The CMU Baryon Amplitude Analysis Program
JLab N* Analysis Workshop

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Nov. 4th, 2006
Outline

1. What are we looking for?
2. Where are we looking?
3. How are we looking?
   - Overview example
   - What physics do we put in?
   - How do we code up the physics?
   - How do we fit the data?
4. How far have we gotten?
5. Summary
What are we looking for?

Where are we looking?

How are we looking?

Overview example

What physics do we put in?

How do we code up the physics?

How do we fit the data?

How far have we gotten?

Summary
ATTACKING THE MISSING BARYONS PROBLEM

Big motivation is the missing baryons issue. Two competing explanations:
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Two competing explanations:

- 1969, Lichtenberg
  - Di-quark coupling amongst the 3 constituent quarks.
  - Removes a degree of freedom when calculating masses.
  - Very good agreement with observed resonances.
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Two competing explanations:

- **1969, Lichtenberg**
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  - Removes a degree of freedom when calculating masses.
  - Very good agreement with observed resonances.

- **1980, Koniuk and Isgur**
  - 1980, Harmonic oscillator model with QCD-inspired corrections
  - Calculate decay couplings
  - Found that many missing states couple relatively weakly to $N\pi$!
  - But much of experimental data is $N\pi \rightarrow N\pi$ scattering
<table>
<thead>
<tr>
<th>State</th>
<th>Width (MeV/c²)</th>
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<tr>
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Experimentally measure differential cross sections and polarization observables. Publish these and let theorists and phenomenologists try to fit to these.
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- Try to describe

\[ \pi N \rightarrow \pi N \]
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- We are applying the meson approach to our baryon analysis.
CURRENT ANALYSIS MODEL

Experiment
**Current analysis model**

- Experiment
- Observables, $d\sigma$
Current analysis model

Experiment

Theory

observables, $d\sigma$
**PROPOSED ANALYSIS MODEL**

*Experiment*

*Theory*
PROPOSED ANALYSIS MODEL

Experiment

Models

Theory
**Proposed analysis model**

Experiment → Models → 4-vectors

Theory → Models
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Analyze reactions with no $N\pi$ (e.g. photoproduction)

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Currently analyzing
Up on deck...
Pros/Cons of each channel

- $\gamma p \rightarrow p\pi^+\pi^-$
  - Multiple isobars: $\Delta^{++}\pi^-$, $\Delta^0\pi^+$, $p\rho$

However, ability to measure final state in different ways (reconstructing any missing track) give you a systematic check on the acceptance of the detector.
Pros/Cons of each channel

- $\gamma p \rightarrow p\pi^+\pi^-$
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  - Accesses both $N^*$'s and $\Delta$'s.
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- $\gamma p \rightarrow p\omega$
- $\gamma p \rightarrow p\eta/\eta'$
- $\gamma p \rightarrow \Lambda K^+$
  - Single decay paths

Isospin filters: access only $N^*$'s.
Can key on $S_{11}$ resonances for $p\eta$ decays.
Significant contribution from $t^-$ channel $\omega$ production ($\pi$ exchange).
$\eta/\eta'$ production ($\rho/\omega$ exchange).
Contribution from $u^-$ channel processes.
Background under each process.
Limited angular information with which to extract resonance parameters.
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What are we going to?

- Goal is to perform an event-based, energy-independent amplitude analysis.
What are we going to?

- Goal is to perform an event-based, energy-independent amplitude analysis.
- A lot of stuff goes into this.
- In order to orient ourselves, we take a look at a cartoon example of an ideal procedure...
A toy example

Total cross section for $\gamma p \rightarrow p\pi^+\pi^-$ as measured in previous experiments.
A TOY EXAMPLE

- First step: describe the physics in a single W-bin.
- Let the fit find optimal mix of physics processes which describes the data in this small energy range.
- We do not put in shapes (e.g. Breit-Wigner) for the resonances for which we are looking.
Dalitz plot data shows structure...

...which is not seen in the accepted Monte Carlo.
**A TOY EXAMPLE**

- Dalitz plot data shows structure...
- ...but which we do get out of the fit results.
A TOY EXAMPLE

- Fit results allow calculation of differential and total cross sections.

The differential cross sections are not the end results of this analysis. In a sense though, we get them for free.

CMU PWA Group (CMU)
A TOY EXAMPLE

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- The differential cross sections are not the end results of this analysis.
- In a sense though, we get them for free.
A TOY EXAMPLE

- Really want to look at contributions of the various physics processes in our fit.
- In this toy example, we show contributions from:
  - **Non-resonant term**
  - $\gamma p \rightarrow \frac{1}{2}^- \rightarrow \Delta \pi$
  - $\gamma p \rightarrow \frac{3}{2}^+ \rightarrow \Delta \pi$
- Can also look at the phase difference of $\frac{1}{2}^-$ and $\frac{3}{2}^+$ terms.
A TOY EXAMPLE

- We go to some other bin and perform an *independent fit* with the same.
A TOY EXAMPLE

- We go to some other bin and perform an *independent fit* with the same.
- While we may constrain parameters in our non-resonant terms across $W$-bins, keep in mind we *do not* constrain this for the waves which could describe resonant processes.
A TOY EXAMPLE

- Repeat this across the full energy range...
A TOY EXAMPLE

Repeat this across the full energy range...
A TOY EXAMPLE

How are we looking?

Repeat this across the full energy range...
A TOY EXAMPLE

- Repeat this across the full energy range...
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- Repeat this across the full energy range...
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- Repeat this across the full energy range...
If there are waves which describe resonant processes, this should show up in intensity and phase differences even though this information was not put into the fit a priori.
Outline

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Writing our amplitudes

- What do we do with this?
- Use Rarita-Schwinger formalism.
- Calculate $\psi$ as function of...

\[ \gamma \rightarrow \pi^+ \pi^- \pi^0 \]

\[ p_i \rightarrow p_f \]
How are we looking?  What physics do we put in?

**Writing our amplitudes**

- What do we do with this?
- Use Rarita-Schwinger formalism.
- Calculate $\psi$ as function of...
  - $X$: isobar (e.g. $\omega, \eta, \eta'$)
  - Intermediate state:
    - $I$: isospin
    - $J$: spin
    - $P$: parity
  - $E/M$: production multipole
  - $L$: final state orbital angular momentum
- Also a function of initial/final spin-projections
  - $m_\gamma, m_i, m_f$
Writing our amplitudes

We do not put in Breit-Wigner shapes for the $IJ^P$ intermediate state

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  - $m_\gamma, m_i, m_f$
**Fit Parameters**

We do not put in Breit-Wigner shapes for the $IJ^P$ intermediate state

- **Fit $\alpha$**
  - Ratio of $E/M$ multipole.
  - Production phase - in future mass dependant analysis, this maps onto a Breit-Wigner phase.
  - Indexed by $I, J, P$

- **Fit $\beta$**
  - Decay strength and overall scale
  - Indexed by $I, J, P, \text{isobar, } L$
t— AND u—CHANNEL DIAGRAMS

- We have our pick of propagator (Regge, Feynman, etc.)
- Can write out our $u$-channel terms in the same way.
- Fit the $t$—channel coupling for the higher photon energies, and then fix the value everywhere.
- Can put in couplings directly from previous measurements.
**For example:** $\gamma p \rightarrow p \omega$

Can have the same production couplings, but different decay couplings for different $L$'s in final state.
**For example:** \( \gamma p \rightarrow p \omega \)

Can have the same production couplings, but different decay couplings for different \( L \)'s in final state.
For example: $\gamma p \rightarrow p\omega$

Can have the same production couplings, but different decay couplings for different $L$’s in final state.
Coupled channel

- Most immediate application of this is to apply to coupled channel analysis.
- Production is same coupling...only decay coupling varies
- $2\pi$ production ($\Delta^{++}\pi^-, \Delta^0\pi^+, p\rho^0$)
- Currently in the process of analyzing $\eta/\eta'$ in this way.
Access to multiple channels lets us use different analysis in conjunction with one another.

For example, given \( \omega \) production with \( \eta \) exchange, there are two couplings that are fit in the \( t \)-channel process.

- \( g_{\gamma \omega \eta} \)
- \( g_{pp\eta} \)
Access to multiple channels lets us use different analysis in conjunction with one another.

When we analyze $\eta$ production with $\omega$ exchange, there is a common coupling with the $\omega$ production.

- $g_{\gamma\omega\eta}$
- $g_{pp\omega}$
How are we looking? What physics do we put in?

**t—CHANNEL DIAGRAMS ACROSS CHANNELS**

- Access to multiple channels lets us use different analysis in conjunction with one another.
- When we analyze $\eta$ production with $\omega$ exchange, there is a common coupling with the $\omega$ production.
  - $g_{\gamma\omega\eta}$
  - $g_{pp\omega}$
- Right now, we try to measure these in one channel, and put in by hand to the other.
- Next step is to fit the datasets simultaneously.
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**Amplitude Generation**

- Requires tensor algebra
  - CPU intensive
  - $L_{\mu_1 \ldots \mu_\ell}(p_1, p_2)$ is an $\ell$th-rank tensor
  - Need general, fast code.
Amplitude generation

- Requires tensor algebra
  - CPU intensive
  - \( L_{\mu_1...\mu_\ell}(p_1, p_2) \) is an \( \ell \)th-rank tensor
  - Need general, fast code.

- C++ classes
  - Matrix, Tensor
    - DiracGamma
    - DiracSpinor
    - PolVector
    - LeviCivitaTensor
    - OrbitalTensor
  - \( \gamma^\mu \) : The Dirac Gamma matricies
  - \( u_{\mu_1\mu_2...\mu_{5-1/2}}(p, m) \) : Spin-S spinors, half-integer spin
  - \( \epsilon_{\mu_1\mu_2...\mu_5} \) : Spin-S polarization vectors, integer spin
  - \( \epsilon_{\mu\nu\rho\sigma} \) : Totally anti-symmetric Levi-Civita tensor
  - \( L^{(\ell)}_{\mu_1\mu_2...\mu_\ell} \) : Orbital angular momentum tensors
C O D I N G  t h e  p r o c e s s e s

Start with some process...

\[ \gamma \rightarrow J^P p \eta \sim \bar{u} \gamma (p) \]

\[ \rightarrow \gamma (p) \bar{m}_\gamma (p) P_{1/2} \gamma (p) \mu \gamma (5) \bar{u} \gamma (m) \gamma (p) \Theta (P) \]
Coding the processes

Start with some process...

\[ A_{\gamma p \to \frac{1}{2}^- \to p\eta} \sim \bar{u}_{mp_f} (p_{pf}) P \frac{1}{2} (P) \gamma_{\mu} \gamma^5 u_{mp_i} (p_{pi}) \epsilon_{m\gamma}^{\mu} (p_{\gamma}) \Theta(P) \]
CODY THE PROCESSES

...we can turn this *directly* into usable C++ code.

\[ A_{\gamma P \rightarrow \frac{1}{2}^{-} \rightarrow P\eta} \sim \bar{u}_{mp_f}(p_{pf})P^2(P)\gamma_\mu \gamma^5 u_{mp_i}(p_{pi})\epsilon_{m_\gamma}(p_\gamma)\Theta(P) \]

DiracSpinor _u_f; ///< final nucleon spinor
DiracSpinor _u_i; ///< initial nucleon spinor
PolVector _eps_g; ///< photon polarization vector
LeviCivitaTensor _lct; ///< Levi-Civita tensor
DiracGamma _gamma; ///< Dirac gamma matrices
vector<int> _Ls; ///< list of ALL L’s that will be used
vector<OrbitalTensor> _Lf; ///< Lf[L] will be rank-L orbital tensor

Li[l].SetP4(p4_phot,p4_targ);
_m_jp_cm[mg_ind] = psi.Projector() | (Li[l]*_gamma)*(_gamma*_eps_g(m_g));
Coding the processes

Take 4-vectors of tracks measured in CLAS...
**Coding the Processes**

Take 4-vectors of tracks measured in CLAS... ...pass them into our code...

```cpp
int main()
{
    Awesome amp
generation code

    return 0;
}
```
Coding the processes

Take 4-vectors of tracks measured in CLAS...

...pass them into our code...

```c
int main()
{
    Awesome amp generation code
    return 0;
}
```

...and get out the amplitude for these events for a given process.

(0.298813,0.694998)
(-0.840490,-0.701032)
(-0.941817,0.453170)
(1.102081,-0.109626)
(-1.055170,-0.492774)
(0.647787,0.015414)
(0.570040,0.570075)
(-0.075149,-0.643757)
(-0.012921,-0.835250)
(-0.286855,0.344333)
(-0.002072,0.812462)
(-0.638055,-0.125474)
...
Coding the processes

- This code makes things *much* easier on the whole process.
- Calculation no longer relies on simplifying the original amplitude or boosting and working out dependance on particular differential cross sections.
- We can take almost any processes from any model and apply *directly* to the data.
- [http://www-meg.phys.cmu.edu/pwa/wiki-qft++]
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Maximum likelihood method

- Currently our fits primarily use an event-based, Maximum Likelihood procedure.
- Essential for $p\pi^+\pi^-$ channel where you have multiple isobars.
- Perhaps not quite as necessary for $\omega$ or $\eta, \eta'$ channels where there is limited physics information (e.g. $\cos(\theta)_{CM}$).
  - We have recently done some preliminary fits, fitting to differential cross sections for these channels.
Maximum likelihood method

Construct likelihood function from the intensity

\[ \mathcal{L} \propto \left[ \frac{\bar{n}^n e^{-\bar{n}}}{n!} \right] \prod_{i} l(T_i) \int \frac{l(\tau)\eta(\tau)d\tau}{\int l(\tau)\eta(\tau)d\tau} \]
**Maximum likelihood method**

Construct likelihood function from the intensity

\[ \mathcal{L} \propto \left[ \frac{\bar{n}^n}{n!} e^{-\bar{n}} \right] \prod_{i}^{n} \frac{l(\tau_i)}{\int l(\tau) \eta(\tau) d\tau} \]

\[ \ln \mathcal{L} = \sum_{i}^{n} \ln \left[ \sum_{\alpha \alpha'} g_{1\alpha} g_{1\alpha'}^{*} g_{2\alpha} g_{2\alpha'}^{*} \psi_{\alpha}(\tau) \psi_{\alpha'}^{*}(\tau) \right] \]

\[ -n \sum_{\alpha \alpha'} g_{1\alpha} g_{1\alpha'}^{*} g_{2\alpha} g_{2\alpha'}^{*} \left[ \frac{1}{N_{AMC}} \sum_{i_{AMC}} \psi_{\alpha}(\tau) \psi_{\alpha'}^{*}(\tau) \right] \]

- \( \alpha \) - index representing the waves (\( IJ^P, E/M, L... \))
- \( \tau \) - the kinematics of the event
**INTENSITY FOR ONE WAVE**

\[ I_\alpha = |\Psi_\alpha(\tau)|^2 \]
**Intensity for one wave**

\[ I_\alpha = |\psi_\alpha(\tau)|^2 \]

\[ = |g_{1\alpha}g_{2\alpha}\psi_\alpha(\tau)|^2 \]
**Intensity for one wave**

\[ I_\alpha = |\psi_\alpha(\tau)|^2 \]
\[ = |g_{1\alpha}g_{2\alpha}\psi_\alpha(\tau)|^2 \]
\[ = \sum_{m_\gamma} \sum_{m_i} \sum_{m_f} |g_{1\alpha}g_{2\alpha}\psi_\alpha(\tau)|^2 \]
INTENSITY FOR ONE WAVE - SPINS

\[ I_\alpha = \left| g_{1\alpha} g_{2\alpha} \psi_\alpha; m_\gamma = +1, m_i = +\frac{1}{2}, m_f = +\frac{1}{2}(\tau) \right|^2 \]
INTENSITY FOR ONE WAVE - SPINS

\[ I_\alpha = \left| g_1 \alpha g_2 \psi_\alpha; m_\gamma = +1, m_i = +\frac{1}{2}, m_f = +\frac{1}{2}(\tau) \right|^2 + \left| g_1 \alpha g_2 \psi_\alpha; m_\gamma = +1, m_i = +\frac{1}{2}, m_f = -\frac{1}{2}(\tau) \right|^2 + \left| g_1 \alpha g_2 \psi_\alpha; m_\gamma = +1, m_i = -\frac{1}{2}, m_f = +\frac{1}{2}(\tau) \right|^2 + \left| g_1 \alpha g_2 \psi_\alpha; m_\gamma = +1, m_i = -\frac{1}{2}, m_f = -\frac{1}{2}(\tau) \right|^2 + \left| g_1 \alpha g_2 \psi_\alpha; m_\gamma = -1, m_i = +\frac{1}{2}, m_f = +\frac{1}{2}(\tau) \right|^2 + \left| g_1 \alpha g_2 \psi_\alpha; m_\gamma = -1, m_i = +\frac{1}{2}, m_f = -\frac{1}{2}(\tau) \right|^2 + \left| g_1 \alpha g_2 \psi_\alpha; m_\gamma = -1, m_i = -\frac{1}{2}, m_f = +\frac{1}{2}(\tau) \right|^2 + \left| g_1 \alpha g_2 \psi_\alpha; m_\gamma = -1, m_i = -\frac{1}{2}, m_f = -\frac{1}{2}(\tau) \right|^2 \]
**Intensity for one wave - spins**

\[
I_{\alpha} = \left| g_{1\alpha} g_{2\alpha} \psi_{\alpha; m_{\gamma}=+1, m_i=+\frac{1}{2}, m_f=+\frac{1}{2}}(\tau) \right|^2 + \left| g_{1\alpha} g_{2\alpha} \psi_{\alpha; m_{\gamma}=+1, m_i=+\frac{1}{2}, m_f=-\frac{1}{2}}(\tau) \right|^2 \\
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+ \left| g_{1\alpha} g_{2\alpha} \psi_{\alpha; m_{\gamma}=-1, m_i=+\frac{1}{2}, m_f=+\frac{1}{2}}(\tau) \right|^2 + \left| g_{1\alpha} g_{2\alpha} \psi_{\alpha; m_{\gamma}=-1, m_i=+\frac{1}{2}, m_f=-\frac{1}{2}}(\tau) \right|^2 \\
+ \left| g_{1\alpha} g_{2\alpha} \psi_{\alpha; m_{\gamma}=-1, m_i=-\frac{1}{2}, m_f=+\frac{1}{2}}(\tau) \right|^2 + \left| g_{1\alpha} g_{2\alpha} \psi_{\alpha; m_{\gamma}=-1, m_i=-\frac{1}{2}, m_f=-\frac{1}{2}}(\tau) \right|^2 \\
+ \left| \text{non-interfering background term} \right|^2
\]
**Writing the Code**

- What is needed out of the fitting code?
  - Easily construct our likelihood function.
    - Define who interferes with whom.
    - Add and subtract waves.
    - Fix couplings to known values, or zero out entirely.

- Constrain couplings across different waves/datasets.
- Look at projections of the data after the fit. ($d\sigma$)
- Do systematic checks.

We have a suite of packages utilizing C++, Ruby and XML that handles all of the above.
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How far have we gotten?

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How far have we gotten?

**WHERE ARE WE?** \( \gamma p \rightarrow p\pi^+\pi^- \)

Can compare *measured* differential yields. **PRELIMINARY!**

**Black** is SAPHIR and **blues** are CLAS.

\( W=1416-1480 \text{ Mev/c}^2 \)

\[ \frac{\text{yield}}{dt} t(\gamma - (\pi^+\pi^-)) \]

\( W=1542-1603 \text{ Mev/c}^2 \)

Our binning is finer and we get pretty good agreement.

**CMU PWA Group (CMU)**

**CMU Baryon Amplitude Analysis**

Nov. '06 61 / 66
γp → pω: Differential Cross Sections

- We can compare our results to previous CLAS results: Battaglieri, et al
- We have 21 bins overlapping their 4 W bins (2.62 < W < 2.87)
- Very good agreement
- Notice that the cross section appears to be dominated by non-resonant (t-, u-channel) processes
How far have we gotten?

$\eta/\eta' \frac{5}{2}^-$ PWA prod. strength and phase results

Production strength of the $\frac{5}{2}^-$ partial wave for the $\eta$ and $\eta'$

eta phase diff.

etaprime phase diff.

PRELIMINARY
For $\eta$ and $\eta'$ we have started extracting coupling constants!

Will soon be coupling this analysis with the $\omega$ where they share many couplings.

Stay tuned...
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5. **Summary**
Summary

Non-strange baryon spectroscopy very promising...but not easy.
Non-strange baryon spectroscopy very promising...but not easy. We have put together a complete analysis package which should allow us to tackle this comprehensive program.
Non-strange baryon spectroscopy very promising...but not easy. We have both the data and tools to perform a mass-independent, event based PWA.
SUMMARY

Non-strange baryon spectroscopy *very* promising...but not easy. Will be able to perform coupled channel analysis.
Non-strange baryon spectroscopy very promising...but not easy. Can naturally apply theory directly to the CLAS data analysis.
Non-strange baryon spectroscopy very promising...but not easy. Would be able to naturally include data from other experiments and models!
Non-strange baryon spectroscopy very promising...but not easy. Generality makes this program extensible to other analysis (GlueX, E852, CLEO Dalitz analysis etc.).

**EBAC models**

**FSU, etc. models**

---

**CMU Machinery**

**CLAS data**

**CB-ELSA, etc. data**
Non-strange baryon spectroscopy very promising...but not easy.

Very excited about near-future developments!