Quark and Gluon Angular Momentum in the Nucleon

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**Motivation**

- polarized DIS: *only ~ 30% of the proton spin due to quark spins*

  - ‘spin crisis’ → ‘spin puzzle’, because \( \Delta \Sigma \) much smaller than the quark model result \( \Delta \Sigma = 1 \)

  - quest for the remaining 70%
    - quark orbital angular momentum (OAM)
    - gluon spin
    - gluon OAM

  - How are the above quantities defined?
  - How can the above quantities be measured
example: angular momentum in QED

\[ \vec{J}_\gamma = \int d^3 r \, \vec{r} \times \left( \vec{E} \times \vec{B} \right) = \int d^3 r \, \vec{r} \times \left[ \vec{E} \times \left( \vec{\nabla} \times \vec{A} \right) \right] \]

- use \( \vec{E} \times \left( \vec{\nabla} \times \vec{A} \right) = E^j \vec{\nabla} A^j - \left( \vec{E} \cdot \vec{\nabla} \right) \vec{A} \)

and integrate \( \int d^3 r \, \vec{r} \times \left( \vec{E} \cdot \vec{\nabla} \right) \vec{A} \) by parts

\[ \rightarrow \vec{J}_\gamma = \int d^3 r \, \left[ E^j \left( \vec{r} \times \vec{\nabla} \right) A^j + \left( \vec{r} \times \vec{A} \right) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A} \right] \]

- replace 2\textsuperscript{nd} term (eq. of motion \( \vec{\nabla} \cdot \vec{E} = e j^0 = e \psi^\dagger \psi \)), yielding

\[ \vec{J}_\gamma = \int d^3 r \, \left[ \psi^\dagger \vec{r} \times e \vec{A} \psi + E^j \left( \vec{r} \times \vec{\nabla} \right) A^j + \vec{E} \times \vec{A} \right] \]

- \( \psi^\dagger \vec{r} \times e \vec{A} \psi \) cancels similar term in electron OAM \( \psi^\dagger \vec{r} \times (\vec{p} - e \vec{A}) \psi \)

\[ \rightarrow \text{decomposing } \vec{J}_\gamma \text{ into spin and orbital also shuffles angular momentum from photons to electrons!} \]
total angular momentum of isolated system uniquely defined

ambiguities arise when decomposing $\vec{J}$ into contributions from different constituents

gauge theories: changing gauge may also shift angular momentum between various degrees of freedom

decomposition of angular momentum in general depends on ‘scheme’ (gauge & quantization scheme)

does not mean that angular momentum decomposition is meaningless, but

one needs to be aware of this ‘scheme’-dependence in the physical interpretation of exp/lattice/model results in terms of spin vs. OAM

and, for example, not mix ‘schemes’, e.t.c.
Outline

- Ji decomposition
- Jaffe decomposition
- recent lattice results (Ji decomposition)
- model/QED illustrations for Ji v. Jaffe
- Chen-Goldman decomposition
The nucleon spin pizza(s)

\[ \frac{1}{2} \Delta \Sigma \equiv \frac{1}{2} \sum_q \Delta q \] common to both decompositions!
Angular Momentum Operator

- Angular momentum tensor $M^{\mu \nu \rho} = r^{\mu} T^{\nu \rho} - r^{\nu} T^{\mu \rho}$

- $\partial_{\rho} M^{\mu \nu \rho} = 0$

$\vec{J}^i = \frac{1}{2} \varepsilon^{i j k} \int d^3 r M^{j k 0}$ is conserved

$$\frac{d}{dt} \vec{J}^i = \frac{1}{2} \varepsilon^{i j k} \int d^3 r \partial_0 M^{j k 0} = \frac{1}{2} \varepsilon^{i j k} \int d^3 r \partial_l M^{j k l} = 0$$

- $M^{\mu \nu \rho}$ contains time derivatives (since $T^{\mu \nu}$ does)
  - Use eq. of motion to get rid of time derivatives
  - Integrate total derivatives appearing in $T^{0 i}$ by parts
  - Yields terms where derivative acts on $r^i$ which then 'disappears'

$\vec{J}^i$ usually contains both
  - 'Extrinsic' terms, which have the structure '\( \vec{r} \times \) Operator', and can be identified with 'OAM'
  - 'Intrinsic' terms, where the factor $\vec{r} \times$ does not appear, and can be identified with 'spin'
\[ \vec{J} = \int d^3r \sum_q \left[ \frac{1}{2} q^\dagger \Sigma q + q^\dagger (\vec{r} \times i\vec{D}) q \right] + \vec{r} \times (\vec{E} \times \vec{B}) \]

or, with \( S^\mu = (0, 0, 0, 1) \)

\[ \frac{1}{2} = \sum_q J_q + J_g = \sum_q \left( \frac{1}{2} \Delta q + L_q \right) + J_g \]

\[ \frac{1}{2} \Delta q = \frac{1}{2} \int d^3r \langle P, S| q^\dagger (\vec{r}) \Sigma^3 q(\vec{r}) |P, S\rangle \quad \Sigma^3 = i\gamma^1\gamma^2 \]

\[ L_q = \int d^3r \langle P, S| q^\dagger (\vec{r}) (\vec{r} \times i\vec{D})^3 q(\vec{r}) |P, S\rangle \quad i\vec{D} = i\vec{\partial} - g\vec{A} \]

\[ J_g = \int d^3r \langle P, S| \left[ \vec{r} \times (\vec{E} \times \vec{B}) \right]^3 |P, S\rangle \]
\[ \vec{J} = \int d^3r \sum_q \left[ \frac{1}{2} q^\dagger \Sigma q + q^\dagger \left( \vec{r} \times i \vec{D} \right) q \right] + \vec{r} \times \left( \vec{E} \times \vec{B} \right) \]

applies to each vector component of nucleon angular momentum, but usually applied only to \( \hat{z} \) component where at least quark spin has parton interpretation as difference between number densities

- \( \Delta q \) from polarized DIS
- \( J_q = \frac{1}{2} \Delta q + L_q \) from exp/lattice (GPDs)
- \( L_q \) in principle independently defined as matrix element of \( q^\dagger \left( \vec{r} \times i \vec{D} \right) q \), but in practice easier by subtraction \( L_q = J_q - \frac{1}{2} \Delta q \)
- \( J_g \) in principle accessible through gluon GPDs, but in practice easier by subtraction \( J_g = \frac{1}{2} - J_q \)
- Ji makes no further decomposition of \( J_g \) into intrinsic (spin) and extrinsic (OAM) piece
**L_q for proton from Ji-relation (lattice)**

- lattice QCD ⇒ moments of GPDs (LHPC; QCDSF)
  - insert in Ji-relation
  
  \[ \langle J^i_q \rangle = S^i \int dx \left[ H_q(x, \xi, 0) + E_q(x, \xi, 0) \right] x. \]

- \[ L^z_q = J^z_q - \frac{1}{2} \Delta q \]
  - \( L_u, L_d \) both large!
  - present calcs. show \( L_u + L_d \approx 0 \), but
    - disconnected diagrams ..?
    - \( m^2_\pi \) extrapolation
    - parton interpret. of \( L_q \)...
The Ji-relation (poor man’s derivation)

- What distinguishes the Ji-decomposition from other decompositions is the fact that $L_q$ can be constrained by experiment:

$$\langle \vec{J}_q \rangle = \vec{S} \int_{-1}^{1} dx \ x [H_q(x, \xi, 0) + E_q(x, \xi, 0)]$$

(nucleon at rest; $\vec{S}$ is nucleon spin)

$\rightarrow \ L_q^z = J_q^z - \frac{1}{2} \Delta q$

- derivation (MB-version):
  - consider nucleon state that is an eigenstate under rotation about the $\hat{x}$-axis (e.g. nucleon polarized in $\hat{x}$ direction with $\vec{p} = 0$ (wave packet if necessary)
  - for such a state, $\langle T_q^{00} y \rangle = 0 = \langle T_q^{zz} y \rangle$ and $\langle T_q^{0y} z \rangle = -\langle T_q^{0z} y \rangle$

$\rightarrow \ \langle T_q^{++} y \rangle = \langle T_q^{0y} z - T_q^{0z} y \rangle = \langle J_q^x \rangle$

$\rightarrow \$ relate 2nd moment of $\perp$ flavor dipole moment to $J_q^x$
derivation (MB-version):

consider nucleon state that is an eigenstate under rotation about the \(\hat{x}\)-axis (e.g. nucleon polarized in \(\hat{x}\) direction with \(\vec{p} = 0\) (wave packet if necessary)

for such a state, \(\langle T_{q}^{00}y \rangle = 0 = \langle T_{q}^{zz}y \rangle\) and \(\langle T_{q}^{0y}z \rangle = -\langle T_{q}^{0z}y \rangle\)

\(\langle T_{q}^{++}y \rangle = \langle T_{q}^{0y}z - T_{q}^{0z}y \rangle = \langle J_{q}^{x} \rangle\)

relate 2\(^{nd}\) moment of \(\perp\) flavor dipole moment to \(J_{q}^{x}\)

effect sum of two effects:

\(\langle T_{q}^{++}y \rangle\) for a point-like transversely polarized nucleon

\(\langle T_{q}^{++}y \rangle\) for a quark relative to the center of momentum of a transversely polarized nucleon

2\(^{nd}\) moment of \(\perp\) flavor dipole moment for point-like nucleon

\[
\psi = \left( \frac{f(r)}{\vec{\sigma} \cdot \vec{p}} \right) \chi \quad \text{with} \quad \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}
\]
The Ji-relation (poor man’s derivation)

**derivation (MB-version):**

- \( T^0_{qz} = i\bar{q} \left( \gamma^0 \partial^z + \gamma^z \partial^0 \right) q \)

- since \( \psi^\dagger \partial_z \psi \) is even under \( y \rightarrow -y \), \( i\bar{q}\gamma^0 \partial^z q \) does not contribute to \( \langle T^{0z} y \rangle \)

\[ \leftrightarrow \text{using } i\partial_0 \psi = E\psi, \text{ one finds} \]

\[
\langle T^{0z} b_y \rangle = E \int d^3r \psi^\dagger \gamma^0 \gamma^z \psi y = E \int d^3r \psi^\dagger \begin{pmatrix} 0 & \sigma^z \\ \sigma^z & 0 \end{pmatrix} \psi y \\
= \frac{2E}{E + M} \int d^3r \chi^\dagger \sigma^z \sigma^y \chi f(r)(-i)\partial^y f(r)y = \frac{E}{E + M} \int d^3r f^2(r)
\]

- consider nucleon state with \( \vec{p} = 0 \), i.e. \( E = M \) & \( \int d^3r f^2(r) = 1 \)

\[ \leftrightarrow 2^{nd} \text{ moment of } \perp \text{ flavor dipole moment } \langle T_{qz}^{-+} y \rangle = \langle T^{0z} b_y \rangle = \frac{1}{2} \]

\[ \leftrightarrow \text{‘overall shift’ of nucleon COM yields contribution} \]

\[ \frac{1}{2} \int dx x H_q(x, 0, 0) \text{ to } \langle T_{qz}^{-+} y \rangle \]
The Ji-relation (poor man’s derivation)

- spherically symmetric wave packet for Dirac particle with \( J_x = \frac{1}{2} \) centered around the origin has \( \perp \) center of momentum \( \frac{1}{M} \langle T^{++} b_y \rangle \) not at origin, but at \( \frac{1}{2M} \)!
- consistent with

\[
\frac{1}{2} = \langle J_x \rangle = \langle (T^0 z b^y - T^0 y b^z) \rangle = 2 \langle T^0 z b^y \rangle = \langle T^{++} b^y \rangle
\]

- ‘overall shift of \( \perp \) COM yields \( \langle T^{++} b_y \rangle = \frac{1}{2} \int dx x H_q(x, 0, 0) \)
- intrinsic distortion adds \( \frac{1}{2} \int dx x E_q(x, 0, 0) \) to that

- So far: only unpolarized (or long. pol.) nucleon! In general ($\xi = 0$):

$$\int \frac{dx^-}{4\pi} e^{ip^+ x^- - x} \left\langle P + \Delta, \uparrow \right| \bar{q}(0) \gamma^+ q(x^-) \left| P, \uparrow \right\rangle = H(x,0,-\Delta^2)$$

$$\int \frac{dx^-}{4\pi} e^{ip^+ x^- - x} \left\langle P + \Delta, \uparrow \right| \bar{q}(0) \gamma^+ q(x^-) \left| P, \downarrow \right\rangle = -\Delta_x \Delta_y e^{i\Delta y} E(x,0,\Delta^2).$$

- Consider nucleon polarized in $x$ direction (in IMF)

$$|X\rangle \equiv |p^+, \mathbf{R}_\perp = 0_\perp, \uparrow\rangle + |p^+, \mathbf{R}_\perp = 0_\perp, \downarrow\rangle.$$  

$\rightarrow$ unpolarized quark distribution for this state:

$$q(x, b_\perp) = \mathcal{H}(x, b_\perp) - \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} E(x, 0, -\Delta^2) e^{-i b_\perp \cdot \Delta_\perp}$$

- Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from $j^3$!

[X.Ji, PRL 91, 062001 (2003)]
p polarized in $+\hat{x}$ direction

\[ u(x, b_\perp) \]
\[ d(x, b_\perp) \]

\[ \vec{p}_{\gamma} \]

lattice results (Hägler et al.)
The Ji-relation (poor man’s derivation)

- ‘overall shift of ⊥ COM yields $\langle T^+ q^+ b_y \rangle = \frac{1}{2} \int dx \ xH_q(x, 0, 0)$
- intrinsic distortion adds $\frac{1}{2} \int dx \ xE_q(x, 0, 0)$ to that
- $J^x_q = \frac{1}{2} \int dx \ x[H_q(x, 0, 0) + E_q(x, 0, 0)]$
- rotational invariance: should apply to each vector component, but parton interpretation (transverse shift) only for ⊥ pol. nucleon
Angular Momentum in QCD (Jaffe & Manohar)

- define OAM on a light-like hypersurface rather than a space-like hypersurface

\[ \tilde{J}^3 = \int d^2 x_\perp \int dx^- M^{12+} \]

where \( x^- = \frac{1}{\sqrt{2}} (x^0 - x^3) \) and \( M^{12+} = \frac{1}{\sqrt{2}} (M^{120} + M^{123}) \)

- Since \( \partial_\mu M^{12\mu} = 0 \)

\[ \int d^2 x_\perp \int dx^- M^{12+} = \int d^2 x_\perp \int dx^3 M^{120} \]

(compare electrodynamics: \( \vec{\nabla} \cdot \vec{B} = 0 \) \( \Rightarrow \) flux in = flux out)

- use eqs. of motion to get rid of ‘time’ \( (\partial_+ \text{ derivatives}) \) & integrate by parts whenever a total derivative appears in the \( T^{i+} \) part of \( M^{12+} \)
Jaffe/Manohar decomposition

in light-cone framework & light-cone gauge
\( A^+ = 0 \) one finds for \( J^z = \int dr d^2 r_\perp M^{+xy} \)

\[
\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g
\]

where \( (\gamma^+ = \gamma^0 + \gamma^z) \)

\[
\mathcal{L}_q = \int d^3 r \langle P, S | \bar{q}(\vec{r}) \gamma^+ (\vec{r} \times i\vec{\partial})^\dagger q(\vec{r}) | P, S \rangle
\]

\[
\Delta G = \varepsilon^{+ - ij} \int d^3 r \langle P, S | \text{Tr} F^{+i} A^j | P, S \rangle
\]

\[
\mathcal{L}_g = 2 \int d^3 r \langle P, S | \text{Tr} F^{+j} (\vec{r} \times i\vec{\partial})^\dagger A^j | P, S \rangle
\]
Jaffe/Manohar decomposition

\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g \]

- \( \Delta \Sigma = \sum_q \Delta q \) from polarized DIS (or lattice)
- \( \Delta G \) from \( \overrightarrow{p} \overleftarrow{p} \) or polarized DIS (evolution)
- \( \Delta G \) gauge invariant, but local operator only in light-cone gauge
- \( \int dxx^n \Delta G(x) \) for \( n \geq 1 \) can be described by manifestly gauge inv. local op. (\( \rightarrow \) lattice)
- \( \mathcal{L}_q, \mathcal{L}_g \) independently defined, but
  - no exp. identified to access them
  - not accessible on lattice, since nonlocal except when \( A^+ = 0 \)
- parton net OAM \( \mathcal{L} = \mathcal{L}_g + \sum_q \mathcal{L}_q \) by subtr. \( \mathcal{L} = \frac{1}{2} - \frac{1}{2} \Delta \Sigma - \Delta G \)
- in general, \( \mathcal{L}_q \neq \mathcal{L}_q \) \( \mathcal{L}_g + \Delta G \neq J_g \)
- \( J_g - \Delta G \sim \int d^3r F^{+j} \left( \vec{r} \times i \vec{\partial} \right)^z A^j + \psi^\dagger \vec{r} \times g \vec{A} \psi \) Interpretation??
$L_q \neq L_q$

- $L_q$ matrix element of

$$q^\dagger \left[ \vec{r} \times \left( i\vec{\partial} - g\vec{A} \right) \right] \stackrel{\sim}{z} q = \overline{q}\gamma^0 \left[ \vec{r} \times \left( i\vec{\partial} - g\vec{A} \right) \right] \stackrel{\sim}{z} q$$

- $L_q^z$ matrix element of $(\gamma^+ = \gamma^0 + \gamma^z)$

$$\overline{q}\gamma^+ \left[ \vec{r} \times i\vec{\partial} \right] \stackrel{\sim}{z} q \bigg|_{A^+=0}$$

- (for $p = 0$) matrix element of $\overline{q}\gamma^z \left[ \vec{r} \times \left( i\vec{\partial} - g\vec{A} \right) \right] \stackrel{\sim}{z} q$ vanishes (parity!)

$\leftrightarrow L_q$ identical to matrix element of $\overline{q}\gamma^+ \left[ \vec{r} \times \left( i\vec{\partial} - g\vec{A} \right) \right] \stackrel{\sim}{z} q$ (nucleon at rest)

$\leftrightarrow$ even in light-cone gauge, $L_q^z$ and $L_q^z$ still differ by matrix element of $q^\dagger \left( \vec{r} \times g\vec{A} \right) \stackrel{\sim}{z} q \bigg|_{A^+=0} = q^\dagger (xgA^y - ygA^x) q \bigg|_{A^+=0}$
Summary (part 1):

- Ji: \( J^z = \frac{1}{2} \Delta \Sigma + \sum_q L_q + J_g \)
- Jaffe: \( J^z = \frac{1}{2} \Delta \Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g \)

\( \Delta G \) can be defined without reference to gauge (and hence gauge invariently) as the quantity that enters the evolution equations and/or represented by simple (i.e. local) operator only in LC gauge and corresponds to the operator that one would naturally identify with ‘spin’ only in that gauge

In general \( L_q \neq \mathcal{L}_q \) or \( J_g \neq \Delta G + \mathcal{L}_g \), but

How significant is the difference between \( L_q \) and \( \mathcal{L}_q \), etc.?
OAM in scalar diquark model

[MB + H. Budhathoki Chhetri (BC), 2009]

- toy model for nucleon where nucleon (mass $M$) splits into quark (mass $m$) and scalar ‘diquark’ (mass $\lambda$)

- light-cone wave function for quark-diquark Fock component

$$\psi_{+\frac{1}{2}}(x, \mathbf{k}_\perp) = \left(M + \frac{m}{x}\right) \phi$$

$$\psi_{-\frac{1}{2}} = -\frac{k^1 + i k^2}{x} \phi$$

with $\phi = \frac{c/\sqrt{1-x}}{M^2 - \frac{k^2 + m^2}{x} - \frac{k^2 + \lambda^2}{1-x}}$.

- quark OAM according to JM: $L_q = \int_0^1 dx \int \frac{d^2 k_\perp}{16\pi^3} (1-x) \left| \psi_{-\frac{1}{2}} \right|^2$

- quark OAM according to Ji: $L_q = \frac{1}{2} \int_0^1 dx \ x \left[ q(x) + E(x, 0, 0) \right] - \frac{1}{2} \Delta q$

- (using Lorentz inv. regularization, such as Pauli Villars subtraction) both give identical result, i.e. $L_q = \mathcal{L}_q$

- not surprising since scalar diquark model is not a gauge theory
But, even though $L_q = \mathcal{L}_q$ in this non-gauge theory

$$\mathcal{L}_q(x) \equiv \int \frac{d^2k_\perp}{16\pi^3} (1-x) \left| \psi_{-\frac{1}{2}}^\uparrow \right|^2 \neq \frac{1}{2} \left\{ x \left[ q(x) + E(x, 0, 0) \right] - \Delta q(x) \right\} \equiv L_q(x)$$

\[\text{‘unintegrated Ji-relation’ does not yield x-distribution of OAM}\]
light-cone wave function in $e\gamma$ Fock component

$$\Psi_{+\frac{1}{2}+1}^{\uparrow}(x, k_{\perp}) = \sqrt{2} \frac{k_{1} - ik_{2}}{x(1 - x)} \phi$$

$$\Psi_{+\frac{1}{2}-1}^{\uparrow}(x, k_{\perp}) = -\sqrt{2} \frac{k_{1} + ik_{2}}{1 - x} \phi$$

$$\Psi_{-\frac{1}{2}+1}^{\uparrow}(x, k_{\perp}) = \sqrt{2} \left( \frac{m}{x} - m \right) \phi$$

$$\Psi_{-\frac{1}{2}-1}^{\uparrow}(x, k_{\perp}) = 0$$

OAM of $e^-$ according to Jaffe/Manohar

$$L_{e} = \int_{0}^{1} dx \int d^{2}k_{\perp} (1 - x) \left[ \left| \Psi_{+\frac{1}{2}-1}^{\uparrow}(x, k_{\perp}) \right|^{2} - \left| \Psi_{+\frac{1}{2}+1}^{\uparrow}(x, k_{\perp}) \right|^{2} \right]$$

$e^-$ OAM according to Ji $L_{e} = \frac{1}{2} \int_{0}^{1} dx \ x \left[ q(x) + E(x, 0, 0) \right] - \frac{1}{2} \Delta q$

$\sim \ L_{e} = L_{e} + \frac{\alpha}{4\pi} \neq L_{e}$

Likewise, computing $J_{\gamma}$ from photon GPD, and $\Delta \gamma$ and $L_{\gamma}$ from light-cone wave functions and defining $\hat{L}_{\gamma} \equiv J_{\gamma} - \Delta \gamma$ yields

$$\hat{L}_{\gamma} = L_{\gamma} + \frac{\alpha}{4\pi} \neq L_{\gamma}$$

$\frac{\alpha}{4\pi}$ appears to be small, but here $L_{e}$, $L_{e}$ are all of $O(\frac{\alpha}{\pi})$
1-loop QCD: \[ L_q - L_q = \frac{\alpha_s}{3\pi} \] (for \( j_z = +\frac{1}{2} \))

recall (lattice QCD): \( L_u \approx -0.15; \quad L_d \approx +0.15 \)

QCD evolution yields negative correction to \( L_u \) and positive correction to \( L_d \)

evolution suggested (A.W. Thomas) to explain apparent discrepancy between quark models (low \( Q^2 \)) and lattice results \( (Q^2 \sim 4\text{GeV}^2) \)

above result suggests that \( L_u > L_u \) and \( L_d < L_d \)

additional contribution (with same sign) from vector potential due to spectators (MB, to be published)

possible that lattice result consistent with \( L_u > L_d \)
inclusive $\overrightarrow{e\bar{p}}/\overrightarrow{p\bar{p}}$ provide access to
- quark spin $\frac{1}{2}\Delta q$
- gluon spin $\Delta G$
- parton grand total OAM $\mathcal{L} \equiv \mathcal{L}_g + \sum_q \mathcal{L}_q = \frac{1}{2} - \Delta G - \frac{1}{2} \sum_q \Delta q$

DVCS & polarized DIS and/or lattice provide access to
- quark spin $\frac{1}{2}\Delta q$
- $J_q$ & $L_q = J_q - \frac{1}{2}\Delta q$
- $J_g = \frac{1}{2} - \sum_q J_q$

$J_g - \Delta G$ does not yield gluon OAM $\mathcal{L}_g$

$L_q - \mathcal{L}_q = \mathcal{O}(0.1 \ast \alpha_s)$ for $\mathcal{O}(\alpha_s)$ dressed quark
Chen, Goldman et al.: integrate by parts in $J_g$ only for term involving $A_{phys}$, where

$$A = A_{pure} + A_{phys} \quad \text{with} \quad \nabla \cdot A_{phys} = 0 \quad \nabla \times A_{pure} = 0$$

$$\frac{1}{2} = \sum_q J_q + J_g = \sum_q \left( \frac{1}{2} \Delta q + L'_q \right) + S'_g + L'_g \quad \text{with} \ \Delta q \ \text{as in JM/Ji}$$

$$L'_q = \int d^3 x \langle P, S | q^\dagger(\vec{x}) \left( \vec{x} \times i\vec{D}_{pure} \right)^3 q(\vec{x}) | P, S \rangle$$

$$S'_g = \int d^3 x \langle P, S | \left( \vec{E} \times \vec{A}_{phys} \right)^3 | P, S \rangle$$

$$L'_g = \int d^3 x \langle P, S | E^i \left( \vec{x} \times \vec{\nabla} \right)^3 A^i_{phys} | P, S \rangle$$

$$i\vec{D}_{pure} = i\vec{\partial} - g\vec{A}_{pure}$$

only $\frac{1}{2} \Delta q$ accessible experimentally
**example: angular momentum in QED**

- consider now, QED with electrons:

\[
\vec{J}_\gamma = \int d^3r \vec{x} \times (\vec{E} \times \vec{B}) = \int d^3r \vec{x} \times [\vec{E} \times (\vec{\nabla} \times \vec{A})]
\]

- integrate by parts

\[
\vec{J} = \int d^3r \left[ E^j \left( \vec{x} \times \vec{\nabla} \right) A^j + \left( \vec{x} \times \vec{A} \right) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A} \right]
\]

- replace 2\textsuperscript{nd} term (eq. of motion \( \vec{\nabla} \cdot \vec{E} = e j^0 = e \psi^\dagger \psi \)), yielding

\[
\vec{J}_\gamma = \int d^3r \left[ \psi^\dagger \vec{r} \times e \vec{A} \psi + E^j \left( \vec{x} \times \vec{\nabla} \right) A^j + \vec{E} \times \vec{A} \right]
\]

- \( \psi^\dagger \vec{r} \times e \vec{A} \psi \) cancels similar term in electron OAM \( \psi^\dagger \vec{r} \times (\vec{p} - e \vec{A}) \psi \)

\( \leftrightarrow \) decomposing \( \vec{J}_\gamma \) into spin and orbital also shuffles angular momentum from photons to electrons!
Chen, Goldman et al.: integrate by parts in $J_g$
only for term involving $A_{pure}$, where

$$A = A_{pure} + A_{phys} \quad \text{with} \quad \nabla \cdot A_{phys} = 0 \quad \nabla \times A_{pure} = 0$$
B.L.T. pizza?

- Bakker, Leader, Trueman:
  - JM only applies for \( s = \hat{p} \) (helicity sum rule)
  - Ji applies to any component, but parton interpretation only for \( S_z \)
  - For \( p \neq 0 \), Ji only applies to helicity
  - ‘sum rule’ \( s \perp \hat{p} \)

\[
\frac{1}{2} = \frac{1}{2} \sum_{a \in q, \bar{q}} \int dx h_1^a(x) + \sum_{a \in q, \bar{q}, g} \langle L_{sT}^a \rangle
\]

where \( L_{sT}^a \) component of \( L^a \) along \( s_T \)

- note: \( \sum_{a \in q, \bar{q}} \int dx h_1^a(x) \) not tensor charge (latter is: ‘\( q - \bar{q} \)’)
- \( L^a \sim \psi^\dagger k \times \nabla_k \psi \)
- distinction between transversity and transverse spin obscure in two-component formalism used
‘B.L.T. sum rule’ s \perp \hat{p} 
\frac{1}{2} = \frac{1}{2} \sum_{a \in q, \bar{q}} \int dx h^a_1(x) + \sum_{a \in q, \bar{q}, s} \langle L^a_{sT} \rangle 

should already be suspicious as \( T^{\mu\nu} \) is chirally even (\( m_q = 0 \)) and so should \( \vec{J} \)...

\( \langle L^a_{sT} \rangle \) not accessible experimentally, i.e. B.L.T. not experimentally falsifiable, but

studies (diquark model) under way to test B.L.T. ...

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