Probing the quark mass in elastic and transition form factors

Ian Cloët
(University of Washington)

Collaborators
Craig Roberts (ANL)

NSTAR2011 – The 8th International Workshop on the Physics of Excited Nucleons
JLab, May 2011
The Quark Mass Function

- Quark propagator is fundamental when building models of QCD
  - In QCD $M(p^2)$ must run & give perturbative limit
- A constant constituent mass is phenomenologically successful
  - Constituent quark models for spectroscopy
  - NJL models for meson and baryon static properties
- Can we identify observables that are sensitive to $M(p^2)$
  - If so can experiment help constrain $M(p^2)$ within DSE framework
\[ \frac{1}{m_G^2} g_{\mu\nu} \gamma^\nu \]

\[ g^2 D_{\mu\nu}(p - k)\Gamma^\nu(p, k) \]

\[ S(p) = \frac{Z(p^2)}{[i\not{\phi} + M(p^2)]} \]

QCDs Gap Equation:
\[ \frac{1}{m_G^2} g_{\mu\nu} \gamma^\nu \]

\[ g^2 D_{\mu\nu}(p - k) \Gamma^\nu(p, k) \]

- Compare observables within one framework with different interactions
- Experiment will constrain interaction \( \leftrightarrow \) quark–gluon vertex
- Knowledge of quark–gluon vertex provides \( \alpha_s(Q^2) \) within DSEs
  - also gives \( \beta \)-function and may shed light on confinement
Given a quark–gluon vertex we can solve for $M(p^2)$.
Quark DSE $\iff$ Gap Equation

Quark propagator:

$\frac{1}{\Box} = \frac{1}{\Box} + \frac{1}{\Box}$

Ghost propagator:

$\frac{1}{\Phi} = \frac{1}{\Phi} + \frac{1}{\Phi}$

Ghost-gluon vertex:

$\frac{1}{\Phi} = \frac{1}{\Phi} + \frac{1}{\Phi}$

Quark-gluon vertex:

$\frac{1}{\Phi} = \frac{1}{\Phi} + \frac{1}{\Phi}$
DSE and the Maris–Tandy Model

- Clearly need a sensible truncation scheme
  - must maintain symmetries of theory
  - rainbow-ladder truncation is one such scheme

- Maris–Tandy – ansätze for gluon propagator and quark-gluon vertex

\[
\frac{1}{4\pi} g^2 D_{\mu\nu}(p-k) \Gamma_\nu(p,k) \rightarrow \alpha_{\text{eff}}(p-k) D_{\mu\nu}^{\text{free}}(p-k) \gamma_\nu
\]

- Build in the correct perturbative limit

\[
\alpha_{\text{eff}}(k^2) \xrightarrow{k^2 \rightarrow \infty} \frac{\pi \gamma_m}{\ln \left( k^2 / \Lambda_{\text{QCD}}^2 \right)}
\]

Mesons and the Bethe-Salpeter Equation

- Mesons show up as poles in the two-body $T$-matrix
- What is the BSE kernel: must preserve symmetries
  - e.g. Axial–Vector Ward–Takahashi Identity
  \[
  q_\mu \Gamma_{\frac{5}{2},i}^\mu(p',p) = S^{-1}(p') \gamma_5 \frac{1}{2} \tau_i + \frac{1}{2} \tau_i \gamma_5 S^{-1}(p) + 2m \Gamma_{\pi}^i(p',p)
  \]
- Kernels of gap and BSE must be intimately related
- Maris–Tandy: excellent description of light pseudoscalar and vector mesons – 31 masses/couplings/radii with rms error of 15%
Pion form factor

- Pion BSE vertex has the general form

\[ \Gamma_\pi(p, k) = \gamma_5 \left[ E_\pi(p, k) + \not{p} F_\pi(p, k) + \not{k} k \cdot p G(p, k) + \sigma^{\mu\nu} k_\mu p_\nu H(p, k) \right] \]

- Use Ball-Chiu Ansatz for quark–photon vertex: satisfies WTI

\[ \Gamma^{\mu}_{BC}(p', p) = \gamma^\mu \Sigma_A(p'^2, p^2) + P^\mu \Delta_B(p'^2, p^2) + P^\mu \not{p} \Delta_A(p'^2, p^2) \]
Some Consequences of Running Quark Mass


In gap equation use simpler kernel

\[ g^2 D_{\mu\nu}(p - k)\Gamma^\nu(p, k) \to \frac{1}{m_G^2} g_{\mu\nu} \gamma^\nu \]

Quark no longer has a running mass

- Pion PDF \( x \to 1 \): contact – \( q(x) \sim (1 - x)^1 \); DSE – \( q(x) \sim (1 - x)^2 \)
Toward a General Quark–Gluon Vertex

- Maris–Tandy has been successful, however it does breakdown
  ✦ e.g. excited states, $\rho - a_1$ mass splitting, …

- Clear signal that the Maris–Tandy quark–gluon vertex is too simple

- Inability to construct new Bethe–Salpeter kernel blocked progress

- However, it is now possible to formulate an Ansatz for Bethe-Salpeter kernel given any form for the dressed-quark-gluon vertex


- This enables direct connection between experiment and a general quark–gluon vertex with DSE framework
Quark–Gluon and Quark–Photon Vertices

- Quark–gluon and quark–photon vertices have same Lorentz structure

\[ \Gamma_\mu(p', p) = \sum_{i=1}^{12} \lambda_i^\mu f_i(p'^2, p^2, q^2) = \Gamma_\mu^L + \Gamma_\mu^T \]

- Coupling of photon to quark is given by inhomogeneous BSE
  - properties dictated by quark propagator and quark–gluon vertex

- Ward-Takahashi identity constrains \( \Gamma_\mu^L \) for quark–photon vertex

\[ q_\mu \Gamma_{\gamma qq}^\mu = q_\mu \Gamma_\mu^L = \hat{Q} \left[ S^{-1}(p') - S^{-1}(p) \right], \quad q_\mu \Gamma_{\gamma qq}^T = 0 \]

- Constituent quarks are strongly dressed by gluons
  - therefore expect sizable transverse form factors – c.f. nucleon
Dressed Quark Anomalous Magnetic Moment

- Include $\sigma^{\mu\nu} q_\nu \tau_5 (p', p)$ [anomalous chromomagnetic] term in quark–gluon vertex
  - has been absent from previous calculations
- Generates anomalous electromagnetic term in quark–photon vertex
- Confined quarks $\Rightarrow$ no mass shell – anomalous mm ill defined
  - however associate with $i\sigma^{\mu\nu} q_\nu$ piece of quark–photon vertex

- Investigate effect on nucleon form factors
**Nucleon and the Faddeev Equation**

- Consistency $\implies$ solve Faddeev Equation with DSE kernel

\[ \Gamma = \Gamma + \Gamma + \Gamma \]


- Instead we approximate nucleon as a quark–diquark bound state

\[ \Gamma = \Gamma \]

- Include scalar and axial-vector diquarks

- For masses quark–diquark approx results agree to within 5%

- Equation has discrete solutions at $p^2 = m_i^2$; nucleon, roper, etc

- Yields Faddeev amplitude describes quark-diquark relative motion
Current conservation requires the following diagrams:

- Dressed quark–photon vertex
  - longitudinal piece, \( \Gamma^\mu_L \), constrained by WTI
  - transverse piece, \( \Gamma^\mu_T \), include \( i\sigma^{\mu\nu}q_\nu \) term

Predictions for nucleon form factors to \( Q^2 \sim 10 - 15 \text{ GeV}^2 \)
Nucleon Form Factors Results

\[ F_{1p}(Q^2) \] with \( \tau_5 \) term
\[ F_{2p}(Q^2) \] with \( \tau_5 \) term
\[ F_{1n}(Q^2) \] with \( \tau_5 \) term
\[ F_{2n}(Q^2) \] with \( \tau_5 \) term

- \( \tau_5 \) is the anomalous magnetic moment term in quark–photon vertex
DSE results now include the anomalous electromagnetic term
   - important for low to moderate $Q^2$

Reasonable description of nucleon form factors

DSE model for nucleon can be improved
   - need to include $\rho$ and $\omega$ contribution to $\Gamma_{\gamma qq}^{\mu}$

DSE prediction agrees with this recent data
Comparison with Constant Mass Function

- Find that at $Q^2 = 0$ two results agree rather well
- Reinforces the notion that a constant constituent mass is a reasonable approximation to low energy QCD
  - provided symmetries are preserved
  - good for calculating static properties: mag. moments, PDFs, etc
- However for $Q^2 \neq 0$ operators – running mass is important
The $N^*$ (Roper) Resonance

- $N^*$ manifests as second pole in Faddeev equation kernel
  - $M_N = 0.940 \text{ GeV}$ and $M_{N^*} = 1.8 \text{ GeV}$
  - Agrees very well with EBAC value for quark core mass

- “Wavefunction” is given by eigenvector at pole: $p^2 = m_i^2$

- For contact model $N$, $N^*$ “wavefunction” has the simple form

$$
\Gamma(p) = \left[ \frac{\alpha_1}{\alpha_2 \frac{p^\mu}{M} \gamma_5 + \alpha_3 \gamma^\mu \gamma_5} \right] u(p)
$$

- For the nucleon: $\alpha_1 = 0.43$, $\alpha_2 = 0.024$, $\alpha_3 = -0.45$
- For the Roper: $\alpha_1 = 0.0011$, $\alpha_2 = 0.94$, $\alpha_3 = -0.051$

- For nucleon scalar and axial–vector diquarks equally dominant

- However, $N^*$ is completely dominated by the axial–vector diquark
A Radial Excitation

1. Nucleon and Roper angular momentum must satisfy:

\[ J = \frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta L_q + \Delta J_g \]

2. For nucleon experiment gives

\[ \Delta \Sigma = 0.33 \pm 0.03(\text{stat.}) \pm 0.05(\text{syst.}) \quad [\text{COMPASS & HERMES}] \]

3. Contact interaction gives:

\[ \Delta \Sigma_N = 0.68 - 0.21 = 0.47, \quad \Delta \Sigma_{N^*} = -0.02 + 0.01 \simeq 0.0 \]

4. Result \implies subtle cancellation between quark and diquark spin states

- e.g. axial–vector diquark now has greater probability to have spin opposite nucleon
Nucleon and $N^*$ Form Factors

- Note these results are obtained within the constant mass function framework
  - therefore moderate to large $Q^2$ behaviour is poor
- Pion cloud effects have been ignored
  - expect magnetic moments and radii to be too small
- However we find $N^*$ radii are 10% larger than the nucleons
- Find a zero in both $F_1$ and $F_2$ for Roper
Nucleon and $N^*$ – $F_1$ Form Factors Results

- Contributions originate from the following diagrams

- Find that $N^*$ form factors are axial–vector diquark dominated
Contributions originate from the following diagrams

Find that $N^*$ form factors are axial–vector diquark dominated
$N \rightarrow N^*$ Transition Form Factors Results

$F_{p \rightarrow N^*}(Q^2)$

$F_{2p \rightarrow N^*}(Q^2)$

CLAS – $F_1$

CLAS – $F_2$

PDG

$p \rightarrow N^*$ – diagrams

$F_{sq}^1(Q^2)$

$F_{aq}^1(Q^2)$

$F_{sd}^1(Q^2)$

$F_{ad}^1(Q^2)$

$F_{m}^1(Q^2)$

$q \rightarrow \mu$

$p'$

$F_{p' \rightarrow N^*}(Q^2)$

$F_{sq}^2(Q^2)$

$F_{aq}^2(Q^2)$

$F_{sd}^2(Q^2)$

$F_{ad}^2(Q^2)$

$F_{m}^2(Q^2)$

$Q^2 (GeV^2)$

$F_{p \rightarrow N^*}$ – diagrams

$F_{1}^{ad}(Q^2)$

$F_{1}^{aq}(Q^2)$

$F_{1}^{sq}(Q^2)$

$F_{1}^{sm}(Q^2)$

$F_{2}^{ad}(Q^2)$

$F_{2}^{aq}(Q^2)$

$F_{2}^{sq}(Q^2)$

$F_{2}^{sm}(Q^2)$

$Q^2 (GeV^2)$
The photon–axial-vector diquark vertex has the form

\[ \Lambda_{\alpha \beta}^{\mu} = g_{\alpha \beta} F_1(Q^2) - \frac{q_{\alpha} q_{\beta}}{2M_a^2} F_2(Q^2) (p + p')^\mu - (q^\alpha g^{\mu \beta} - q^\beta g^{\mu \alpha}) F_3(Q^2) \]

The three axial-vector diquark form factors are positive definite.

Cancellations between pieces of diagram give zero in \( F_{2p \rightarrow N^*} \).

This zero is directly related to the zeros in the \( N^* \) form factors.
Conclusion

- A thorough understanding of hadron structure requires a nonperturbative, symmetry preserving framework
  - Poincaré covariance, chiral symmetry, current conservation, etc
- Dyson–Schwinger equations provides such a framework
  - single approach that combines UV and IR physics
  - incorporates both quarks AND gluons
- Confronting experiment within the DSE framework will hopefully shed light on the non–perturbative structure of QCD
- Tried to highlight that form factors possibly provide the best empirical constraints on non–perturbative structure within the DSE framework
  - In particular the dressed quark–gluon vertex
- We have outlined a simple but intuitive picture regarding $N \rightarrow N^*$ transition form factors $\leftrightarrow$ axial–vector diquark dominance
  - however much work still remains before a robust picture emerges