Precision measurement of $\pi\pi$ scattering lengths at NA48/2

Andrea Bizzeti
University of Modena and Reggio Emilia and I.N.F.N. Sezione di Firenze, Italy

on behalf of the NA48/2 collaboration:
Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Vienna

Introduction

The NA48/2 experiment at CERN SPS: setup and data

$K^\pm \rightarrow \pi^+\pi^- e^{\pm}\nu (K_{e4})$ decays: Form Factors and $\pi\pi$ scattering lengths

NA48/2 $K_{e4}$ and $K_{3\pi}$ results: comparison

Conclusions
Introduction

The NA48/2 experiment at CERN SPS: setup and data

\[ K^\pm \rightarrow \pi^+\pi^- e^{\pm}\nu \ (K_{e4}) \] decays: Form Factors and \( \pi\pi \) scattering lengths

NA48/2 \( K_{e4} \) and \( K_{3\pi} \) results: comparison

Conclusions
Why to measure $\pi\pi$ scattering lengths

- $a_0$ and $a_2$ are the S-wave $\pi\pi$ scattering lengths in isospin states $I=0$ and $I=2$
- They are related to fundamental parameters of Chiral Perturbation Theory (ChPT)
- Theoretical calculations based on ChPT provide accurate predictions for their values
- A precise measurement of $a_0$ and $a_2$ allows to test the theory and provides important constraints for the ChPT Lagrangian parameters
How to measure $\pi\pi$ scattering lengths

3 kinds of measurements have been developed:

**Pionium atoms:**

DIRAC (CERN SPS) ($\pi\pi$) lifetime

$PLB$ 619 (2005) 50-60

**$K_{3\pi}$ modes (cusp):**

$BR(\, K^\pm \rightarrow \pi^\pm \pi^0\pi^0\,) = (1.757 \pm 0.024) \cdot 10^{-2}$

$\rightarrow$ NA48/2 (CERN SPS): 60 \cdot 10^6

$EPJC$ 64 (2009) 589-608

$BR(\, K_L^0 \rightarrow \pi^0\pi^0\pi^0\,) = (19.56 \pm 0.14) \cdot 10^{-2}$

KTeV (FNAL Tevatron). 68 \cdot 10^6

NA48 (CERN SPS): 100 \cdot 10^6

**$K_{e4}$ decays:**

$BR(\, K^\pm \rightarrow \pi^+\pi^- e^\pm \nu\,) = (4.09 \pm 0.09) \cdot 10^{-5}$

Very clean environment, but limited statistics:

S118 (CERN PS): 0.03 \cdot 10^6

E685 BNL experiment 0.4 \cdot 10^6

$\rightarrow$ NA48/2 (CERN SPS): 1.1 \cdot 10^6
Introduction

The NA48/2 experiment at CERN SPS: setup and data

\[ K^\pm \rightarrow \pi^+\pi^- e^{\pm}\nu (K_{e4}) \text{ decays: Form Factors and } \pi\pi \text{ scattering lengths} \]

NA48/2 \( K_{e4} \) and \( K_{3\pi} \) results: comparison

Conclusions
The NA48/2 experiment at CERN SPS
The NA48/2 beams

- **Simultaneous** \( P = (60 \pm 3) \text{ GeV/c} \) \( K^+ \) and \( K^- \) beams
  \( \Rightarrow \) large charge symmetrization of experimental conditions
- Beams coincide within \( \sim 1 \text{ mm} \) along the 114 m decay volume.
- Flux ratio \( K^+/K^- \sim 1.8 \).
The NA48/2 detectors

- LKr electromagnetic calorimeter: quasi-homogeneous, high granularity
  \[ \sigma[M(\pi^\pm \pi^0 \pi^0)] = 1.4 \text{ MeV/c}^2 \]

- Magnetic spectrometer:
  4 DCH + dipole magnet
  \[ \sigma[M(3\pi^\pm)] = 1.7 \text{ MeV/c}^2 \]
  \( \Rightarrow \) e/\pi discrimination \((E/p)\)

- Scintillator hodoscope for charged fast trigger:
  \( \sigma(t) = 150 \text{ ps} \)

- hadron calorimeter
- muon counters
- photon vetoes
The NA48/2 Data sample

Main goal of NA48/2: search for CP violation in $K^\pm \rightarrow 3\pi$ Dalitz plots

Two years of data taking: 2003 run ($\sim$ 50 days) + 2004 run ($\sim$ 60 days)

Total statistics:

- $K^\pm \rightarrow \pi^+ \pi^- \pi^\pm : \sim 4 \cdot 10^9$
- $K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm : \sim 1 \cdot 10^8$
- $K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu : \sim 1.13 \cdot 10^6$

$\pi \pi$ scattering lengths measurement from $K_{3\pi}$ decays ("cusp")

already published $\rightarrow$ EPJ C 64 (2009) 589-608
Introduction

The NA48/2 experiment at CERN SPS: setup and data

\[ K^\pm \rightarrow \pi^+\pi^- e^{\pm}\nu \ (K_{e4}) \] decays: Form Factors and \(\pi\pi\) scattering lengths

NA48/2 \(K_{e4}\) and \(K_{3\pi}\) results: comparison

Conclusions
Ke4 decays: kinematics and formalism

Five kinematic variables (Cabibbo-Maksymowicz 1965):

\[ s_\pi = M^2_{\pi\pi}, \quad s_e = M^2_{e\nu}, \]
\[ \cos \theta_\pi, \quad \cos \theta_e, \quad \phi \]

Partial Wave expansion of the amplitude into s and p waves
+ Watson theorem for \( \delta^I_l \)
⇒ \( \delta_0^0 = \delta_s \) and \( \delta_1^1 = \delta_p \)

2 Axial Form Factors (\( F \) and \( G \)):
\[ F = F_s e^{i\delta_s} + F_p e^{i\delta_p} \cos \theta_\pi \]
\[ G = G_p e^{i\delta_p} \]

1 Vector Form Factor (\( H \)):
\[ H = H_p e^{i\delta_p} \]

\( K^+ \) (at rest) → \( \pi^+ \pi^- e^+ \nu \)

Fit the distribution in the five-dimensional space of the Ca.Ma. variables with 4 form factors and only one phase shift, assuming identical phases for the p-wave form factors

The fit parameters are:
\[ F_p, F_p, G_p, H_p \] and \( \delta = \delta_s - \delta_p \)
\( (F_p, F_p, G_p, H_p, \delta \) are real)
Ke4 decay: Event selection and background rejection

Signal \((\pi^+ \pi^- e^{\pm} \nu)\) topology:

- 3 charged tracks and a “good” vertex
- 2 opposite sign pions, 1 electron \([E_{LK}\!/p \simeq 1]\)
- some missing energy and \(p_T (\nu)\)
- good reconstructed \(P_K\) (missing \(\nu\) hypothesis)
Ke4 decay: Event selection and background rejection

Signal \((\pi^+\pi^-e^±\nu)\) topology:

- 3 charged tracks and a “good” vertex
- 2 opposite sign pions, 1 electron \([E_{LKr}/p \simeq 1]\)
- some missing energy and \(p_T(\nu)\)
- good reconstructed \(P_K\) (missing \(\nu\) hypothesis)

Background main sources:

- \(K^+ \rightarrow \pi^+\pi^-\pi^+\) (\(\pi^+ \rightarrow e^+\nu\) or \(\pi^+\) mis-ID)
- \(K^+ \rightarrow \pi^+\pi^0\) (\(\pi^0 \rightarrow e^+e^-\gamma\) and \(e^-\) mis-ID)
Ke4 decay: Event selection and background rejection

Signal \((\pi^+\pi^-e^{\pm}\nu)\) topology:

- 3 charged tracks and a “good” vertex
- 2 opposite sign pions, 1 electron \([E_{LKr}/p \simeq 1]\)
- some missing energy and \(p_T(\nu)\)
- good reconstructed \(P_K\) (missing \(\nu\) hypothesis)

Background main sources:

- \(K^+ \rightarrow \pi^+\pi^-\pi^+\) \((\pi^+ \rightarrow e^+\nu\) or \(\pi^+\text{ mis-ID})\)
- \(K^+ \rightarrow \pi^+\pi^0\) \((\pi^0 \rightarrow e^+e^-\gamma\) and \(e^-\text{ mis-ID})\)

Control sample from data (assuming \(\Delta S = \Delta Q\)):

- \(K^\pm \rightarrow \pi^{\pm}\pi^{\pm}e^{\mp}\nu\) (“Wrong-Sign” events)
  2 same sign pions + 1 electron
- Ratio \(RS/WS\) (“Right-Sign”/“Wrong-Sign”) events =
  - 2/1 if coming from \(K_{3\pi}\) (dominant)
  - 1/1 if coming from \(K_{2\pi}\)
Ke4 decay: background rejection

Total background level $\sim 2 \times 0.3\%$

- estimated from WS events in Data
- checked with Monte Carlo simulation of background processes

![Graph showing data and Monte Carlo comparison](image)

Data/MC ratio
- black: no BKG subtraction
- red: subtraction = $2 \times$ WS
Ke4 decay: fitting procedure

Total (2003+2004) 1.13 million $K_{e4}$ decays

Using iso-populated boxes in the 5-D space of the Ca.Ma variables $(M_{\pi\pi}, M_{e\nu}, \cos \theta_{\pi}, \cos \theta_{e} \text{ and } \phi)$ we define a grid of

\[10 \times 5 \times 5 \times 5 \times 12 = 15000 \text{ variable size boxes}\]
Ke4 decay: fitting procedure

Total (2003+2004) 1.13 million $K_{e4}$ decays

Using iso-populated boxes in the 5-D space of the Ca.Ma variables ($M_{\pi\pi}$, $M_{e\nu}$, $\cos \theta_{\pi}$, $\cos \theta_{e}$ and $\phi$) we define a grid of

$$10 \times 5 \times 5 \times 5 \times 12 = 15000 \text{ variable size boxes}$$

In each $M_{\pi\pi}$ “slice” (containing 1500 boxes):

- a set of 4 fit parameters ($F_p$, $G_p$, $H_p$, $\delta$) is extracted
- the normalization $F_s^2$ is obtained by the ratio Data / MC

<table>
<thead>
<tr>
<th></th>
<th>Data sample (events)</th>
<th>Monte Carlo sample (events)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{e4}^+$</td>
<td>726 400 (48 / box)</td>
<td>17.4 million (1160 / box)</td>
</tr>
<tr>
<td>$K_{e4}^-$</td>
<td>494 400 (27 / box)</td>
<td>9.7 million (650 / box)</td>
</tr>
</tbody>
</table>
Ke4 decay: fitting procedure

Total (2003+2004) 1.13 million $K_{e4}$ decays

Using iso-populated boxes in the 5-D space of the Ca.Ma variables ($M_{\pi\pi}$, $M_{e\nu}$, $\cos \theta_{\pi}$, $\cos \theta_{e}$ and $\phi$) we define a grid of

$$10 \times 5 \times 5 \times 5 \times 12 = 15000 \text{ variable size boxes}$$

In each $M_{\pi\pi}$ “slice” (containing 1500 boxes):

- a set of 4 fit parameters ($F_p$, $G_p$, $H_p$, $\delta$) is extracted
- the normalization $F_s^2$ is obtained by the ratio Data / MC

<table>
<thead>
<tr>
<th></th>
<th>Data sample (events)</th>
<th>Monte Carlo sample (events)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+_{e4}$</td>
<td>726 400 ( 48 / box )</td>
<td>17.4 million ( 1160 / box )</td>
</tr>
<tr>
<td>$K^-_{e4}$</td>
<td>494 400 ( 27 / box )</td>
<td>9.7 million ( 650 / box )</td>
</tr>
</tbody>
</table>

$K^+$ and $K^-$ samples fitted separately in 10 independent $M_{\pi\pi}$ slices, then combined in each slice according to their statistical error.

No assumption is made on the variation of the phase $\delta$ (and FF) from one $M_{\pi\pi}$ slice to the next (i.e. “model independent” analysis)
Ke4 decay: Data / MC comparison (after the fit)

Points = Data
Histogram = Simulation after fit
Yellow hist. = WS background
($\times 10$ to be visible)
Ke4 Form Factors: fit results

A series expansion in \( q^2 = \left[ M_{\pi \pi}^2 / (4m_{\pi}^2) - 1 \right] \) and \( M_{e\nu}^2 / (4m_{\pi}^2) \) is used to describe the FF variations in the isospin symmetry limit:

\[
F_s^2 = f_s^2 \left[ 1 + f_s'/f_s \ M_{e\nu} / (4m_{\pi}^2) \right] + f_s''/f_s \ q^4 + f_e'/f_s \ M_{e\nu} / (4m_{\pi}^2) \]

\[
G_p/f_s = g_p/f_s + g_p'/f_s \ q^2
\]

\[
F_p = f_p \quad H_p = h_p
\]

Systematics:

- mostly from background + acceptance control
- comparable or smaller than statistical error

<table>
<thead>
<tr>
<th>Total statistics (2003+2004)</th>
<th>value</th>
<th>stat</th>
<th>syst</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_s'/f_s )</td>
<td>0.152</td>
<td>±0.007</td>
<td>±0.005</td>
</tr>
<tr>
<td>( f_s''/f_s )</td>
<td>-0.073</td>
<td>±0.007</td>
<td>±0.006</td>
</tr>
<tr>
<td>( f_e'/f_s )</td>
<td>0.068</td>
<td>±0.006</td>
<td>±0.007</td>
</tr>
<tr>
<td>( f_p/f_s )</td>
<td>-0.048</td>
<td>±0.003</td>
<td>±0.004</td>
</tr>
<tr>
<td>( g_p/f_s )</td>
<td>0.868</td>
<td>±0.010</td>
<td>±0.010</td>
</tr>
<tr>
<td>( g_p'/f_s )</td>
<td>0.089</td>
<td>±0.017</td>
<td>±0.013</td>
</tr>
<tr>
<td>( h_p/f_s )</td>
<td>-0.398</td>
<td>±0.015</td>
<td>±0.008</td>
</tr>
</tbody>
</table>
Ke4 decays: phase shifts and $\pi\pi$ scattering lengths

The relation between the fitted $\delta = \delta_s - \delta_p$ phase shift and the $\pi\pi$ scattering length $a_0$ and $a_2$ can be predicted from data above 0.8 GeV using Roy equations (unitarity, analicity and crossing symmetries). Numerical solutions have been developed, valid only in the Isospin symmetry limit.

⇒ Need to take into account isospin breaking.

Electromagnetic effects

- Gamow-Sommerfeld factor:
  “classical” Coulomb attraction between 2 charged pions

- PHOTOS generator:
  real photon(s) emitted and tracked in the simulation

Mass effects ($m_u \neq m_d$, $m_{\pi^+} \neq m_{\pi^-}$)

- recently computed as a correction to the measurements
- larger than current experimental precision!
  (CGR, EPJ C59(2009) 777)
Ke4 decays: phase shifts and scattering lengths

Black: **without** isospin breaking corrections
Red: **with** isospin breaking corrections

Red ellipse: 2–parameter fit
Blue band: ChPT constraint
CGL, NPB 603 (2001) 125
CGL, PRL 86 (2001) 5008

Green ellipse: 1-parameter fit (using ChPT constraint)

\[ \times \left( \frac{1}{m_{\pi^+}} \right) \]
Ke4 decays: comparison with theoretical predictions

NA48/2 experimental results (in $1/m_{\pi^+}$ units)

2-par. Fit: $a_0 = 0.2220 \pm 0.0128_{\text{stat}} \pm 0.0050_{\text{syst}} \pm 0.0037_{\text{th}}$
(correl. $\rho=0.967$) $a_2 = -0.0432 \pm 0.0086_{\text{stat}} \pm 0.0034_{\text{syst}} \pm 0.0028_{\text{th}}$

1-par. Fit (ChPT): $a_0 = 0.2206 \pm 0.0049_{\text{stat}} \pm 0.0018_{\text{syst}} \pm 0.0064_{\text{th}}$

Theory prediction

Assuming more inputs from ChPT and low energy constants (CGL NPB603(2001), PRL86(2001))

$a_0 = 0.220 \pm 0.005$
$a_2 = -0.0444 \pm 0.0008$
Introduction

The NA48/2 experiment at CERN SPS: setup and data

$K^\pm \rightarrow \pi^+\pi^- e^{\pm}\nu$ ($K_{e4}$) decays: Form Factors and $\pi\pi$ scattering lengths

NA48/2 $K_{e4}$ and $K_{3\pi}$ results: comparison

Conclusions
$K_{e4}$ and $K_{3\pi}$(cusp) results comparison

Two independent measurements

- $K_{e4}$: $1.13 \cdot 10^6$ events
- Cusp: $60 \cdot 10^6$ $K_{3\pi}$ ($K^\pm \rightarrow \pi^0\pi^0\pi^\pm$) events

Different systematics

- $K_{e4}$: electron mis-ID and Background
- Cusp: Calorimeter and Trigger

Different theoretical inputs

- $K_{e4}$: Roy equations and Isospin breaking corrections
- Cusp: Final state rescattering and ChPT expansion
Combined NA48/2 results from K3pi(cusp) and Ke4

<table>
<thead>
<tr>
<th></th>
<th>stat</th>
<th>syst</th>
<th>theo</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_0 )</td>
<td>0.2210 ± 0.0047 ± 0.0015 ± 0.0049</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a_2 )</td>
<td>-0.0429 ± 0.0044 ± 0.0016 ± 0.0030</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation</td>
<td>0.910</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Including ChPT constraint:

\( a_2 = -0.0444 ± 0.0007 ± 0.0005 (±0.0012) \) stat/syst/(theo)

\( a_0 = 0.2195 ± 0.0027 ± 0.0021 (±0.0048) \) or \( a_0 - a_2 = 0.2640 ± 0.0020 ± 0.0017 (±0.0035) \)

Total exp. errors:

\( \Delta a_0 = ±0.0034, \quad \Delta a_2 = ±0.0009, \quad \Delta (a_0 - a_2) = ±0.0026 \)
Introduction

The NA48/2 experiment at CERN SPS: setup and data

\[ K^\pm \rightarrow \pi^+\pi^- e^{\pm}\nu \ (K_{e4}) \text{ decays: Form Factors and } \pi\pi \text{ scattering lengths} \]

NA48/2 \(K_{e4}\) and \(K_{3\pi}\) results: comparison

Conclusions
Conclusions

- The kaon decays give the possibility to study the low energy hadronic interaction with good precision.

- Thanks to large statistics and high data quality, NA48/2 can check ChPT predictions with very high accuracy.

- $\pi\pi$ scattering lengths from $K_{e4}$ and $K_{3\pi}$ are fully consistent.

- The achieved experimental precision on $a_0$ is now competitive with the theoretical precision ($\pm 0.005$).

- The two precise and independent measurements of $\pi\pi$ scattering lengths provide a very strong test of the theory.