New definition of TMD parton densities

(corrected)

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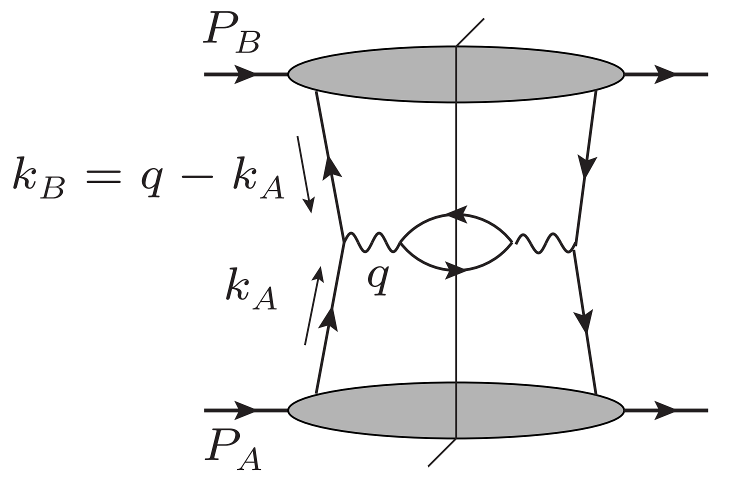
Overview

- Genesis of TMD pdfs à la parton model
- Complications for defining TMD pdfs in QCD
- New definition
- Implications

References:

JCC, "Foundations of Perturbative QCD" (Cambridge University Press, May 2011)
M. Aybat & T.C. Rogers arXiv:1101.5057

Whence TMD pdfs? Parton model for Drell-Yan



- Use cancellation of spectator-spectator interactions
- Assume other topologies unimportant
- Assume limited k_T and virtuality

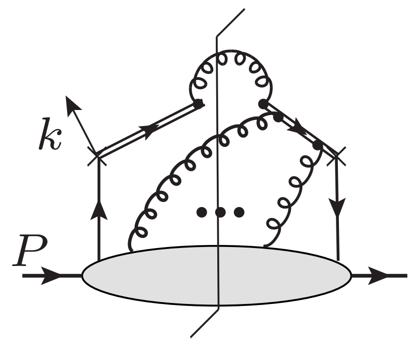
Get TMD factorization

$$\frac{d\sigma}{d^4q\,d\Omega} \stackrel{?}{=} \sum_j \int d^2\boldsymbol{k}_{A\,\mathsf{T}}\; f_{j/h_A}(x_A, \boldsymbol{k}_{A\,\mathsf{T}})\; f_{\bar{j}/h_B}(x_B, \boldsymbol{q}_{\mathsf{T}} - \boldsymbol{k}_{A\,\mathsf{T}})\; \frac{d\hat{\sigma}_{j\bar{j}}}{d\Omega}$$

Explicit definition of TMD pdf: complications in QCD

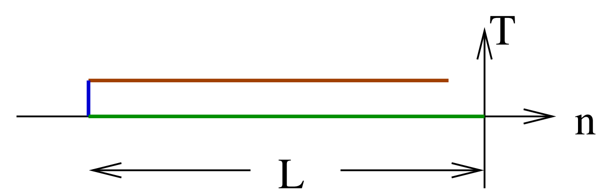
First attempt: $A^+ = 0$ gauge or use Wilson line in direction $n = (0, 1, \mathbf{0}_\mathsf{T})$:

$$f_{j/h}(\xi, \boldsymbol{k}_\mathsf{T}) \stackrel{?}{=} \int \frac{\mathrm{d}k^-}{(2\pi)^4} \operatorname{Tr} \frac{\gamma^+}{2}$$



$$= \text{F.T.} \left\langle P \right| \overline{\psi}_j(0, w^-, \boldsymbol{w}_\mathsf{T}) \, W[w,n]^\dagger \frac{\gamma^+}{2} \, [\text{tr. link}] \, W[0,n] \psi_j(0) \left| P \right\rangle_\mathsf{c}$$

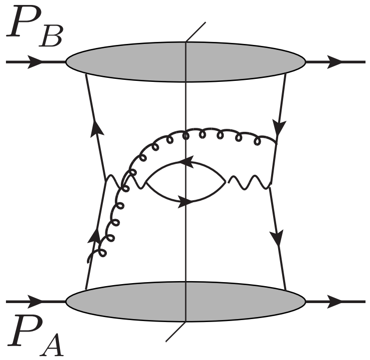
Wilson line from side: $\lim_{L\to\infty}$



Complications:

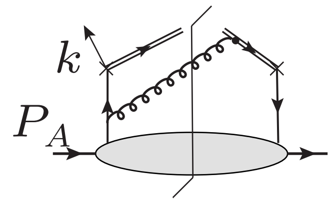
- (UV divergences)
- Rapidity divergences
- Wilson line self energies, . . .

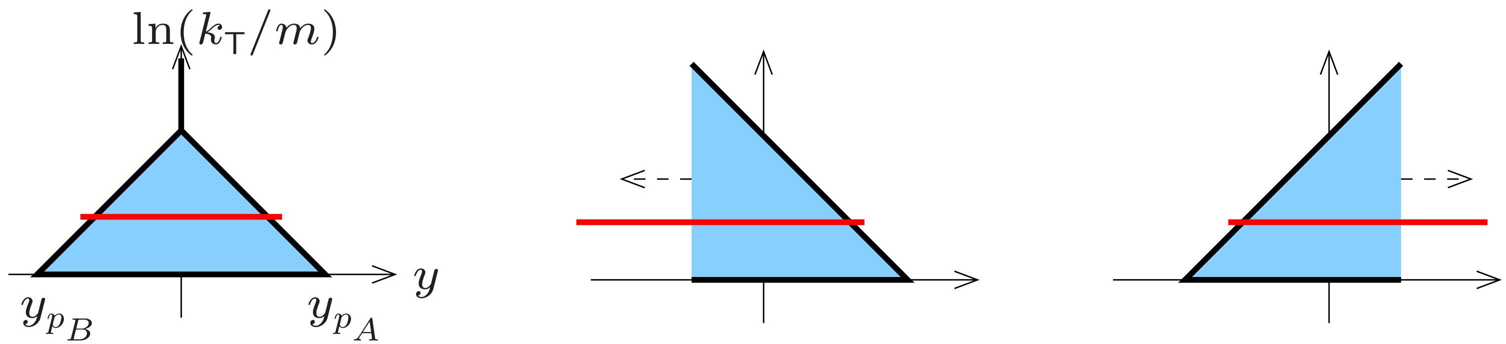
Example: Regions for one gluon



B-pdf

A-pdf:





Results:

- TMD pdf correctly gives its own collinear region (by Ward identity)
- Fails badly in opposite region
- Double counting of gluon contributions

TMD factorization in QCD

- Basic form (CSS):

  $\mathrm{d}\sigma = H \times$ convolution of ABS + high-k_T correction (Y) + power-suppressed

  – Double-counting subtractions in definitions of factors;
  – Suitable cut-offs in definitions of TMD pdfs;
  – (CSS) evolution equations for Q-dependence of A, B and S.

  But soft factor S has no independent experimental access.

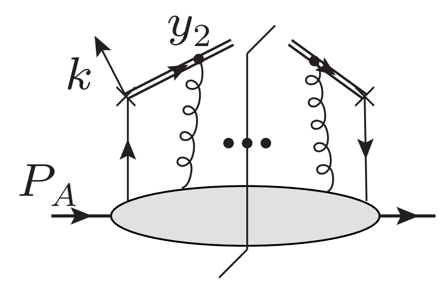
  N.B.

$$\text{Errors in} \left\{ \begin{array}{c} \text{collinear factorization} \\ \text{TMD factorization} \\ \text{TMD factorization} + Y \end{array} \right\} \text{are a power of} \left\{ \begin{array}{c} \dfrac{\Lambda}{q_\mathsf{T}} \\ \dfrac{\Lambda}{Q}, \dfrac{q_\mathsf{T}}{Q} \\ \dfrac{\Lambda}{Q} \end{array} \right.$$

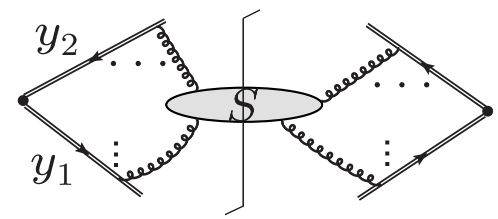
New definition of TMD pdf

Use non-light-like Wilson lines in basic unsubtracted definitions

$$\tilde{f}^{\text{unsub}}_{f/H_A}(x, \boldsymbol{b}_{\mathsf{T}}; y_{P_A} - y_2) \stackrel{\text{def}}{=} \underset{\text{color}}{\text{Tr}} \underset{\text{Dirac}}{\text{Tr}} \frac{\gamma^+}{2} \int \frac{\mathrm{d}k^- \, \mathrm{d}^{2-2\epsilon}\boldsymbol{k}_{\mathsf{T}}}{(2\pi)^n} \, e^{-i\boldsymbol{k}_{\mathsf{T}}\cdot\boldsymbol{b}_{\mathsf{T}}}$$



$$\tilde{S}(\boldsymbol{b}_{\mathsf{T}}) \stackrel{\text{def}}{=} \frac{1}{N_c} \int \frac{\mathrm{d}^{4-2\epsilon}k_S}{(2\pi)^{4-2\epsilon}} e^{-i\boldsymbol{k}_{S\,\mathsf{T}}\cdot\boldsymbol{b}_{\mathsf{T}}}$$

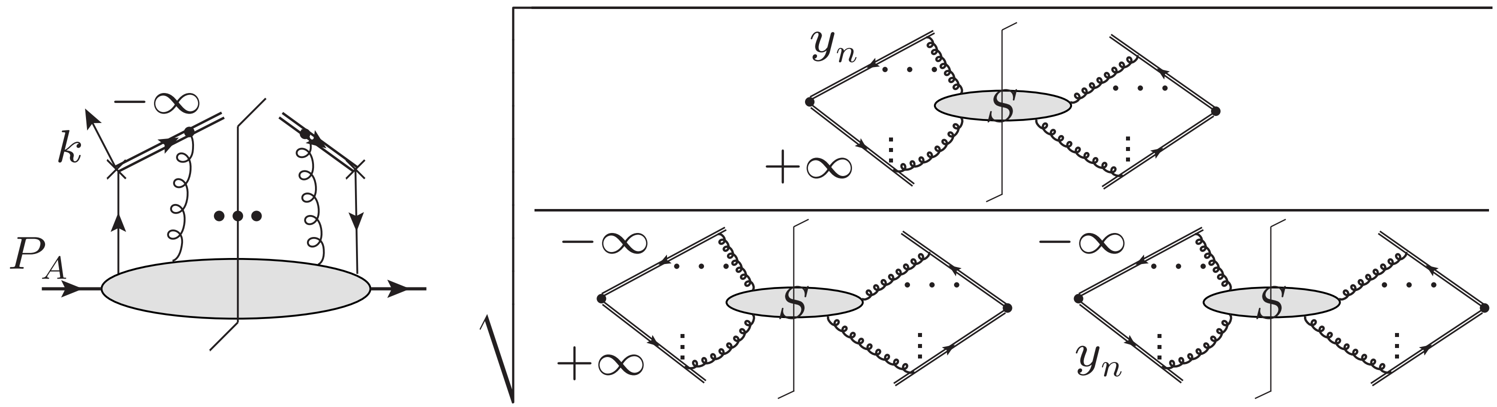


Define TMD pdf (UV renormalization implicit)

$$\tilde{f}_{f/H_A}(x, \boldsymbol{b}_{\mathsf{T}}; \zeta_A; \mu) \stackrel{\text{def}}{=} \lim_{\substack{y_1 \to +\infty \\ y_2 \to -\infty}} \tilde{f}^{\text{unsub}}_{f/H_A}(x, \boldsymbol{b}_{\mathsf{T}}; y_{P_A} - y_2) \sqrt{\frac{\tilde{S}(b_{\mathsf{T}}, y_1, y_n)}{\tilde{S}(b_{\mathsf{T}}, y_1, y_2) \; \tilde{S}_{(0)}(b_{\mathsf{T}}, y_n, y_2)}}$$

$$= \tilde{f}^{\text{unsub}}_{f/H_A}\big(x, \boldsymbol{b}_{\mathsf{T}}; y_{p_A} - (-\infty)\big) \sqrt{\frac{\tilde{S}(b_{\mathsf{T}}; +\infty, y_n)}{\tilde{S}(b_{\mathsf{T}}; +\infty, -\infty) \; \tilde{S}_{(0)}(b_{\mathsf{T}}; y_n, -\infty)}}$$

Why this strange definition?

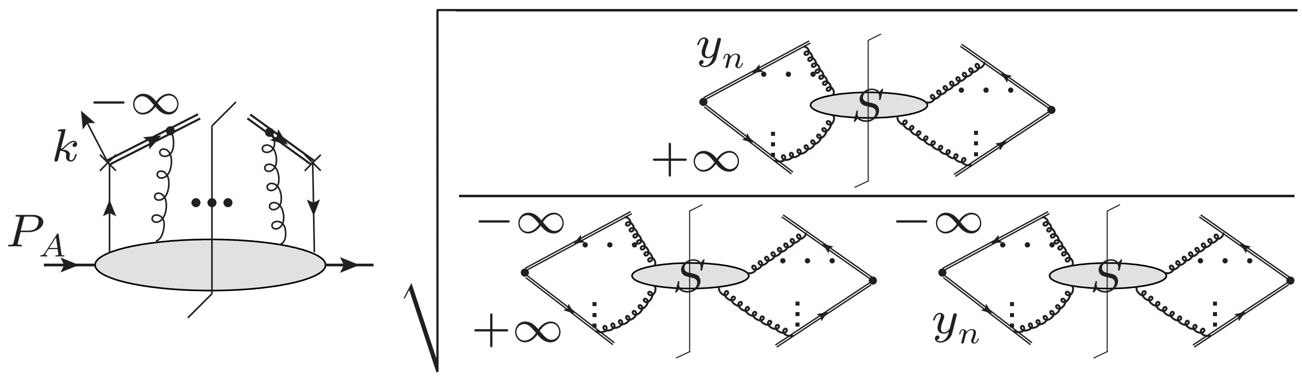


Unique (up to UV prescription) given:

- Product of basic pdf and powers of soft factor (in $\boldsymbol{b}_\mathrm{T}$ space).
- Light-like Wilson lines if possible:
  - Basic pdf has only light-like Wilson line
  - Soft factors have at most one non-light-like line
- Rapidity and $L \to \infty$ divergences cancel

N.B. Only factorization-compatible definitions should be considered.

Consequences of the strange definition



- Factorization, without soft factor
- Rapidity divergences and Wilson-line self-energy divergences cancel
- Link-at-infinity contributions cancel in Feynman gauge
- Effective cutoff on gluon rapidity at y_n
- Energy-dependence $\Longrightarrow$ y_n-dependence $\Longrightarrow$ CSS evolution, etc
- CSS evolution equations are simpler.
- They are without power-law corrections.

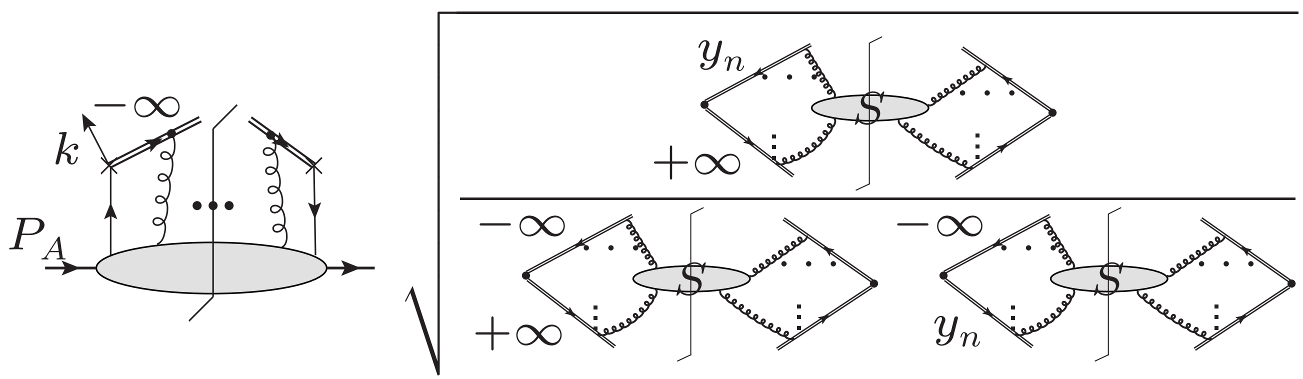
Evolution with respect to y_n

Gluons (etc) far from target rapidity are factored

Convolution in $\boldsymbol{k}_{\mathsf{T}} \Longleftrightarrow$ product in $\boldsymbol{b}_{\mathsf{T}}$

$$\frac{\partial}{\partial y_n}\tilde{f}_{f/H_A}(x,\boldsymbol{b}_{\mathsf{T}};\zeta_A;\mu)$$

$$= \frac{\partial}{\partial y_n}$$



$$= \tilde{f}_{f/H_A}(x,\boldsymbol{b}_{\mathsf{T}};\zeta_A;\mu) \times \text{kernel associated with soft factors}$$

Evolution, etc for TMD pdfs

CSS:

$$\frac{\partial \ln \tilde{f}_{f/H_A}(x, b_\mathsf{T}; \zeta; \mu)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_\mathsf{T}; \mu) \qquad \text{(with } \zeta = M^2 x^2 e^{2(y_p - y_n)} \sim Q^2\text{)}$$

RG:

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\ln\mu} = -\gamma_K(g(\mu))$$

$$\frac{\mathrm{d}\ln \tilde{f}_{f/H}(x, b_\mathsf{T}; \zeta; \mu)}{\mathrm{d}\ln\mu} = \gamma_f\big(g(\mu); \zeta/\mu^2\big)$$

$$\gamma\big(g(\mu); \zeta/\mu^2\big) = \gamma(g(\mu); 1) - \frac{1}{2}\gamma_K(g(\mu)) \ln\frac{\zeta}{\mu^2}$$

Small-b_T:

$$\tilde{f}_{f/H}(x, b_\mathsf{T}; \zeta; \mu) = \sum_j \int_{x-}^{1+} \frac{\mathrm{d}\hat{x}}{\hat{x}}\, \tilde{C}_{f/j}(x/\hat{x}, b_\mathsf{T}; \zeta, \mu, g(\mu))\; f_{j/H}(\hat{x}; \mu) + O[(m b_\mathsf{T})^p]$$

$\Longrightarrow$ See Aybat's talk for results at level of TMD pdfs

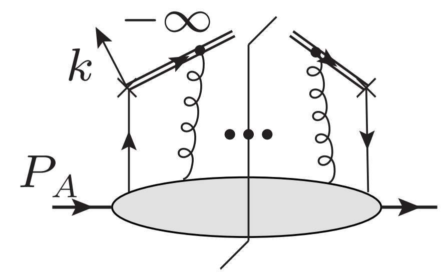
Comparisons I

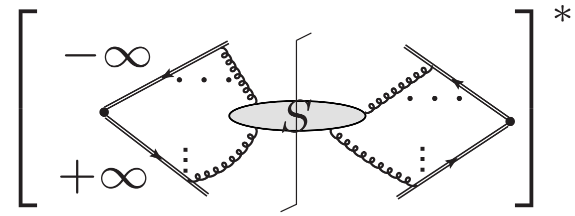
- CSS-style:
  - CSS originally used non-light-like $n \cdot A = 0$ gauge
  - $\Rightarrow$ Lack of actual proof for TMD factorization for Drell-Yan (Glauber region is difficult in "physical gauge")
  - Can convert to gauge-invariant form with non-light-like Wilson lines: space-like to get factorization
  - Separate soft factor
  - Implicit Wilson-line self-energy problems (. . .)
  - Ignored error terms in evolution, etc
  - Ji et al.: Multiple non-light-like Wilson lines
  - New method: non-trivial limit with many Wilson lines light-like
- Simple light-cone-gauge definition: rapidity divergences
- Small x/BFKL-related approaches: See Avsar's talk

Comparisons II: Cherednikov & Stefanis

- Their definition is (slightly modified from)

$$\tilde{f}^{\text{Cher.-Stef.}}_{f/H_A}(x, \boldsymbol{b}_{\mathsf{T}}) \;=$$

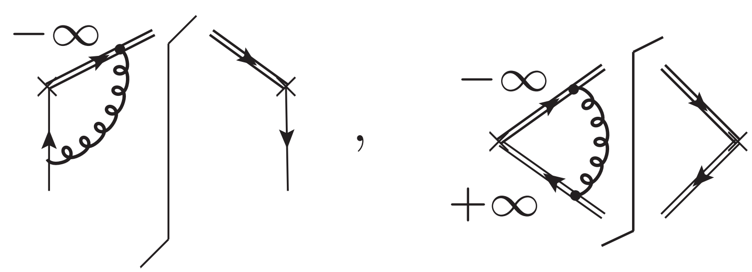




- Gauge-invariant definition, but light-cone gauge is used in calculations, with cutoff in gluon propagator

$$\frac{i}{q^2}\left[-g^{\mu\nu} + (q^\mu n^\nu + n^\mu q^\nu)\frac{1}{2}\left(\frac{1}{q^+ + i\eta} + \frac{1}{q^+ - i\eta}\right)\right]$$

- Definition has uncanceled rapidity divergences from uncalculated graphs. E.g.,



Comparisons III: SCET

- Becher & Neubert
  - Use Smirnov subtraction method in SCET style
  - Define product AB but not individual TMD pdfs
  - Rest of structure is same/close

- Mantry & Petriello
  - "Fully unintegrated pdf" $=$ "impact parameter beam function" (iBF) instead of "TMD pdf"
  - iBFs have zero bin subtractions: something like my soft factors
  - What is relation to operator definitions in QCD?

Implications of new definitions and associated derivations

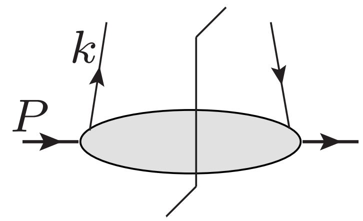
- Precise definition that can be taken literally
- Gauge links at infinity not needed in Feynman gauge
- Much better subtraction methods
- Full proof of TMD factorization for Drell-Yan

  (CSS gave up! . . .)
- Hence: unambiguous calculation of hard scattering
- Clean formalism
- Fully specified relation between exact and approximated parton kinematics.
- Need to relate to other definitions, including those in SCET
- . . .

APPENDIX

Parton model definition of TMD pdf

$$f_{j/h}(\xi, \boldsymbol{k}_\mathsf{T}) \overset{\text{p.m.}}{=} \int \frac{\mathrm{d}k^-}{(2\pi)^4}\, \mathrm{Tr}\, \frac{\gamma^+}{2}$$



$$= \int \frac{\mathrm{d}w^-\, \mathrm{d}^2\boldsymbol{w}_\mathsf{T}}{(2\pi)^3}\, e^{-i\xi P^+ w^- + i\boldsymbol{k}_\mathsf{T}\cdot\boldsymbol{w}_\mathsf{T}}\, \langle P|\, \overline{\psi}_j(0, w^-, \boldsymbol{w}_\mathsf{T})\, \frac{\gamma^+}{2}\, \psi_j(0)\, |P\rangle_\mathsf{c}$$

- Need for TMD factorization:

  Errors in $\left\{ \begin{array}{c} \text{collinear} \\ \text{TMD} \end{array} \right\}$ factorization are a power of $\left\{ \begin{array}{c} \Lambda/q_\mathsf{T} \\ \Lambda/Q \end{array} \right.$

- Importance of a definition as an operator matrix element:
  - Knowing what we're talking about
  - Unambiguous prescription for use in perturbative calculations
  - Unambiguous specification for non-perturbative calculations/analysis

Three views of factorization

1. Main TMD factorization formula
2. After evolution of TMD functions to fixed scales
3. Maximum perturbative content

Main factorization formula

$$W^{\mu\nu} = \frac{8\pi^2 s}{Q^2} \sum_f H_f^{\mu\nu}(\hat{k}_A, \hat{k}_B) \int \mathrm{d}^2\boldsymbol{b}_\mathsf{T}\, e^{i\boldsymbol{q}_{h\,\mathsf{T}}\cdot\boldsymbol{b}_\mathsf{T}} \tilde{f}_{f/H_A}(x_A, \boldsymbol{b}_\mathsf{T}; \zeta_A; \mu)\, \tilde{f}_{\bar{f}/H_B}(x_B, \boldsymbol{b}_\mathsf{T}; \zeta_B; \mu)$$

$+$ polarized terms $+$ large $q_{h\,\mathsf{T}}$ correction, Y.

$+$ power suppressed

where

$$x_A = \frac{Q e^{y}}{\sqrt{s}}, \quad x_B = \frac{Q e^{-y}}{\sqrt{s}},$$

$$\zeta_A = M_A^2 x_A^2 e^{2(y_{P_A} - y_n)}, \quad \zeta_B = M_B^2 x_B^2 e^{2(y_n - y_{P_B})},$$

Implement Y by

$W = F(q_\mathsf{T})$low-q_T term $+$ Collinear factorization on $(W - F(q_\mathsf{T})$low-q_T term$)$

$+$ power suppressed

Evolution, etc for TMD pdfs

CSS:

$$\frac{\partial \ln \tilde{f}_{f/H_A}(x, b_\mathsf{T}; \zeta; \mu)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_\mathsf{T}; \mu).$$

RG:

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\ln \mu} = -\gamma_K(g(\mu))\,.$$

$$\frac{\mathrm{d}\ln \tilde{f}_{f/H}(x, b_\mathsf{T}; \zeta; \mu)}{\mathrm{d}\ln \mu} = \gamma_f\big(g(\mu); \zeta/\mu^2\big).$$

$$\gamma\big(g(\mu); \zeta/\mu^2\big) = \gamma(g(\mu); 1) - \frac{1}{2}\gamma_K(g(\mu)) \ln \frac{\zeta}{\mu^2}.$$

Small-b_T:

$$\tilde{f}_{f/H}(x, b_\mathsf{T}; \zeta; \mu) = \sum_j \int_{x-}^{1+} \frac{\mathrm{d}\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_\mathsf{T}; \zeta, \mu, g(\mu)) \; f_{j/H}(\hat{x}; \mu) + O[(m b_\mathsf{T})^p]$$

Factorization with fixed TMD pdfs

$$W^{\mu\nu} = \frac{8\pi^2 s}{Q^2} \sum_f H_f^{\mu\nu}(\hat{k}_A, \hat{k}_B) \int \mathrm{d}^2\boldsymbol{b}_\mathsf{T}\, e^{i\boldsymbol{q}_{h\,\mathsf{T}}\cdot\boldsymbol{b}_\mathsf{T}} e^{-S(b_\mathsf{T};Q;\mu_Q,\mu_0)} \times$$

$$\times\, \tilde{f}_{f/H_A}(x_A, \boldsymbol{b}_\mathsf{T}; m^2, \mu_0)\, \tilde{f}_{\bar{f}/H_B}(x_B, \boldsymbol{b}_\mathsf{T}; m^2, \mu_0)$$

$$+ \text{ polarized terms} + \text{large } q_{h\,\mathsf{T}} \text{ correction, } Y + \text{p.s.c.}$$

$$e^{-S(b_\mathsf{T};Q;\mu_Q,\mu_0)} = \exp\left\{ \ln\frac{Q^2}{m^2}\tilde{K}(b_\mathsf{T};\mu_0) \right\} \times$$

$$\times \exp\left\{ \int_{\mu_0}^{\mu_Q} \frac{\mathrm{d}\mu'}{\mu'} \left[2\gamma(g(\mu');1) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(g(\mu')) \right] \right\}$$

Factorization with maximum perturbative content

$$
\begin{aligned}
W^{\mu\nu} = \frac{8\pi^2 s}{Q^2} \sum_{f,j_A,j_B} & H_f^{\mu\nu}(Q; g(\mu_Q), \mu_Q) \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{-i\boldsymbol{q}_{h\,T}\cdot\boldsymbol{b}_T} \\
& \times \int_{x_A}^{1} \frac{d\hat{x}_A}{\hat{x}_A} f_{j_A/H_A}(\hat{x}_A; \mu_b)\ \tilde{C}_{f/j_A}\left(\frac{x_A}{\hat{x}_A}, b_*; \mu_b^2, \mu_b, g(\mu_b)\right) \\
& \times \int_{x_B}^{1} \frac{d\hat{x}_B}{\hat{x}_B} f_{j_B/H_B}(\hat{x}_B; \mu_b)\ \tilde{C}_{\bar{f}/j_B}\left(\frac{x_B}{\hat{x}_B}, b_*; \mu_b^2, \mu_b, g(\mu_b)\right) \\
& \times \exp\left[-g_{f/H_A}(x_A, b_T) - g_{\bar{f}/H_B}(x_B, b_T)\right] \\
& \times \exp\left[-\ln\frac{Q^2}{m^2} g_K(b_T) + \ln\frac{Q^2}{\mu_b^2}\tilde{K}(b_*; \mu_b)\right] \\
& \times \exp\left\{\int_{\mu_b}^{\mu_Q} \frac{d\mu'}{\mu'}\left[2\gamma(g(\mu'); 1) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(g(\mu'))\right]\right\}
\end{aligned}
$$

$+$ polarized terms $+$ large-$q_{h\,T}$ correction, Y $+$ p.s.c.

where: $\mathbf{b}_* = \boldsymbol{b}_T/\sqrt{1 + b_T^2/b_{\max}^2}$, $\mu_b = C_1/b_*(b_T)$, $\mu_Q = C_2 Q$