Resonance/Parton Duality in Electroproduction of Pions

Murat Kaskulov, Ulrich Mosel
**General Motivation**

- Extraction of Resonance properties (and pion formfactor) relies on model: how good is it?
  - QCD expect: \( \sigma_L / \sigma_T \propto 1/Q^2 \) but \( \sigma_T \) is large, and seems to grow

- Where does pQCD start to work: at JLAB@6 or JLAB@12?

- Pion production in JLAB, HERMES, Cornell expts.

- Role of Resonances?
Electro-Pion production

N* parameters from analyses of exclusive electroproduction channels!

Resonant amplitudes

Non-resonant amplitudes

Electrocouplings are proportional to the helicity amplitudes for transition between the initial photon-proton state of the initial particle helicities, and the final state with unstable N* resonances.

Electrocouplings are well defined, directly related to N* structure and are part of observable quantities, e.g. cross sections.

Both needed for gauge invariance

V. Mokeev
Pion e.m. Formfactor

- Pion e.m. FF at larger $Q^2$ mainly from JLAB, extracted from model for long. X-section

\[ \sigma_L \propto \left[ \frac{F_\pi(Q^2)}{t - m_\pi^2 + i0^+} \right]^2 \]

VGL model: $t$-channel exchange + Born-graph and $F_p = F_\pi$
Extended VGL Model

- **Current for** $\pi^+$ **production**

\[ -i J_s^\mu (\gamma^* p \rightarrow \pi^+ n) = \sqrt{2} g_{NN} \bar{u}_s (p') \gamma_5 \left[ F_{\gamma\pi\pi} (Q^2, t) \frac{(k + k')^\mu}{t - m_\pi^2 + i0^+} + F_s (Q^2, s, t) \frac{(p + q)_\sigma \gamma^\sigma \gamma^\mu + M_p \gamma^\mu}{s - M_p^2 + i0^+} \right. \]

\[ \left. + [F_{\gamma\pi\pi} (Q^2, t) - F_s (Q^2, s, t)] \frac{(k - k')^\mu}{Q^2} \right] u_s (p), \]

- **Reggeize propagator**

\[
D(t) = \frac{1}{t - m_\pi^2 + i0^+} \Rightarrow \mathcal{R}[\alpha_\pi (t)] \]

\[
= \frac{1 + e^{-i\pi \alpha_\pi (t)}}{2} (-\alpha'_\pi) \Gamma [-\alpha_\pi (t)] e^{g_\pi (t) \ln (\alpha'_\pi)}, \]

Different e.m. formfactors for $\pi$ and $p$, still gauge invariant (Gross-Riska)
Extended VGL Model

- Extended VGL model very good for $L$

- still dramatically bad for $T$

Data: Jlab, Horn et al.
Partons in Electro-Pionproduction

- Observe
  1. $\sigma_T(e, e'\pi)(Q^2) \sim \sigma_{\text{DIS}}(Q^2)$
  2. $\sigma_L/\sigma_T \to 0$ for $Q^2 \to \infty$

- Take excl. limit of DIS:

$p(e, e'\pi^+)X \to p(e, e'\pi^+)n$ for $z \to 1$

$\sigma = \sigma(t - \text{channel}) + \sigma(\text{Born}) + \sigma(\text{DIS})$

Bebek et al, 1978
Partons in Electro-Pionproduction

- Use PYTHIA to calculate excl. DIS
- Stringbreak $\rightarrow$ DIS
- $\text{DIS} = N^* (W > 2 \text{ GeV}, \text{resonances overlap})$
\[ \frac{d\sigma}{dt} = \frac{d\sigma_T}{dt} + \varepsilon \frac{d\sigma_L}{dt} \]

- **JLAB Data:**
  - Perfect fit!

Data: JLAB, Quian et al

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'exclusive' DIS describes $T$

- Excellent Description of $L$, determined by $t$-channel + Born (red)
- Excellent description of $T$, determined by DIS (green)

![Graphs showing data for different $Q^2$ and $W$ values for $\frac{d\sigma}{dt}$]
'exclusive' DIS describes $T$
**L and T at JLAB and HERMES**

- **JLAB@12**: \( L \) wins at forward angles, \( F_\pi \) extract. less contaminated by transverse (partonic) part

![Graph showing the comparison between transverse and longitudinal modes with W = 2 GeV and W = 3 GeV.](image)

solid: transverse
dashed: longitudinal
Improvement: amplitudes

- Model works very well, but cannot give mixed $LTX$-sections

- String breaking gives only $X$-sections, not amplitudes
Improvement: amplitudes

- Exps. (JLAB, HERMES) cover \( W \sim 2 - 4 \text{ GeV} \) region of overlapping nucleon resonances

- Add sum over (many) \( N^* \)s to Born-term

- Use duality to obtain infos on coupling of high \( N^* \) and to connect \( N^* \) with partons
N* contribution to $\pi^+$ production

- **Current for $\pi^+$ production**

$$-i J^\mu_s(\gamma^* p \rightarrow \pi^+ n) = \sqrt{2} g_{\pi NN} \bar{u}_s(p') \gamma_5 \left[ F_{\gamma \pi \pi}(Q^2, t) \frac{(k + k')^\mu}{t - m^2_\pi + i0^+} 
+ \mathcal{F}_s(Q^2, s, t) \frac{(p + q)_\sigma \gamma^\sigma \gamma^\mu + M_p \gamma^\mu}{s - M^2_p + i0^+} 
+ \left[ F_{\gamma \pi \pi}(Q^2, t) - \mathcal{F}_s(Q^2, s, t) \right] \frac{(k - k')^\mu}{Q^2} \right] u_s(p),$$

- **Replace**

$$\frac{F_s(Q^2, M_p)}{s - M^2_p + i0^+} \rightarrow \sum r(M_i) c(M_i) \frac{F(Q^2, M_i^2)}{s - M_i^2 + i0^+},$$

and

$$\sum_i \rightarrow \int_{M^2_p}^\infty dM^2_i \rho(M^2_i).$$

e.m. Coupling strong

Density of N* resonances

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N* contribution to $\pi^+$ production

- Local Bloom-Gilman duality:
  $$F_2^p(x_B, Q^2) = \sum (M_i^2 - M_p^2 + Q^2) W(Q^2, M_i) \delta(s - M_i^2),$$
  with
  $$W(Q^2, M_i) = r^2(M_i)[F(Q^2, M_i)]^2, \quad F(0, M_i) = 1.$$  

- Integrate over $M_i$
  $$F_2^p(x_B, Q^2) = (s - M_p^2 + Q^2)r^2(s)[F(Q^2, s)]^2 \rho(s).$$

Relation between $F_2$ and Formfactor $F$
N* contribution to $\pi^+$ production

Duality

\[ F_2^p(\omega') \propto (\omega' - 1)^3, \]
\[ \omega' = 1 + W^2/Q^2 \]

\[ F(Q^2, M_i^2) = \left( \frac{1}{1 + \xi \frac{Q^2}{M_i^2}} \right)^2, \]

\[ (\omega' - 1)^3 \propto Q^2(\omega' - 1)^4 \xi^4 r^2(s) \rho(s). \]

\[ r^2(s) \rho(s) \propto \frac{1}{Q^2(\omega' - 1)} = \frac{1}{s}. \]

e.m. coupling $r(s)$ decreases with $s$

\[-\rightarrow \text{Integral over all resonances converges}\]
N* contribution to $\pi^+$ production

Absorb all N*s into Born term:

$$\sum_i r(M_i)c(M_i) \frac{F(Q^2, M_i^2)}{s - M_i^2 + i0^+}$$

$$\Rightarrow \int_{M_p^2}^{\infty} dM_i^2 \rho(M_i^2) r(M_i^2)c(M_i^2) \frac{F(Q^2, M_i^2)}{s - M_i^2 + i0^+}$$

$$= \int_{M_p^2}^{\infty} ds_i \frac{s_i^{-\beta}}{\lambda} \frac{F(Q^2, s_i)}{s - s_i + i0^+} \equiv \frac{F_s(Q^2, s)}{s - M_p^2 + i0^+},$$

with

$$\rho(s_i)r(s_i)c(s_i) = \frac{1}{\lambda} s_i^{-\beta}.$$
N* contribution to $\pi^+$ production

- 'effective Born-Term FF'

\[
F_s(Q^2, s) = \frac{\int_{M_p^2}^{\infty} ds_i \frac{s_i^{-\beta}}{s-s_i+i0^+} \left( \frac{1}{1+iQ^2/s_i} \right)^2}{\int_{M_p^2}^{\infty} ds_i \frac{s_i^{-\beta}}{s-s_i+i0^+}},
\]

harder for higher resonances: consequence of BG duality

W = 2.2 GeV

Dashed: model
Solid: free proton

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N* contribution to $\pi^+$ production

Perfect agreement over wide range of $Q^2$

Data from F-$\pi2$, $\pi$-CT expts.

Green: $t$-channel
dashed: $t$ + Born
solid: $t$ + Born + res
\[ \pi_0 \text{ photoproduction} \]

Dash-dash-dotted: resonance, dashed: t-channel omega

\( E_\gamma = 6 \, \text{GeV} \)
\( E_\gamma = 9 \, \text{GeV} \)
\( E_\gamma = 12 \, \text{GeV} \)
\( E_\gamma = 15 \, \text{GeV} \)

$\pi_0$ photo- + electroproduction

data: HERMES

dash-dash-dotted: resonance, dash-dotted: $t$-channel, dashed: $L$-contrib

Benchmark for JLAB@12

\[ Q^2 = 1.2 \text{ GeV}^2 \]
\[ W = 3.1 \text{ GeV} \]
\[ \varepsilon = 0.825 \]

\[ -45 \text{ deg} < \phi < 45 \text{ deg} \]

W $\sim$ 3 GeV

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JLAB@12 Predictions for \((e,e'\pi^+)\)

\(\sigma_L\) dominates at forward angles

\(-\) cleaner determination of \(F_\pi\)

\(\sigma_T\) grows with \(Q^2\) relative to \(\sigma_L\)

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**Q^2 scaling of L and T**

**T** very different from hard scattering prediction (Horn et al)

Res/parton model predicts increase with Q^2 only for smaller x

Hard scattering Prediction: \( \sigma_L/\sigma_T \sim Q^2 \)

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Summary

- Any good reaction model or pion production has to describe both t-channel and s-channel (N*)
- Transverse pion production does not follow QCD scaling law at JLAB@5 and JLAB@12
- Transverse strength from resonance contributions
- Duality fixes e.m. formfactors of high-lying resonances
- Cornell, DESY and JLAB data are all described, with same model, same parameters
- QCD scaling for $L/T$ holds only for small $x < 0.5$
References

- Neutral pion electroproduction in $p(e,e'\pi^0)p$ above $s\sqrt{s}>2$ GeV
- Exclusive pion electroproduction off nucleons and nuclei.
- Beam spin asymmetry in deeply virtual $\pi$ production.
- Deep exclusive charged $\pi$ electroproduction above the resonance region.
- Deep exclusive electroproduction of $\pi^+$ from data measured with the HERMES detector at DESY.
- Deeply inelastic pions in the exclusive reaction $p(e, e' \pi^+)p$ above the resonance region.
Backups
Long and *Transv Pion Production*

- VGL model
  - very good for $L$
  - dramatically bad for $T$

Nagging thought: can $F_\pi$ be reliably extracted from a model that fails for $T$?

Data from $\pi$-CT Exp. at JLAB, T. Horn et al., PR C78, 058201 (2008)
Extended VGL Model

- Extends VGL model, allows for gauge-invariant treatment of different FFs for pion and proton (Gross-Riska)
Extended VGL Model

Regge trajectories in t-channel

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Prediction for $\pi^-$ on $n$

- **For $L$:**
  - $\pi$-pole dominance: $\sigma_L^n/\sigma_L^p \approx 1$

- **For $T$:**
  - Dominated by structure functions:
    $$\sigma_T^n/\sigma_T^p \approx F_1^n/F_1^p \approx F_2^n/F_2^p < 1$$
Resonance/Parton connection

- Use BG duality to keep strong transverse contribs of partons: $N^* \sim \text{partons}$

From:
W. Melnitchouk, Proc. MENU 2007
N* contribution to $\pi^+$ production

- $\phi$ dependence determined by N*

Data from JLAB, T. Horn et al, V. Tadevosyan et al

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$Q^2$, $\varepsilon$-dependence

JLAB

$Q^2 = 1.1 \text{ GeV}^2$
$W = 2.26 \text{ GeV}$
$\varepsilon = 0.49$

$Q^2 = 2.17 \text{ GeV}^2$
$W = 2.22 \text{ GeV}$
$\varepsilon = 0.55$

$Q^2 = 4 \text{ GeV}^2$
$W = 2.14 \text{ GeV}$
$\varepsilon = 0.25$

$Q^2 = 2.2 \text{ GeV}^2$
$W = 2.13 \text{ GeV}$
$\varepsilon = 0.27$

$Q^2 = 3.9 \text{ GeV}^2$
$W = 2.25 \text{ GeV}$
$\varepsilon = 0.39$

$Q^2 = 4.7 \text{ GeV}^2$
$W = 2.25 \text{ GeV}$
$\varepsilon = 0.26$

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$T, L X$-sections at DESY, JLAB
$\pi^-/\pi^+$ ratio

JLAB@5

HERMES

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DESY Data

\[ Q^2 = \{0.70 \text{ GeV}^2, 1.35 \text{ GeV}^2 \} \]

\[ W = \{2.19 \text{ GeV}, 2.19 \text{ GeV} \} \]

\[ \epsilon = \{0.86, 0.84 \} \]

\[ p(\gamma, \pi^+) n \]

\[ n(\gamma, \pi^-) p \]

\[ \text{d} \sigma / \text{d}t \text{ [mb/GeV]} \]

\[ -t \text{ [GeV}^2] \]
HERMES Data

\[ 1 < Q^2 < 2 \text{ GeV}^2 \]

\[ p(\gamma^*, \pi^+) n \]
π+ production at HERMES
TT\textsubscript{(solid)} and LT\textsubscript{(dashed)} at HERMES

\begin{align*}
1 < Q^2 < 2 \text{ GeV}^2 \\
0.02 < x_B < 0.15
\end{align*}

\begin{align*}
2 < Q^2 < 3 \text{ GeV}^2 \\
0.06 < x_B < 0.23
\end{align*}

\begin{align*}
3 < Q^2 < 4 \text{ GeV}^2 \\
0.11 < x_B < 0.31
\end{align*}

\begin{align*}
4 < Q^2 < 11 \text{ GeV}^2 \\
0.15 < x_B < 0.55
\end{align*}
Beam Spin Azimuthal Moment

JLAB@5.7 GeV

\[ p(e,e'\pi^+)n \]

\[ n(e,e'\pi^-)p \]

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