Uses of $Q^2$ evolution in GPD phenomenology

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- GPD definitions and one trivial remark on $Q^2$ evolution
- Uses of conformal symmetry
- Modeling GPDs at the initial scale and their $Q^2$ evolution
- Is evolution needed to describe present and future hard exclusive photon and meson electroproduction data?

based on collaborations with

A. Belitsky (98-01)
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Field theoretical GPD definition

GPDs are defined as matrix elements of renormalized light-ray operators:

\[ F(x, \eta, \Delta^2, \mu^2) = \int_{-\infty}^{\infty} d\kappa \, e^{i\kappa \cdot x \cdot P} \langle P_2 | \mathcal{R} T : \phi(-\kappa n)[(-\kappa n), (\kappa n)] \phi(\kappa n) : | P_1 \rangle, \, n^2 = 0 \]

momentum fraction \( x \), skewness \( \eta = \frac{n \cdot \Delta}{n \cdot P} \) \( \Delta = P_2 - P_1 \) \( P = P_1 + P_2 \) \( \Delta^2 \equiv t \)

For a nucleon target we have four chiral even twist-two GPDs:

\[ \bar{\psi}_i \gamma_+ \psi_i \Rightarrow i_q^V = \bar{U}(P_2, S_2) \gamma_+ U(P_1, S_1) H_i + \bar{U}(P_2, S_2) \frac{i\sigma + \nu \Delta^\nu}{2M} U(P_1, S_1) E_i \]

\[ \bar{\psi}_i \gamma_+ \gamma_5 \psi_i \Rightarrow i_q^A = \bar{U}(P_2, S_2) \gamma_+ \gamma_5 U(P_1, S_1) \tilde{H}_i + \bar{U}(P_2, S_2) \frac{\gamma_5}{2M} U(P_1, S_1) \tilde{E}_i \]

**shorthands:**

chiral even GPDs: \( F = \{ H, E, \tilde{H}, \tilde{E} \} \) & CFFs: \( \mathcal{F} = \{ \mathcal{H}, \mathcal{E}, \tilde{\mathcal{H}}, \tilde{\mathcal{E}} \} \)

chiral odd GPDs: \( F_T = \{ H_T, E_T, \tilde{H}_T, \tilde{E}_T \} \) \( \mathcal{F}_T = \{ \mathcal{H}_T, \mathcal{E}_T, \tilde{\mathcal{H}}_T, \tilde{\mathcal{E}}_T \} \)
"two-body" operators possess apart from self-energy insertions no singularities (in a generic scalar theory)

\[ \mathcal{O}(x, y) = z_\phi T : \phi^{\bar{\text{bar}}}(y) \phi^{\bar{\text{bar}}}(x) : (x - y)^2 \neq 0 \]

usually a minimal subtraction (MS) scheme is used, e.g.

\[ z_\phi = 1 + \frac{1}{\epsilon} \left( \frac{\alpha_s}{2\pi} z_\phi^{1, (0)} + O(\alpha_s^2) \right) + \frac{1}{\epsilon^2} O(\alpha_s^2) + \cdots \]

scale dependence is governed by anomalous dimensions

\[ \left[ \mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial g} \right] \mathcal{O}(x, y) = -2\gamma_\phi \mathcal{O}(x, y), \quad \gamma_\phi = -\frac{1}{2} g \frac{\partial}{\partial g} z_\phi^{(1)} \]

leading twist operators on the light cone possess logarithmic singularities

\[ \left[ \mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial g} \right] \mathcal{O}(n, -n) = -\int d\kappa_1 \int d\kappa_2 \gamma(\kappa_1, \kappa_2) \mathcal{O}(\kappa_1 n, \kappa_2 n), \ n^2 = 0 \]
Conformal operator basis

irreducible representations:

\[ \Phi_{j,l} = \partial^l \Phi_j(0) , \quad j = (d + s)/2 \]

\[ [j_1] \otimes [j_2] = \bigoplus_{n \geq 0} [j_n] , \quad j_n = j_1 + j_2 + n. \]

\[ O^{j,j}_{n,l} \propto \partial_+^{n+l} \left[ \Phi_j C_n^{(2j-1)} \left( \frac{\delta^l - \delta^l}{\partial_+ + \partial_+} \right) \Phi_j \right] \]

✓ conformal symmetry is preserved at tree-level

► diagonal LO anomalous dimensions [Ohrndorf 82, DM 91]

! conformal symmetry is broken by the trace anomaly in \( d=4-2\varepsilon \) dimensions

► apart from \( \beta \)-proportional term it is also broken by the renormalization scheme

✓ conformal renormalization scheme exist so that the breaking appears only due to the \( \beta \) proportional trace anomaly in \( d=4 \) dimensions [DM (97)]

➢ anomalous dimensions and DVCS hard-scattering part @NLO [Belitsky, DM (98)]

➢ constructing all 12 twist-two NLO evolution kernels [Belitsky, DM, Freund (00)]
  (two explicit calculated NLO kernels [Radyushkin et al (~85); Mikhailov, Vladimirov (09)])
conformal PW expansion of DAs

Conformal symmetry in LO pQCD suggest Gegenbauer expansion

\[ \phi_M(v, Q^2) = f_M \sum_{n=0}^{\infty} 6(1 - v)vC_n^{3/2}(2v - 1) E_n(Q, Q_0) a_n(Q_0^2) \]

(eigenfunction of the LO evolution operator)

- LO evolution equation is trivially solved

\[ E_n(Q, Q_0) = \left( \frac{\ln(Q^2/\Lambda_{QCD}^2)}{\ln(Q_0^2/\Lambda_{QCD}^2)} \right)^{-\gamma_n^{(0)}/\beta_0} \]
\[ \gamma_n^{(0)} = \frac{4}{3} \left( 4S_{n+1} - \frac{2}{(n + 1)(n + 2)} - 3 \right) \]

- inverse moment enters in LO descriptions of form factors

\[ \mathcal{I}_M(Q^2) = \frac{1}{3f_M} \int_0^1 du \frac{\phi_M(u, Q^2)}{u} = \sum_{n=0}^{\infty} E_n(Q, Q_0) a_n(Q_0^2) \]
Effective model for DAs

three conformal moments, two free parameters

\[ a_0 = 1 \text{ (fixed by normalization), } a_2, a_4, \text{ for } \mu^2 = Q_0^2 \]

suppose we have a “measurement”, there is still freedom left

\[ \mathcal{I}_M(Q^2 = Q_0^2) = 1 + a_2 + a_4, \text{ fixed by data} \]

\[ \mathcal{I}_M(Q^2 \to \infty) = 1, \text{ asymptotic limit is slowly reached} \]

suppose \[ \mathcal{I}_M(Q^2 = Q_0^2) = 1: \text{ Can one practically pin down such a model?} \]
a GPD can be expanded with respect to conformal partial waves of the
collinear conformal group SO(2,1) (similar to SO(3) expansion)

expansion in terms of discrete conformal spin \( j+2 \) for
\[ \eta > 1, \ |x/\eta| \leq 1 \]

\[
F(x, \eta, t) = \sum_{j=0}^{\infty} (-1)^j p_j(x, \eta) F_j(\eta, t)
\]

conformal moments (partial wave amplitudes) are polynomials:
\[
F_j(x, \eta) = \frac{\Gamma(3/2)\Gamma(1+j)}{2^j\Gamma(3/2+j)} \int_{-1}^{1} dx \eta^{j+1} C_j^{3/2} \left( \frac{x}{\eta} \right) F(x, \eta, t)
\]

conformal partial waves ensure the polynomiality condition:
\[
p_j(x, \eta) = \frac{\Gamma(5/2+j)}{j!\Gamma(1/2)\Gamma(2+j)} \frac{d^j}{dx^j} \int_{-1}^{1} du (1 - u^2)^{j+1} \delta(x - u\eta)
\]

\textbf{crossing symmetry} allows for a more convenient representation
(technicality, e.g., Sommerfeld-Watson transform, numerous failures in the literature)

\textbf{partial waves evolve autonomously} \quad \textbf{trivial implementation of evolution}
Summing up conformal PWs

• GPD support is a consequence of Poincaré invariance (polynomiality)

\[ H_j(\eta, t, \mu^2) = \int_{-1}^{1} dx \, c_j(x, \eta) H(x, \eta, t, \mu^2) , \quad c_j(x, \eta) = \eta^j C_j^{3/2}(x/\eta) \]

• conformal moments evolve autonomous (to LO and beyond in a special scheme)

\[ \mu \frac{d}{d\mu} H_j(\eta, t, \mu^2) = -\frac{\alpha_s(\mu)}{2\pi} \gamma_j^{(0)} H_j(\eta, t, \mu^2) \]

• inverse relation is given as series of mathematical distributions:

\[ H(x, \eta, t) = \sum_{j=0}^{\infty} (-1)^j p_j(x, \eta) H_j(\eta, t) , \quad p_j(x, \eta) \propto \theta(|x| \leq \eta) \frac{\eta^2 - x^2}{\eta^{j+3}} C_j^{3/2}(-x/\eta) \]

• various ways of resummation were proposed:
  • smearing method [Radyushkin (97); Geyer, Belitsky, DM., Niedermeier, Schäfer (97/99)]
  • mapping to a kind of forward PDFs [A. Shuvaev (99), J. Noritzsch (00)]
  • dual parameterization [M. Polyakov, A. Shuvaev (02)]
  • based on conformal light-ray operators [Balitsky, Braun (89); Kivel, Mankewicz (99)]
  • **Mellin-Barnes integral** [DM, Schäfer (05); A. Manashov, M. Kirch, A. Schäfer (05)]
Sommerfeld-Watson transform

✓ rewrite sum as an integral around the real axis:

\[ F(x, \eta, \Delta^2) = \frac{1}{2i} \int_{(0)}^{(\infty)} dj \frac{1}{\sin(\pi j)} p_j(x, \eta) F_j(\eta, \Delta^2) \]

✓ find appropriate analytic continuation of \( p_j \) and \( F_j \) (Carlson’s theorem)

\[ p_j(x, \eta) = \theta(\eta - |x|) \eta^{-j-1} \mathcal{P}_j \left( \frac{x}{\eta} \right) + \theta(x - \eta) \eta^{-j-1} \mathcal{Q}_j \left( \frac{x}{\eta} \right) \]

\[ \mathcal{P}_j(x) = \frac{2^{j+1} \Gamma(5/2 + j)}{\Gamma(1/2) \Gamma(1 + j)} (1 + x) \ _2F_1 \left( \begin{array}{c} -j - 1, j + 2 \\ 2 \end{array} \right| \frac{1 + x}{2} \right) \]

\[ \mathcal{Q}_j(x) = -\frac{\sin(\pi j)}{\pi} x^{-j-1} \ _2F_1 \left( \begin{array}{c} (j + 1)/2, (j + 2)/2 \\ 5/2 + j \end{array} \right| \frac{1}{x^2} \right) \]

✓ change integration path so that singularities remain on the l.h.s.

\[ F(x, \eta, \Delta^2) = \frac{i}{2} \int_{c-i\infty}^{c+i\infty} dj \frac{1}{\sin(\pi j)} p_j(x, \eta) F_j(\eta, \Delta^2) \]
Advantages of the Mellin-Barnes integral

- another possibility to parameterize GPDs [similar to the dual parameterization] (basic properties are implemented, essential for flexible fitting routines)

- (LO) solution of the evolution equation is trivial implemented

\[
F(x, \eta, \Delta^2, Q^2) = \frac{i}{2} \int_{c-i\infty}^{c+i\infty} dj \frac{p_j(x, \eta)}{\sin(\pi j)} \exp \left\{ -\frac{\gamma_j^{(0)}}{2} \int_{Q^2_0}^{Q^2} \frac{d\sigma}{\sigma} \frac{\alpha_s(\sigma)}{2\pi} \right\} F_j(\eta, \Delta^2, Q^0_2)
\]

- fast and robust numerical evaluation

- simple representation of amplitudes

\[
\mathcal{F}(\xi, \Delta^2, Q^2) = \int_{-1}^{1} dx \left[ \frac{e^2}{\xi - x - i\epsilon} \mp \frac{e^2}{\xi + x - i\epsilon} \right] F(x, \xi, \Delta^2, Q^2)
\]

\[
\mathcal{F} = \frac{e^2}{2i} \int_{c-i\infty}^{c+i\infty} dj \xi^{-j-1} \frac{2j+1}{\Gamma(3/2)\Gamma(3+j)} \left( i - \frac{\cos(\pi j) \mp 1}{\sin(\pi j)} \right) F_j(\xi, \Delta^2, Q^2)
\]

- MS factorization conventions can be implemented at NLO

- CS factorization conventions enable us to explore NNLO corrections
What is `dual'' GPD parameterization?

- t-channel scattering angle and skewness parameter are related: \( \cos \theta \approx -1/\eta \)
- labeling the conformal moments by the t-channel **angular momentum** \( J \) (conjugated variable to \( \theta \) or in some sense to \( \eta \))

\[
F_j(\eta, t) = \eta^{j+1} \sum_{J=J_{\text{min}}}^{j+1} f_{j,J}(t) \, d_J(1/\eta)
\]

Partial wave amplitudes depending on \( j \) and \( J \)
- reduced Wigner rotation matrices

- primary `quantum numbers' are \( j+2 \) and the difference \( \nu = j+1-J \)
- in `dual'' parameterization \( j+2 \) is replaced by conjugate momentum fraction \( z \)

\[
F(x, \eta, t) = \sum_{\nu=0}^{\infty} \int_0^1 d\nu \, K_\nu(x, \eta|z)Q_\nu(z, t)
\]

- GPD model building in terms of \( f_{j,j+1-\nu}(t) \) or \( Q_\nu(z,t) \) (one-to-one to DDs)

`dual'' parameterization [Guzey, Teckentrup (06)] effectively took \( \nu=0 \) [Polyakov (07)]
A flexible GPD model

• take three effective SO(3) partial waves
  \[ F_j(\eta, t) = \hat{d}_j(\eta) f_j^{j+1}(t) + \eta^2 \hat{d}_{j-2}(\eta) f_j^{j-1}(t) + \eta^4 \hat{d}_{j-4}(\eta) f_j^{j-3}(t), \quad j \geq 4 \]
  \[ f_j^{j-k}(\eta, t) = s_k f_j^{j+1}(\eta, t), \quad k = 2, 4, \ldots \]

• rewrite Mellin-Barnes integral
  \[ \mathcal{F} = \frac{1}{2i} \sum_{k=0}^{4} \int_{c-i\infty}^{c+i\infty} \xi^{-j-1} \frac{2^{j+1+k} \Gamma(5/2 + j + k)}{\Gamma(3/2) \Gamma(3 + j + k)} \left( i - \frac{\cos(\pi j)}{\sin(\pi j)} \right) \]
  \[ \times s_k E_{j+k}(Q^2) f_j^{j+1}(t) \hat{d}_j(\xi), \quad s_0 = 1 \]

NOTE:

➢ first partial wave amplitude is fixed by PDFs (if they exist) and FFs
➢ “Regge poles” should be in the angular momentum J-plane (not in the j-plane)

\[ H(x, x, t = 0, Q^2) \xrightarrow{x \to 0} \sum_{k=0}^{4} s_k \frac{2^{\alpha+k} \Gamma(3/2 + \alpha + k)}{\Gamma(3/2) \Gamma(2 + \alpha + k)} q(x, Q^2) \]

➢ a J-pole is associated with a series of spurious poles in the j-plane
Is the conformal ratio supported?

associating “Regge poles” with the $j$-plane yields "erroneous small $x$-claim" that GPDs are “tied” to PDFs:

$$ r = \frac{H(x,x,t=0,Q^2)}{q(x,Q^2)} $$

by the conformal (Shuvaev) ratio:

$$ r_{\text{con}} = \frac{2^\alpha \Gamma(3/2+\alpha)}{\Gamma(3/2)\Gamma(2+\alpha)} $$

counter example (non-singlet case)

[Martin, Ryskin, Shuvaev et al.]

meson-like DA for $J=1$ ($t$-channel)

asymptotic GDA

skewness ratio $r(Q^2)$

conformal ratio

$Q^2 \text{ [GeV}^2\text{]}$
Modeling & evolution in x-space

• “Dispersion relation” can be used at twist-two level:

\[
\Re e F(\xi, t, Q^2) = \frac{1}{\pi} \text{PV} \int_0^1 d\xi' \left( \frac{1}{\xi - \xi'} \mp \frac{1}{\xi + \xi'} \right) \Im e F(\xi', t, Q^2) + C(t, Q^2)
\]

\[
\frac{1}{\pi} \Im e F(\xi = x, t, Q^2) \overset{\text{LO}}{=} F(x, x, t, Q^2) \mp F(-x, x, t, Q^2)
\]

• outer region governs the evolution at the cross-over trajectory

\[
\mu^2 \frac{d}{d\mu^2} H(x, x, t, \mu^2) = \int_x^1 \frac{dy}{x} V(1, x/y, \alpha_s(\mu)) H(y, x, \mu^2)
\]

GPD at \( \eta = x \) is `measurable‘ (LO)

net contribution of outer + central region is governed by a sum rule:

\[
\text{PV} \int_0^1 dx \frac{2x}{\eta^2 - x^2} H^-(x, \eta, t) = \text{PV} \int_0^1 dx \frac{2x}{\eta^2 - x^2} H^-(x, x, t) + C(t)
\]
good DVCS fits to H1 and ZEUS data at LO, NLO, and NNLO with flexible GPD ansatz
large $Q^2$ lever arm and the “pomeron” pole in the gluonic sector allow to ask for gluon contributions in DVCS at small $x$

conformal ratio excluded at LO or in a DVCS scheme

transverse distribution in impact parameter space
“pomeron pole” related NLO and NNLO corrections

- drastically reduction of perturbative corrections at NNLO for the hard part
- reduction of renormalization scale dependence
- but perturbative predictions for the evolution is unstable
- no improvement of factorization scale dependence
Evolution is not needed to analyze fixed target DVCS data (HERMES, JLAB) uses of “dispersion relation” approach (modeling accessible degrees of freedom).

- Fits to HALL A harmonics are fine for unexpected large $\hat{H}$ or $\hat{E}$ contribution.
- Large $\hat{H}$ KM09 scenario is excluded from longitudinal TSA (HERMES, CLAS).

HALL-A data: neglected at all ratios of moments cross sections.
Can one use evolution to pin down valence GPDs in a future EIC measurement?

all four models are compatible with present DVCAS data (HALL A excluded)

differences due to evolution

it will be a challenge to discriminate between models
Summary

- pQCD formalism for hard exclusive production is available at NLO
  - for DVCS even at NNLO in a specific subtraction scheme
  - pQCD@NLO will be needed for a global analysis of photon and meson data

- NLO evolution kernels where obtained from the understanding that conformal symmetry is broken by the normalization conditions
  - for $\beta=0$ restoration of conformal symmetry is possible in any order
  - formally proved from conformal algebra and Ward identities

- evolution operator in the flavor singlet and parity even sector becomes unstable in the small $x$-region
  - fortunately, this is a universal feature

- a high luminosity machine with dedicated experiments is desired
  - to resolve the transverse degrees of freedom
  - within the discussed EIC it might be possible to employ evolution effects to explore GPDs apart from the cross-over line