Excited Baryons in Holographic QCD

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1 Introduction

Gauge Gravity Correspondence and Light-Front QCD

- The AdS/CFT correspondence [Maldacena (1998)] between gravity on AdS space and conformal field theories in the light front (LF) has led to an analytical semiclassical approximation for QCD, which provides physical insights into its non-perturbative dynamics.

- LF quantization is the ideal framework to describe hadronic structure in terms of constituents: simple vacuum structure allows to compute matrix elements diagonal in particle number.

- Calculation of matrix elements $\langle P + q | J | P \rangle$ requires boosting the hadronic bound state from $| P \rangle$ to $| P + q \rangle$: boosts are trivial in LF.

- Invariant Hamiltonian equation for bound states similar structure of AdS equations of motion: direct connection of QCD and AdS/CFT possible.

- Isomorphism of $SO(4, 2)$ group of conformal transformations with generators $P^\mu, M^{\mu\nu}, K^\mu, D$, with the group of isometries of AdS$_5$, a space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space (Dim isometry group of AdS$_{d+1}$ is $(d + 1)(d + 2)/2$).
• AdS$_5$ metric:

$$ds^2_{L_{\text{AdS}}} = \frac{R^2}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right)_{L_{\text{Minkowski}}}$$

• A distance $L_{\text{AdS}}$ shrinks by a warp factor $z/R$ as observed in Minkowski space ($dz = 0$):

$$L_{\text{Minkowski}} \sim z \frac{L_{\text{AdS}}}{R}$$

• Since the AdS metric is invariant under a dilatation of all coordinates $x^\mu \to \lambda x^\mu$, $z \to \lambda z$, the variable $z$ acts like a scaling variable in Minkowski space.

• Short distances $x_\mu x^\mu \to 0$ map to UV conformal AdS$_5$ boundary $z \to 0$.

• Large confinement dimensions $x_\mu x^\mu \sim 1/\Lambda_{\text{QCD}}^2$ map to large IR region of AdS$_5$, $z \sim 1/\Lambda_{\text{QCD}}$, thus there is a maximum separation of quarks and a maximum value of $z$.

• Use the isometries of AdS to map the local interpolating operators at the UV boundary of AdS into the modes propagating inside AdS.
2 Light Front Dynamics

- Different possibilities to parametrize space-time [Dirac (1949)]
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different “times” and has its own Hamiltonian, but should give the same physical results
- Forms of Relativistic Dynamics: dynamical vs. kinematical generators [Dirac (1949)]
- *Instant form*: hypersurface defined by \( t = 0 \), the familiar one
  \[
  H, \, K \text{ dynamical,} \quad L, \, P \text{ kinematical}
  \]
- *Point form*: hypersurface is an hyperboloid
  \[
  P^\mu \text{ dynamical,} \quad M^{\mu\nu} \text{ kinematical}
  \]
- *Front form*: hypersurface is tangent to the light cone at \( \tau = t + z/c = 0 \)
  \[
  P^-, \, L^x, \, L^y \text{ dynamical,} \quad P^+, \, P_\perp, \, L^z, \, K \text{ kinematical}
  \]
Light-Front Fock Representation

• Construct LF Lorentz invariant Hamiltonian equation for the relativistic bound state

\[ P_\mu P^\mu |\psi(P)\rangle = (P^- P^+ - P^2) |\psi(P)\rangle = \mathcal{M}^2 |\psi(P)\rangle \]

• State \( |\psi(P)\rangle \) is expanded in multi-particle Fock states \( |n\rangle \) of the free LF Hamiltonian

\[ |\psi\rangle = \sum_n \psi_n |n\rangle, \quad |n\rangle = \{ |uud\rangle, |uudg\rangle, |uud\bar{q}q\rangle, \cdots \} \]

with \( k_i = (k_i^+, k_i^-, k_{\perp i}) \), \( k_i^2 = m_i^2 \), \( k^-_i = \frac{k^2_i + m_i^2}{k_i^+} \) for each constituent \( i \) in state \( n \)

• Fock components \( \psi_n(x_i, k_{\perp i}, \lambda^z_i) \) independent of \( P^+ \) and \( P^\perp \) and depend only on relative partonic coordinates: momentum fraction \( x_i = k_i^+ / P^+ \), transverse momentum \( k_{\perp i} \) and spin \( \lambda^z_i \)

\[ \sum_{i=1}^n x_i = 1, \quad \sum_{i=1}^n k_{\perp i} = 0. \]
Semiclassical Approximation to QCD in the Light Front

[GdT and S. J. Brodsky, PRL 102, 081601 (2009)]

• Compute $\mathcal{M}^2$ from hadronic matrix element

$$\langle \psi(P')|P_\mu P^\mu|\psi(P)\rangle = \mathcal{M}^2 \langle \psi(P')|\psi(P)\rangle$$

• Relevant variable in the limit of zero quark masses (variable dual to the invariant mass)

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j b_{\perp j} \right|$$

the $x$-weighted transverse impact coordinate of the spectator system (active quark)

• For a two-parton system $\zeta^2 = x(1-x)b_{\perp}^2$

• To first approximation LF dynamics depend only on the invariant variable $\zeta$, and hadronic properties are encoded in the hadronic mode $\phi(\zeta)$ from

$$\psi(x, \zeta, \varphi) = e^{iLz\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

factoring angular $\varphi$, longitudinal $X(x)$ and transverse mode $\phi(\zeta)$

($P^+, P_\perp$ and $J_z$ commute with $P^-$)
• Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple ($L = |L^z|$)

$$\mathcal{M}^2 = \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left( -\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta)$$

where the confining forces from the interaction terms are summed up in the effective potential $U(\zeta)$

• LF eigenvalue equation $P_\mu P^\mu |\phi\rangle = \mathcal{M}^2 |\phi\rangle$ is a LF wave equation for $\phi$

$$\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

• Effective light-front Schrödinger equation: relativistic, frame-independent and analytically tractable

• Eigenmodes $\phi(\zeta)$ determine the hadronic mass spectrum and represent the probability amplitude to find $n$-massless partons at transverse impact separation $\zeta$ within the hadron at equal light-front time

• Semiclassical approximation to light-front QCD does not account for particle creation and absorption
3 Light-Front Holographic Mapping of Wave Equations

Higher Spin Modes in AdS Space

- Description of higher spin modes in AdS space (Frondsal, Fradkin and Vasiliev)
- Spin-\(J\) in AdS represented by totally symmetric rank \(J\) tensor field \(\Phi_{M_1...M_J}\)
- Action for spin-\(J\) field in AdS\(_{d+1}\) in presence of dilaton background \(\varphi(z)\) \((x^M = (x^\mu, z))\)

\[
S = \frac{1}{2} \int d^d x \, d z \, \sqrt{g} \, e^{\varphi(z)} \left( g^{NN'} \, g^{M_1 M_1'} \cdots g^{M_J M_J'} \, D_N \Phi_{M_1...M_J} \, D_{N'} \Phi_{M_1'...M_J'} \right. \\
- \mu^2 \, g^{M_1 M_1'} \cdots g^{M_J M_J'} \Phi_{M_1...M_J} \Phi_{M_1'...M_J'} + \cdots
\]

where \(D_M\) is the covariant derivative which includes parallel transport

\[
[D_N, D_K] \Phi_{M_1...M_J} = -R^L_{M_1 N K} \Phi_{L...M_J} - \cdots - R^L_{M_J N K} \Phi_{M_1...L}
\]

- Physical hadron has plane-wave and polarization indices along \(3+1\) physical coordinates

\[
\Phi_P(x, z)_{\mu_1...\mu_J} = e^{-i P \cdot x} \Phi(z)_{\mu_1...\mu_J}, \quad \Phi_{z\mu_2...\mu_J} = \cdots = \Phi_{\mu_1\mu_2...z} = 0
\]

with four-momentum \(P_\mu\) and invariant hadronic mass \(P_\mu P^\mu = M^2\)
• Construct effective action in terms of spin-$J$ modes $\Phi_J$ with only physical degrees of freedom

[ H. G. Dosch, S. J. Brodsky and GdT (in preparation)]

• Introduce fields with tangent indices

$$\hat{\Phi}_{A_1 A_2 \cdots A_J} = e_{A_1}^M e_{A_2}^M \cdots e_{A_J}^M \Phi_{M_1 M_2 \cdots M_J} = \left( \frac{z}{R} \right)^J \Phi_{A_1 A_2 \cdots A_J}$$

• Find effective action

$$S = \frac{1}{2} \int d^d x \, d z \, \sqrt{g} \, e^{\varphi(z)} \left( g^{N N'} \eta_{\mu_1 \mu'_1} \cdots \eta_{\mu_J \mu'_J} \partial_N \hat{\Phi}_{\mu_1 \cdots \mu_J} \partial_{N'} \hat{\Phi}_{\mu'_1 \cdots \mu'_J} - \mu^2 \eta_{\mu_1 \mu'_1} \cdots \eta_{\mu_J \mu'_J} \hat{\Phi}_{\mu_1 \cdots \mu_J} \hat{\Phi}_{\mu'_1 \cdots \mu'_J} \right)$$

upon $\mu$-rescaling

• Variation of the action gives AdS wave equation for spin-$J$ mode $\Phi_J = \Phi_{\mu_1 \cdots \mu_J}$

$$\left[ - \frac{z^{d-1-2J}}{e^{\varphi(z)}} \frac{d}{dz} \left( \frac{e^{\varphi(z)}}{z^{d-1-2J}} \frac{d}{dz} \right) + \left( \frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = M^2 \Phi_J(z)$$

with $\hat{\Phi}_J(z) = \left( z/R \right)^J \Phi_J(z)$ and all indices along 3+1
Dual QCD Light-Front Wave Equation

[GdT and S. J. Brodsky, PRL 102, 081601 (2009)]

Upon substitution $z \to \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2} + J e^{\varphi(z)/2} \Phi_J(\zeta)$ in AdS WE

$$\left[ -\frac{z^{d-1-2J}}{e^{\varphi(z)}} \frac{\partial}{\partial z} \left( \frac{e^{\varphi(z)}}{z^{d-1-2J}} \frac{\partial}{\partial z} \right) + \left( \frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)$$

find LFWE ($d = 4$)

$$\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

with

$$U(\zeta) = \frac{1}{2} \varphi''(z) + \frac{1}{4} \varphi'(z)^2 + \frac{2J - 3}{2z} \varphi'(z)$$

and $(\mu R)^2 = -(2 - J)^2 + L^2$

AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$

Scaling dimension $\tau$ of AdS mode $\hat{\Phi}_J$ is $\tau = 2 + L$ in agreement with twist scaling dimension of a two parton bound state in QCD and determined by QM stability condition
Bosonic Modes and Meson Spectrum

Soft wall model: linear Regge trajectories [Karch, Katz, Son and Stephanov (2006)]

• Dilaton $\varphi(z) = +\kappa^2 z^2$ (Minkowski metrics), $\varphi(z) = -\kappa^2 z^2$ (Euclidean metrics)

• Effective potential: $U(z) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$

• Normalized eigenfunctions $\langle \phi | \phi \rangle = \int d\zeta |\phi(z)|^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L} (\kappa^2 \zeta^2)$$

• Eigenvalues $M_{n,L,S}^2 = 4\kappa^2 (n + L + S/2)$

![Graphs of dilaton modes](image)

LFWFs $\phi_{n,L}(\zeta)$ in physical space time for dilaton $\exp(\kappa^2 z^2)$: a) orbital modes and b) radial modes
Regge trajectories for the \( \pi \) (\( \kappa = 0.6 \text{ GeV} \)) and the \( I = 1 \) \( \rho \)-meson and \( I = 0 \) \( \omega \)-meson families (\( \kappa = 0.54 \text{ GeV} \))
Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL 94, 201601 (2005)]

• Nucleon LF modes

\[
\psi_+ (\zeta)_{n,L} = \kappa^{2+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{3/2+L} e^{-\kappa^2 \zeta^2/2} L_{n+1}^{L+1} \left( \kappa^2 \zeta^2 \right)
\]

\[
\psi_- (\zeta)_{n,L} = \kappa^{3+L} \frac{1}{\sqrt{n+L+2}} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{5/2+L} e^{-\kappa^2 \zeta^2/2} L_n^{L+2} \left( \kappa^2 \zeta^2 \right)
\]

• Normalization

\[
\int d\zeta \psi_+^2 (\zeta) = \int d\zeta \psi_-^2 (\zeta) = 1
\]

• Eigenvalues

\[
\mathcal{M}_{n,L,S=1/2}^2 = 4\kappa^2 (n + L + 1)
\]

• “Chiral partners”

\[
\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}
\]
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Same multiplicity of states for mesons and baryons!

Regge trajectories for positive parity $N$ and $\Delta$ baryon families ($\kappa = 0.5$ GeV)

$4\kappa^2$ for $\Delta n = 1$

$4\kappa^2$ for $\Delta L = 1$

$2\kappa^2$ for $\Delta S = 1$
Regge trajectories for $N$ and $\Delta$ baryon families ($\kappa = 0.5$ GeV): upper curve $s = 3/2$, lower $s = 1/2$
4 Light-Front Holographic Mapping of Current Matrix Elements

[S. J. Brodsky and GdT, PRL 96, 201601 (2006)], PRD 77, 056007 (2008)]

• EM transition matrix element in QCD: local coupling to pointlike constituents

\[ \langle \psi(P') | J^\mu | \psi(P) \rangle = (P + P')^\mu F(Q^2) \]

where \( Q = P' - P \) and \( J^\mu = e_{q\bar{q}} \gamma^\mu q \)

• EM hadronic matrix element in AdS space from coupling of external EM field propagating in AdS with extended mode \( \Phi(x, z) \)

\[ \int d^4x \, dz \sqrt{g} \, e^{\varphi(z)} A^M(x, z) \Phi^*_P(x, z) \leftrightarrow \partial_M \Phi_P(x, z) \]

\[ \sim (2\pi)^4 \delta^4 (P' - P) \epsilon_\mu (P + P')^\mu F(Q^2) \]

• How to recover hard pointlike scattering at large \( Q \) out of soft collision of extended objects?

[Polchinski and Strassler (2002)]

• Mapping of \( J^+ \) elements at fixed light-front time: \( \Phi_P(z) \leftrightarrow |\psi(P)\rangle \)
Mapping Form-Factors

- Drell-Yan-West electromagnetic FF in impact space [Soper (1977)]

\[
F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2b_{\perp j} \sum e_q \exp \left( i q_{\perp} \cdot \sum_{k=1}^{n-1} x_k b_{\perp k} \right) |\psi_n(x_j, b_{\perp j})|^2
\]

- Consider a two-quark $\pi^+$ Fock state $|ud\rangle$ with $e_u = \frac{2}{3}$ and $e_d = \frac{1}{3}$

\[
F_{\pi^+}(q^2) = \int_0^1 dx \int d^2b_{\perp} e^{i q_{\perp} \cdot b_{\perp} (1-x)} |\psi_{ud/\pi}(x, b_{\perp})|^2
\]

with normalization $F_{\pi^+}(q=0) = 1$

- Integrating over angle

\[
F_{\pi^+}(q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int \zeta d\zeta J_0 \left( \zeta q \sqrt{\frac{1-x}{x}} \right) |\psi_{ud/\pi}(x, \zeta)|^2
\]

where $\zeta^2 = x(1-x)b_{\perp}^2$
• Compare with electromagnetic FF in AdS space [Polchinski and Strassler (2002)]

\[ F(Q^2) = R^3 \int \frac{dz}{z^3} V(Q, z) \Phi_{\pi^+}^2(z) \]

where \( V(Q, z) = zQ K_1(zQ) \)

• Use the integral representation

\[ V(Q, z) = \int_0^1 dx J_0(\zeta Q \sqrt{\frac{1-x}{x}}) \]

• Compare with electromagnetic FF in LF QCD for arbitrary \( Q \). Expressions can be matched only if LFWF is factorized

\[ \psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}} \]

• Find

\[ X(x) = \sqrt{x(1-x)}, \quad \phi(\zeta) = \left( \frac{\zeta}{R} \right)^{-3/2} \Phi(\zeta), \quad z \to \zeta \]
• Form factor in soft-wall model expressed as $\tau - 1$ product of poles along vector radial trajectory
  (twist $\tau = N + L$) [Brodsky and GdT, Phys.Rev. D77 (2008) 056007]

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M^2_\rho}\right) \left(1 + \frac{Q^2}{M^2_{\rho'}}\right) \cdots \left(1 + \frac{Q^2}{M^2_{\rho^\tau-2}}\right)}$$

where $M^2_{\rho_n} \to 4\kappa^2(n + 1/2)$

• Correct scaling incorporated in the model

• “Free current” $V(Q, z) = zQK_1(zQ) \to$ infinite radius (mauve), no pole structure in time-like region

• “Dressed current” non-perturbative sum of an infinite number of terms $\to$ finite radius (blue)
Nucleon Elastic Form Factors

- Light Front Holographic Approach [Brodsky and GdT]

- EM hadronic matrix element in AdS space from non-local coupling of external EM field in AdS with fermionic mode $\Psi_P(x, z)$

$$
\int d^4 x \ d z \ \sqrt{g} \ e^{\varphi(z)} \overline{\Psi}_P(x, z) \ e_A^M \Gamma^A A_M(x, z) \Psi_P(x, z)
$$

$$
\sim (2\pi)^4 \delta^4 \left( P' - P \right) \epsilon_\mu \langle \psi(P'), \sigma' | J^\mu | \psi(P), \sigma \rangle
$$

- Effective AdS/QCD model: additional term in the 5-dim action


$$
\eta \int d^4 x \ d z \ \sqrt{g} \ e^{\varphi(z)} \overline{\Psi} \ e_A^M e_B^N \left[ \Gamma^A, \Gamma^B \right] F_{MN} \Psi
$$

Couplings $\eta$ determined by static quantities

- Generalized Parton Distributions in gauge/gravity duals

[Vega, Schmidt, Gutsche and Lyubovitskij, Phys.Rev. D83 (2011) 036001]

[Nishio and Watari, arXiv:1105.290]
• Compute Dirac proton form factor using SU(6) flavor symmetry

\[ F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} V(Q, z) \Psi_+^2(z) \]

• Nucleon AdS wave function

\[ \Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}} z^{7/2+L} L_n^{L+1} (\kappa^2 z^2) e^{-\kappa^2 z^2/2} \]

• Normalization \( (F_1^p(0) = 1, \ V(Q = 0, z) = 1) \)

\[ R^4 \int \frac{dz}{z^4} \Psi_+^2(z) = 1 \]

• Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

\[ V(Q, z) = \kappa^2 z^2 \int_0^1 dx \frac{Q^2}{(1-x)^2} x \frac{Q^2}{4\kappa^2} e^{-\kappa^2 z^2 x/(1-x)} \]

• Find

\[ F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M^2_\rho}\right) \left(1 + \frac{Q^2}{M^2_{\rho'}}\right)} \]

with \( M_{\rho n}^2 \rightarrow 4\kappa^2(n + 1/2) \)
Nucleon Transition Form Factors

• Compute spin non-flip EM transition $N(940) \rightarrow N^*(1440)$

$$n = 0, L = 0, s_z = 1/2 \rightarrow n = 1, L = 0, s_z = 1/2 : \quad \Psi_{n=0,L=0}^+ \rightarrow \Psi_{n=1,L=0}^+$$

• Transition form factor

$$F_{1^p_{N \rightarrow N^*}}(Q^2) = R^4 \int \frac{dz}{z^4} \Psi_{n=1,L=0}^+(z)V(Q,z)\Psi_{n=0,L=0}^+(z)$$

• Orthonormality of Laguerre functions

$$\left( F_{1^p_{N \rightarrow N^*}}(0) = 0, \quad V(Q = 0, z) = 1 \right)$$

$$R^4 \int \frac{dz}{z^4} \Psi_{n',L}^+(z)\Psi_{n,L}^+(z) = \delta_{n,n'}$$

• Find

$$F_{1^p_{N \rightarrow N^*}}(Q^2) = \frac{2\sqrt{2}}{3} \frac{Q^2}{M_P^2} \left( 1 + \frac{Q^2}{M_{\rho}^2} \right) \left( 1 + \frac{Q^2}{M_{\rho'}^2} \right) \left( 1 + \frac{Q^2}{M_{\rho''}^2} \right)$$

with $\mathcal{M}_{\rho_n}^2 \rightarrow 4\kappa^2(n + 1/2)$
Data from I. Aznauryan, et al. CLAS (2009)
“Working with a front is a process that is unfamiliar to physicists. But still I feel that the mathematical simplification that it introduces is all-important. I consider the method to be promising and have recently been making an extensive study of it. It offers new opportunities, while the familiar instant form seems to be played out”

P.A.M. Dirac (1977)