Extraction of Resonance Parameter

T. Sato

Osaka U.
Characterize Resonances:

- Excitation spectrum Baryon: $J^P, T, M, \Gamma$
- Coupling constant: $g_\alpha, g_\beta$, Branching ratio, electromagnetic form factor
Partial wave amplitude (PWA) $\rightarrow$ Resonance parameters

GW-VPI, Bonn-Gatchina, Jlab-Yerevan, MAID, CMB/Pitt-ANL, Zagreb, Giessen, KENT Julich-Georgia, DMT, KVI, EBAC

- Breit-Wigner formula and pole of S-matrix
- Extraction of resonance parameters
- Simple exercise for extracting resonance parameter from ideal PWA
- Understanding resonance parameters
Breit-Wigner formula and pole of S-matrix
Breit-Wigner Formula

Resonance: \( \frac{d\delta}{dE} \) has sharp maximum (elastic scattering)

Breit-Wigner formula

\[
T = \frac{e^{2i\delta_b} \Gamma_{BW}/2}{M_{BW} - E - i\Gamma_{BW}/2} + B
\]

- resonance mass \( M_{BW} \), width \( \Gamma_{BW} \)
- \( \Gamma_{BW}, M_{BW}, B, \delta_b \) are \( E \)-independent constants
Extension for Multi-channel: Generalized BW formula

(Davies Baranger, McVoy)

\[ T_{\beta,\alpha} = \frac{\gamma_{BW,\beta} \gamma_{BW,\alpha}}{M_{BW} - E - i\Gamma_{BW}/2} + B_{\alpha,\beta} \]

- Require unitarity assuming E-independent parameters

\[ \gamma_{BW,\alpha} = e^{i\delta_{\alpha}} \sqrt{\Gamma_{BW,\alpha}/2} \]

\[ \Gamma_{BW,\alpha} : \text{partial width, } \Gamma_{BW} = \sum_{\alpha} \Gamma_{BW,\alpha} \]
Resonance: pole of S-matrix on unphysical sheet
elastic scattering amplitude near pole position (Laurent expansion)

\[ T = \frac{R}{M_P - E - i\Gamma_P/2} + B(E) \]

- Resonance mass \( M_P \) and width \( \Gamma_P \).
  \[ M_P = M_{BW}, \Gamma_P = \Gamma_{BW} \]
- If \( B(E) \approx B \) is a good approximation
  \[ R \to e^{2i\delta_B} \Gamma_{BW}/2 \]
- multi-channel case:
  \[ R \to \gamma_\alpha \gamma_\beta, \text{ multi-Riemann sheets structure} \]
coupling constant, form factor from residue of amplitude at pole

\[ \gamma_{em} = \langle \psi_{Res} | j_{em} | \psi_{Gr} \rangle \]

- Resonance 'wave function' : 'Eigen state' of Hamiltonian with non-hermite outgoing boundary condition. (Siegert, Dalitz)

\[ \partial \psi_{Res}/\partial r_\beta - ip_\beta \psi_{Res}|_\infty = 0 \]

- \( \gamma_\alpha \) need not be real

\[ |\psi_{Res} > = |'bound' > + |'scattering' > \]

well defined resonance parameters can be a starting point to contact with hadron models.
Extraction of resonance parameters
Mass and Width of $P_{11}$ resonance from PDG

![Graph showing Mass and Width of $P_{11}$ resonance](image-url)
Breit-Wigner parametrization

BW parameters in practice

\[ T = \frac{R(E)}{M_{BW} - E - i\Gamma(E)/2} + B(E) \]

- energy dependent \( B(E), \Gamma(E) \)
  \[ \Gamma(E) = (p/p_0)^{2l+1}\Gamma_{BW} \]
- K-matrix approach: invent recipe to match BW form
- \( M_P < M_{BW}, \Gamma_P < \Gamma_{BW} \) (Lichtenberg, Manley)
  \[ M_P \sim M_{BW} - \Gamma_{BW}/2(1/\alpha^2), \alpha = \Gamma'/2 \]
Pole and residue are automatically obtained from PWA of K-matrix (on-shell), dynamical (full off-shell dynamics) by analytic continuation of the amplitude on unphysical sheet

- **K-matrix**: use appropriate 'on-shell' momentum (Bonn-Gatchina, VPI, Giessen)

\[
T = K \frac{1}{1 - i\rho K}
\]

- **Dynamical model**: choose appropriate path of integration (EBAC, Juelich)

\[
T(p', p; E) = V(p', p) + \int_C dq q^2 V(p', q) G_0(q; E) T(q, p; E)
\]

for un-stable particle final state: need to take care of 3-body intermediate state.
Simple exercise
Stability of resonance parameters extracted from PWA

- **Input:** $T(E_i)$ from $\pi N - \pi N$ amplitude of VPI
- **Output:** $T(E)$ calculate pole and residue without physics input

Calculate $T(E)$ from known $T(E_i) i = 1, \ldots N$ (continued fraction)

$$T(E) = \frac{T(E_1)}{1 + \frac{a_1(E - E_1)}{1 + \frac{a_2(E - E_2)}{1 + \cdots}}}, \quad a_1 = \frac{T(E_1)}{E_2 - E_1} - 1, \ldots$$

(Schlessinger)
\( \pi N \) input amplitude (GW/VPI)

Output:

<table>
<thead>
<tr>
<th>Input</th>
<th>( M_P - i\Gamma_P/2 ) (MeV)</th>
<th>Residue (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-dep [1210-1500] 59pt</td>
<td>1362 -89i</td>
<td>4.9 -44i</td>
</tr>
<tr>
<td>E-dep [1300-1500] 41pt</td>
<td>1362 -90i</td>
<td>4.6 -45i</td>
</tr>
<tr>
<td>E-ind [1217-1500] 17pt</td>
<td>1366 -128i</td>
<td>10 -68i</td>
</tr>
<tr>
<td>E-ind [1289-1500] 13pt</td>
<td>1347 -93i</td>
<td>-11 -52i</td>
</tr>
</tbody>
</table>
Obtained pole position is more stable than residue. In practice, PWA is obtained within certain accuracy. Resonance parameters may not be well determined from PWA such as E-ind PWA alone.

Theoretical input (Tree diagram+ background, K-matrix, Dispersion relation, dynamical approach) is needed in extracting resonance parameters.
Residue of the Pole gives coupling constant ($D_{13}(1520)$)

$$T_{res} = \frac{\gamma_\beta \gamma_\alpha}{M_P - E - i \Gamma_P / 2}$$

<table>
<thead>
<tr>
<th>$B_\alpha$</th>
<th>$\pi N$</th>
<th>$\eta N$</th>
<th>$\pi \Delta$</th>
<th>$\sigma N$</th>
<th>$\rho N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EBAC</td>
<td>65%</td>
<td>0.02</td>
<td>33</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Manley92</td>
<td>59</td>
<td>0</td>
<td>20</td>
<td>0</td>
<td>21</td>
</tr>
<tr>
<td>Vrana00</td>
<td>63</td>
<td>0</td>
<td>26</td>
<td>1</td>
<td>9</td>
</tr>
</tbody>
</table>

$B_\alpha = |\gamma_\alpha|^2 / \Gamma_P$: (effective phase space factor for unstable particle channel $\pi \Delta$..)

EBAC

<table>
<thead>
<tr>
<th></th>
<th>$A_{3/2}$</th>
<th>$A_{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EBAC</td>
<td>125 + 25i</td>
<td>-42 + 8i</td>
</tr>
<tr>
<td>Bonn-Gatchina</td>
<td>130 + 14i</td>
<td>-30 + 8i</td>
</tr>
</tbody>
</table>

$(\text{Gev}^{-1/2} \times 10^3)$
Electromagnetic form factor

$Q^2$ Dependence of $A_{1/2}, A_{3/2}$ (D13)

[Graphs showing the dependence of $A_{1/2}$ and $A_{3/2}$ on $Q^2$ for EBAC and Jlab CLAS.]
Understanding resonance parameters
understanding resonance parameters

Example 1: $N\Delta$ electromagnetic form factor

- meson-nucleon continuum component is part of resonance property
understanding resonance parameters

Example 2: P11 resonance Roper

Pole trajectory

Analysis of EBAC

- reaction mechanism modifies resonance energy

\[ m_{P11} = 1.76 \text{GeV} \] (Dyson-Schwinger H. L. L. Roberts et al.)
Resonances are characterized by the pole and residue of the PWA reaction dynamics, which is part of the resonance properties (mass, coupling constant, and extracted resonance parameters). Resulting coupling constants are complex numbers.

PWA analysis never gives us 100% accurate amplitudes for the whole energy region. Theoretical inputs on reaction dynamics are unavoidable/necessary both in extracting PWA, extraction of resonance parameters and to understand resonance.

Combined analysis of 'reaction theory' + 'structure model of hadron' is one promising approach to understand resonance parameters.