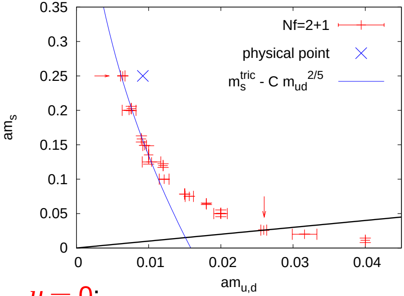
$$\text{with } \frac{T_c(\mu,m)}{T_c(0,m_0^c)} = 1 - 0.667(6)\left(\frac{\mu}{\pi T_c}\right)^2 + \ldots$$

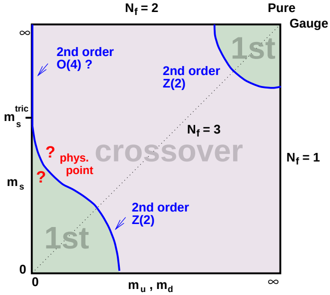
$$\frac{m_c(\mu)}{m_c(\mu{=}0)} = 1 - 0.7(4)\left(\frac{\mu}{\pi T}\right)^2 + \ldots$$

**The first-order region shrinks when $\mu$ is turned on!**

# Extend to $N_f = 2+1$

Line of second-order phase transitions in the quark mass plane $(m_{u,d}, m_s)$




$\mu = 0$:

- data consistent with tricritical point at $m_{u,d} = 0,\ m_s \sim 2.8T_c$
- physical point in crossover region

Fodor & Katz

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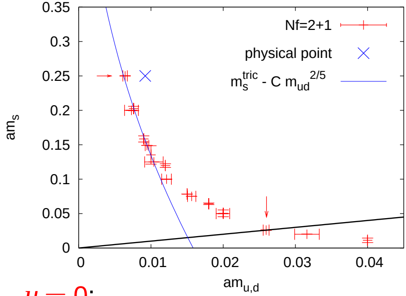
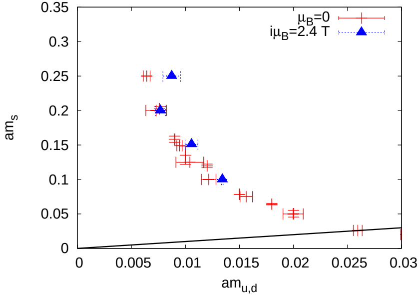



$\mu = 0$:

- data consistent with tricritical point at $m_{u,d} = 0$, $m_s \sim 2.8T_c$
- physical point in crossover region — Fodor & Katz

$\mu \neq 0$ imaginary:

- $am_c$ increases slightly $\Rightarrow$ decreases slightly with real $\mu$

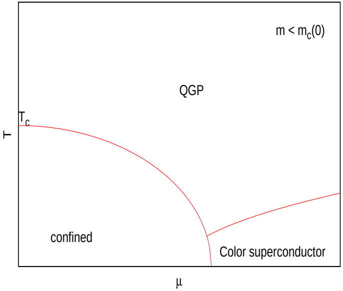
$$\Rightarrow \frac{d \; m_c(\mu)/m_c(0)}{d(\mu/T)^2} \lesssim \frac{d \; T_c(\mu)/T_c(0)}{d(\mu/T)^2} < 0$$

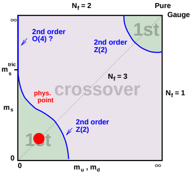
**The first-order region shrinks when $\mu$ is turned on**

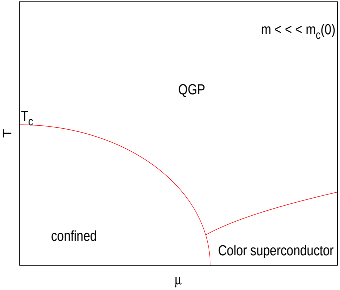
# Resulting phase diagram (simplest possibility)

Standard scenario

Exotic scenario



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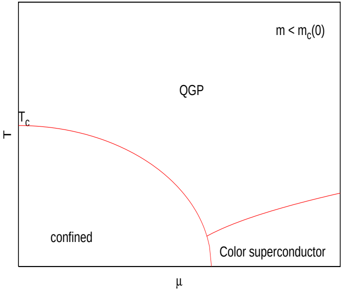
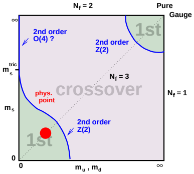
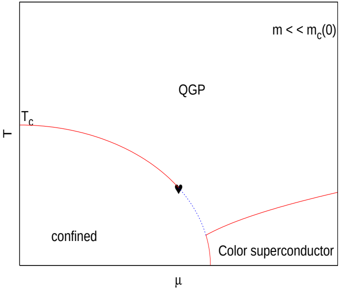
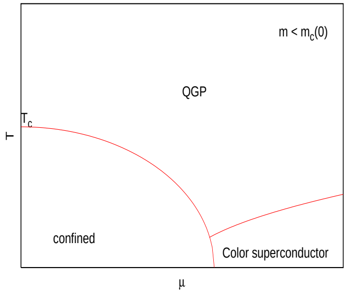


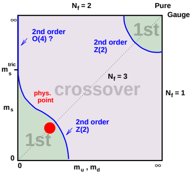



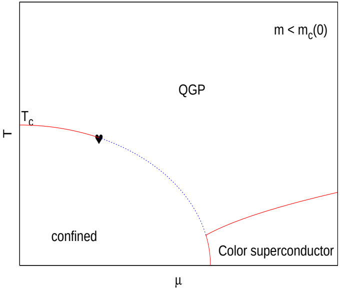
# Resulting phase diagram (simplest possibility)

## Standard scenario



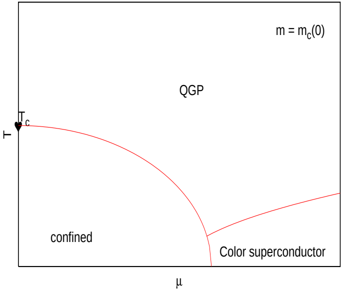

## Exotic scenario

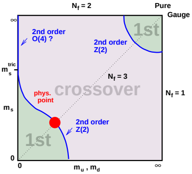


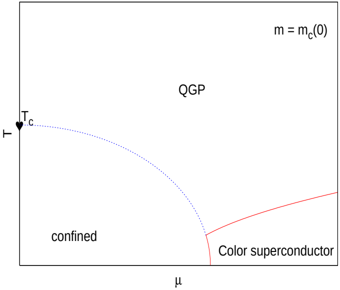
# Resulting phase diagram (simplest possibility)

Standard scenario

Exotic scenario



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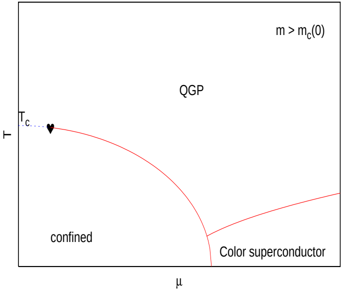
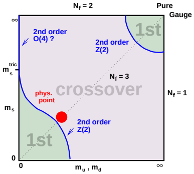
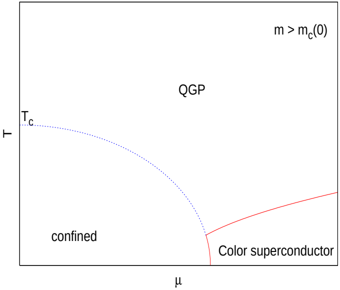
Standard scenario

Exotic scenario



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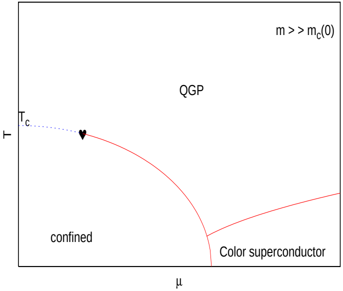
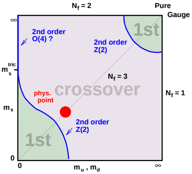
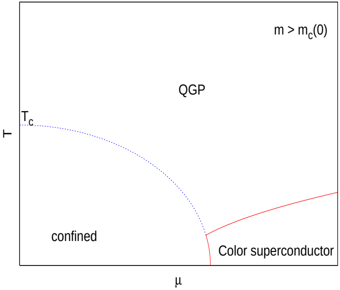


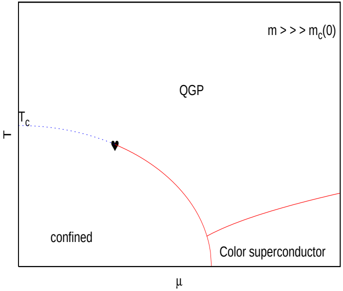



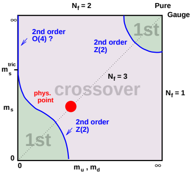



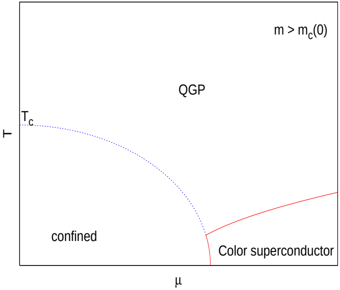
# Resulting phase diagram (simplest possibility)

## Standard scenario

## Exotic scenario

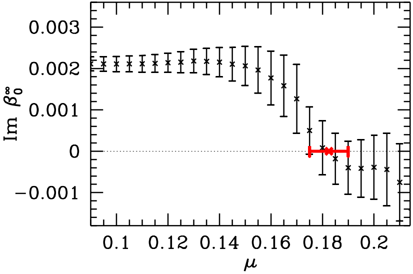


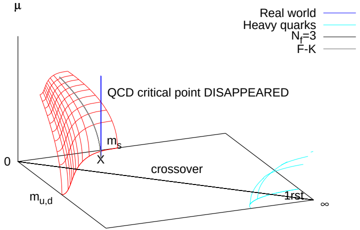



# Contradiction with other lattice studies?

- Fodor & Katz: $(T_E, \mu_E) = (162(2), 120(13))$ MeV ?



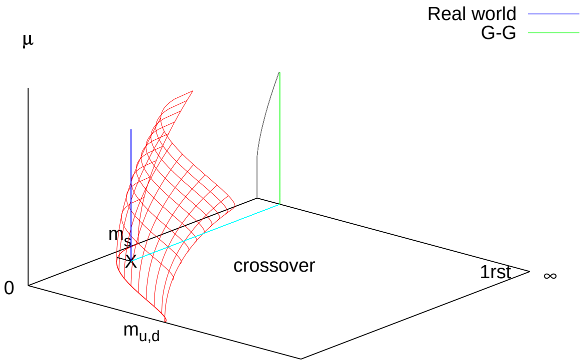




F&K keep $(m_q a)$ fixed, while $a(\beta_c)$ increases with $\mu$ $\rightarrow$ non-const. physics
Lighter quarks at larger $\mu$ may cause the phase transition
(dominant effect in our study)

Fake critical point? In any case, strong underestimate of $\mu_E$

# Contradiction with other lattice studies?

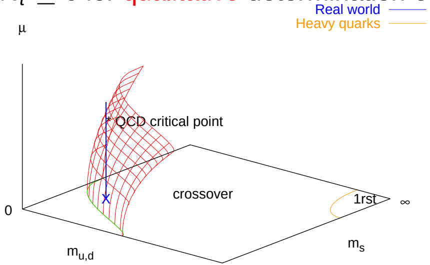
- Gavai & Gupta: $\mu_E / T_E \lesssim 1$ ?
  different theory $N_f = 2$



# Conclusions

Caveat: at $N_f = 3,\ \mu = 0$ critical point, $m_\pi$ varies by **factor** $\sim 4$
with Dirac discretization Bielefeld, MILC

- Need finer lattice $N_t \geq 6$ for qualitative determination of phase diagram




- $\frac{m_c(\mu)}{m_c(\mu=0)} = 1 + \mathcal{O}(1)(\frac{\mu}{\pi T})^2 \rightarrow$ critical surface almost vertical
  - high sensitivity to quark masses (e.g. $m_u \neq m_d$)
  - $\mu_E / T_E \lesssim 1$ requires fine-tuning $\rightarrow$ unnatural
- $1^{\mathrm{st}}$-order region shrinks with RHMC and improved actions/finer lattices
  $\rightarrow$ physical point further from critical surface
  $\rightarrow$ **larger** $\mu_E$