Baryon spectroscopy from lattice QCD

• **Goal:** Determine the hadron mass spectrum of QCD

• **New feature:** Spin identification for N\(^*\) and Δ states
  

• **Comparisons with** \(SU(6) \otimes O(3)\)

• **Conclusions**
Lattice parameters

- $N_f = 2+1$ QCD
  - Gauge action: Symanzik-improved
  - Fermion action: Clover-improved Wilson

- Anisotropic: $a_s = 0.122$ fm, $a_t = 0.035$ fm

<table>
<thead>
<tr>
<th>ensemble</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_\ell$</td>
<td>−.0840</td>
<td>−.0830</td>
<td>−.0808</td>
</tr>
<tr>
<td>$m_s$</td>
<td>−.0743</td>
<td>−.0743</td>
<td>−.0743</td>
</tr>
<tr>
<td>Volume</td>
<td>$16^3 \times 128$</td>
<td>$16^3 \times 128$</td>
<td>$16^3 \times 128$</td>
</tr>
<tr>
<td>Physical volume</td>
<td>(2 fm)$^3$</td>
<td>(2 fm)$^3$</td>
<td>(2 fm)$^3$</td>
</tr>
<tr>
<td>$N_{\text{cfgs}}$</td>
<td>344</td>
<td>570</td>
<td>481</td>
</tr>
<tr>
<td>$t_{\text{sources}}$</td>
<td>8</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>$m_\pi$</td>
<td>0.0691(6)</td>
<td>0.0797(6)</td>
<td>0.0996(6)</td>
</tr>
<tr>
<td>$m_K$</td>
<td>0.0970(5)</td>
<td>0.1032(5)</td>
<td>0.1149(6)</td>
</tr>
<tr>
<td>$m_\Omega$</td>
<td>0.2951(22)</td>
<td>0.3040(8)</td>
<td>0.3200(7)</td>
</tr>
<tr>
<td>$m_\pi$ (MeV)</td>
<td>396</td>
<td>444</td>
<td>524</td>
</tr>
</tbody>
</table>
Tuning of $m_{\ell}$ and $m_s$ yields a good account of hadron masses
Limitations

• **Three-quark operators:**
  - No multiparticle operators
  - No clear evidence for multiparticle states: $\pi N$, etc.

• **One (small) volume and one total momentum** $P = 0$: No extrapolations or $\delta$’s

• $m_\pi = 396, 444, 524$ MeV: Energies generally are high

• **The three-quark states essentially are stable; decays are suppressed.**
Computational Resources

- USQCD allocations
- Jefferson Laboratory GPUs and HPC clusters
- and the Chroma software system (Edwards et al.)
HADRON SPECTRUM COLLABORATION

Standard recipe for lattice spectra

- Use interpolating field operators $B_j^\dagger(x, t)$ to create three-quark baryons.

- Construct operators so that they transform as irreps of cubic group

- Make smooth operators i.e., smear them over many lattice sites
  - Project operators to low eigenmodes of covariant lattice Laplacian

- Matrices of correlation functions: $C_{ij}(t) = \sum_x <0|B_i(x, t)B_j^\dagger(0, 0)|0>$
  - $C_{ij}(t) \sim <i|e^{-Ht}|j>$

- Diagonalize matrices to get principal eigenvalues: $\lambda_n(t, t_0)$
  - Principal eigenvalues separate the decays of N eigenstates: $e^{-m_n(t-t_0)}$

- Fit them & extract masses, $m_n$. 
Contamination from states outside the diagonalization space

Expect \( \lambda_n(t) = e^{-m_n(t-t_0)} + \sum_{k>N} B_k e^{-m_k(t-t_0)} + \cdots \)

Two-exponential fits to principal eigenvalues

\[ \lambda_{fit}(t) = (1 - A_n)e^{-m_n(t-t_0)} + A_n' e^{-m'_n(t-t_0)} \]

Ratio plots to show the goodness of fits

\[ \frac{\lambda_{fit}(t)}{e^{-m_n(t-t_0)}} \]

Ratio tends to constant at large t
Contaminations are fit well by the 2nd exponential
N* spectrum in irreps of cubic group: $m_\pi = 396$ MeV

HADRON SPECTRUM COLLABORATION

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Results of standard recipe

• Lots of states and lots of degeneracies

• Spins are ambiguous
  – Degenerate states in $G_1, H, G_2$ irreps imply a $J = \frac{7}{2}$ state
  – or accidentally degenerate $J = \frac{1}{2}$ and $J = \frac{5}{2}$ states

• Spin identification fails because:
  – there are too many degenerate states to identify the subductions of high spins
  – lattice energies don’t provide sufficient information
New recipe to identify spins

- Use operators with known spins in continuum limit
  - Incorporate covariant derivatives to realize orbital angular momenta

- Subduce the operators to irreps of cubic group

- Use spectral representation of matrices: $C_{ij}(t) = \sum_n Z^*_i Z^n e^{-m_n t}$

- $Z^n_i = \langle n | B^\dagger_i(0,0) | 0 \rangle$ is the overlap of operator $i$ with state $n$

- Use $Z^n_i$ to identify spin: spin of state $n$ is $J$ when largest $Z$’s are for operators subduced from spin $J$
  - The different lattice irreps give approximately the same overlaps
  - $E_n$ is the energy of a state of good $J$. 
Construction of operators with good J in continuum

- **Mesons:** Dudek, *et al.*, Phys.Rev.D80:054506,2009

- **Baryons:** Color singlet structure for 3 quarks, symmetric in space & spin

- \( J = L + S \) with
  - \( S = \frac{1}{2} \) or \( \frac{3}{2} \) from quark spins
  - \( L = 1 \) or 2 from covariant derivatives
  - \( J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \) and \( \frac{7}{2} \)
  - Upper (\( \rho = + \)) and lower (\( \rho = - \)) components of Dirac spinors

- **Lots of operators** \( O^{[J,M]} \) with good spin in continuum limit

- **Feynman, Kislinger and Ravndal formalism** for quark states applied to operator construction, except \( SU(12) \otimes O(3) \)
Subduction to irreps of cubic group

- Cubic group irreps $\Lambda$ and rows $r$ provide orthogonal basis on lattice

- In quantum mechanics, subduction is a change of basis $|J, M\rangle \rightarrow |\Lambda, r; J\rangle$.

- $|\Lambda, r; J\rangle = \sum_M |J, M\rangle \langle J, M|\Lambda, r; J\rangle = \sum_M |J, M\rangle S_{\Lambda, r}^{J, M}$.

- Subduction matrices: $S_{\Lambda, r}^{J, M}$

- Subduced operators: $\mathcal{O}[\Lambda, r; J] = \sum_M \mathcal{O}[J, M] S_{\Lambda, r}^{J, M}$

- When rotational symmetry is broken weakly,

  $\langle 0 | \mathcal{O}[\Lambda, r; J](t) \mathcal{O}[\Lambda, r; J']^\dagger(0) |0 \rangle \approx \delta_{J, J'}$ is block diagonal in $J$. 

HADRON SPECTRUM COLLABORATION

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Matrix $C_{ij}$ is block diagonal approximately

Magnitude of matrix elements in a matrix of correlation functions at timeslice 5.
Reasons for approximate rotational invariance

- Rotational symmetry is broken at $O(a^2)$ by lattice action
- Lattice spacing is 0.12 fm
- Typical hadron size is 1 fm
- Smearing makes operators smooth on the hadron size scale
- Estimate: $O(a^2) \approx \left(\frac{0.12 \text{ fm}}{1.0 \text{ fm}}\right)^2 \approx 0.015$
- For hadrons, rotational symmetry is broken weakly.
Spin identification: $Z_{i}^{n}$ values show which operators dominate each state

\[ (N_{M} \otimes (\frac{1}{2})^{\uparrow} \otimes D_{L=1,M}^{[1]} )^{J=3/2} \]

\[ (N_{M} \otimes (\frac{3}{2})^{\uparrow} \otimes D_{L=1,M}^{[1]} )^{J=3/2} \]

\[ (N_{M} \otimes (\frac{3}{2})^{\downarrow} \otimes D_{L=1,M}^{[1]} )^{J=5/2} \]

\[ (N_{M} \otimes (\frac{1}{2})^{\uparrow} \otimes D_{L=2,S}^{[2]} )^{J=3/2} \]

\[ (N_{M} \otimes (\frac{3}{2})^{\uparrow} \otimes D_{L=2,S}^{[2]} )^{J=5/2} \]

\[ (N_{M} \otimes (\frac{1}{2})^{\downarrow} \otimes D_{L=2,S}^{[2]} )^{J=5/2} \]

\[ (N_{M} \otimes (\frac{3}{2})^{\downarrow} \otimes D_{L=2,S}^{[2]} )^{J=7/2} \]
Spin identification: Nearly the same $Z$ in each lattice irrep that belongs to the subduction of $J$

$J = \frac{5}{2}$

$J = \frac{7}{2}$

$m_{H_u}/m_\Omega = 1.226(6)$
$m_{H_u}/m_\Omega = 1.552(11)$
$m_{G_{2u}}/m_\Omega = 1.198(11)$
$m_{G_{2u}}/m_\Omega = 1.529(14)$

$m_{G_{1u}}/m_\Omega = 1.618(16)$
$m_{H_u}/m_\Omega = 1.633(10)$
$m_{G_{2u}}/m_\Omega = 1.572(23)$
Joint fits of $G_{1u}, H_u, G_{2u}$ principal correlators to a common mass determine the $J = \frac{7}{2}^-$ energies.
Spin identification of baryon excited states

- The spin of a lattice excited state is equal to $J$ when the state is created predominantly by operators subduced from continuum spin $J$.

- Approximately the same $Z$ value is obtained in each lattice irrep that belongs to the subduction of a single $J$ value.

- $Z$ values often are large only for a few operators, allowing interpretation of the states.

- Spin identification is reliable at the scale of hadrons.
Spectral test of approximate rotational invariance

- Rotational invariance implies zero couplings between different \( J \)'s, so \( C \propto \delta_{J,J'} \) is block diagonal

- We find small violations of block diagonality in \( C \).

- Does the spectrum exhibit approximate rotational invariance?

- Calculate energies including \( J \neq J' \) couplings

- Calculate energies omitting \( J \neq J' \) couplings
Approximate rotational invariance in spectrum,

all 48 $H_u$  28 $J=\frac{3}{2}$  16 $J=\frac{5}{2}$  4 $J=\frac{7}{2}$

≈ same energies with and without $J \neq J'$ couplings
Lattice N* excited states vs. $J^P$ : $m_\pi = 396$ MeV
Lattice N* spectrum: bands with $+\,$ and $-\,$ parity.
$3^2 \oplus 1 \rightarrow \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$

$\frac{1}{2} \oplus 1 \rightarrow \frac{1}{2}, \frac{3}{2}$

Pauli spinors

$SU(6) \otimes O(3) : [70, 1^-]$
1\textsuperscript{st} N\textsuperscript{*+} band: $SU(6) \otimes O(3)$ states

Pauli spinors

$SU(6) \otimes O(3)$ : $[56,0^+]$, $[56,2^+]$, $[70,0^+]$, $[70,2^+]$, $[20,1^+]$
Overall pattern of $N^*$ states

Experimental Nucleon Spectrum vs. J

Many more states in the lattice spectrum.
Lattice $\Delta$ excited states vs. $J^P$ : $m_\pi = 396$ MeV
Lattice $\Delta$ spectrum: bands of + and − parity states
$\frac{1}{2} \oplus 1 \rightarrow \frac{1}{2}, \frac{3}{2}$

SU(6) $\otimes$ O(3) : [70,1$^-$]
Pauli spinors

$1^{st}$ $\Delta^-$ band : SU(6) $\otimes$ O(3) states
$SU(6) \otimes O(3)$ : $[56,0^+]$, $[56,2^+]$
$[70,0^+]$, $[70,2^+]$

$1^{st}$ $\Delta^+$ band : $SU(6) \otimes O(3)$ states
Patterns of $\Delta$ states

Expt.  ****  ***  **

Lattice

Many more states in the lattice spectrum.
Comparison of lattice results for Roper resonance

see also D. Leinweber talk in session I-C today at 17:05

Does the Roper resonance have a complex structure?
Conclusions

• **Spins are identified reliably up to** $J = \frac{7}{2}$
  
  – Covariant derivatives **provide orbital angular momenta**
  – Approximate rotational invariance is realized at the scale of hadrons
  – **Spectral overlaps** $Z$ **identify which** $J$ **values dominate a state**

• **Low $N^*$ and $\Delta$ bands**: same states as $SU(6) \otimes O(3)$ based on $\rho = +$ Dirac spinors

• **Patterns of lattice baryonic states are similar to patterns of physical resonance states.**

• **Lots of lattice states**: no signs of chiral restoration
The path forward

- No multiparticle states have been identified so far using three-quark operators

- Multiparticle operators (e.g, $\pi N$, $\pi\pi N$) must be added to realize significant couplings of three-quark states and their decay products.

- Moving operators and larger volumes will allow determination of elastic phase shifts using Luscher’s formalism.

- Much remains to be learned as $m_\pi$ is lowered toward the physical limit
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