Hyperons: Scaling, N* Resonances, and the $\Lambda(1405)$

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Overview

- “Scaling” of the reaction $\gamma + p \rightarrow K^+ + \Lambda$
  - Regge scaling at small $-t$
  - Constituent-counting scaling at high $-t$

- N* Resonances seen in Scaled Cross Sections
  - Strong correlations at large angles $\rightarrow$ interferences
  - Connection to “missing resonance” searches

- Properties of the $\Lambda(1405)$
  - Lineshape reveals compound nature
  - Spin and parity measurement
Regge Scaling at Small $-t$

- How does $d\sigma/dt$ vary with $s$ and $-t$?

\[
\frac{d\sigma}{dt} = D(t) \left( \frac{s}{s_0} \right)^{2\alpha(t)-2}
\]

\[
s = W^2 \quad \text{invariant mass}^2
\]

\[
\alpha(t) = \alpha_{t=t_{\text{min}}} + \alpha' t \quad \text{Regge trajectory}
\]
\[ \gamma + p \rightarrow K^+ + \Lambda \, \text{raw cross section} \]

\[ d \sigma / dt \left( \mu b \times \text{GeV}^2 \right) \]

\[ \cos \theta +.1 \quad 0 \quad -.1 \quad -.2 \quad -.3 \quad -.4 \quad -.5 \]

\[ -t (\text{GeV}^2) \]

M. E. McCracken et al. (CLAS), Phys. Rev. C 81, 025201 (2010)
\[ \frac{d\sigma}{dt} = D(t) \left( \frac{s}{s_0} \right)^{2\alpha(t)-2} \]
Observation of approximate $s^{-2}$ "Regge scaling" implies that
\[ \alpha_{\text{eff}} = \alpha_{K^+} + \alpha_{K^*(892)} \approx 0, \text{ for } t \to 0 \]

Model calculation of $\alpha(t)$ remains as an open task...

We move on to more dramatic phenomenology...
Constituent-Counting Scaling

\[ \frac{d\sigma}{dt} = f\left(\frac{t}{s}\right) s^{2-n} \]

- Constituent counting rules for exclusive scattering
- "Valid" for \( s \rightarrow \infty \) and \( t/s \) fixed
  - \( t/s \sim \cos(\theta_{cm}) \) as \( s \rightarrow \infty \)
- \( n \) = number of point-like constituents
- Follows from pQCD

\[ \sigma = M_{BN} n \]

\( n = 9 \) for \( n = 10 \)

S. J. Brodsky and G. R. Farrar, PRL 31, 1153 (1973)
Scaling Power Determination

- Optimize $N$ in a fit of $s^{-N}$ scaling
- Best fit: $N = 7.1 \pm 0.1$
- $\chi^2_v = 92/60$: fair fit
- Supports hypothesis of photon as a single bare elementary field
- Assume $N = 7$ henceforth...

$\theta_{cm} = 90^\circ$


Scaling in Pion Production

- "perturbative QCD" scaling at SLAC
- $s^{-7}$ scaling found to "work" for $\gamma p \rightarrow \pi^+ n, \pi^0p, \pi^-\Delta^{++}, \rho^0p$, and maybe KY
- The curve is totally ad hoc
- Expect the best evidence for scaling near $90^\circ$

**FIG. 6.** $s^{-7}d\sigma/dt$ versus $\cos \theta^*$ for the reaction $\gamma p \rightarrow \pi^+ n$. The solid line shows the empirical function $(1-z)^{-5}(1+z)^{-4}$ where $z = \cos \theta^*$, which is an empirical fit to the angular distribution.

Evidence for $s^{-7}$ Scaling...

- **CLAS:** 100 MeV wide bins in $W$
  - Green 1.7 GeV
  - Red 2.8 GeV

- **SLAC:**
  - Black 2.9 GeV
  - White 3.5 GeV
  - CLAS & SLAC show good agreement

- $s^{-7}$ scaling happens for $W >$ about 2.3 GeV

- Pions and Kaons scale to same value near 90°
  - Interesting: are the quark mass differences irrelevant?

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Scaling in Pion Production

- Three pion channels at 90° vs. W
- pQCD scaling seen for W > 2.6 GeV
- N* resonances seen below 2 GeV

W. Chen et al. (CLAS), PRL 103, 012301 (2009)
$\gamma + p \rightarrow K^+ + \Lambda$

$pQCD$-like scaling

$N^*$ Resonances?

$\frac{d\sigma}{dt} = s^{2-n} f\left(\frac{t}{s}\right) \rightarrow s^{-7} f\left(\frac{t}{s}\right)$

Forward Angles

Backward Angles

$\cos \theta$

$+1$

$0$

$-1$

$-2$

$-3$

$-4$

$-5$
\[ \gamma + p \rightarrow K^+ + \Lambda \]

\[ d\sigma/dt = s^{2-n} f \left( \frac{t}{s} \right) \rightarrow s^{-7} f \left( \frac{t}{s} \right) \]

\[ \cos \theta \]

\[ +.1 \]
\[ .0 \]
\[ -.1 \]
\[ -.2 \]
\[ -.3 \]
\[ -.4 \]
\[ -.5 \]
Physics Model

- Quantize along beam axis
- Final state amplitude \( \psi_L(J, J_z) \)
- \( \alpha_{1/2, \pm 1/2} \) nucleon spinors
- \( Y_{LM} \) spherical harmonic of final state

Example: \( J = 3/2 \) resonance formed in \( J_z = +1/2 \) substate, decaying to P-wave

\[
\psi_{L=1} \left( J = \frac{3}{2}, J_z = \frac{1}{2} \right) = \left\{ \frac{1}{\sqrt{3}} Y_{1,1}, \alpha_{\frac{1}{2}, \frac{1}{2}} + \frac{2}{\sqrt{3}} Y_{1,0}, \alpha_{\frac{1}{2}, -\frac{1}{2}} \right\} BW_{1/2}(m)
\]

Similar expressions for

\[
\psi_P \left( \frac{3}{2}, \frac{3}{2} \right), \psi_D \left( \frac{3}{2}, \frac{3}{2} \right), \psi_D \left( \frac{3}{2}, \frac{1}{2} \right), \psi_S \left( \frac{1}{2}, \frac{1}{2} \right)
\]
Physics Model

- Each resonance represented as a relativistic Breit-Wigner

\[ BW_{J_z}(m) = \frac{\sqrt{m m_0 \Gamma_{J_z, y^p \rightarrow N^*} \Gamma_{N^* \rightarrow K\Lambda}(q)}}{m^2 - m_0^2 - i m_0 \Gamma_{tot}(q)} \]

\[ \Gamma_{tot}(q) = \Gamma_{N^* \rightarrow K\Lambda}(q) + \Gamma_S(q) \]

\[ \Gamma_{N^* \rightarrow K\Lambda}(q) = \Gamma_0 \left( \frac{q}{q_0} \right)^{2L+1} \quad (L \in 0, 1, 2) \]

\[ \Gamma_S(q) = \Gamma_{S_0} \left( \frac{q}{q_S} \right)^7 \]

- Phenomenological damping of high-mass tail to achieve \( s^{-7} \) scaling
Physics Model

- Compute coherent total amplitude
- Scale cross section
- Fit to optimize observed angular distributions

Total amplitude:

\[ |A(m, \cos \theta_{c.m.})|^2 = \psi_s \left(\frac{1}{2}, \frac{1}{2}\right) + \psi_p \left(\frac{3}{2}, \frac{1}{2}\right) + \psi_p \left(\frac{3}{2}, \frac{3}{2}\right) + \psi_d \left(\frac{3}{2}, \frac{1}{2}\right) + \psi_d \left(\frac{3}{2}, \frac{3}{2}\right) |^2 \]

Cross section to fit:

\[ s^7 \frac{d\sigma}{dt} = s^7 \frac{(hc)^2}{64\pi} s \frac{1}{k^2} |A(m, \cos \theta_{c.m.})|^2 \]
Physics Model

- Resonance combinations tested:
  - Low mass: $S_{11}$
  - Medium mass: $S_{11}, P_{11}, P_{13}$
  - High mass: $S_{11}, P_{11}, P_{13}, D_{15}, D_{13}$

- Free parameters:
  - Masses, widths, couplings

- Not included:
  - Additional near-threshold $P_{11}$ or $P_{13}$ waves
  - Spin observables were not fitted

NStar2011, R. A. Schumacher, Carnegie Mellon University, 5-19-11
$\gamma + p \rightarrow K^+ + \Lambda$

$S_{11}(1690)$  $P_{13}(1920)$  $D_{13}(2100)$

$W$ (GeV)

$\cos \theta$

$-0.5 -0.4 -0.3 -0.2 -0.1 0 +0.1$
NStar2011, R. A. Schumacher, Carnegie Mellon University, 5-19-11

Model Curves

$S_{11}(1690)$
$P_{13}(1920)$
$D_{13}(2100)$

$\cos \theta$

$+1$
$0$
$-1$
$-2$
$-3$
$-4$
$-5$

$W \ (GeV)$

$S^7 \frac{d\sigma}{dt} \times 10^7 (\text{nb} \times \text{GeV}^{12})$

Model Results

<table>
<thead>
<tr>
<th>Resonance &amp; Decay</th>
<th>$m_0$ (GeV)</th>
<th>$\Gamma_0$ (MeV)</th>
<th>$\sqrt{\Gamma_{1/2, \gamma p \rightarrow N^*}}$ (GeV)$^{1/2}$ Phase</th>
<th>$\sqrt{\Gamma_{3/2, \gamma p \rightarrow N^*}}$ (GeV)$^{1/2}$ Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{11}$</td>
<td>1690 ± 10</td>
<td>80 ± 20</td>
<td>1.83 ± .10 ($-142 ± 5)$°</td>
<td>-</td>
</tr>
<tr>
<td>$P_{13}$</td>
<td>1920 ± 10</td>
<td>440 ± 100</td>
<td>1.93 ± .10</td>
<td>1.67 ± .07</td>
</tr>
<tr>
<td>$D_{13}$</td>
<td>2100 ± 20</td>
<td>200 ± 50</td>
<td>0.61 ± .10 ($45 ± 5$)°</td>
<td>1.19 ± .10</td>
</tr>
</tbody>
</table>

\[ \Gamma_s(q) = \Gamma_{s_0} \left( \frac{q}{q_s} \right)^7 \]

\[ \Gamma_{s_0} = 0.50 \text{ GeV} \]

\[ q_s = 0.77 \text{ GeV/c} \]

N* Baryons: Seen & “Missing”

- Relativised CQM
  - Classify oscillator-model states by $I, J, P$

- Possible observation of a “missing” N* state in $K^+\Lambda$

- There is a PDG “**” state $N(2080) D_{13}$
  - Weak evidence in $K\Lambda$
  - Mart & Bennhold: confused with the $P_{13}$ at 1900MeV.

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Schumacher, Carnegie Mellon University, 5-19-11
Next topic…

\[ \Lambda(1405) \]

See talks by: K. Moriya photoproduction III-B
H. Lu electroproduction I-A
What “is” the $\Lambda(1405)$?

- Structure - an issue since its discovery
  - SU(3) singlet 3q state
    $I=0$, $J^\pi = \frac{1}{2}^-$
  - $\bar{K}N$ sub-threshold bound state
  - Gluonic $J^\pi = \frac{1}{2}^+$ hybrid (udsg)
  - Dynamically generated resonance, via unitary meson-baryon channel coupling
The Low-Mass $S=-1$ Hyperons

Energy (Mass) MeV

$\begin{align*}
\Lambda(1670) & : J^\pi = \frac{3}{2}^- \\
\Lambda(1520) & : J^\pi = \frac{1}{2}^-
\end{align*}$

But the mass splitting does not work out...


Isospin = 0  Isospin = 1
The Low-Mass $S=-1$ Hyperons

Energy (Mass) MeV

Mass thresholds:

- $\Lambda(1670)$
- $\Sigma(1670)$
- $\Lambda(1520)$
- $\Sigma(1385)\pi$ (1525)
- $\bar{K}N$ (1432)
- $\pi\pi\Lambda$ (1390)
- $\pi\Sigma$ (1331)
- $\Sigma(1385)$
- $\Sigma(1192)$

Isospin = 0

Isospin = 1
Do the “ground state” mesons and baryons attract strongly enough to form meson-baryon “molecular” bound states or unbound resonances?

\[ \bar{K}N \rightarrow \Lambda(1405) + \Sigma \pi \]

**Channel Coupling:**

\[ K_{12} = \frac{-1}{2f^2} \sqrt{\frac{3}{2}} \left( \sqrt{s} - \frac{M_N + M_\Sigma}{2} \right) \]

\[ K_{11} = \frac{3}{2f^2} \left( \sqrt{s} - M_N \right) \]

\[ K_{22} = \frac{2}{f^2} \left( \sqrt{s} - M_\Sigma \right) \]
Chiral Unitary Models (example 1)

- Mass distribution of the “Λ(1405)” predicted to depend on πΣ decay channel
- Model with I = 0 and I = 1 amplitudes
  - Chiral Lagrangian + Channel Coupling
    - I(π Σ) = {0,1} - not in an isospin eigenstate
      - Neglect I=2
  - Interference between I=0 and I=1 amplitudes modifies mass distributions
  - WT-type interaction: no energy or angle dependence
- Inspired CLAS experiment

Chiral Unitary Models (example 2)

The TWO POLES scenario

- Singularity of $\bar{K}N$ amplitude in the complex energy plane
- Starting point: no channel coupling
- Pole I (dominantly $\bar{K}N$)
- Pole II (dominantly $\pi\Sigma$)
- $z_1(\text{ORB})$, $z_1(\text{BMN})$
- $z_2(\text{ORB})$, $z_2(\text{BNW})$
- $z_2(\text{HNJH})$
- $z_2$ (for $\pi\Sigma$ only)
- $z_1$ (for $\bar{K}N$ only)
- $\pi\Sigma$ resonance
- $\bar{K}N$ bound state
- Channel coupling at work

Graphic: W. Weise
Chiral Unitary Models (example 2)

- SU(3) baryons irreps $1+8_s+8_a$ combine with $0^-$ Goldstone bosons to generate:
  - Two octets and a singlet of $\frac{1}{2}^-$ baryons generated dynamically in SU(3) limit
  - SU(3) breaking leads to two $S=-1$ $I=0$ poles near 1405 MeV
    - $\sim 1420$ mostly $\bar{K}N$
    - $\sim 1390$ mostly $\pi\Sigma$
  - Possible weak $I=1$ pole also predicted

CLAS result for Λ(1405)

- Decay-channel asymmetry of Λ(1405) lineshape confirmed
- Asymmetric among the three charge states → not a pure isospin I=0 process (decomposition in progress...)
- Subtracted backgrounds: Σ(1385), Λ(1520), K*(892)
- Direct Spin-parity measurement: $J^\pi = \frac{1}{2}^-$
- Details:
  - Kei Moriya Session IIIB

Note that “sign” of the charge asymmetry is opposite to Nacher et al prediction
Isospin Decomposition

I=0 contribution

I=1 contributions

Preliminary

$W = 2.10 \text{ GeV}$
Parity and Spin of $\Lambda(1405)$

$J^P$ of $\Lambda(1405)$

no previous direct experimental evidence for the spin and parity 
(PDG assumes $1/2^-$)  "Note on the $\Lambda(1405)$" 1998 PDG, R.H. Dalitz

How do we measure these quantities?

- **spin** – measure distribution into $\Sigma\pi$  
  ▶ flat distribution is best evidence possible for $J = 1/2$
- **parity** – measure polarization of $\Sigma$ from $\Lambda(1405)$  
  ▶ Polarization direction as a function of $\Sigma$ decay angle will be determined by $J^P$ of $\Lambda(1405)$
s-wave, p-wave Scenario

\[
\begin{align*}
L &= 0 \text{ (s-wave)} \\
\vec{P}_\Sigma^+ &= \vec{P}_\Lambda^* \\
\end{align*}
\]

\[
\begin{align*}
\Lambda(1405) \rightarrow \Sigma \pi \text{ is s-wave} \\
\Leftrightarrow J^P = 1/2^- \\
\end{align*}
\]

\[
\begin{align*}
L &= 1 \text{ (p-wave)} \\
\vec{P}_\Sigma^+ &= |\vec{P}_\Lambda^*| \hat{n}(2\theta_{\Sigma^+}) \\
\end{align*}
\]

\[
\begin{align*}
\Lambda(1405) \rightarrow \Sigma \pi \text{ is p-wave} \\
\Leftrightarrow J^P = 1/2^+ \\
\end{align*}
\]
Determination of Parity

polarization of $\Lambda(1405)$ in direction $\perp$ to production plane is measured

- $W = 2.6$ GeV
- forward $K^+$ angles
- use reaction:
  $\Lambda(1405) \rightarrow \Sigma^+ \pi^-$,
  $\Sigma^+ \rightarrow p\pi^0$
- very large hyperon decay parameter
  $\alpha = -0.98$
- bg is $\sim 10\% \Sigma(1385)$

  polarization does not change with $\Sigma^+$ angle ($\theta_{\Sigma^+}$)

  $\Rightarrow J^P = 1/2^-$ is confirmed

  furthermore, this measured $\Sigma^+$ polarization is the $\Lambda(1405)$polarization

  $\Rightarrow \Lambda(1405)$is produced with $\sim +40\%$ polarization

Kei Moriya, C. M. U. PhD thesis, CMU 2010
Conclusions

- Three phenomena in $K^+\Lambda$ photoproduction:
  - Regge scaling $s^{-2}$ small $-t$ – confirmed
  - Constituent-counting $s^{-N}$ – holds for $N = 7$
  - Evidence for $N^*$ production & interference
    - Present best fit has: $S_{11}(1690)$, $P_{13}(1920)$, $D_{13}(2100)$ ← new observation
    - PDG “**” $D_{13}(2080)$ “missing” state possibly seen
  - $\Lambda(1405)$ – mass distributions in $\Sigma \pi$
    - Evidence for $I=0$, $I=1$ interference
    - Spin-Parity $J^P = \frac{1}{2}^-$ confirmed