

# Excited states in QCD

Robert Edwards

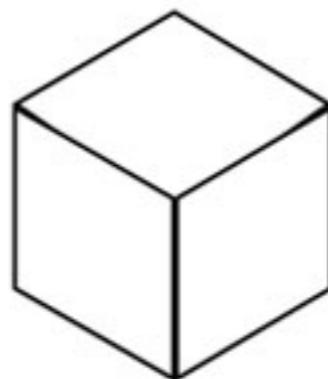


# Finite volume QCD & the hadron spectrum

- Compute correlation functions as an average over field configurations

e.g.  $\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu \bar{\psi} \Gamma \psi(t) \bar{\psi} \Gamma \psi(0) e^{-\int d^4x \mathcal{L}_{\text{QCD}}(\psi, \bar{\psi}, A_\mu)}$

‘sum’      ‘field correlation’    ‘probability weight’



*Field integration within a finite, but continuous, hypercube  
Need some kind of ultraviolet regulator....*

- Spectrum from two-point correlation functions

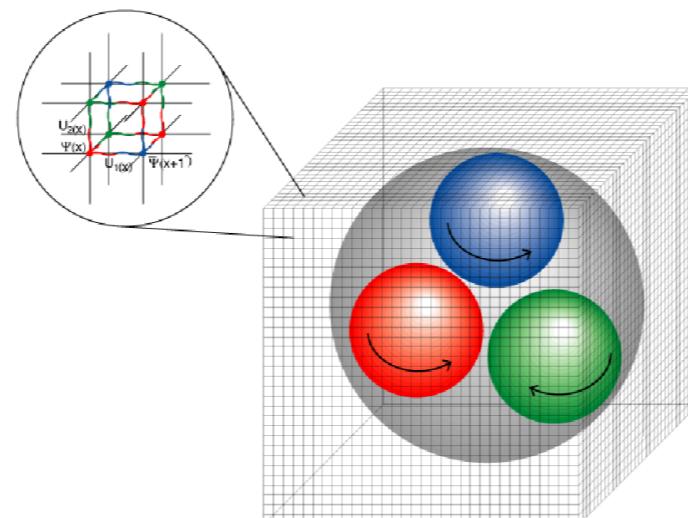
$$\begin{aligned} C(t) &= \langle 0 | \mathcal{O}(t) \mathcal{O}^\dagger(0) | 0 \rangle \\ &= \sum_{\mathbf{n}} e^{-E(\mathbf{n})t} \langle 0 | \mathcal{O}(0) | \mathbf{n} \rangle \langle \mathbf{n} | \mathcal{O}^\dagger(0) | 0 \rangle \end{aligned}$$

# Lattice QCD & the hadron spectrum

- Compute correlation functions as a Monte Carlo average over field configurations

e.g.  $\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu \bar{\psi} \Gamma \psi(t) \bar{\psi} \Gamma \psi(0) e^{-\int d^4x \mathcal{L}_{\text{QCD}}(\psi, \bar{\psi}, A_\mu)}$

‘sum’      ‘field correlation’    ‘probability weight’



*Discretize the action over sites*

*Serves as an ultraviolet regulator*

- Spectrum from two-point correlation functions

$$\begin{aligned} C(t) &= \langle 0 | \mathcal{O}(t) \mathcal{O}^\dagger(0) | 0 \rangle \\ &= \sum_{\mathbf{n}} e^{-E(\mathbf{n})t} \langle 0 | \mathcal{O}(0) | \mathbf{n} \rangle \langle \mathbf{n} | \mathcal{O}^\dagger(0) | 0 \rangle \end{aligned}$$

# Excited states from correlators

- how to get at excited QCD eigenstates ?

- optimal operator for state  $|\mathfrak{n}\rangle$  :  $\Omega_{\mathfrak{n}}^\dagger \sim \sum_i v_i^{(\mathfrak{n})} \mathcal{O}_i^\dagger$

for a basis of  
meson operators  $\{\mathcal{O}_i\}$

- can be obtained (in a variational sense) from the matrix of correlators

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$

- by solving a generalized eigenvalue problem

$$C(t)v^{(\mathfrak{n})} = C(t_0)v^{(\mathfrak{n})} \lambda_{\mathfrak{n}}(t)$$

‘diagonalize the  
correlation matrix’

eigenvalues

$$\lambda_{\mathfrak{n}}(t) \sim e^{-E_{\mathfrak{n}}(t-t_0)}$$

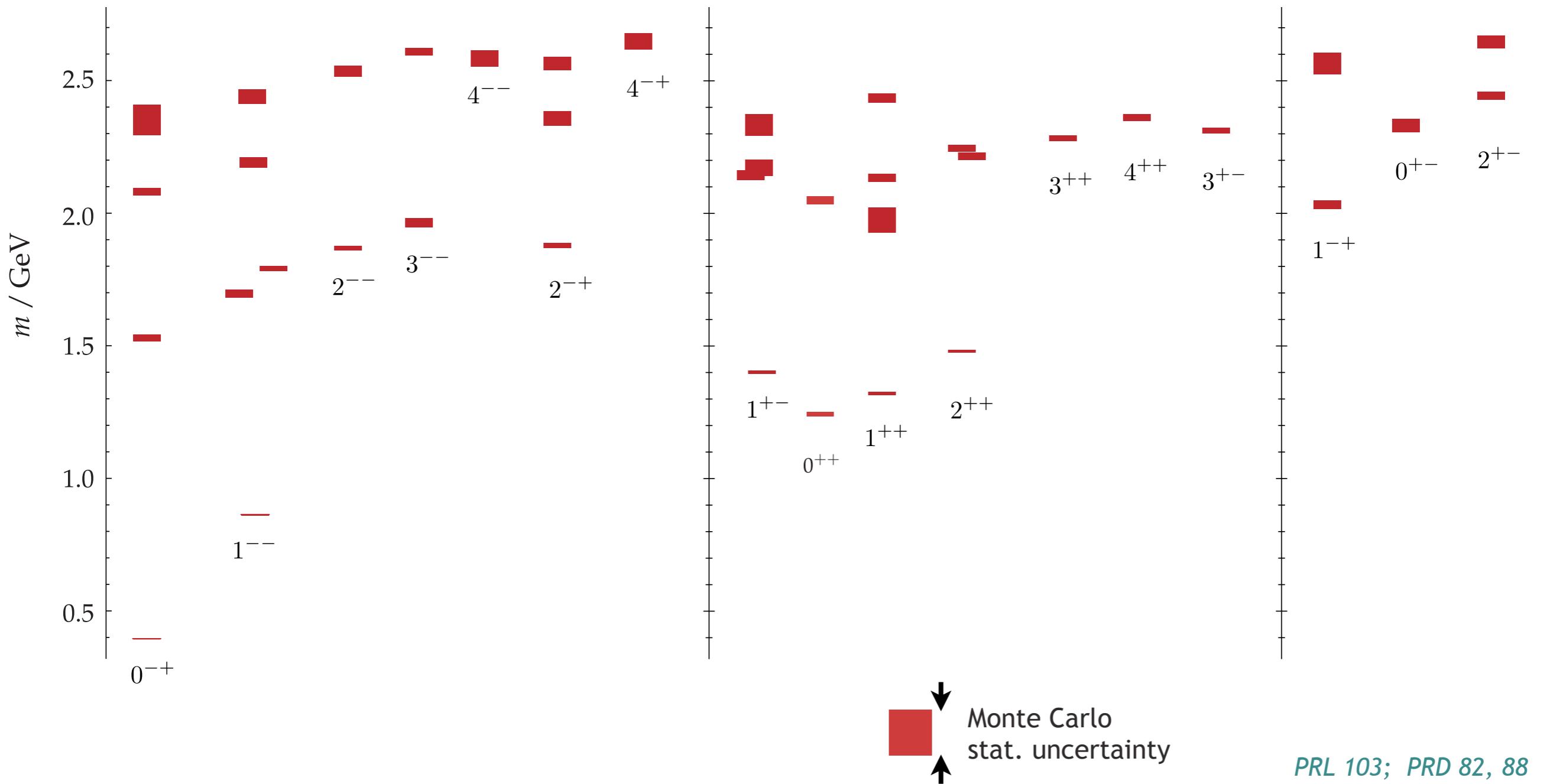
- a large basis can be constructed using covariant derivatives :

$$\mathcal{O} \sim \bar{\psi} \Gamma \overleftrightarrow{D} \dots \overleftrightarrow{D} \psi$$

# Glimpse of meson spectrum from lattice QCD

- Appears to be some  $q\bar{q}$ -like near-degeneracy patterns - isovectors

$m_\pi \sim 391$  MeV

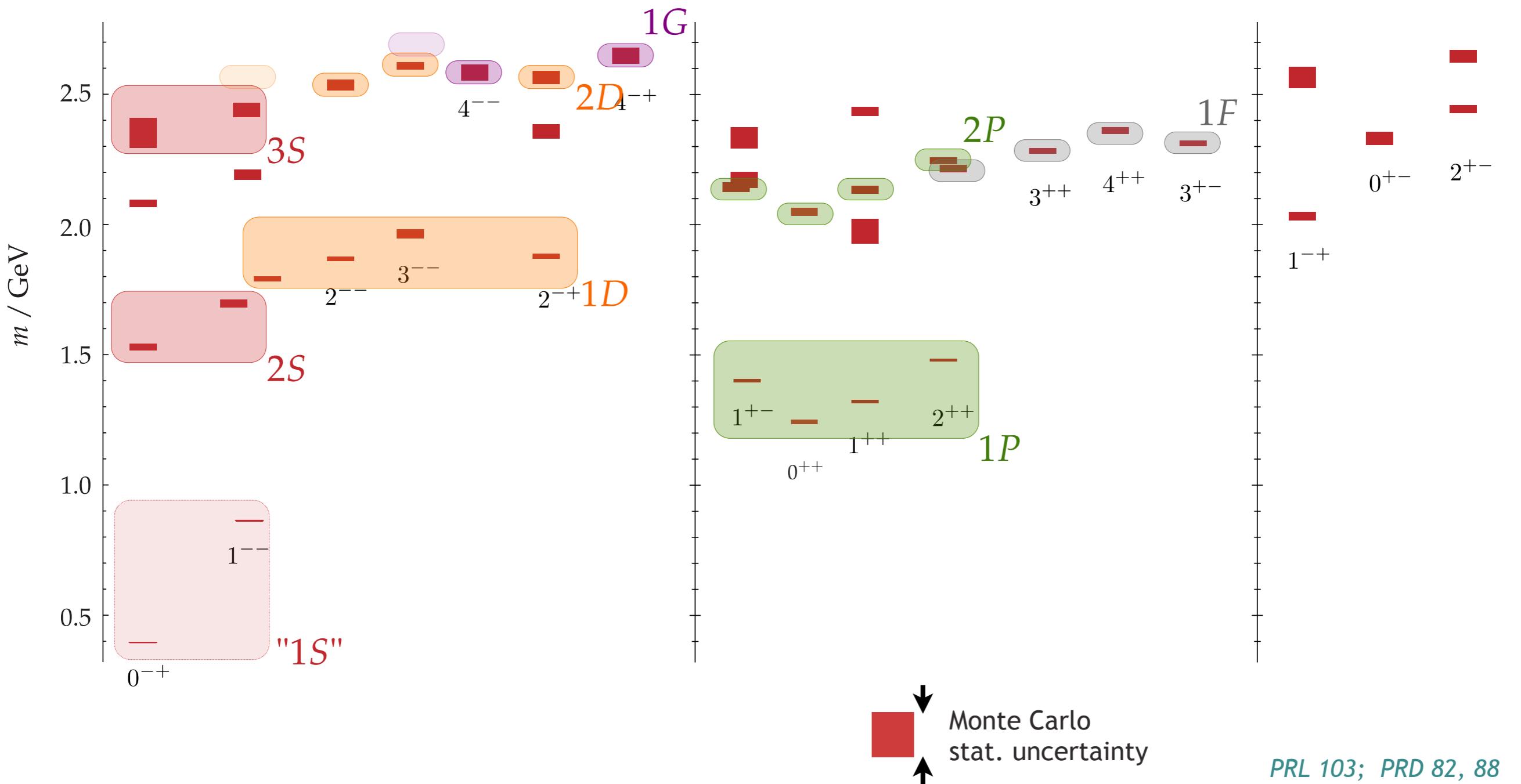


PRL 103; PRD 82, 88

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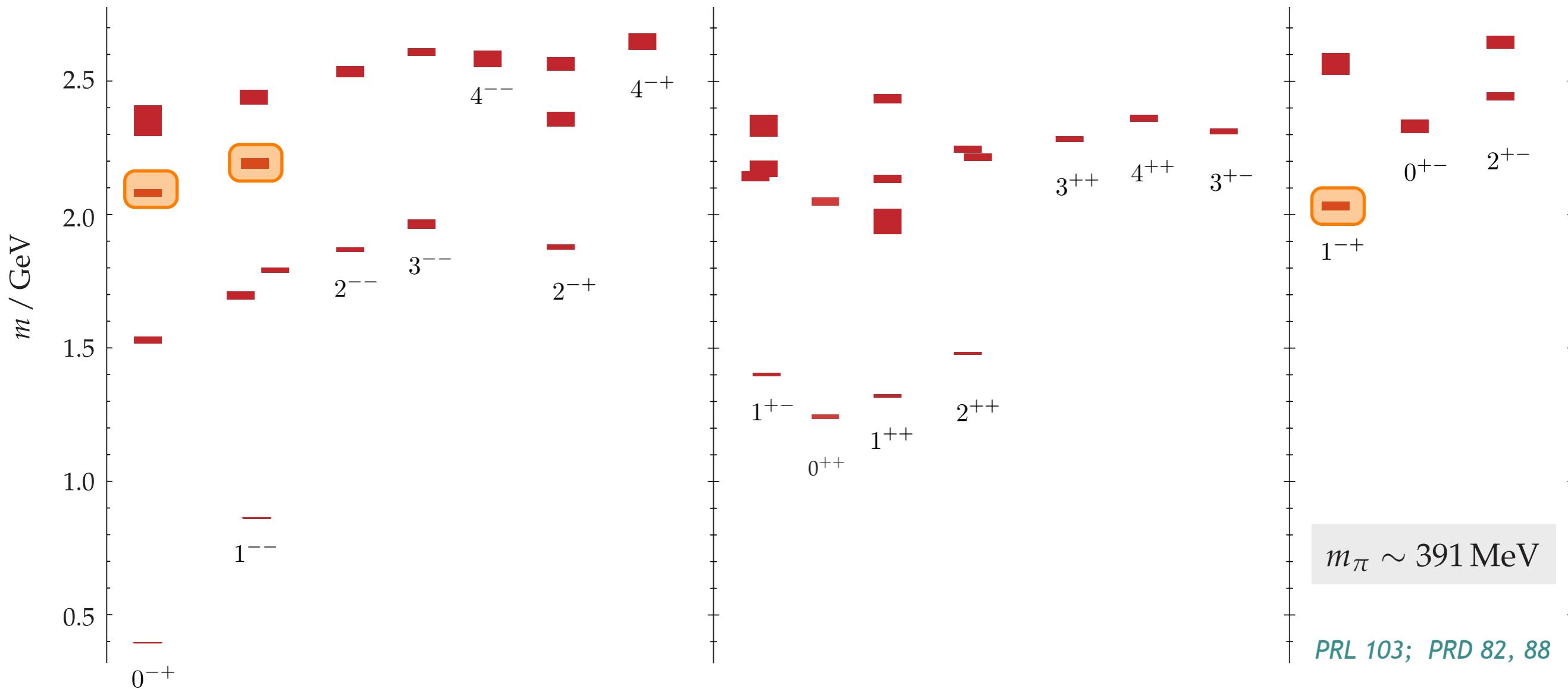


PRL 103; PRD 82, 88

# Glimpse of meson spectrum from lattice QCD

- ‘super’-multiplet of **hybrid mesons** roughly 1.2 GeV above the  $\rho$

$(0, 1, 2)^{-+}, 1^{--}$

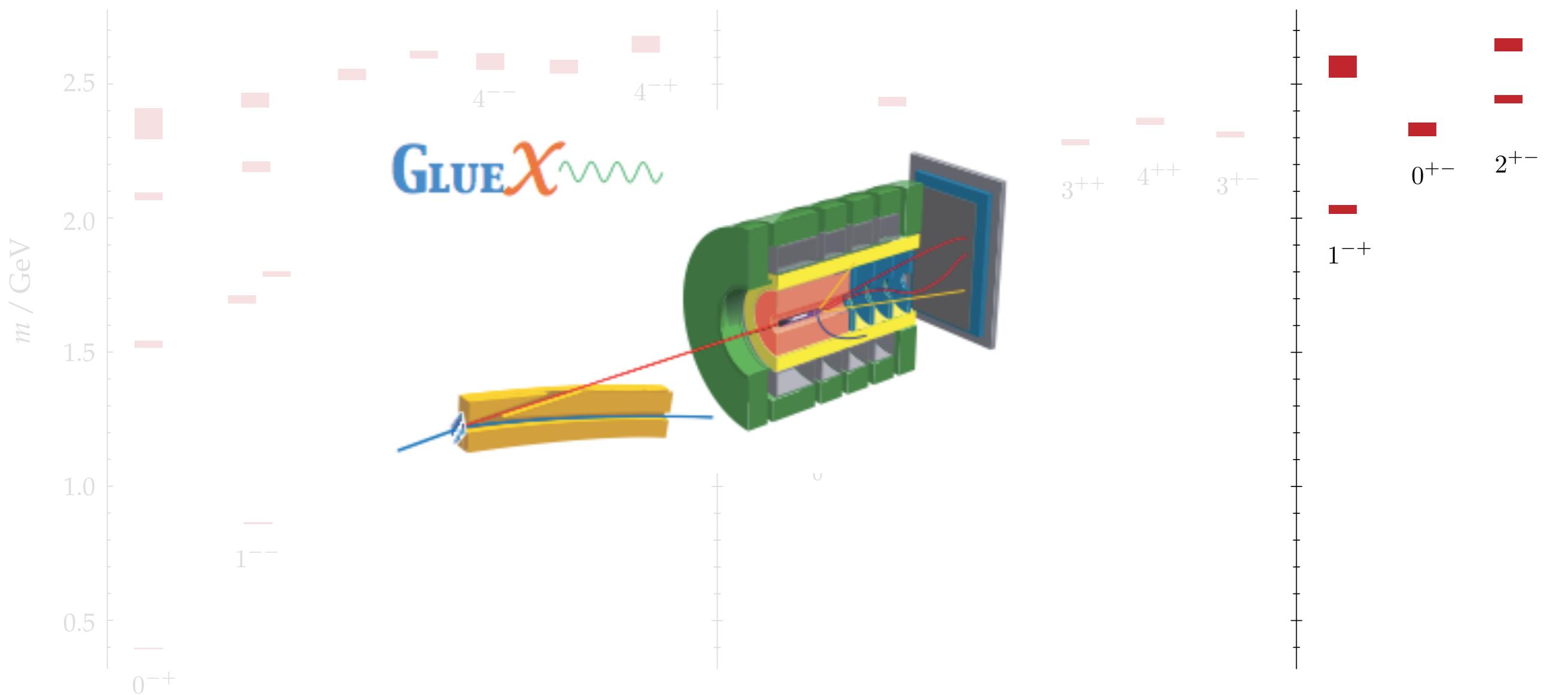


- these states have a dominant overlap onto  $\bar{\psi}\Gamma[D, D]\psi \sim [q\bar{q}]_{8_c} \otimes B_{8_c}$

# Glimpse of meson spectrum from lattice QCD

Multiple exotic mesons within range of GlueX

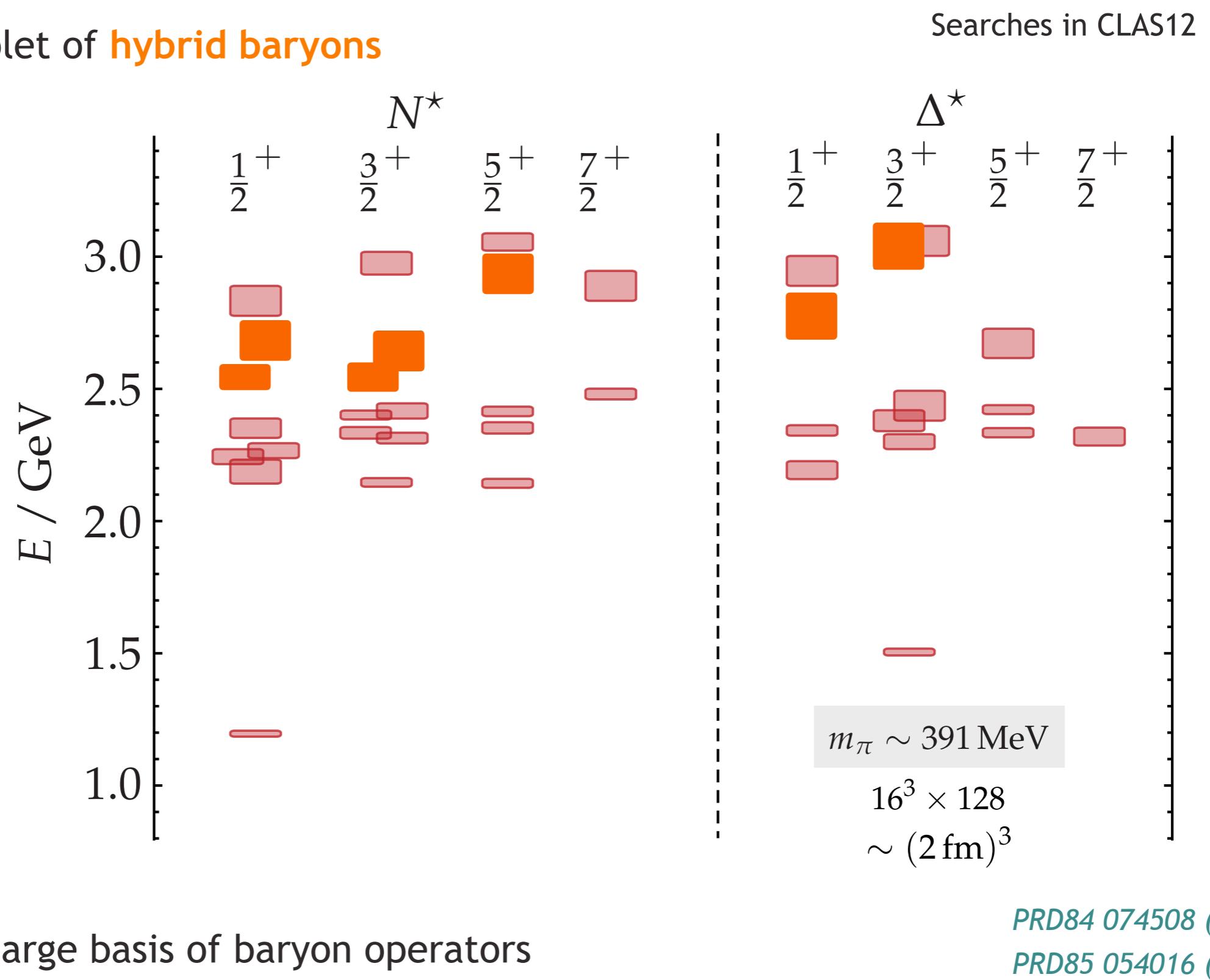
EXOTIC MESONS



PRL 103; PRD 82, 88

# Excited light quark baryons

- A ‘super’-multiplet of **hybrid baryons**

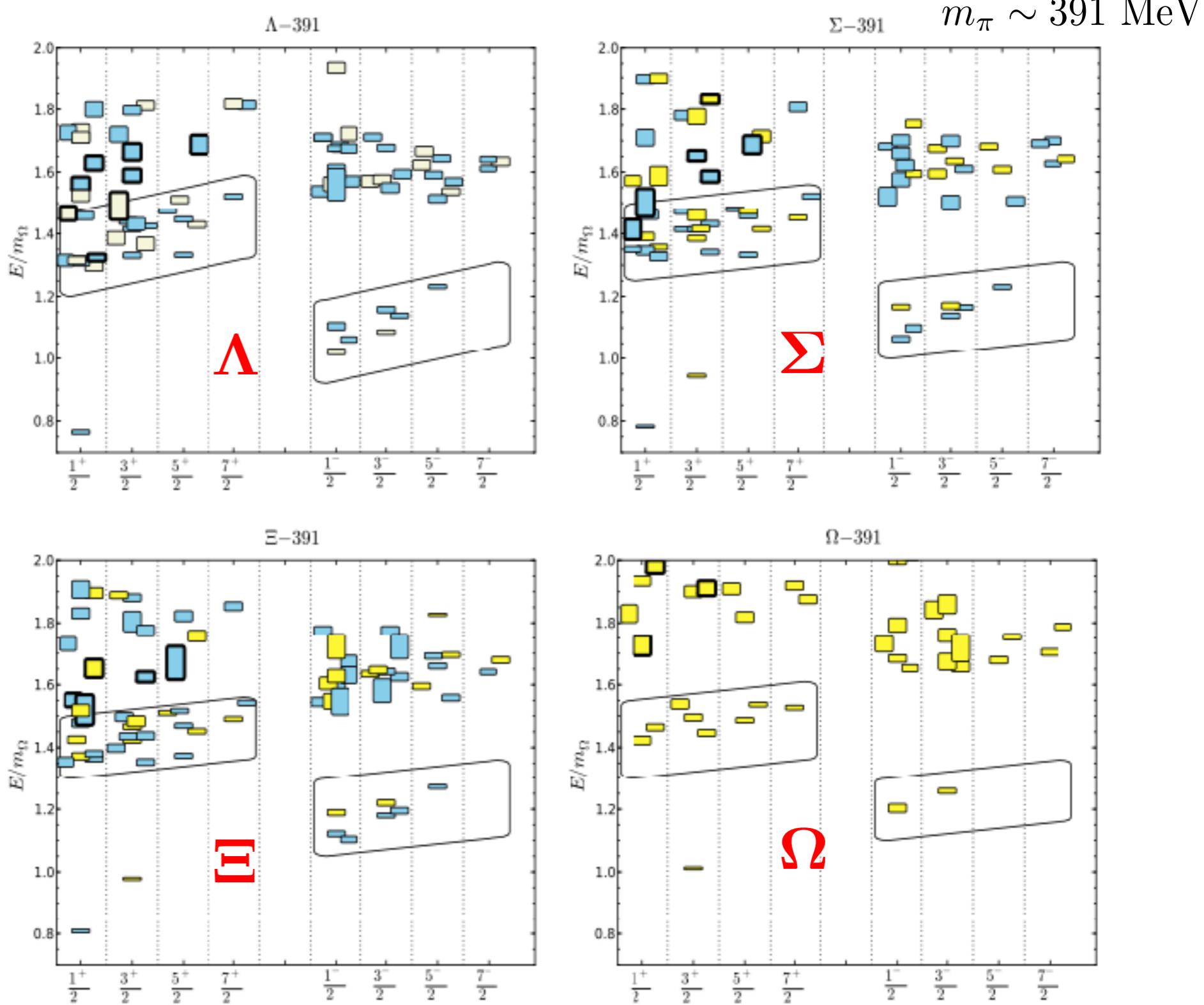


# Excited strange (and charm) quark baryons

Light quarks - SU(3)  
flavor broken

Full non-relativistic  
quark model counting

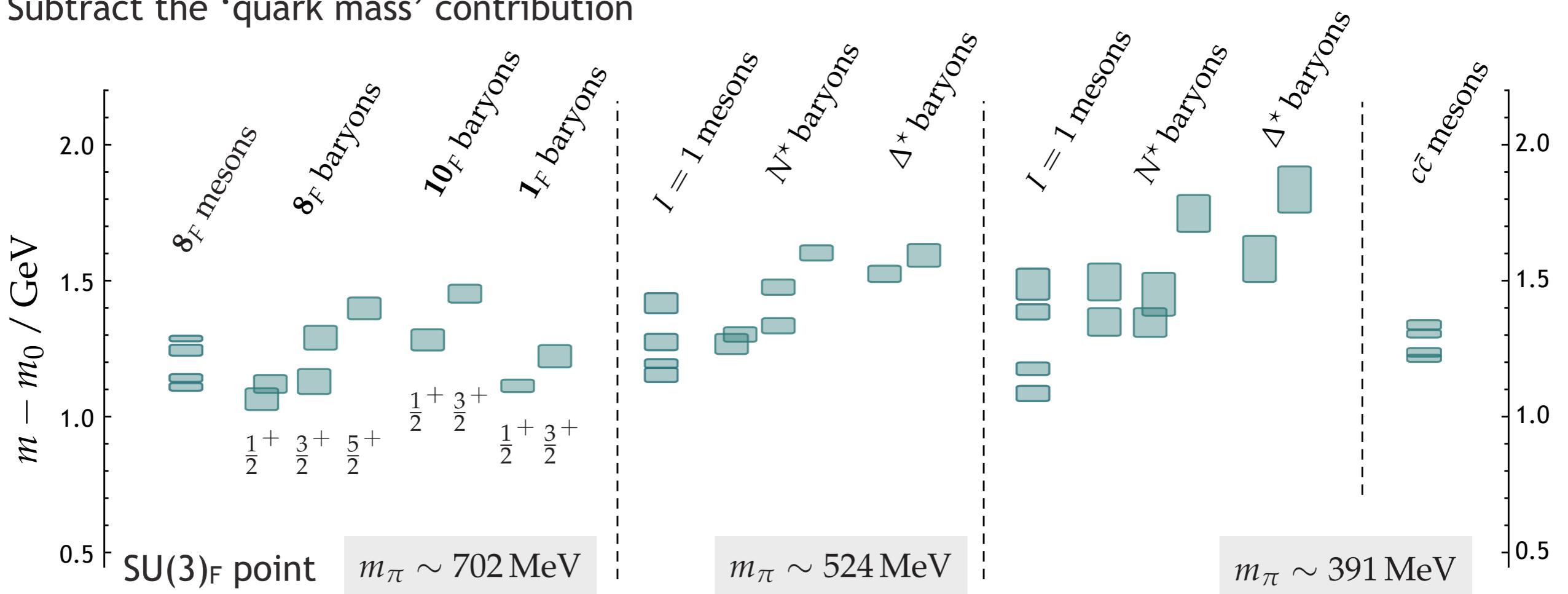
Some mixing of SU(3)  
flavor irreps



PRD87 054506 (2013)  
PRD90 074504 (2014)  
PRD91 054502 (2015)

# Chromo-magnetic excitation

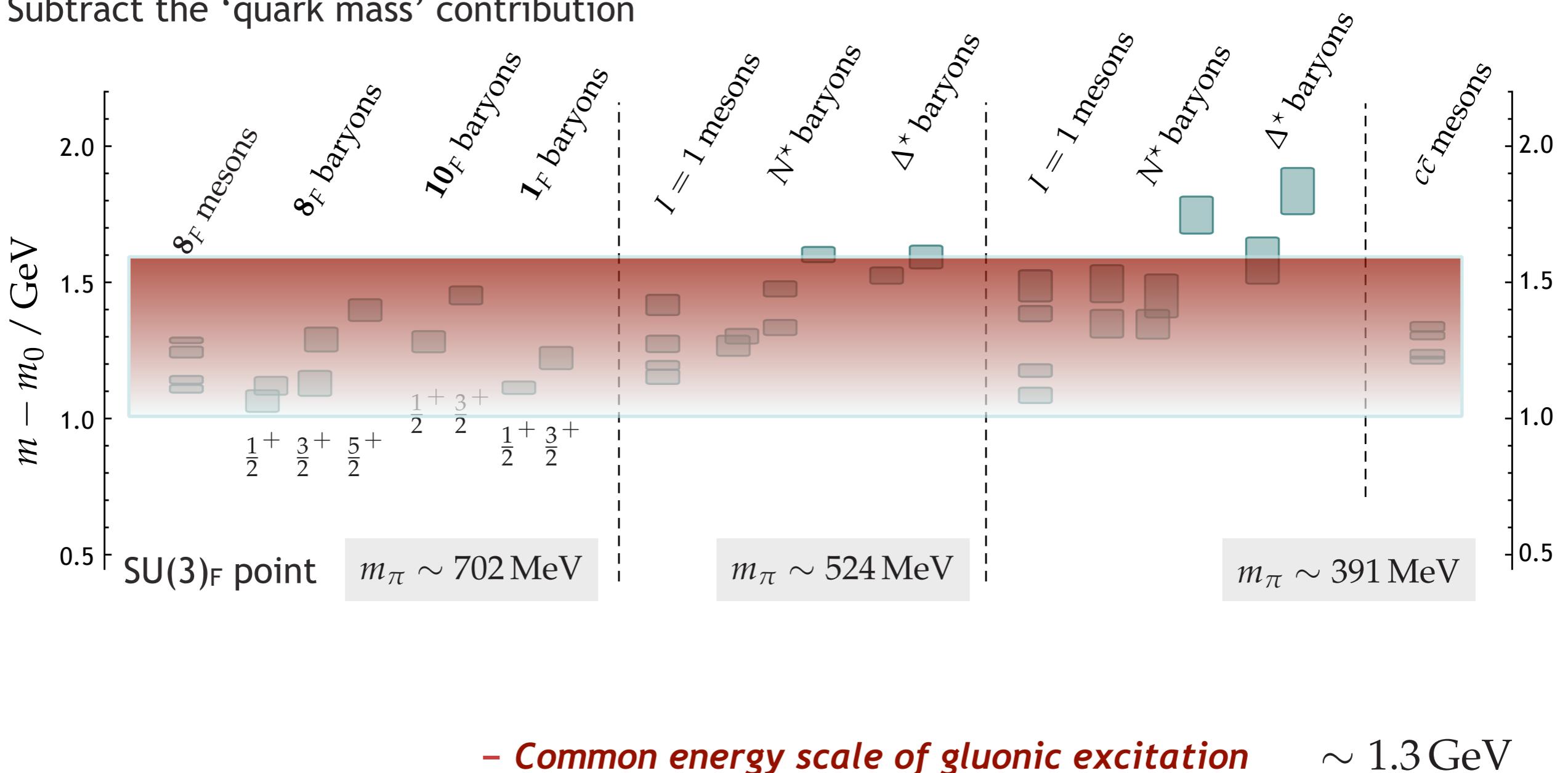
- Subtract the ‘quark mass’ contribution



HADRON SPECTRUM: PRD83 (2011); PRD88 (2013)

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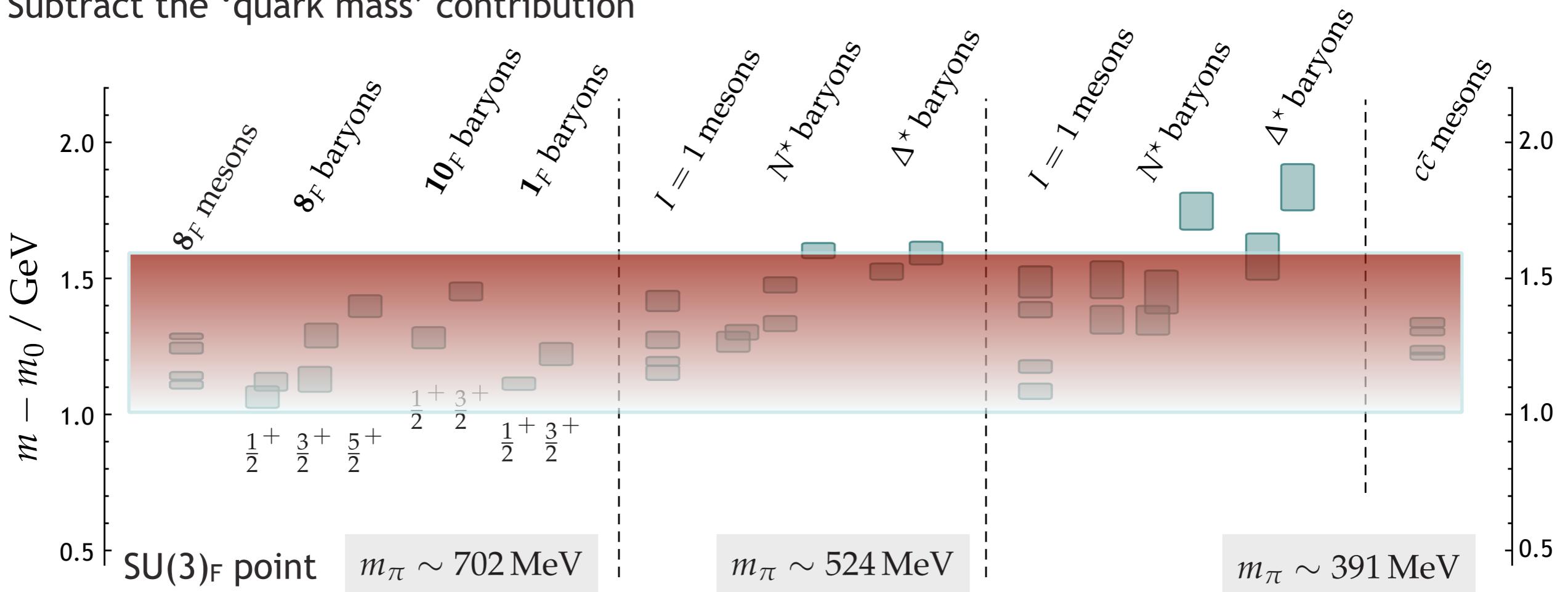
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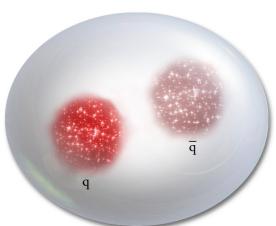
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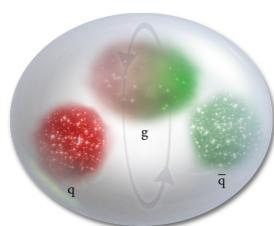
Pattern of states suggest  
gluonic excitations

– *Common energy scale of gluonic excitation*

$\sim 1.3\text{ GeV}$



Conventional Meson

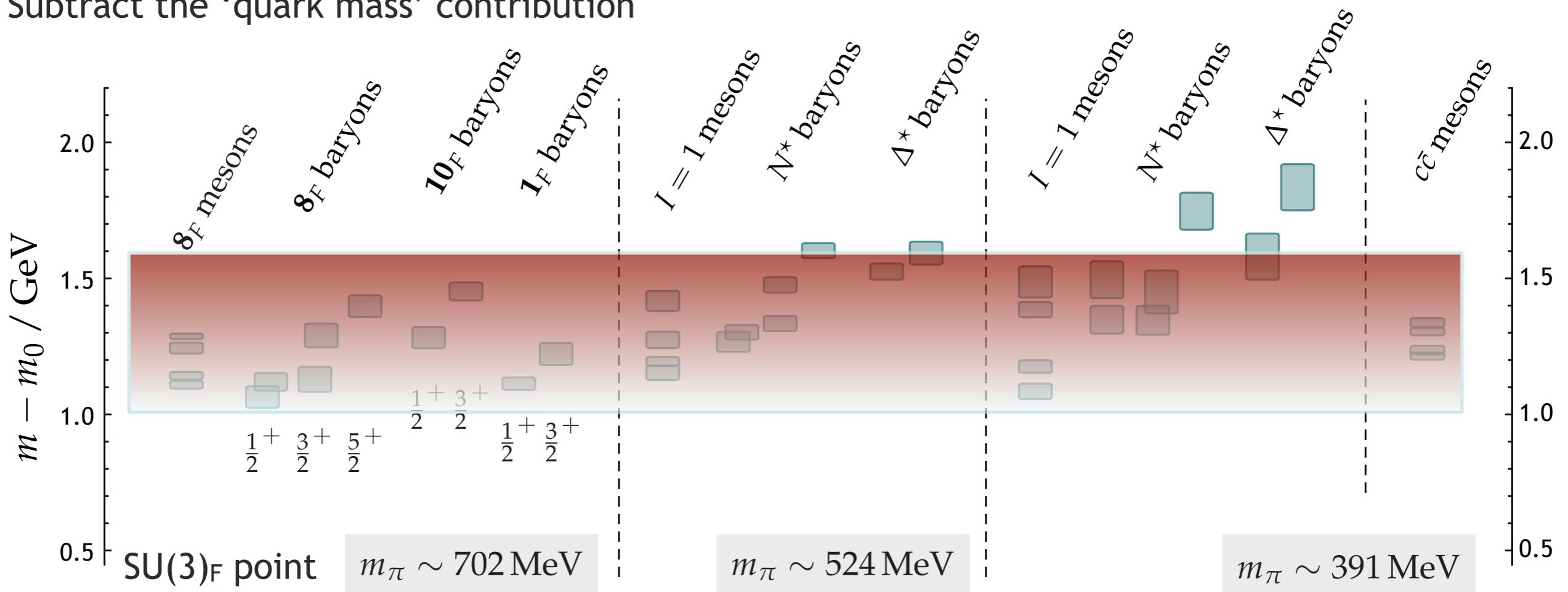


Hybrid Meson

HADRON SPECTRUM: PRD83 (2011); PRD88 (2013)

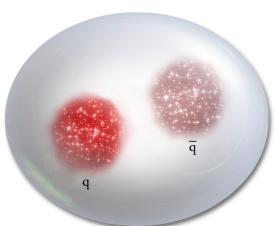
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- Subtract the ‘quark mass’ contribution

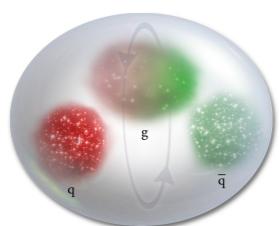


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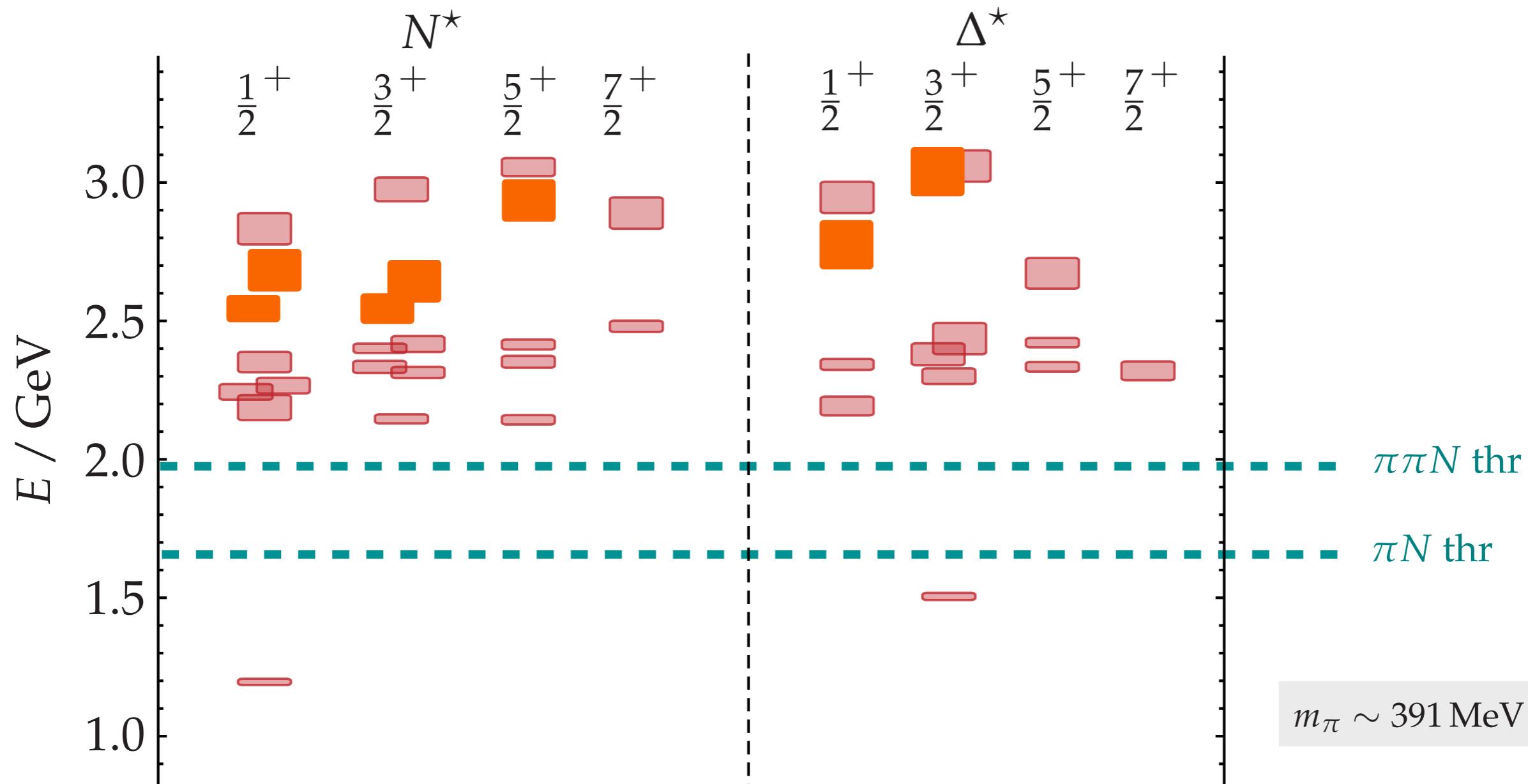
Hybrid Meson

→ Need to know decay modes and rates to compare to expt.

HADRON SPECTRUM: PRD83 (2011); PRD88 (2013)

# Excited states are resonances

- Initial determination of spectrum with only  $qqq$  style operators  
→ missing scattering states



- Some initial results in S11 & P33 have appeared (Graz group)

# What pion mass?

---

- Getting to the physical pion mass **not the most pressing concern here**
- Need to establish feasibility of techniques for resonances

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  - Hard to do with physical kinematics
    - e.g. Some of the simple low-lying resonances:  $f_2(1270)$      $M > 8m_\pi$
    - $a_2(1320)$      $M > 9m_\pi$

the number of open channels  
is too large to start here

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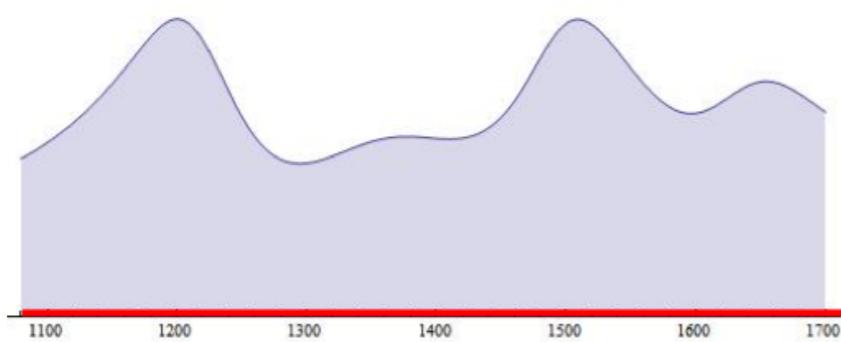
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- Development of three-body formalism required

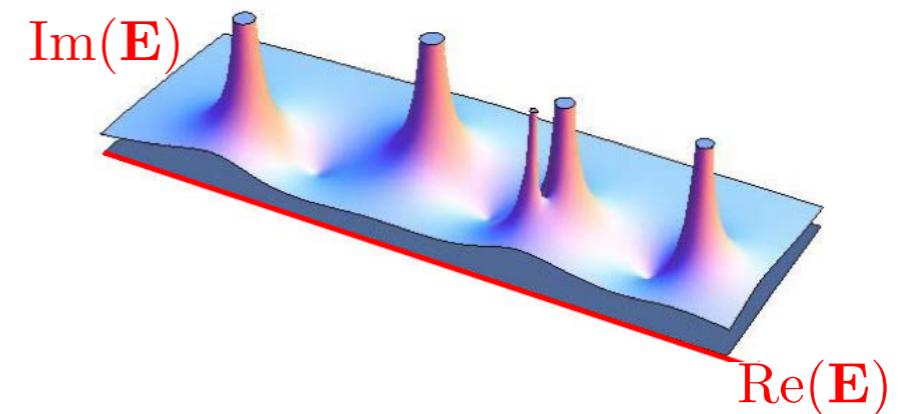
*E.G., HANSEN & SHARPE,  
BEANE/BRICENO/RUSETSKY/SAVAGE,... - PROGRESS*

# Resonances

- Most hadrons are resonances
  - E.g.,  $\pi N \pi N$



$E$  (MeV)



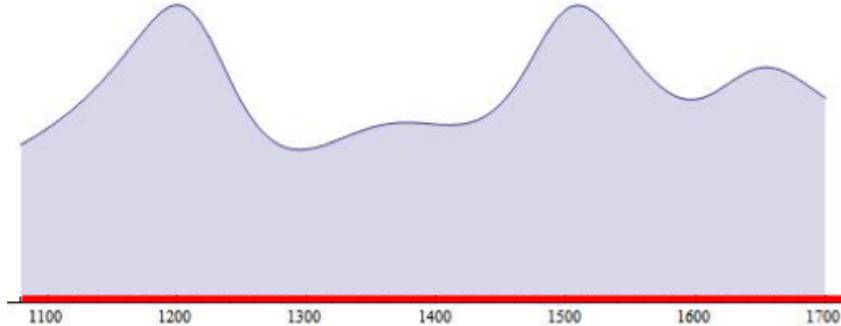
- Formally defined as a pole in a partial-wave scattering amplitude

$$t_l(s) \sim \frac{R}{s_0 - s} + \dots$$

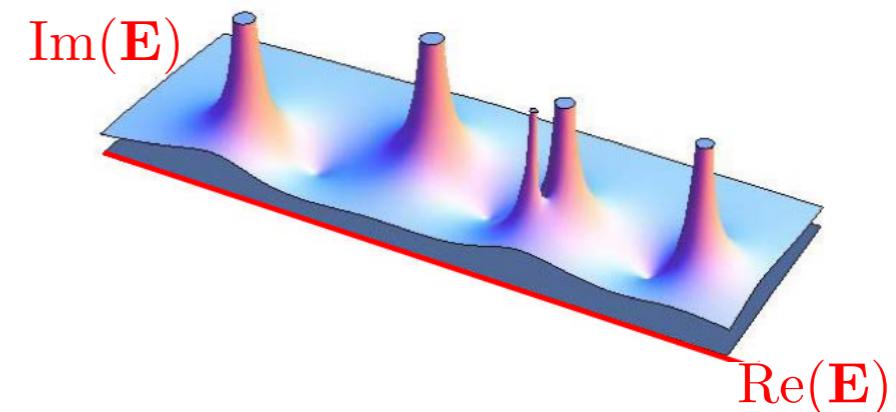
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- Different channels should have same pole location
  - Pole structure gives decay information
- 
- Can we predict hadron properties from first principles?

# Isospin=1 $\pi\pi$ P-wave

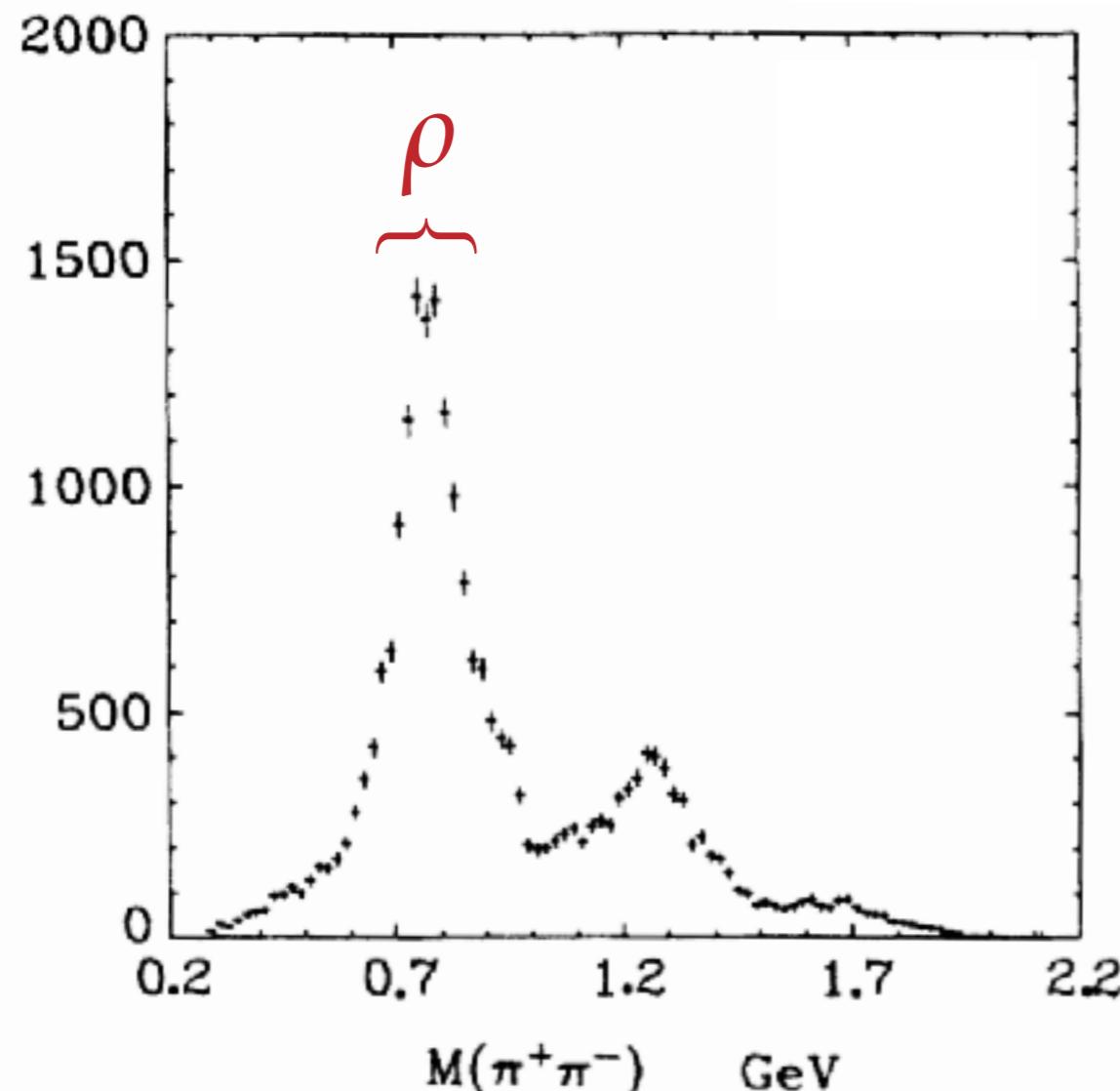
PHYSICAL REVIEW D

VOLUME 1, NUMBER 5

1 MARCH 1973

$\pi\pi$  Partial-Wave Analysis from Reactions  $\pi^+p \rightarrow \pi^+\pi^-\Delta^{++}$  and  $\pi^+p \rightarrow K^+K^-\Delta^{++}$  at 7.1 GeV/c†

S. D. Protopopescu,\* M. Alston-Garnjost, A. Barbaro-Galtieri, S. M. Flatté,†  
J. H. Friedman, I. T. A. Lasinski, G. R. Lynch, M. S. Rabin,|| and F. T. Solmitz  
*Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720*  
(Received 25 September 1972)



expand angular dependence  
in *partial waves*

## PARTIAL WAVE AMPLITUDE

$$f_\ell = \frac{1}{2i} (\eta_\ell e^{2i\delta_\ell} - 1)$$

$\eta = 1$  elastic

$\eta \leq 1$  inelastic

# Isospin=1 $\pi\pi$ P-wave

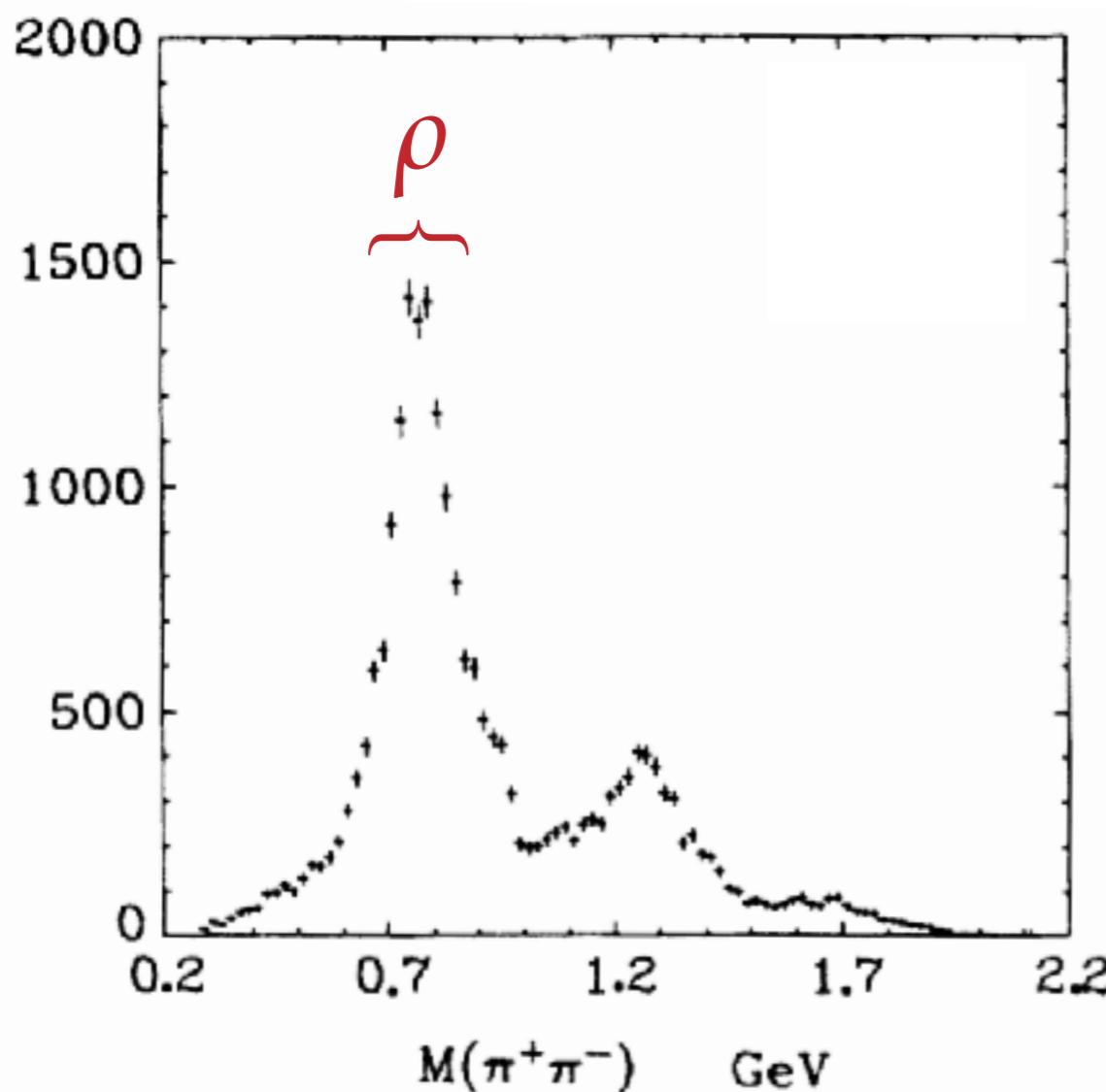
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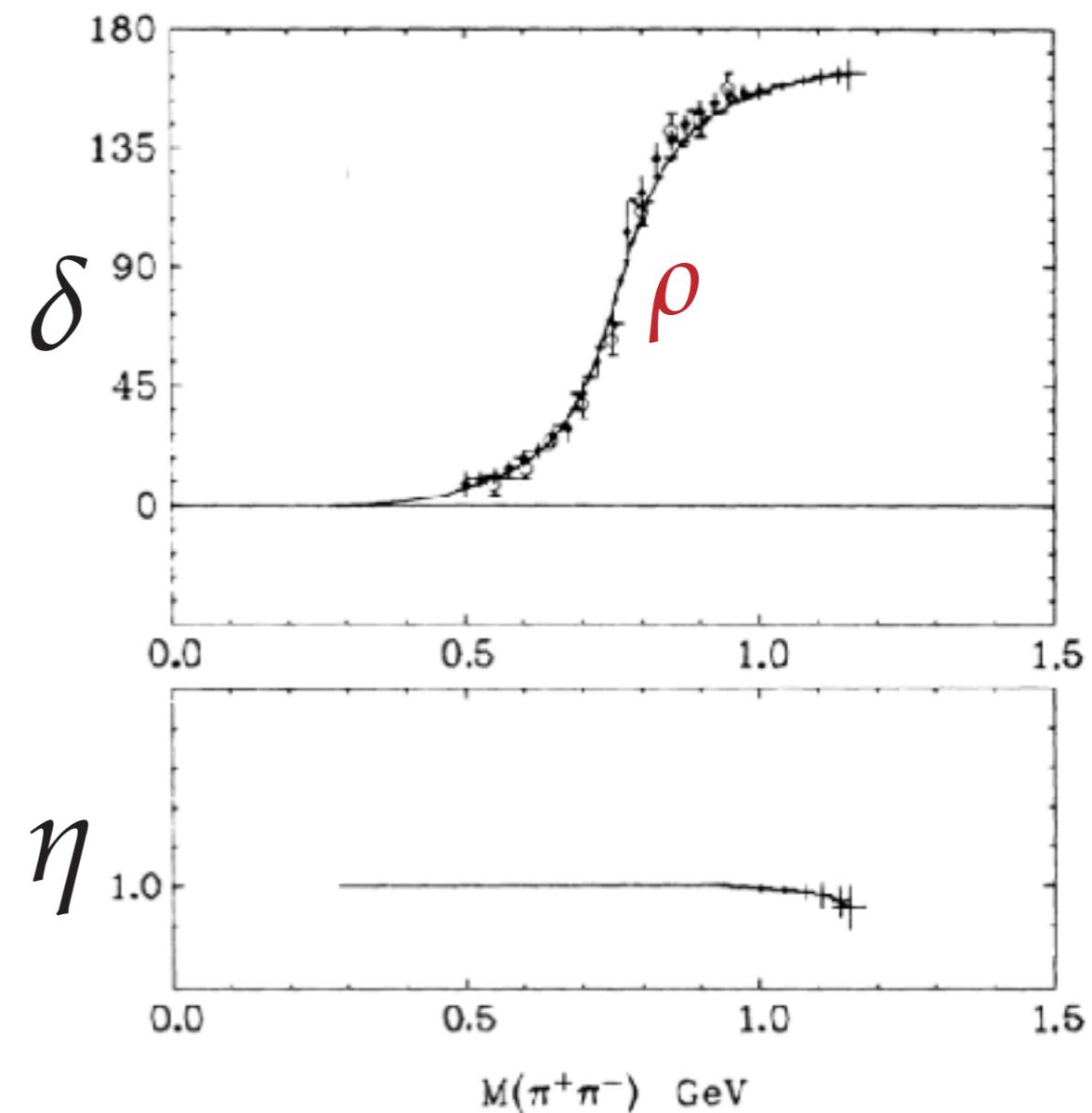
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## RESONANT PHASE SHIFT



# Finite-volume

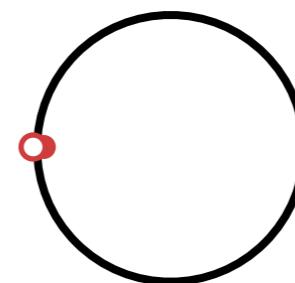
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- Where's the meson-meson continuum ?
  - there isn't one !

# Finite-volume

- Where's the meson-meson continuum ?
  - there isn't one !
  - in a finite-volume the spectrum is discrete

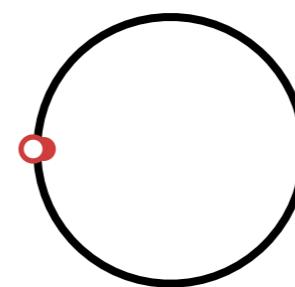
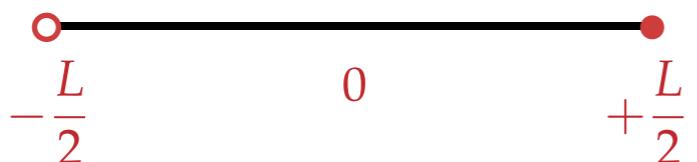
one-dim :



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one-dim :



e.g. a free particle  
 $\psi(x) \sim e^{ipx}$

» periodic boundary condition

$$\begin{aligned}\psi(x) &= \psi(x + L) \\ e^{ipx} &= e^{ip(x+L)} \\ e^{ipL} &= 1\end{aligned}$$

$$p = \frac{2\pi}{L}n$$

discrete  
energy  
spectrum

# Interacting particles in a finite-volume

---

- Two identical bosons **interacting** through a finite-range potential

$$\psi(z) \sim \cos [p|z| + \delta(p)] \quad \text{outside the range of the potential, } |z| > R$$

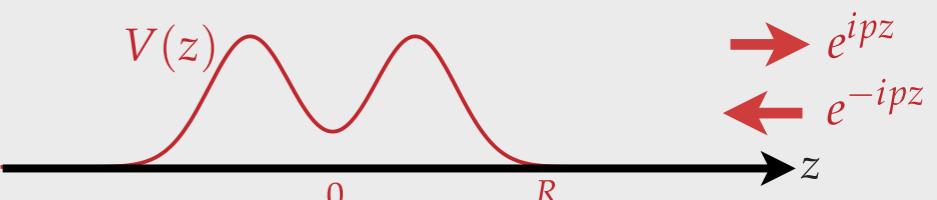
# Interacting particles in a finite-volume

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$$\psi(z > 0) \sim e^{-ipz} + e^{2i\delta(p)} \cdot e^{ipz}$$



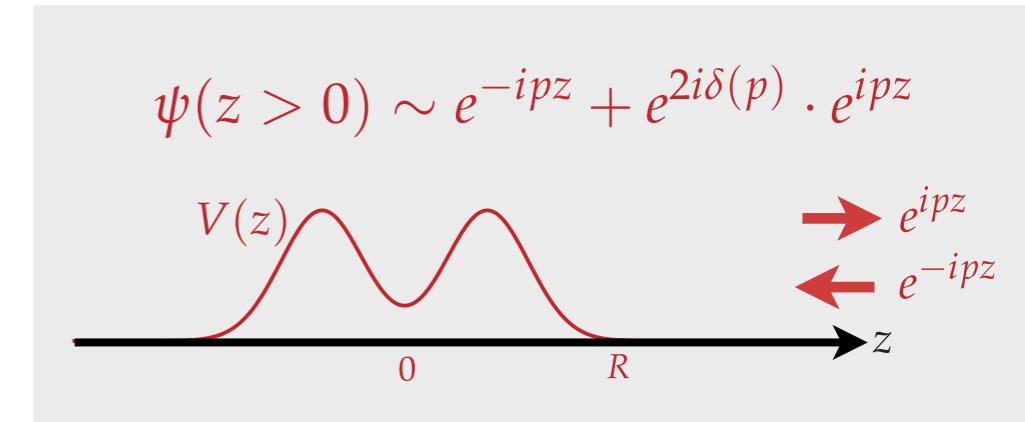
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$$\frac{pL}{2} + \delta(p) = n\pi$$



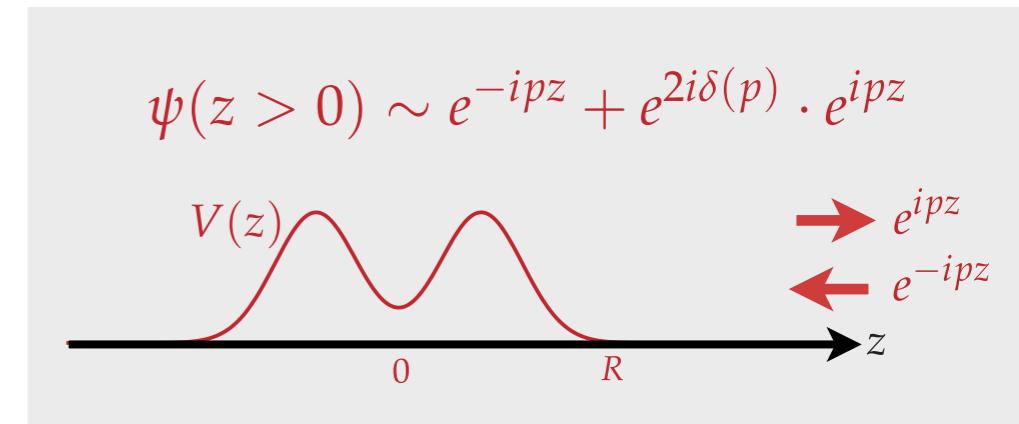
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discrete  
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spectrum

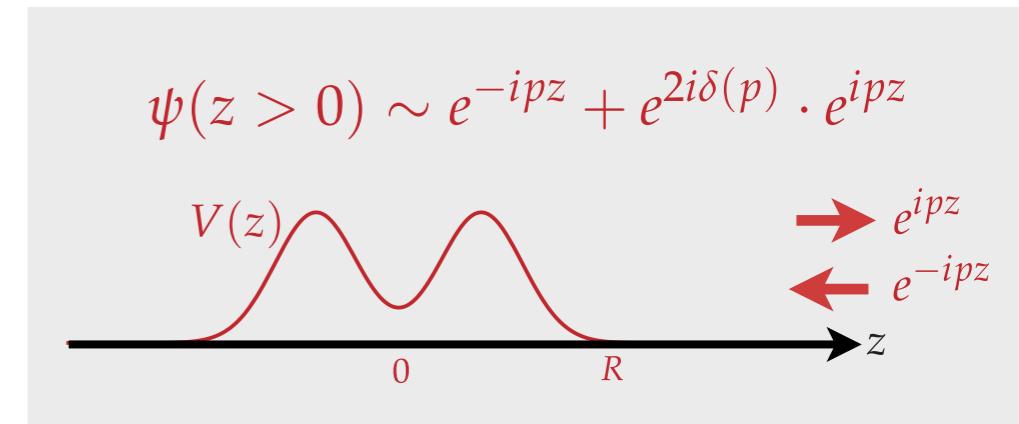
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$$p = \frac{2\pi}{L}n - \frac{2}{L}\delta(p)$$

**discrete  
energy  
spectrum**

discrete energy spectrum is determined by the scattering amplitude

(or vice-versa)

# Scattering in a finite cubic volume

- Expect a discrete spectrum in a finite periodic volume  $\psi(x + L) = \psi(x)$

e.g. free particle  $e^{ip(x+L)} = e^{ipx}$

quantized momentum  $p = \frac{2\pi}{L}n$

- For an interacting theory

$$\cot \delta_\ell(E) = \mathcal{M}_\ell(E, L)$$

LÜSCHER ...

elastic scattering phase-shift

known function

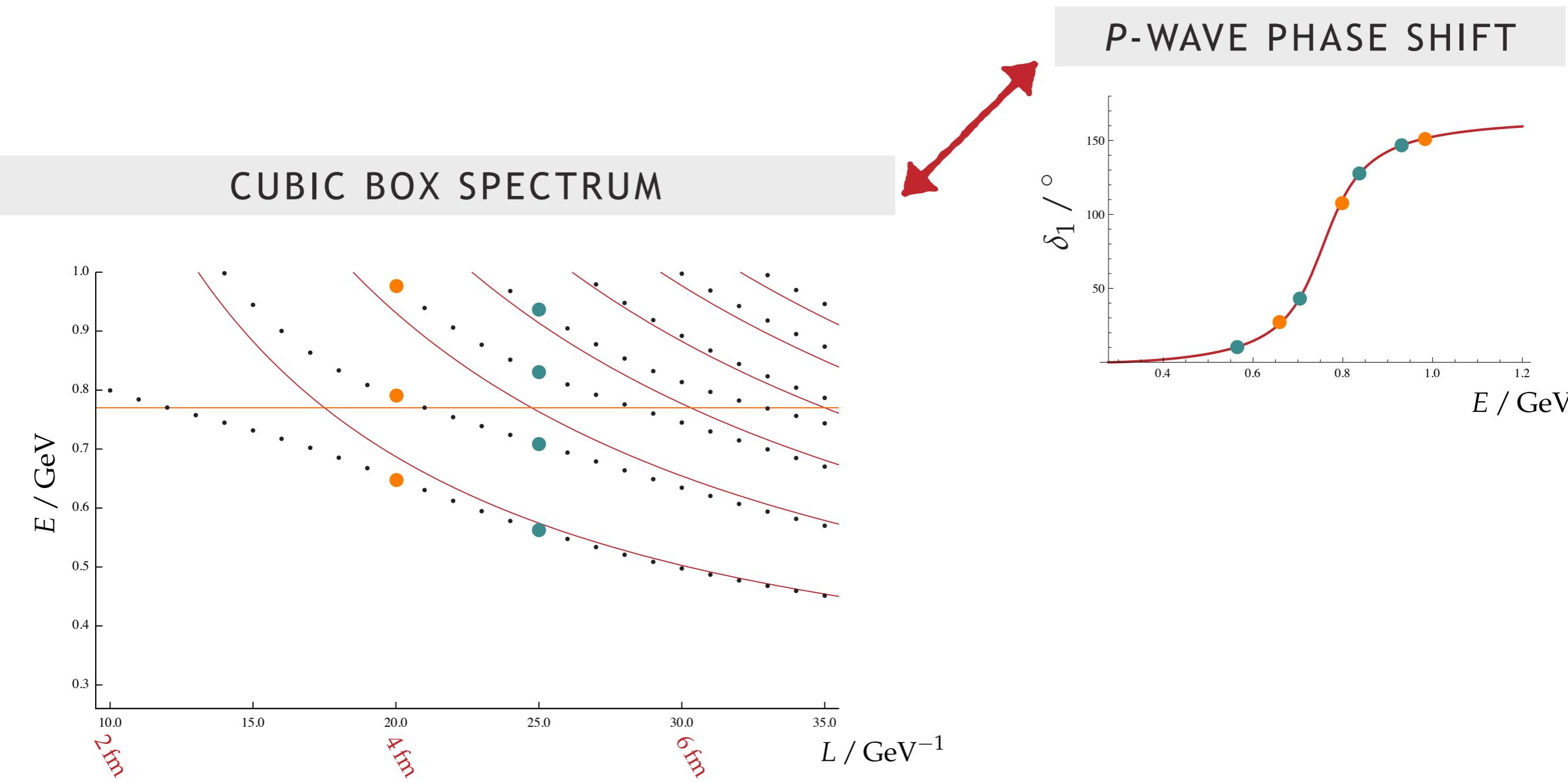
Discrete energies in a finite-volume



Discrete values of the phase-shift

# Scattering in a finite cubic volume

- Experimental  $\pi\pi l=1$   $P$ -wave scattering amplitude



# Coupled-channel scattering

- Finite-volume formalism recently derived (multiple methods)

*HE, JHEP 0507 011  
HANSEN, PRD86 016007  
BRICENO, PRD88 094507  
GUO, PRD88 014051*

$$\det \left[ ([t^{(\ell)}(E)]_{ij}^{-1} + i\rho_i(E) \delta_{ij}) - \delta_{ij} \mathcal{M}_\ell(p_i(E)L) \right] = 0$$

scattering matrix      phase space      known functions      *matrices in partial-wave space ..*

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scattering matrix      phase space      known functions      *matrices in partial-wave space ..*

- However, this is **one equation for multiple unknowns** (per energy level)  $\frac{1}{2}N(N+1)$   
for  $N$  channels

- parameterize the energy dependence of  $t$
  - try to describe a spectrum globally

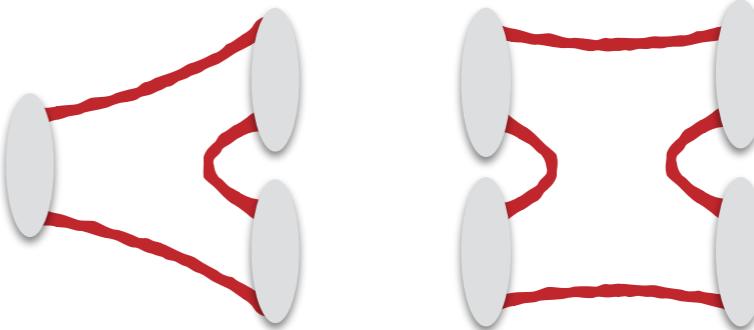
“Energy-dependent” analysis

# An appropriate basis of operators

Form correlator matrix with both  $\bar{\psi}\Gamma\psi$  and hadron-hadron (here mesons)-like operators

e.g.  $\mathcal{O}_{\pi\pi}^{|\vec{p}|} = \sum_{\hat{p}} C(\hat{p}) \mathcal{O}_\pi(\vec{p}) \mathcal{O}_\pi(-\vec{p})$  where  $\mathcal{O}_\pi(\vec{p}) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \bar{\psi}\Gamma\psi(\vec{x})$

now need to evaluate  
diagrams like



**distillation** can handle  
the annihilation lines



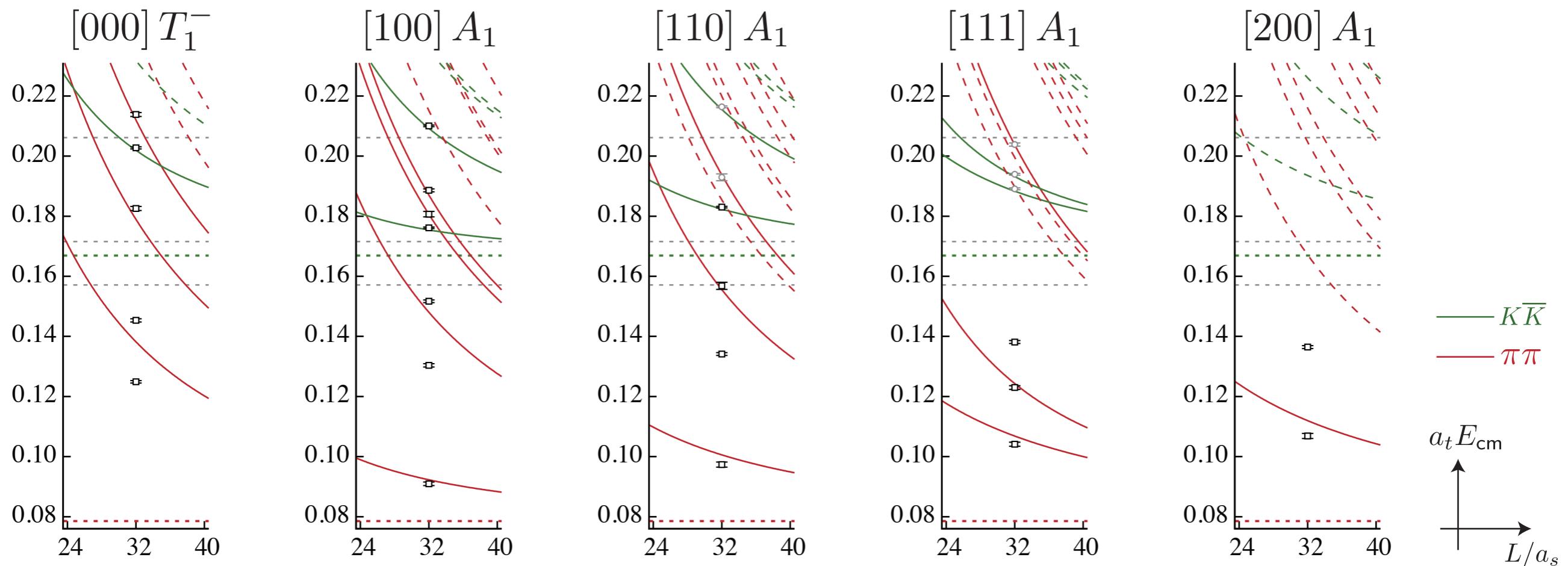
**GPUs** have been the key

# Finite-volume spectrum - moving frames

- Non-interacting thresholds and energies as a function of  $\vec{k}$

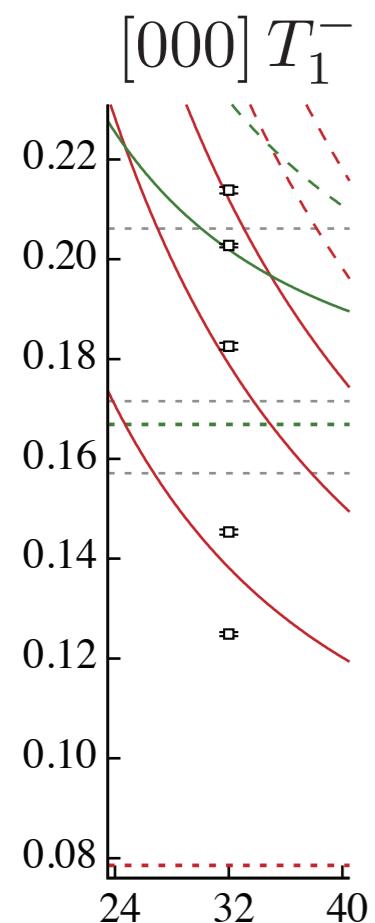
$m_\pi \sim 236$  MeV

Momentum & lattice irrep labels:  $[\vec{k}] \Lambda$



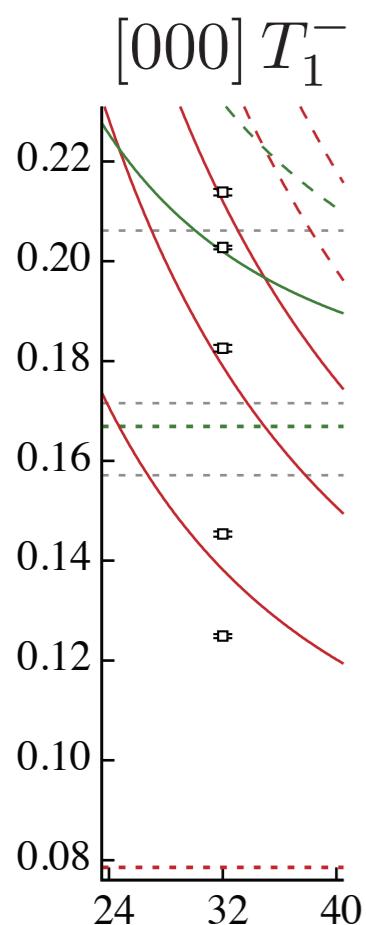
# $\pi\pi/KK$ scattering

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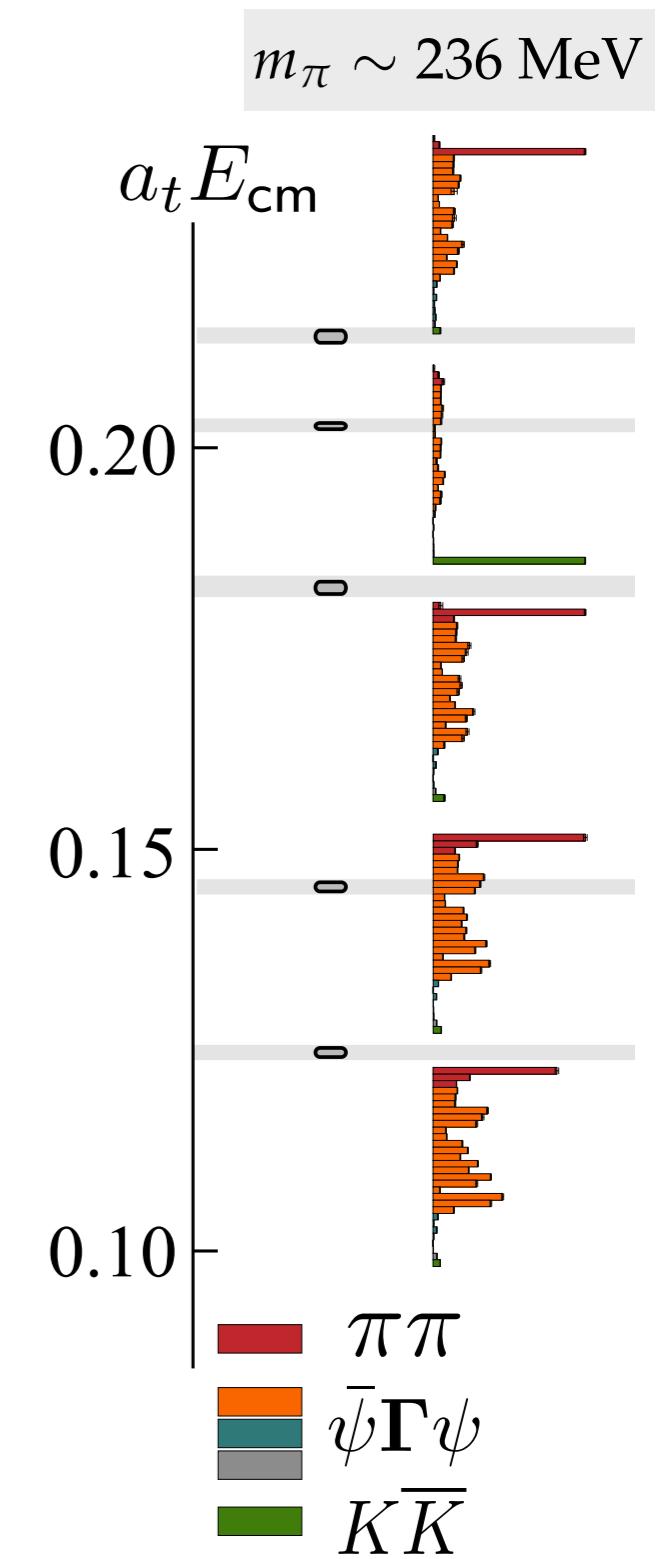
- Consider size of operator overlaps  $\langle n | \mathcal{O}_i^\dagger | \emptyset \rangle$

# $\pi\pi/KK$ scattering

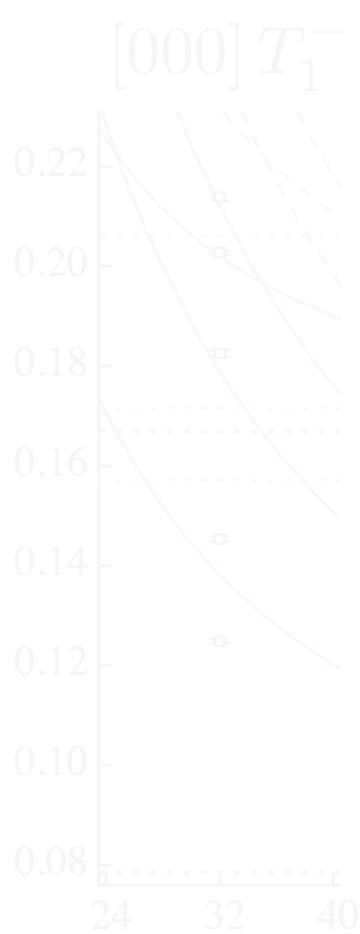


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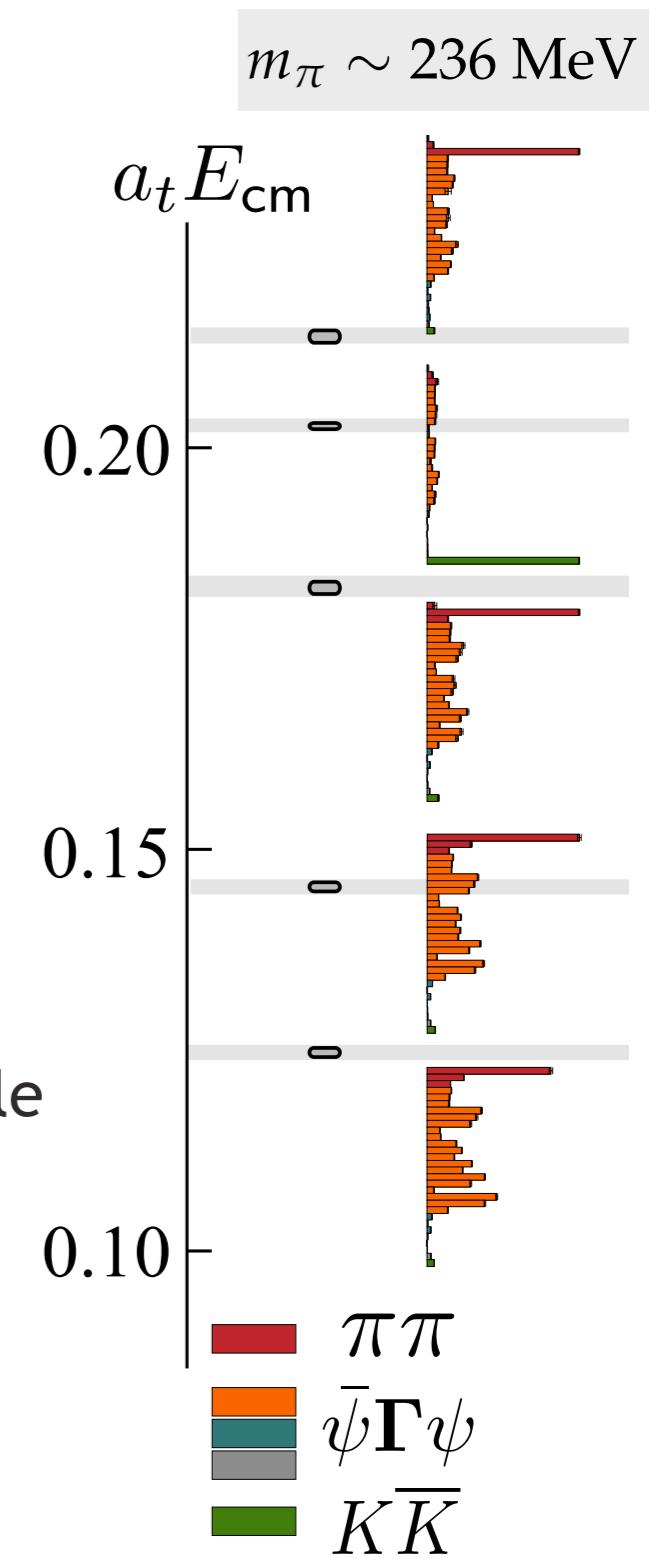
# $\pi\pi/KK$ scattering



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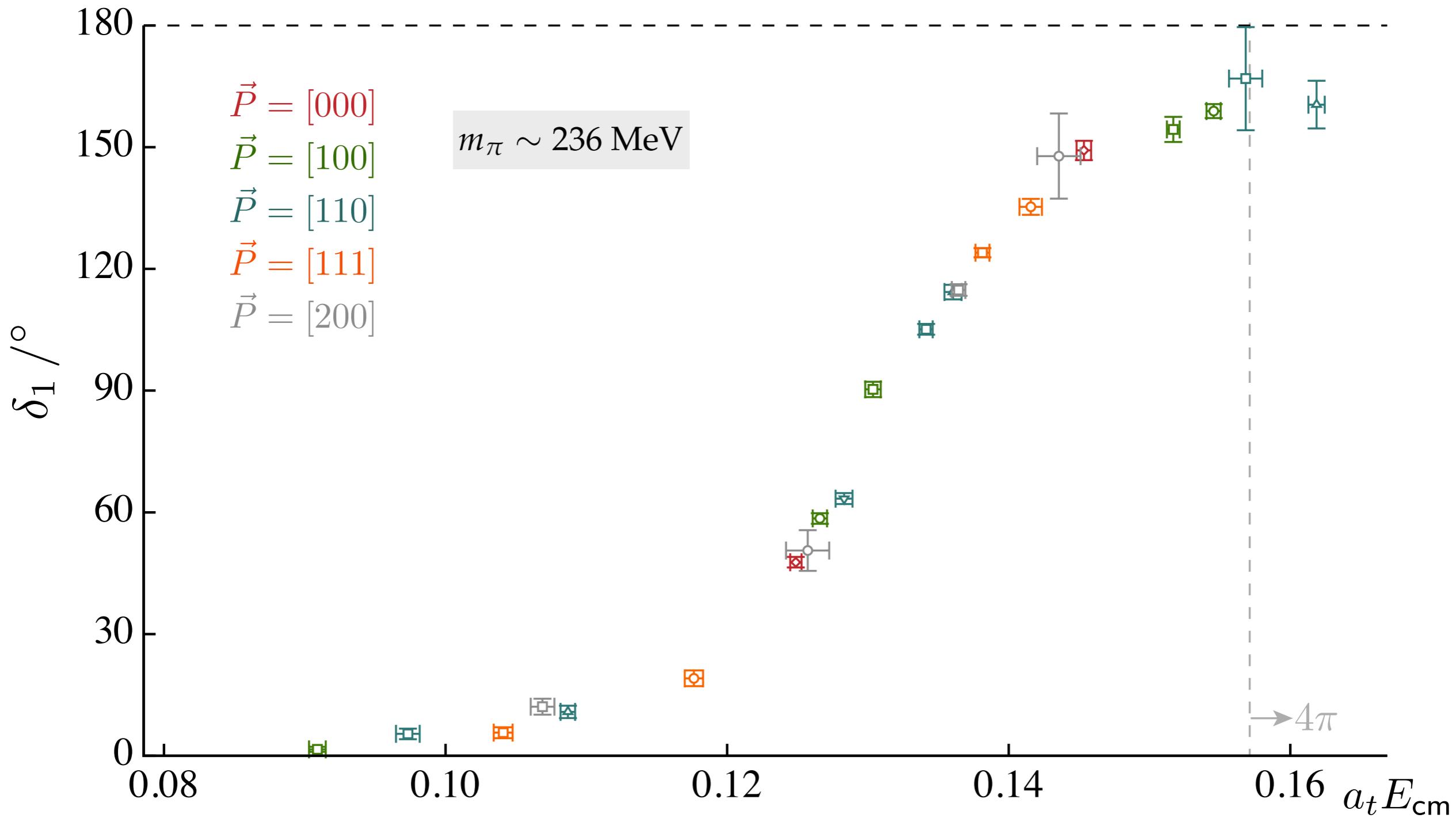
$$\langle n | \mathcal{O}_i^\dagger | \emptyset \rangle$$

» Finite-V: ad-mixture of single & two-particle



# $\pi\pi$ $P$ -wave phase-shift

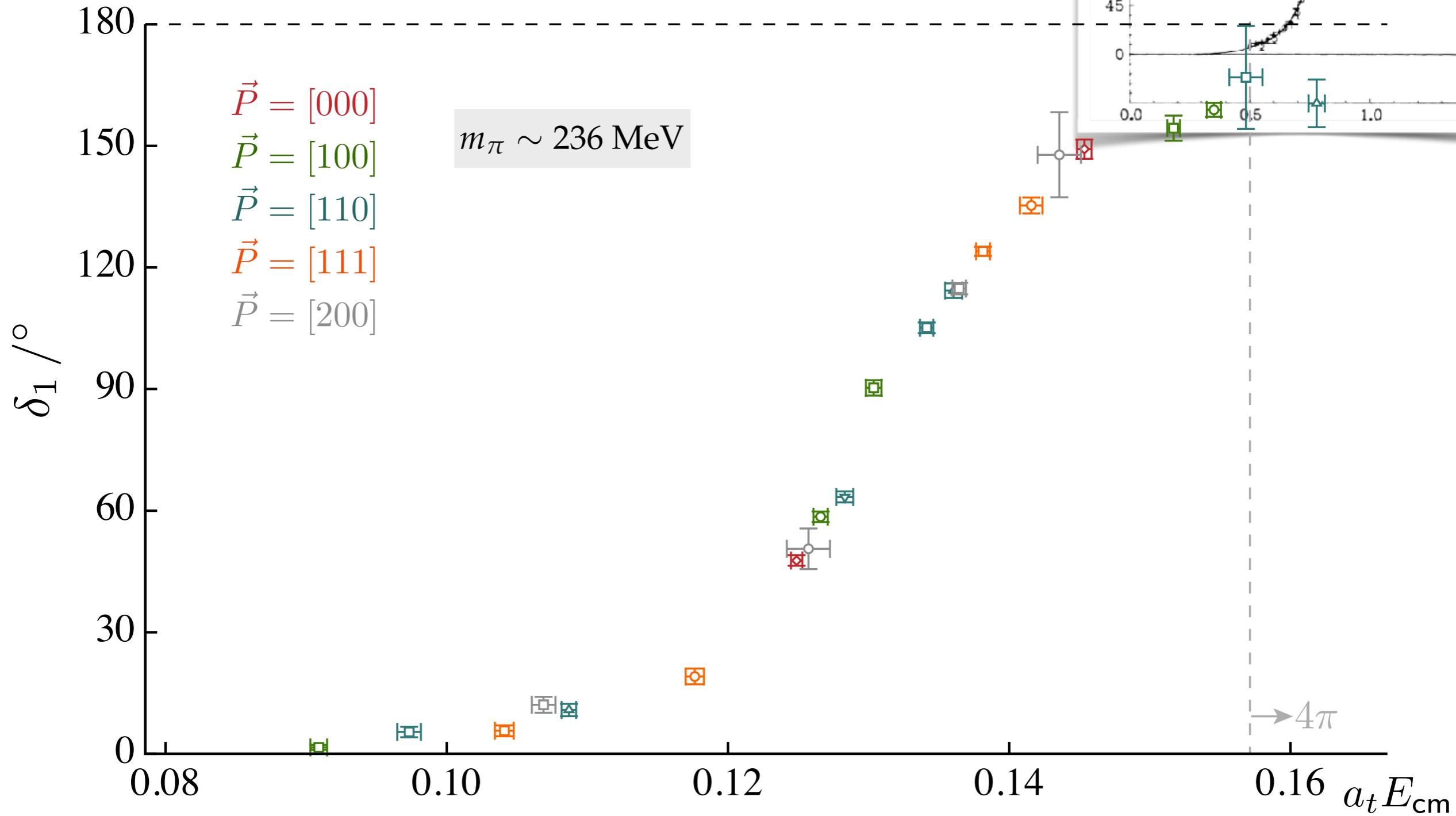
- Restrict to elastic region below  $K\bar{K}$  threshold



To appear very soon...

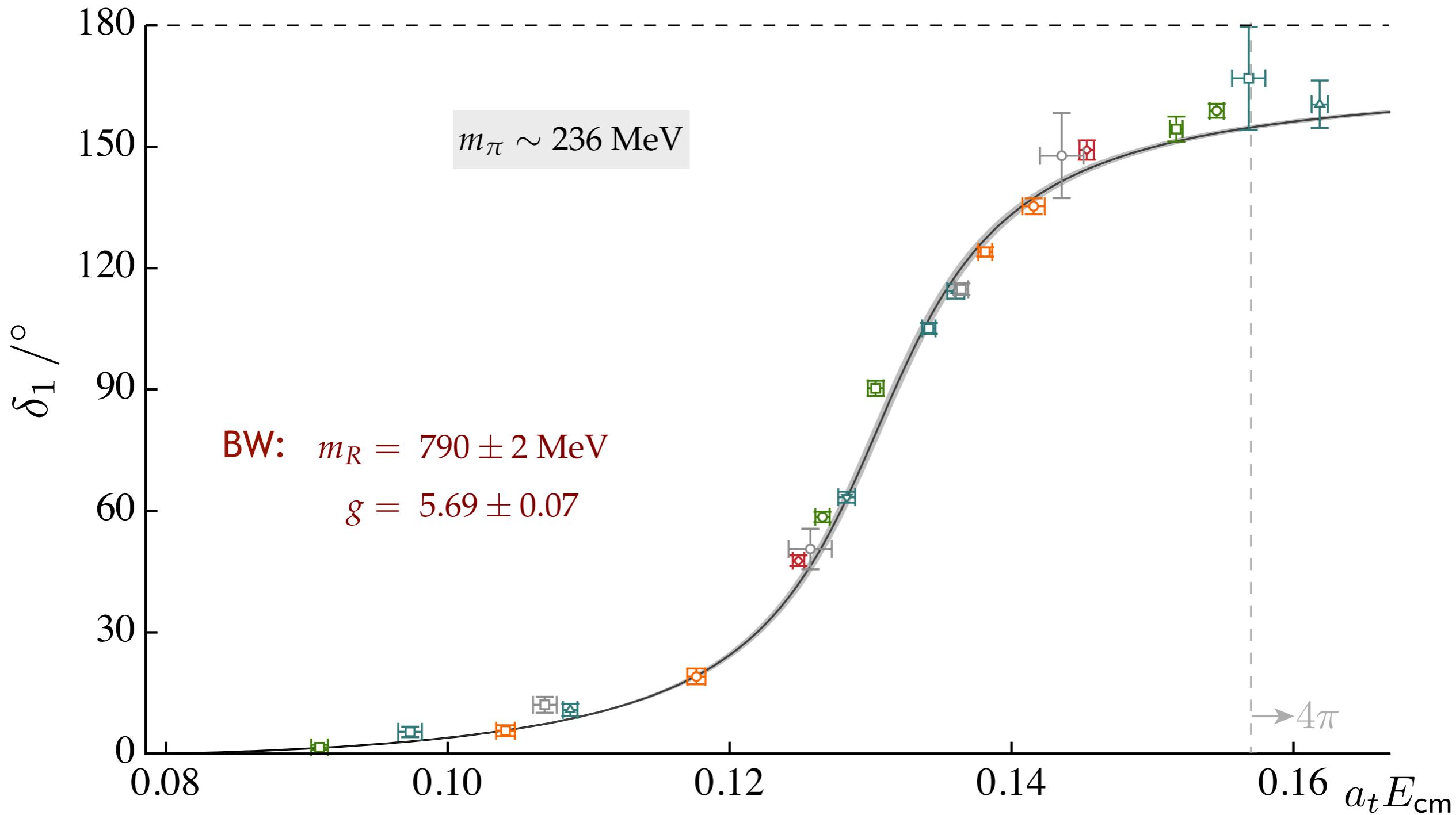
# $\pi\pi$ $P$ -wave phase-shift

- Restrict to elastic region below  $K\bar{K}$  threshold



To appear very soon...

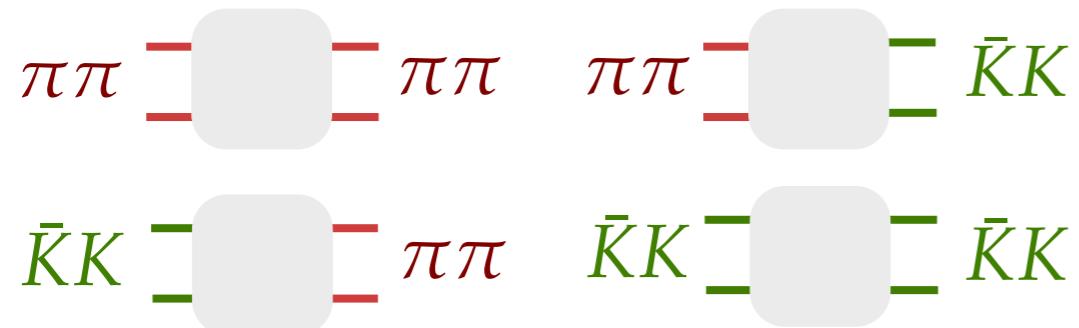
# $\pi\pi$ $P$ -wave phase-shift



To appear very soon...

# $\rho$ resonance as a coupled channel system

- Parameterize the  $t$ -matrix in a unitarity conserving way



- Compute finite-volume spectrum

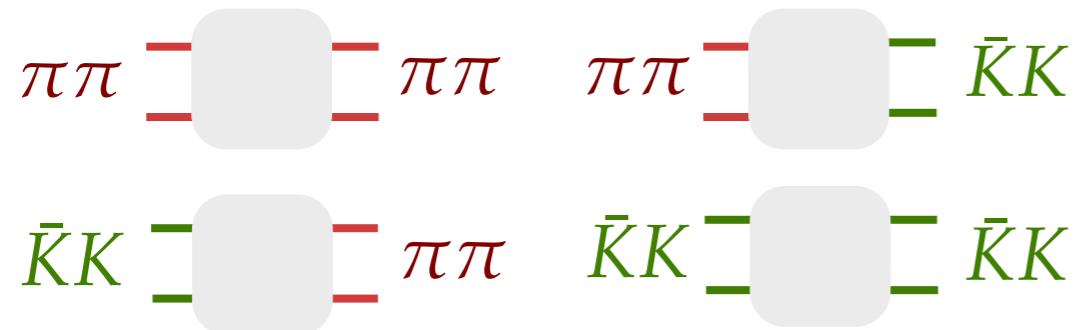
$$\bar{d}\Gamma u$$

$$\sum_{\hat{k}_1, \hat{k}_2} C(\Lambda, \vec{P}, \vec{k}_1, \vec{k}_2) \color{red} \pi^\dagger(\vec{k}_1) \pi^\dagger(\vec{k}_2)$$

$$\sum_{\hat{k}_1, \hat{k}_2} C(\Lambda, \vec{P}, \vec{k}_1, \vec{k}_2) \color{red} K^\dagger(\vec{k}_1) \bar{K}^\dagger(\vec{k}_2)$$

# $\rho$ resonance as a coupled channel system

- Parameterize the  $t$ -matrix in a unitarity conserving way



$$t_{ij}^{-1}(E) = K_{ij}^{-1}(E) + \delta_{ij} I_i(E)$$

$$K_{ij}(E) = \frac{g_i g_j}{m^2 - E^2} + \gamma_{ij}$$

- Vary the parameters, solving

$$\det \left[ ([t^{(\ell)}(E)]_{ij}^{-1} + i\rho_i(E) \delta_{ij}) - \delta_{ij} \mathcal{M}_\ell(E, L) \right] = 0$$

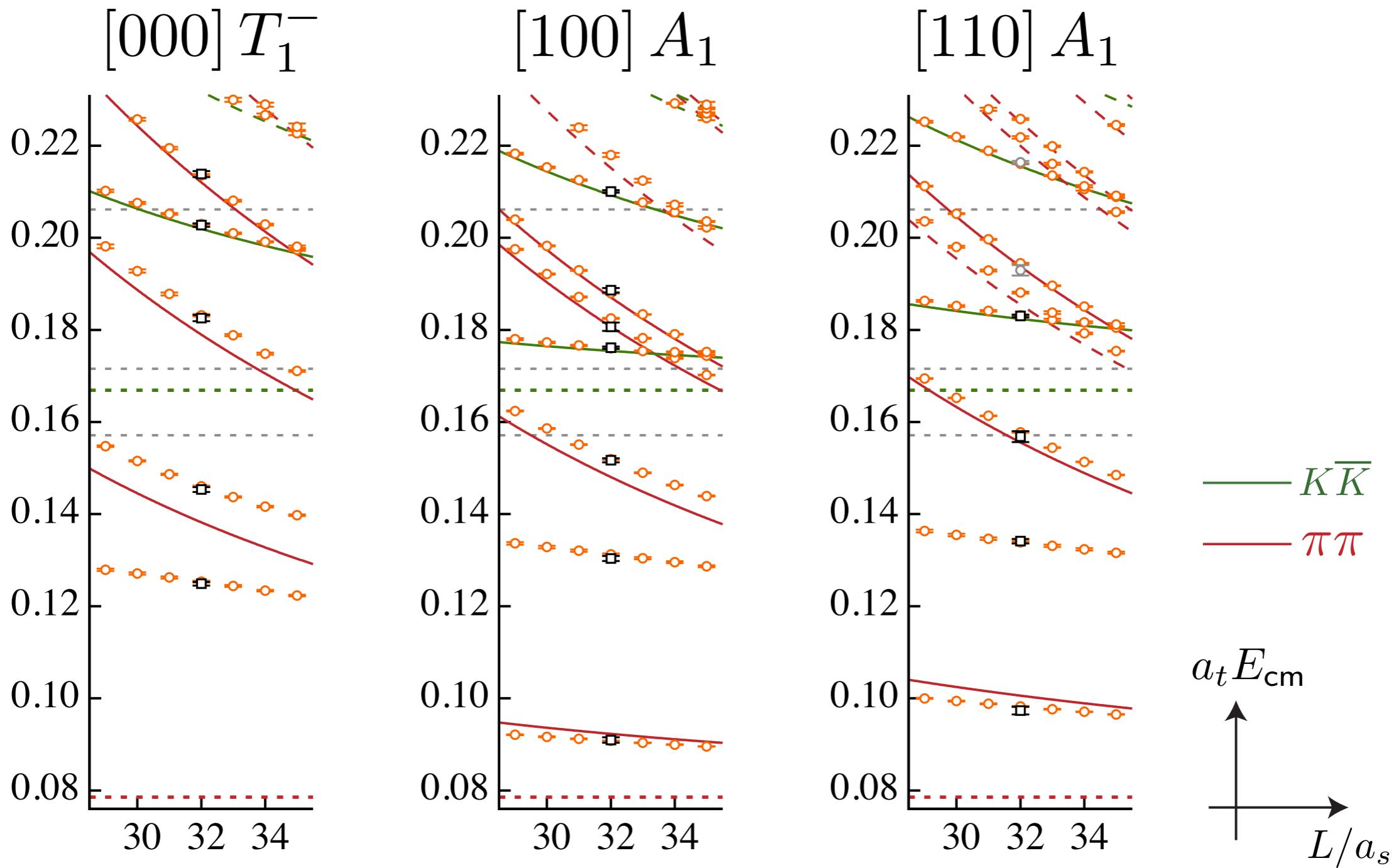
for the spectrum in each irreducible representation & momentum

Want pole mass and couplings of t-matrix

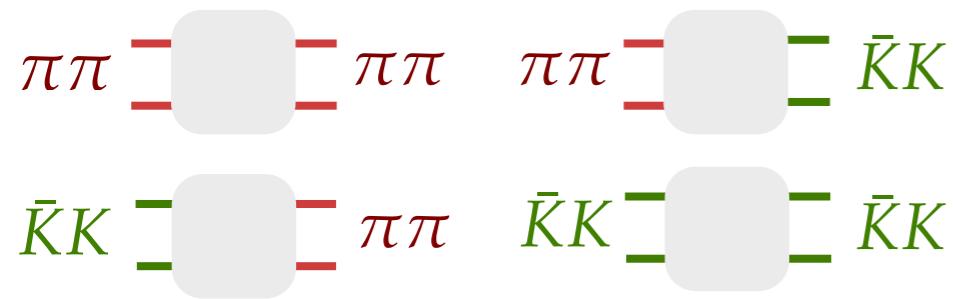
# $\pi\pi/KK$ scattering

- Data points (black) compared to parameterization (gold)

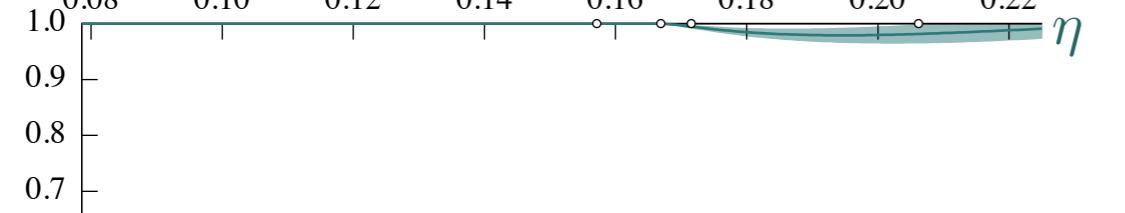
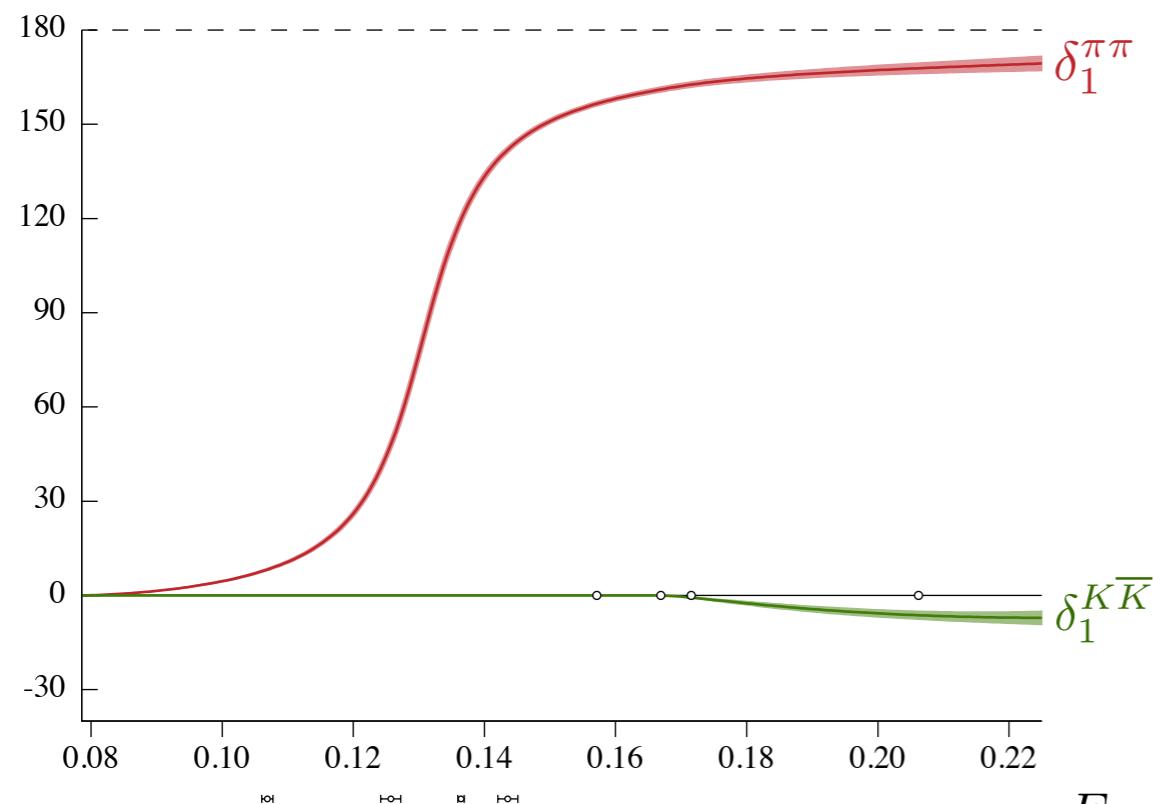
$m_\pi \sim 236$  MeV



# $\rho$ resonance as a coupled channel system

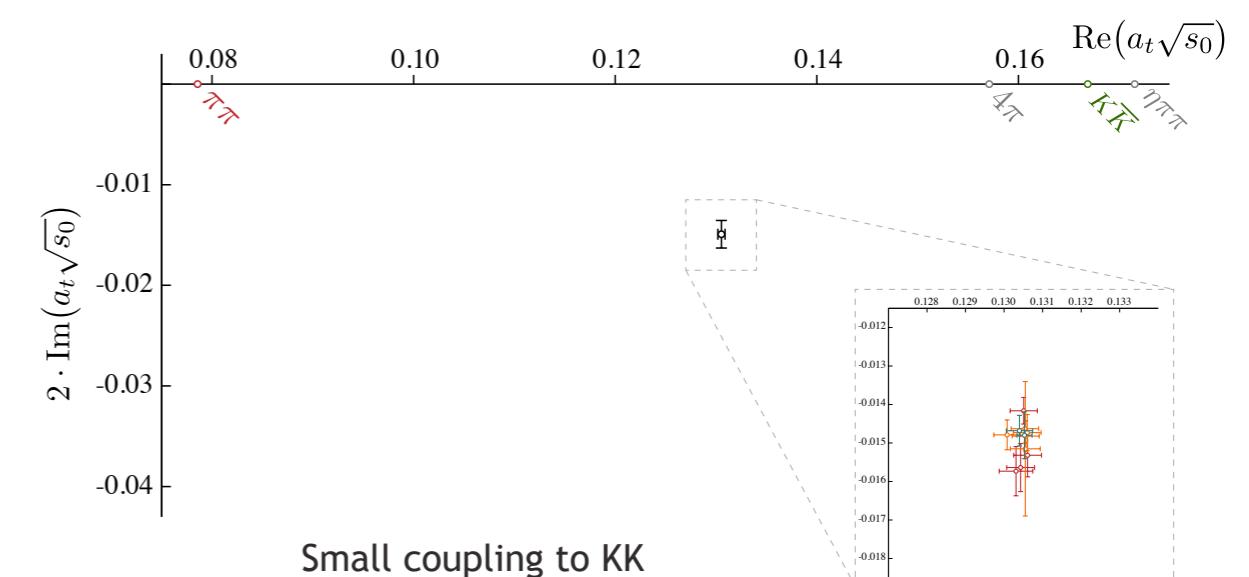


Phase shifts & inelasticity



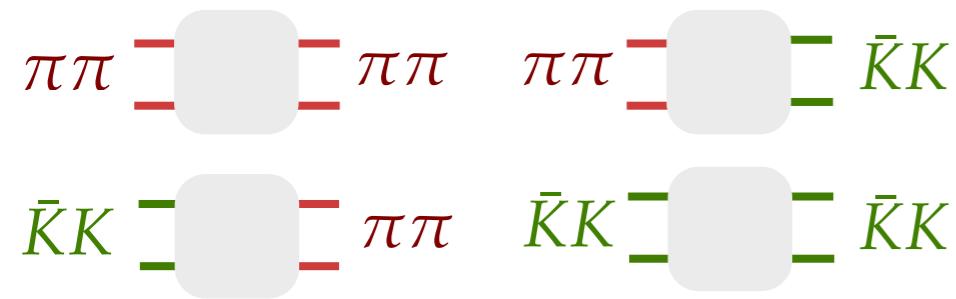
$m_\pi \sim 236 \text{ MeV}$

t-matrix pole location

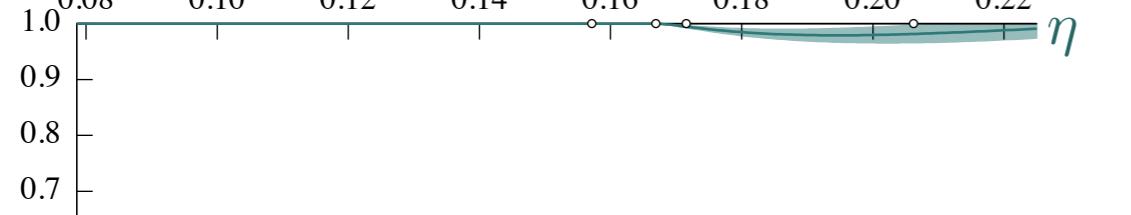
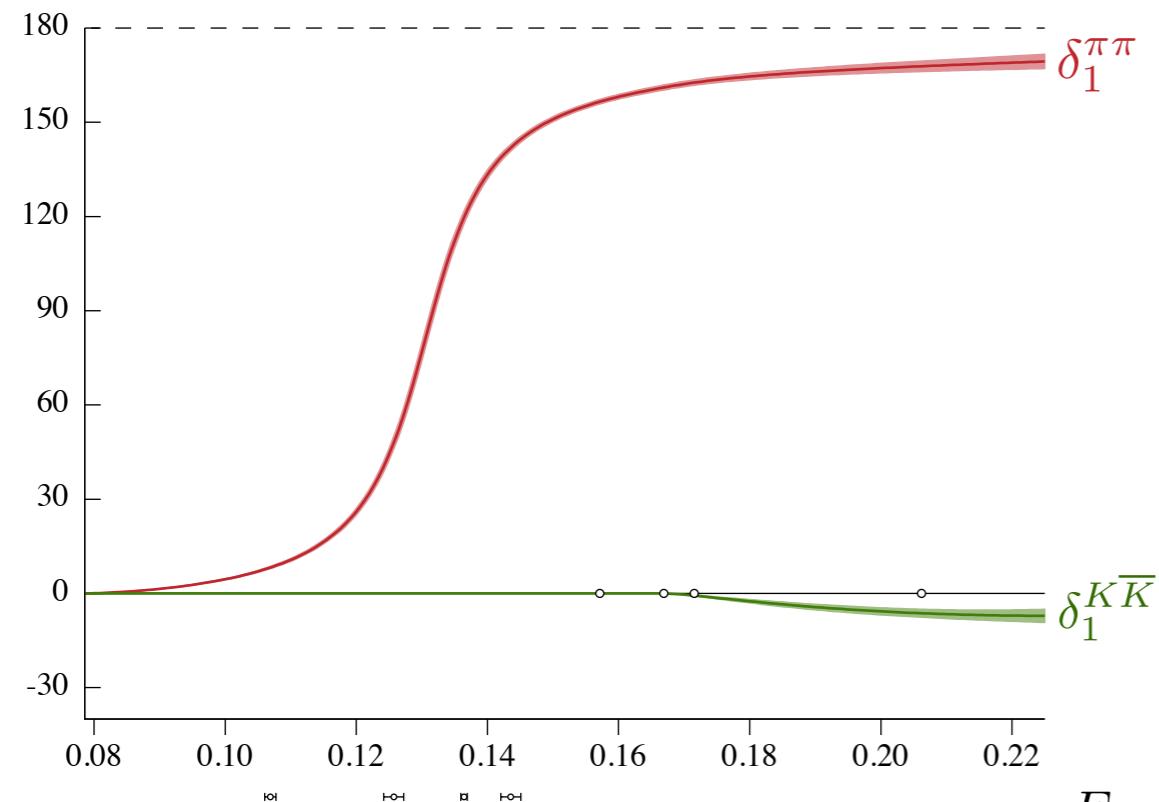


Small coupling to KK

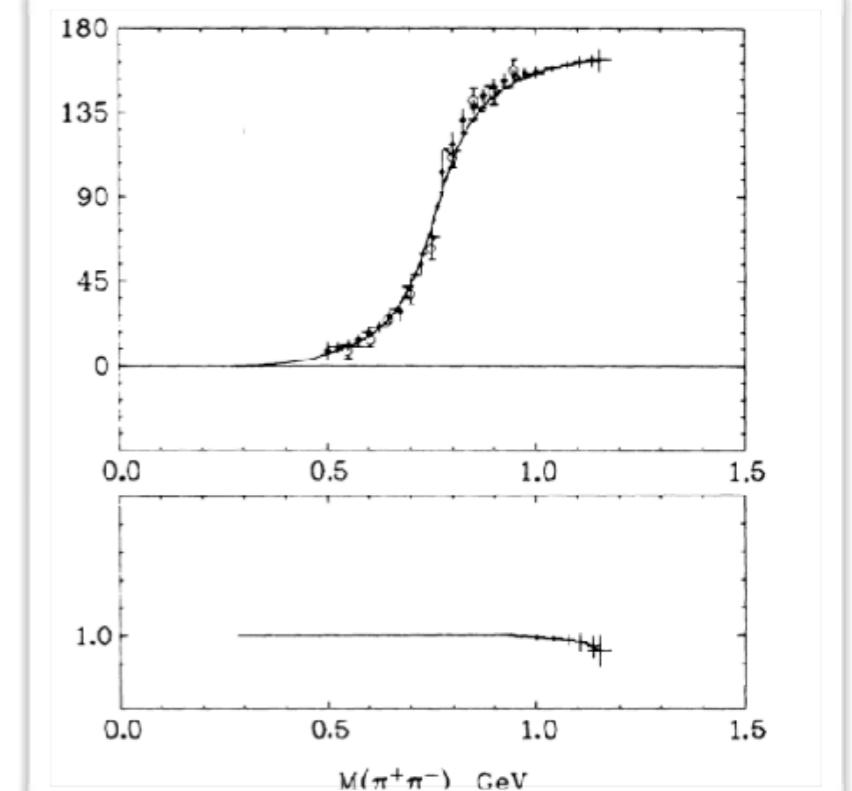
# $\rho$ resonance as a coupled channel system



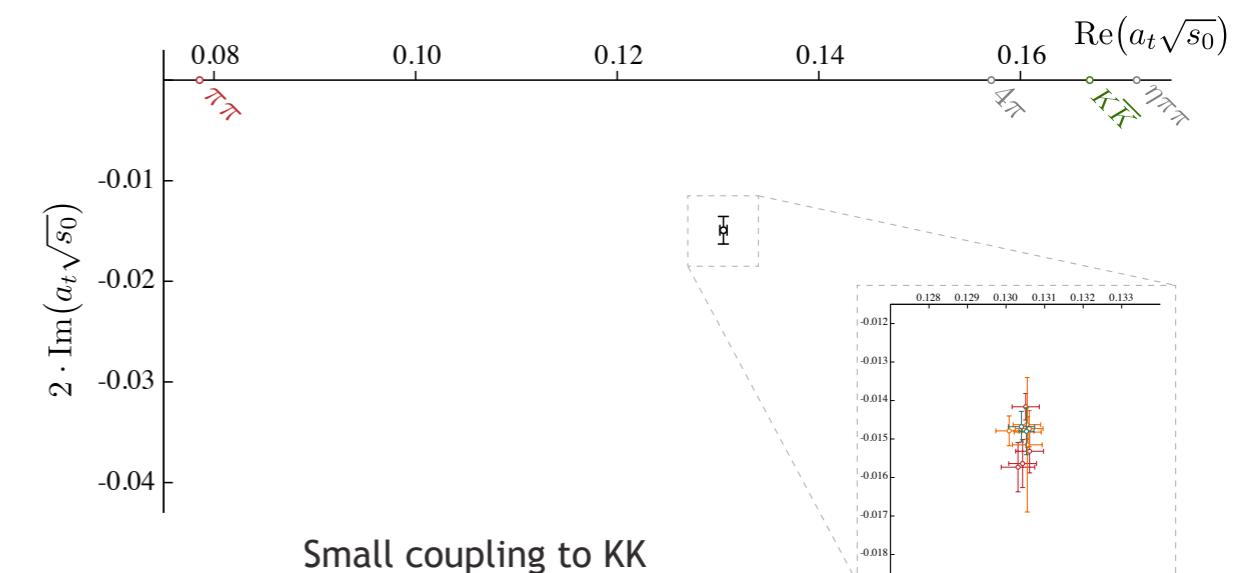
Phase shifts & inelasticity



$m_\pi \sim 236$  MeV



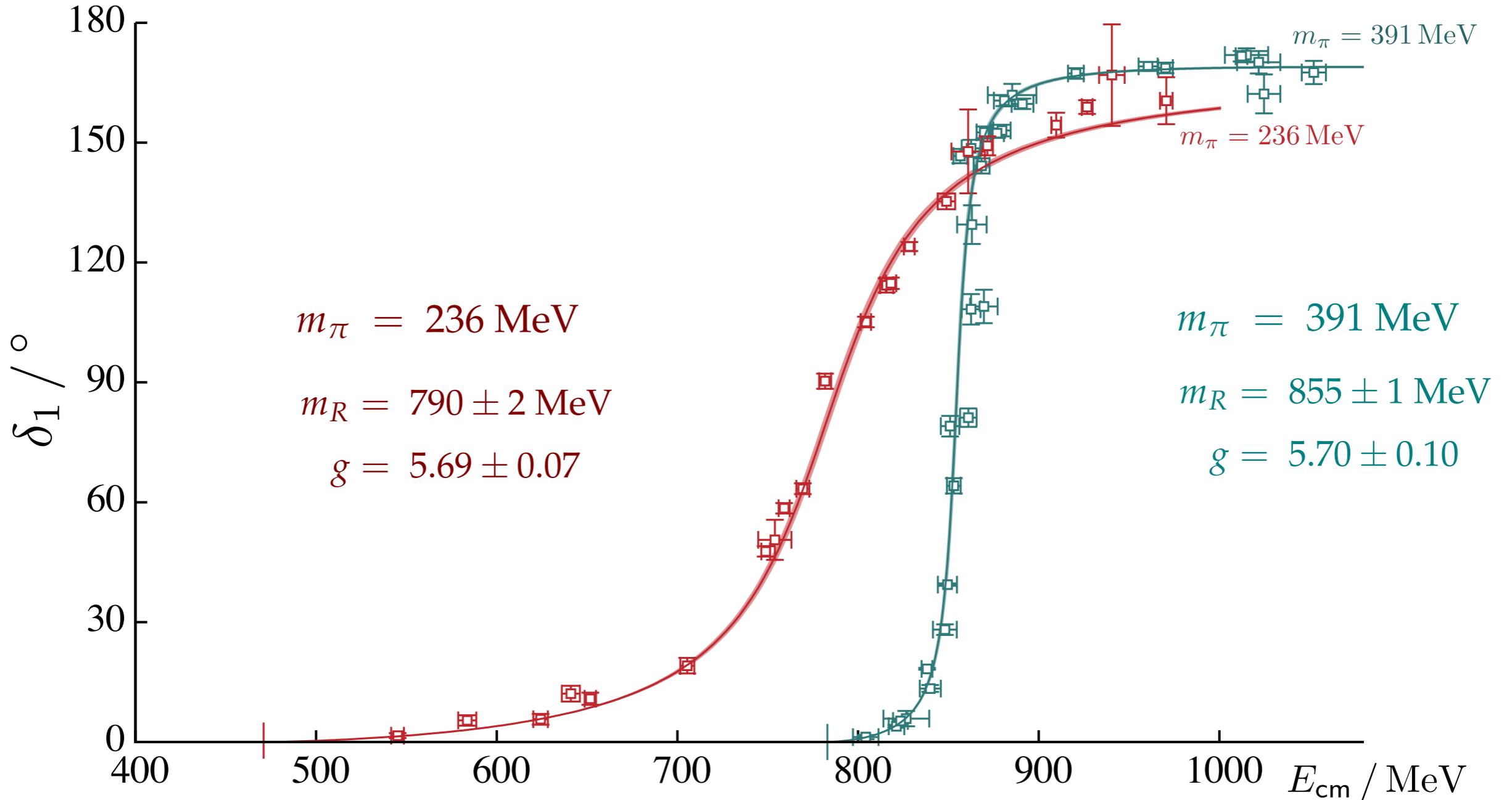
t-matrix pole location



Small coupling to KK

# $\rho$ resonance at different pion masses

- BW couplings nearly constant in pion mass (will come back to this later...)

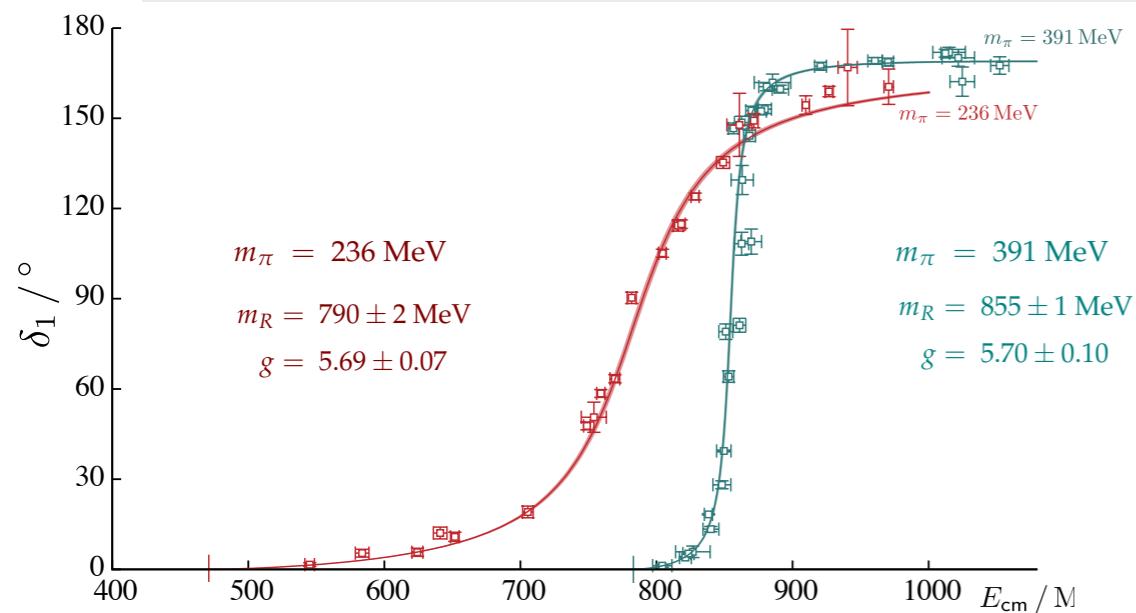


PRD 87 034505

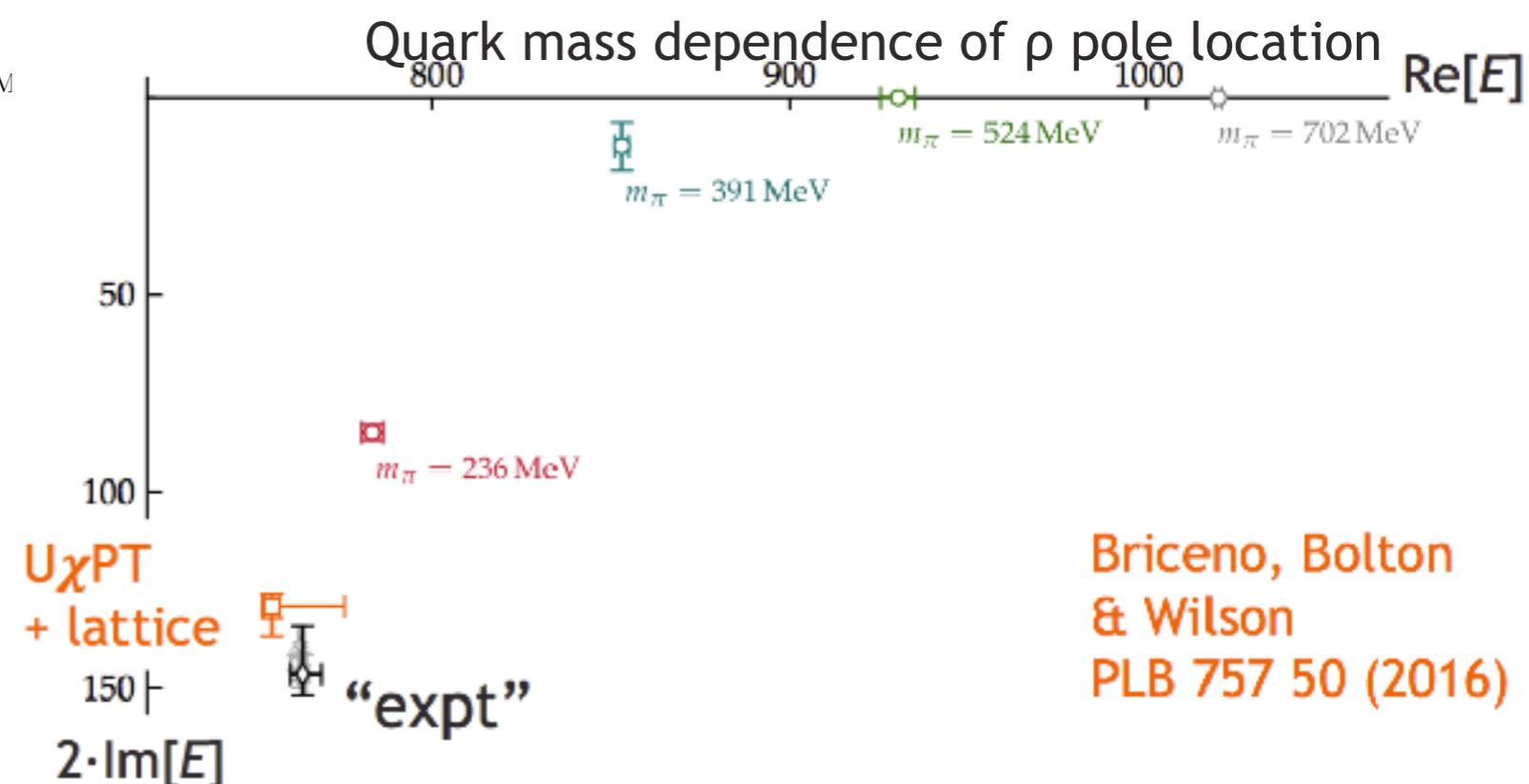
# $\rho$ resonance at different pion masses

Physical point, chiral extrapolations?

## $\pi\pi$ P-WAVE ELASTIC PHASE-SHIFT



Resonance parameters

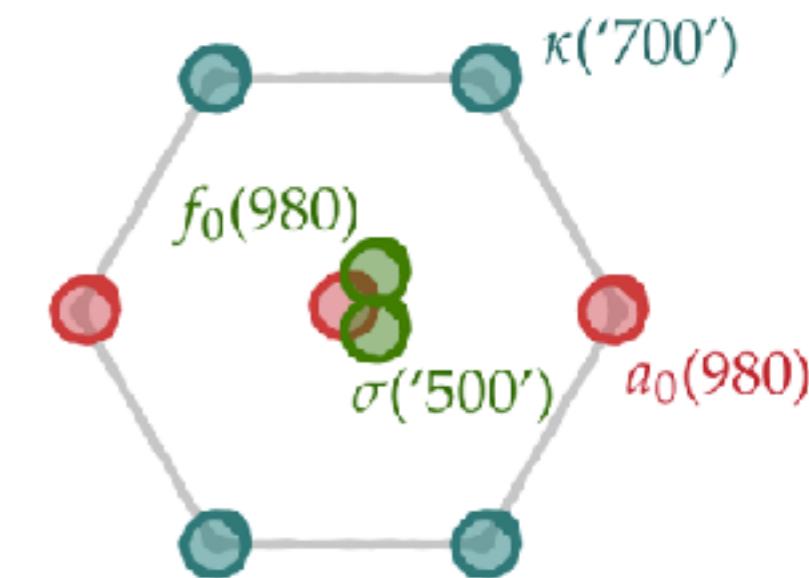


# Mysteries of scalar sector of QCD

**a<sub>0</sub>** in coupled channel  $\pi\eta/\bar{K}\bar{K}$

$m_\pi = 391 \text{ MeV}$

PRD93 094506 (2016)



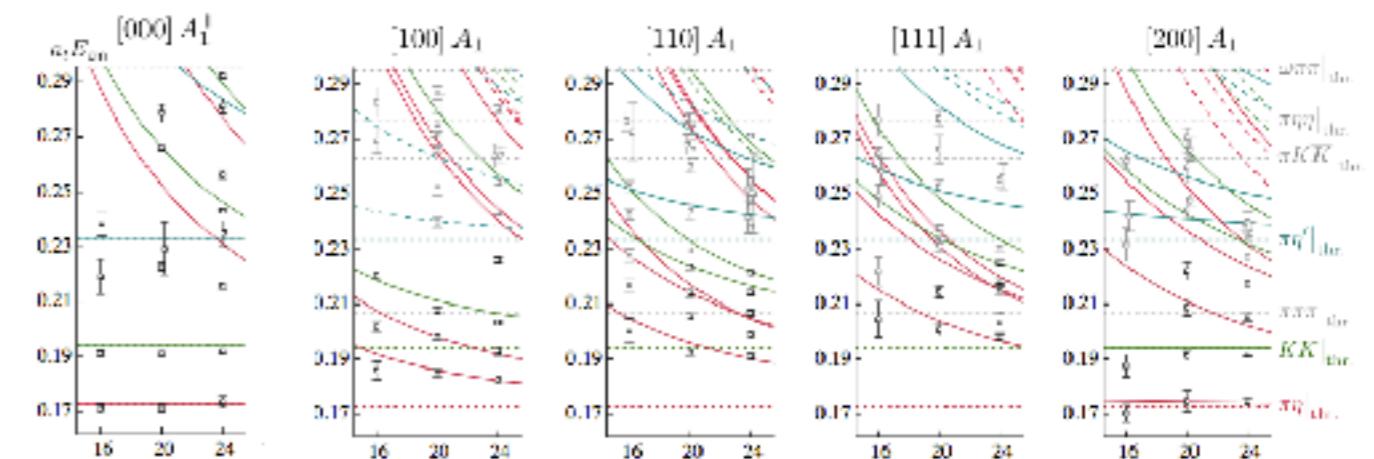
# Mysteries of scalar sector of QCD

$a_0$  in coupled channel  $\pi\eta/\bar{K}\bar{K}$

$m_\pi = 391$  MeV

PRD93 094506 (2016)

LQCD FINITE VOLUME SPECTRUM



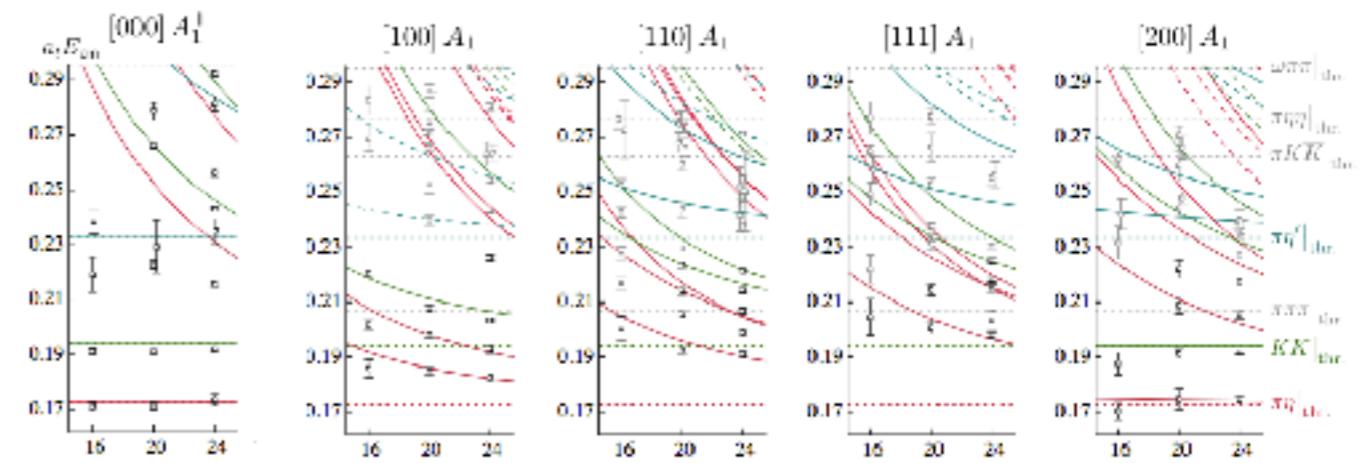
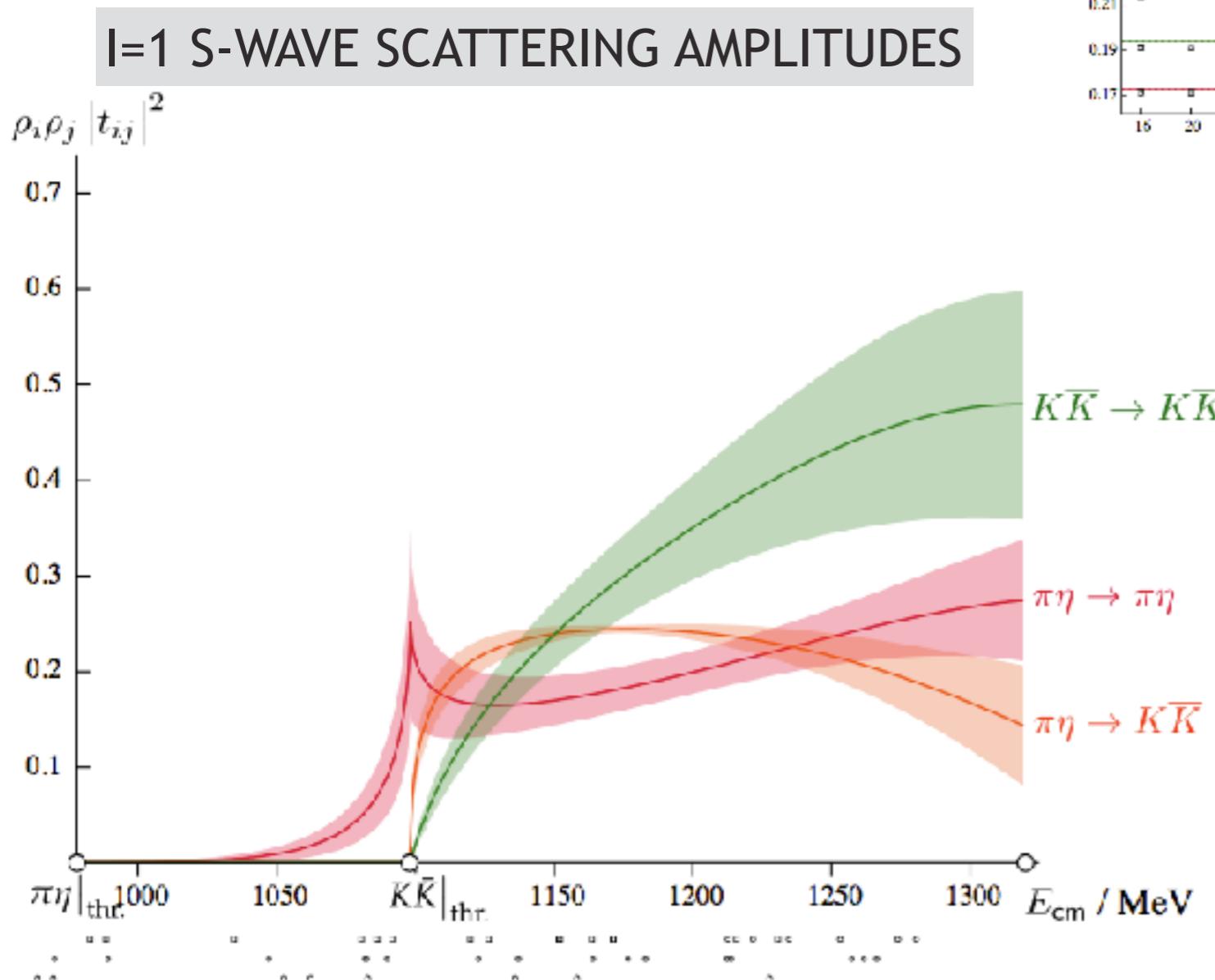
# Mysteries of scalar sector of QCD

## **a<sub>0</sub>** in coupled channel $\pi\eta/\bar{K}\bar{K}$

$$m_\pi = 391 \text{ MeV}$$

PRD93 094506 (2016)

LQCD FINITE VOLUME SPECTRUM



# Mysteries of scalar sector of QCD

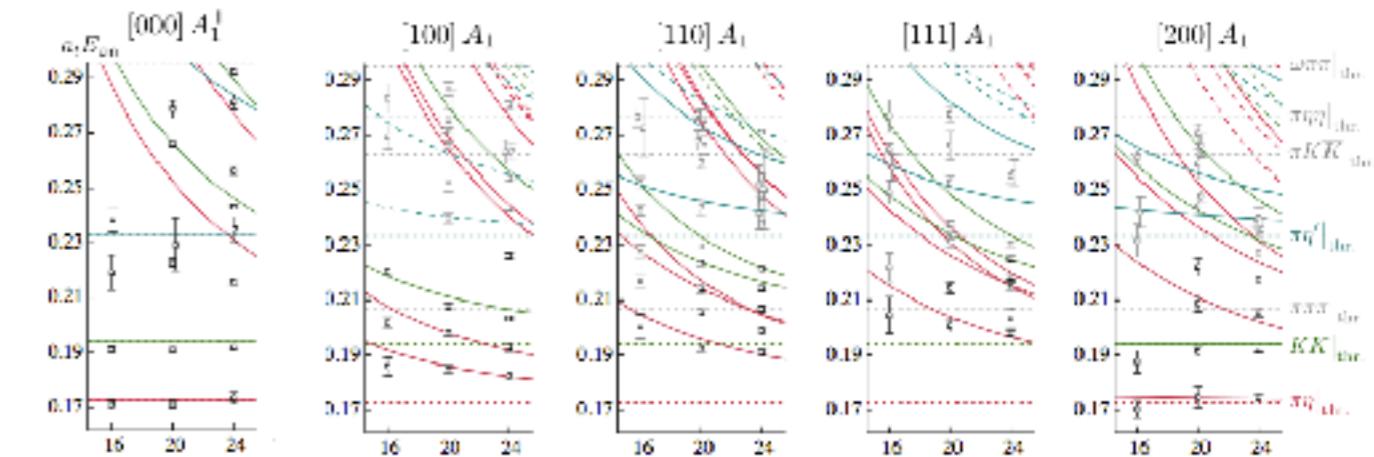
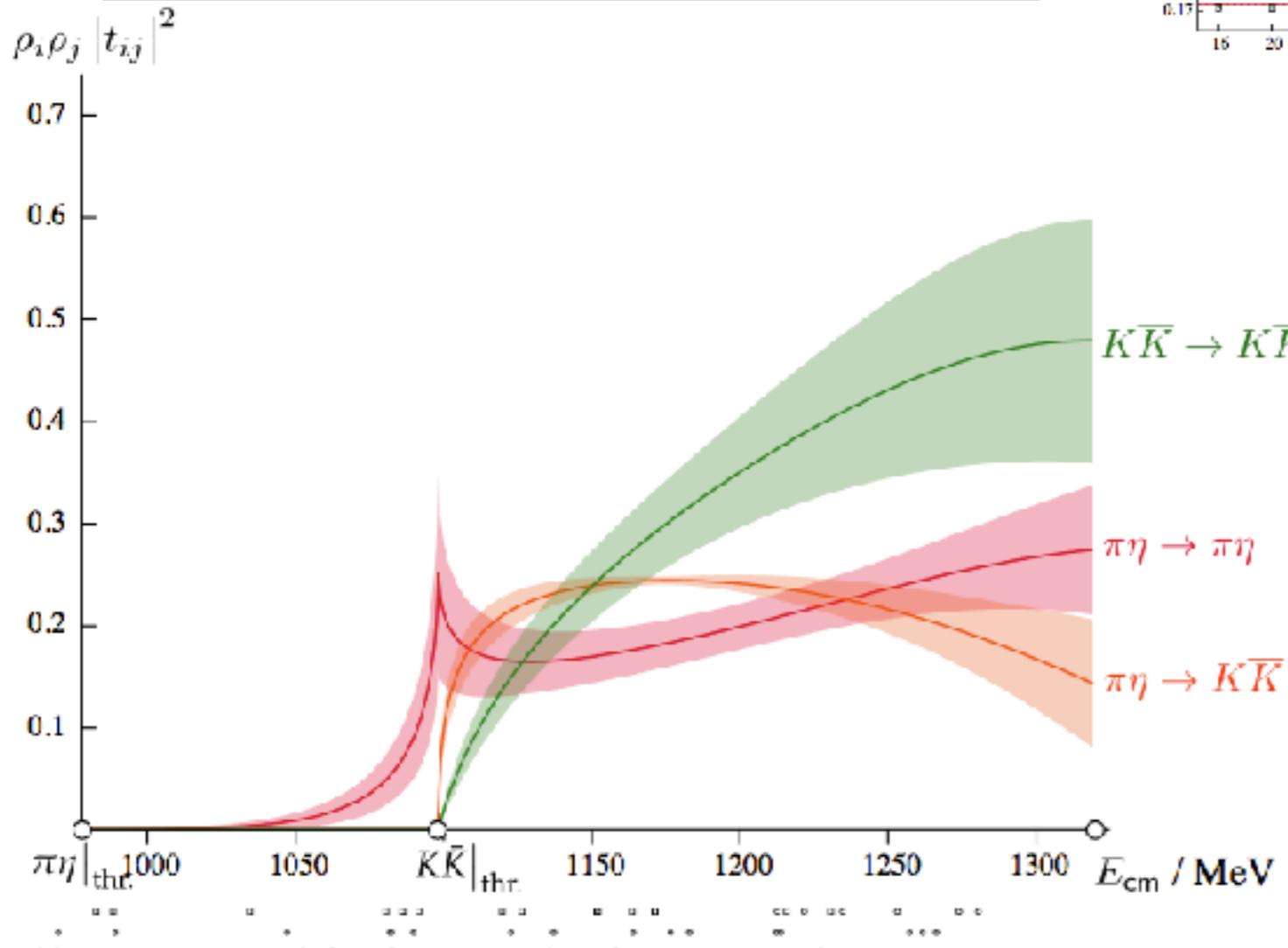
$a_0$  in coupled channel  $\pi\eta/K\bar{K}$

$m_\pi = 391$  MeV

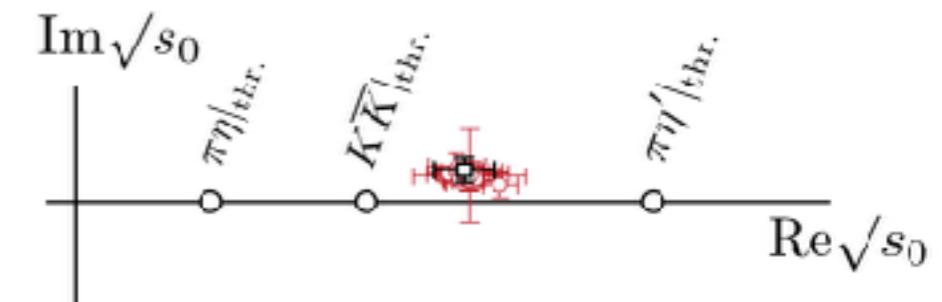
PRD93 094506 (2016)

LQCD FINITE VOLUME SPECTRUM

I=1 S-WAVE SCATTERING AMPLITUDES



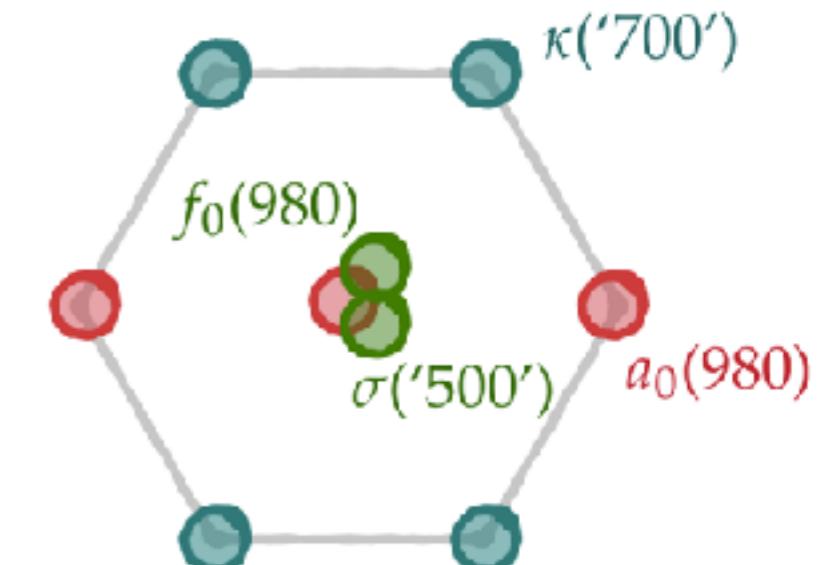
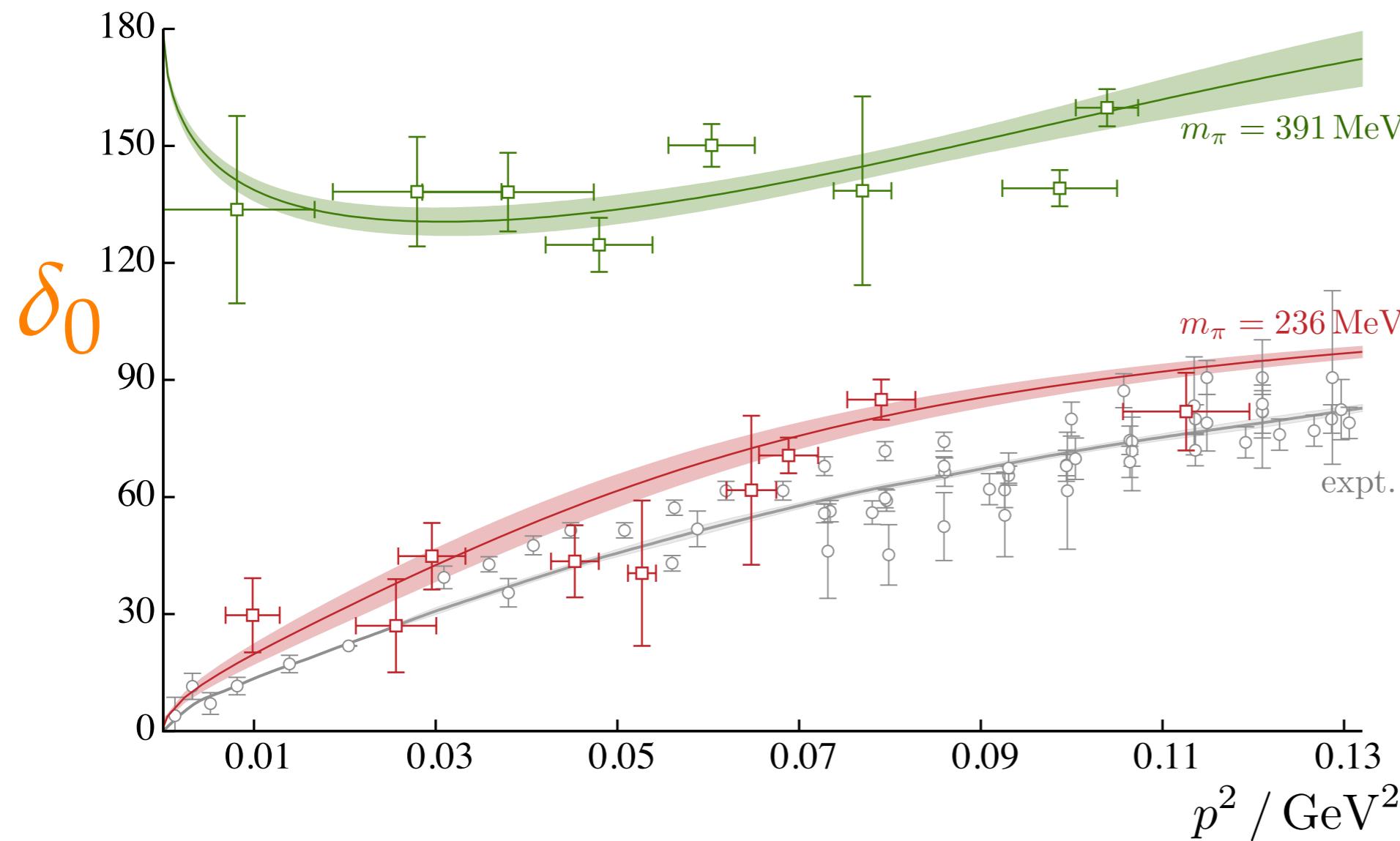
$a_0$  RESONANCE POLE



# Isoscalar-scalar sector

First QCD-based calculation

$\pi\pi \rightarrow \sigma \rightarrow \pi\pi$

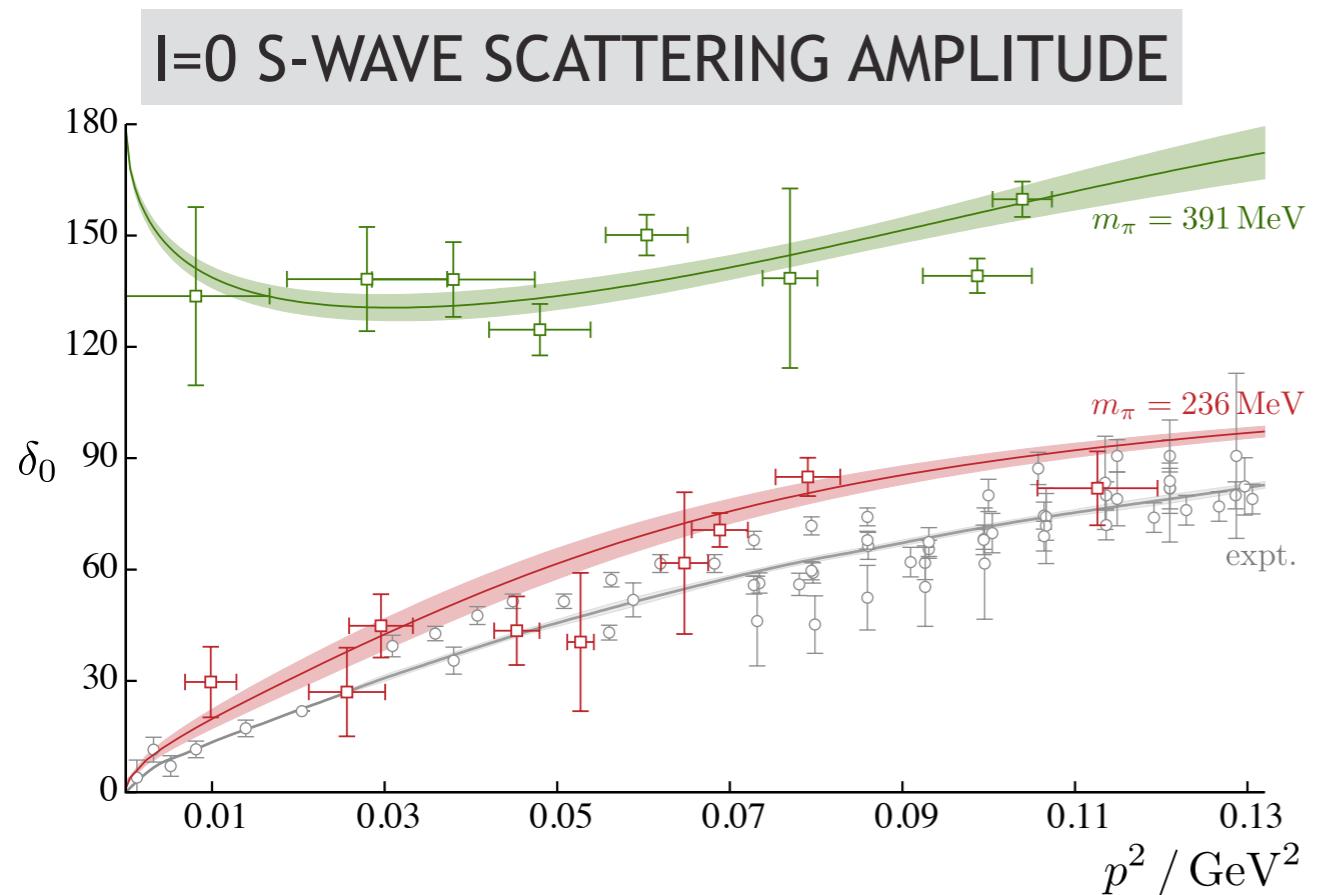


Decreasing  
pion mass

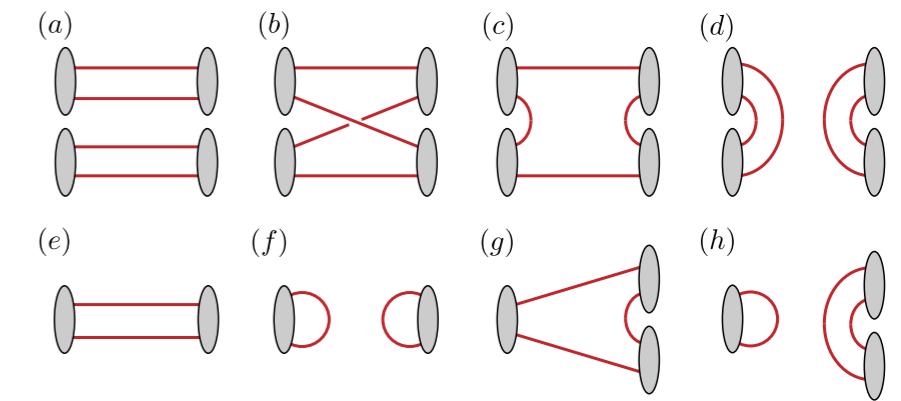
Transition from  
bound-state to  
resonant state

Sensitivity of S-wave  
coupling to quark mass ?

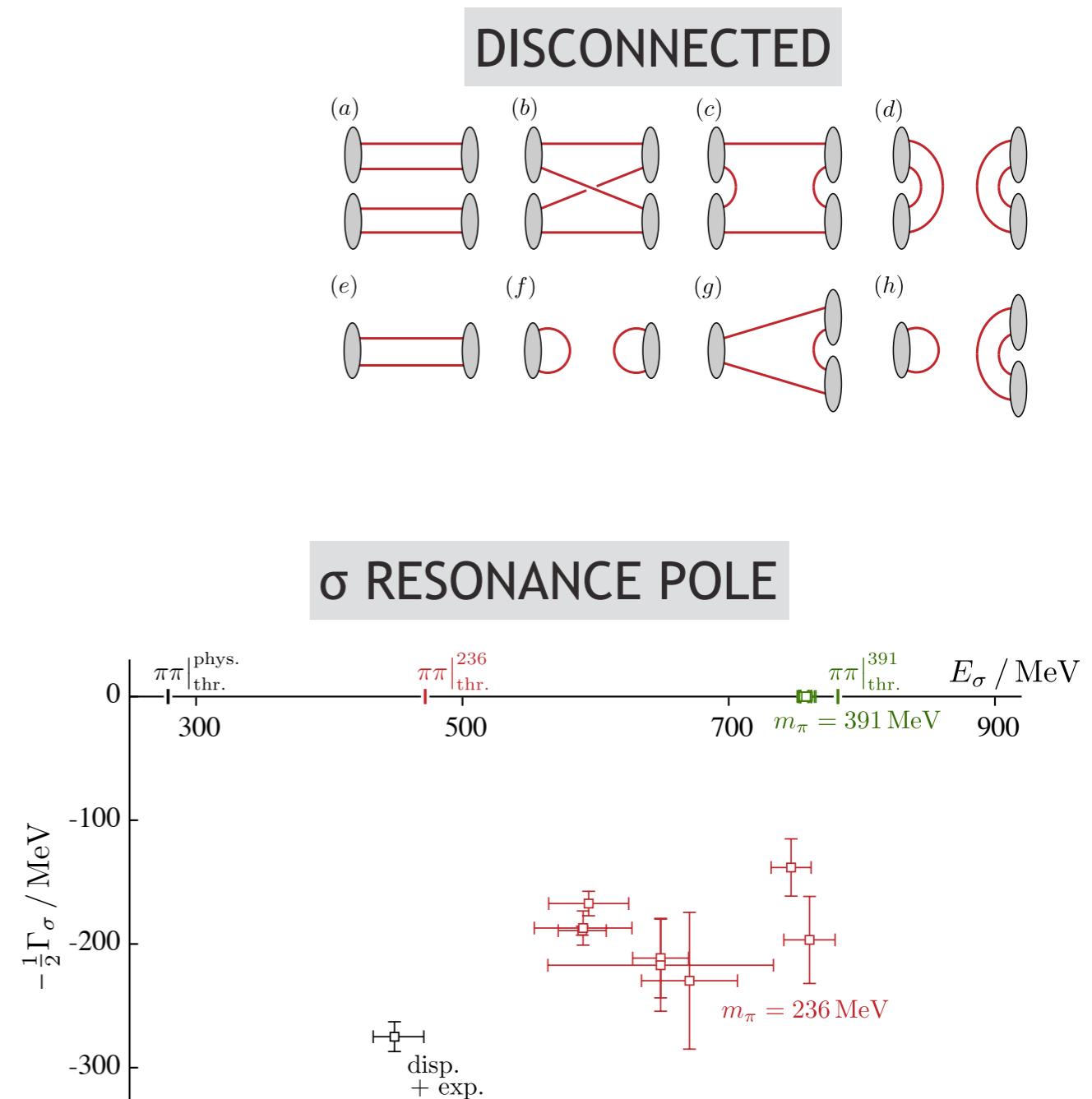
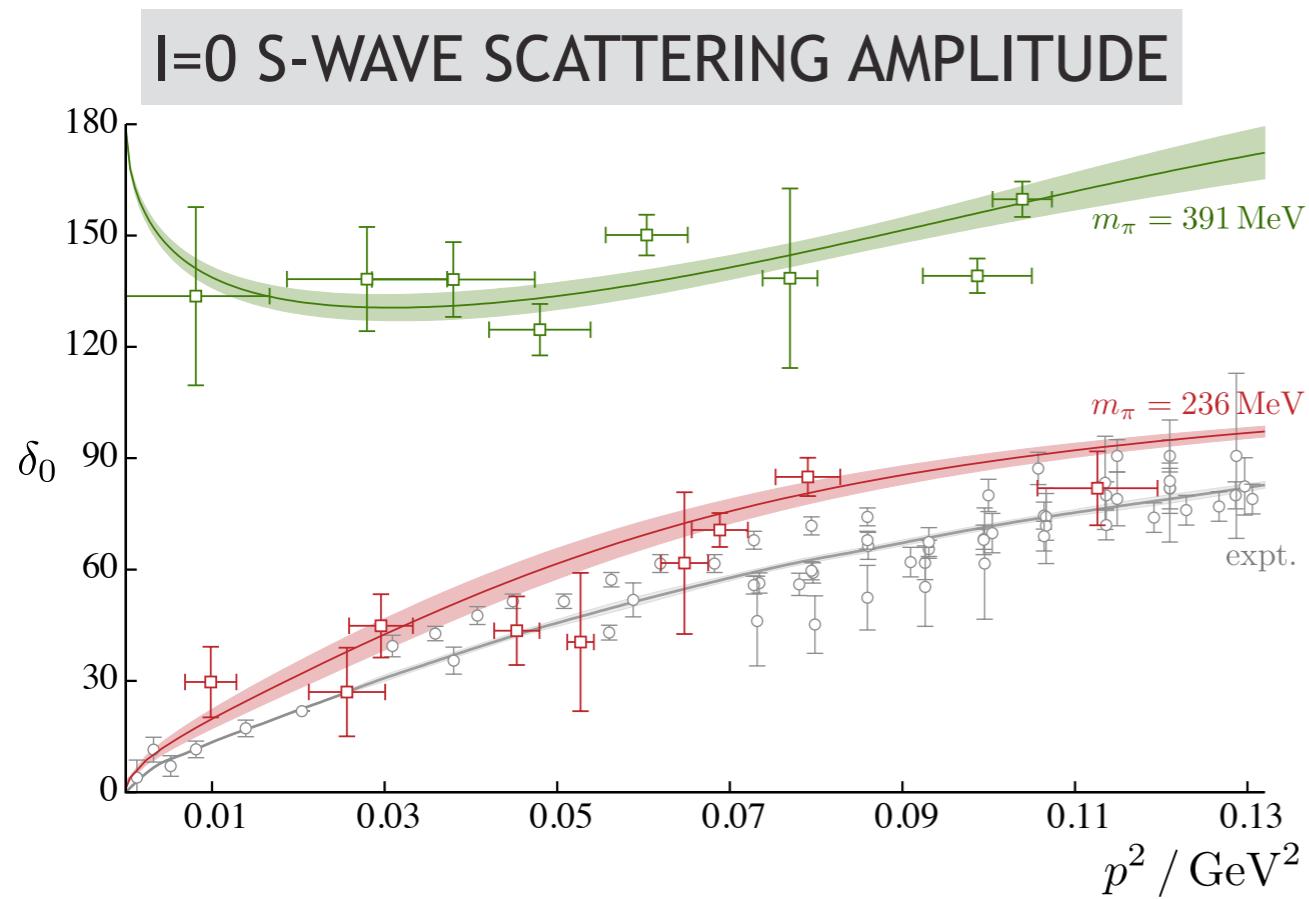
# The $\sigma$ meson



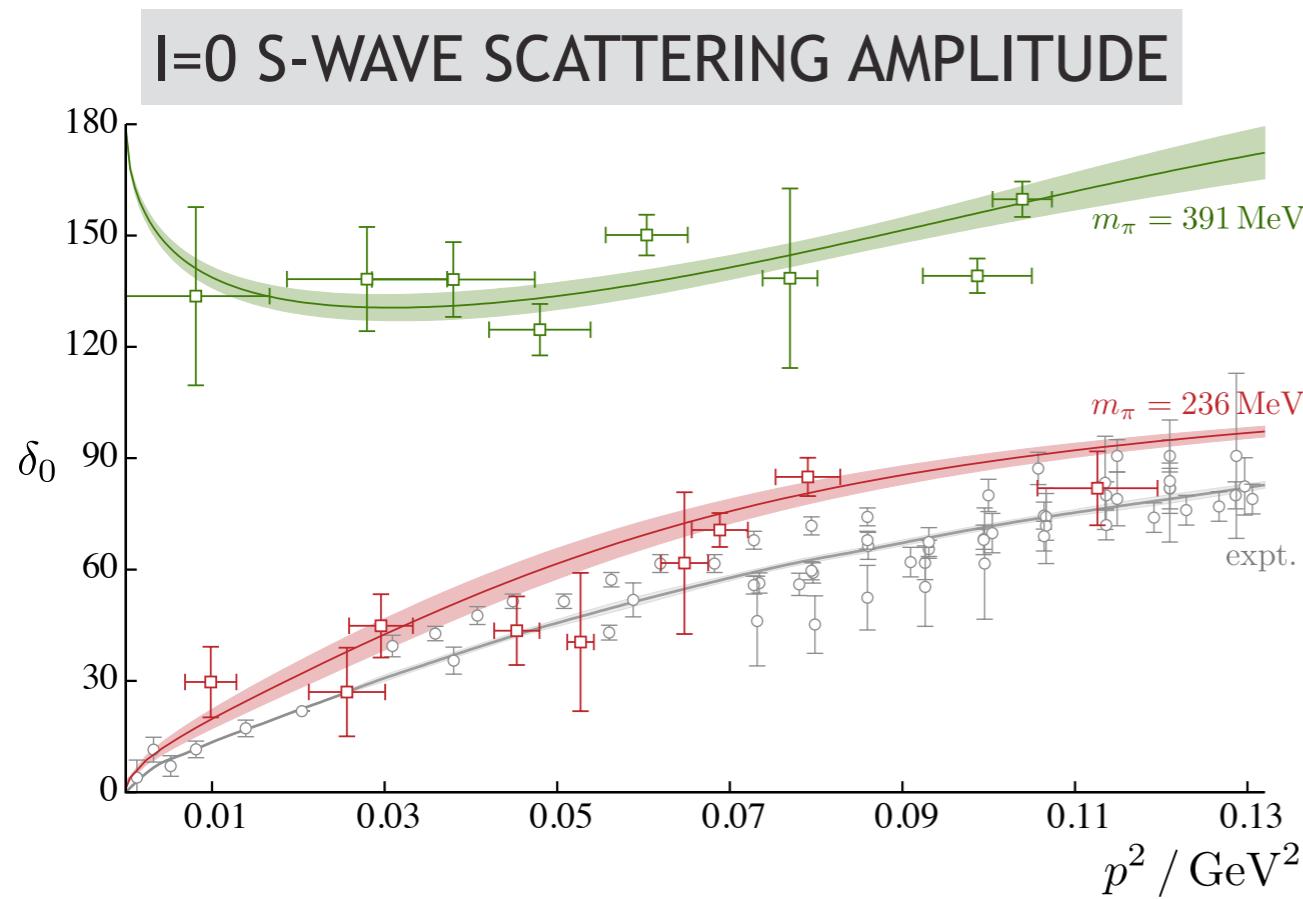
## DISCONNECTED



# The $\sigma$ meson

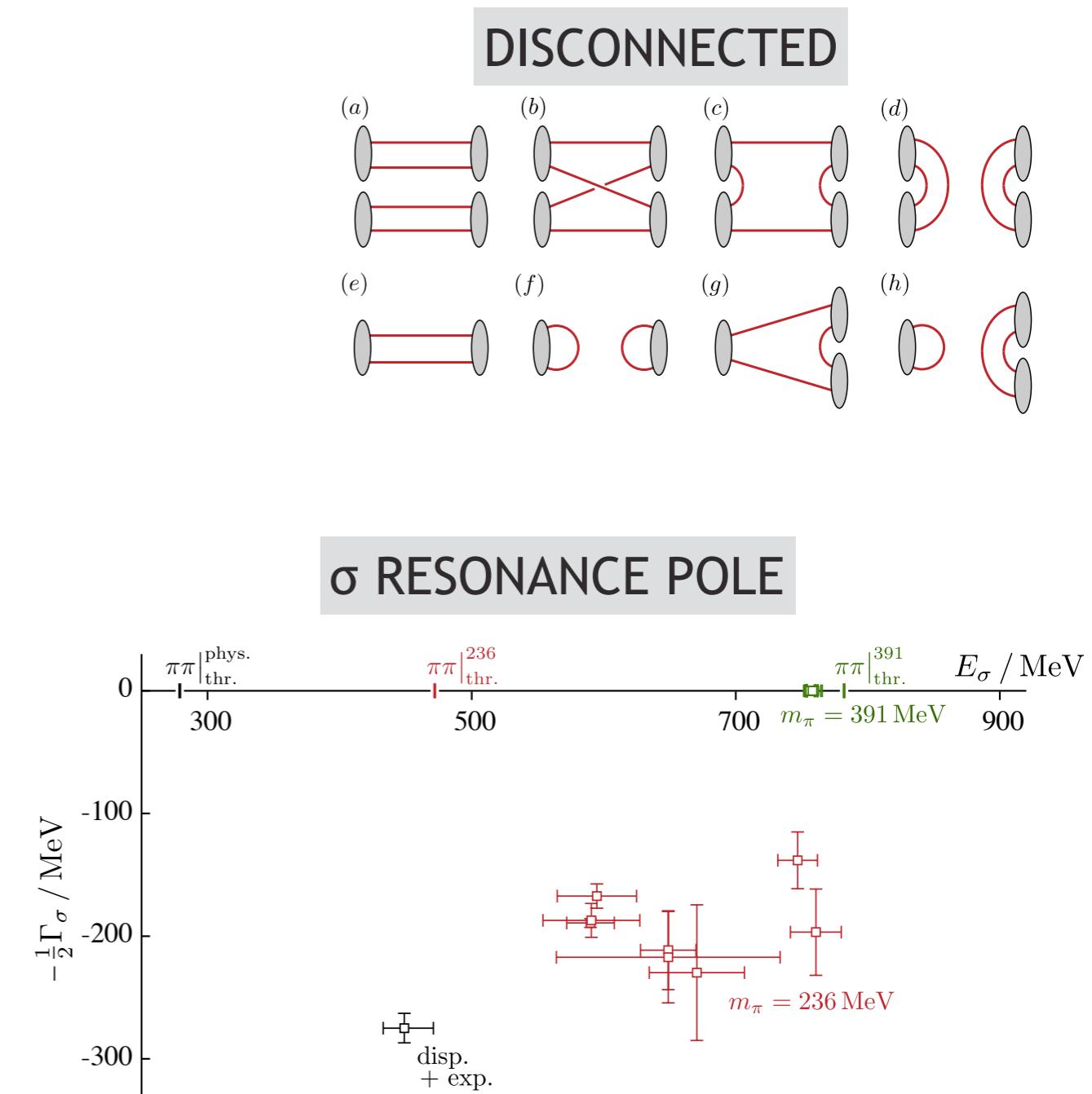


# The $\sigma$ meson



**bound state  $\rightarrow$  broad resonance**

$\rightarrow$  Precise pole determination requires  
more constrained amplitudes



e.g., Roy eqns - cross channels, unitarity, ...

# Scattering with external currents

- E.g.  $\pi\gamma \rightarrow \pi\pi$  in  $P$ -wave : the  $\rho$  appears as a resonance

- The observables are the amplitudes  $A_\ell(E_{\pi\pi}, Q^2)$

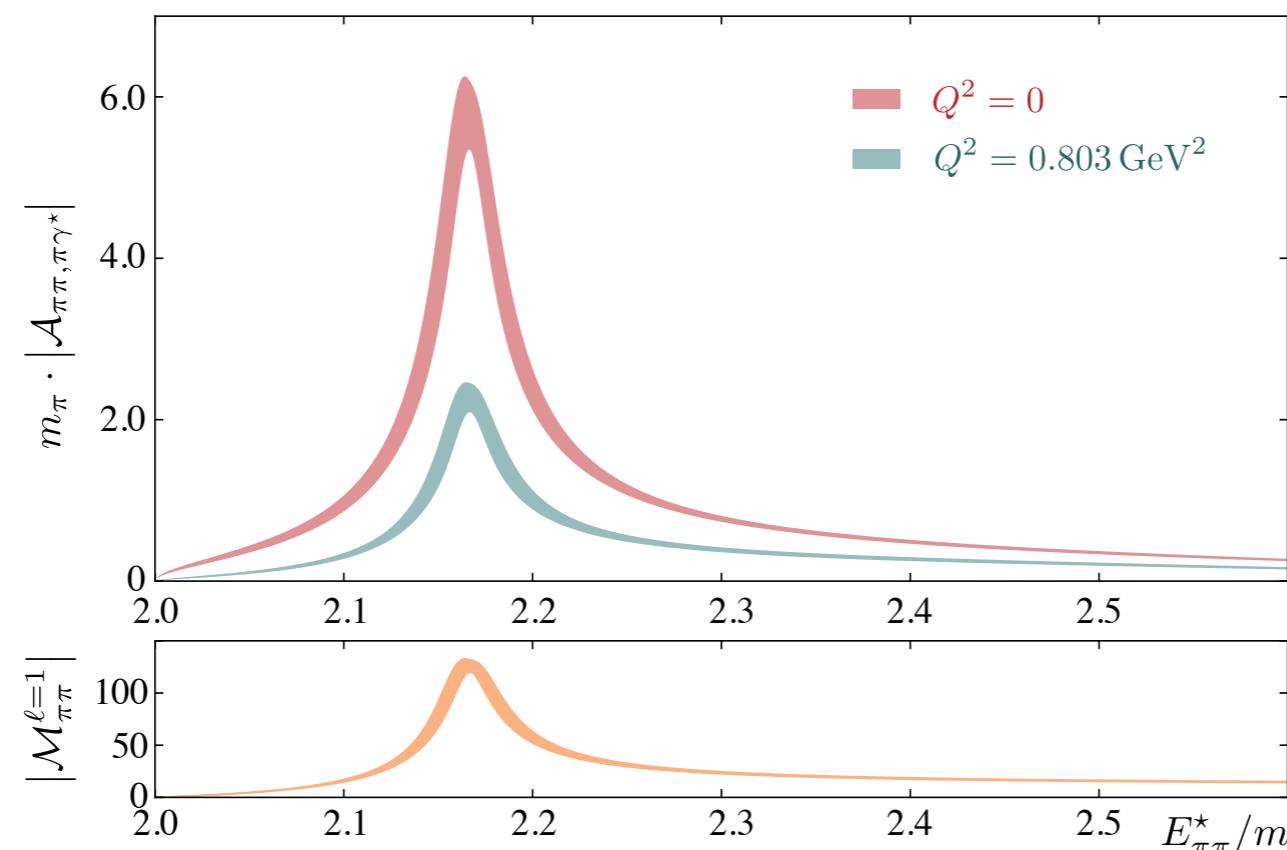
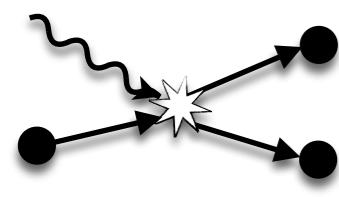
- Can be obtained from correlation functions in finite-volume

$$\langle 0 | \Omega_X(t_f) j^\mu(t) \Omega_\pi^\dagger(0) | 0 \rangle$$

PHYSICAL REVIEW D 91, 034501 (2015)

Multichannel  $1 \rightarrow 2$  transition amplitudes in a finite volume

Raúl A. Briceño,<sup>1,\*</sup> Maxwell T. Hansen,<sup>2,†</sup> and André Walker-Loud<sup>1,3,‡</sup>



$m_\pi = 391 \text{ MeV}$

PRL115 (2015)  
PRD93 (2016)

# Goals

- Compute **properties** of highly excited hadrons, including **hybrid mesons & baryons**  
masses, decays,  
couplings, ...  
searches in  
GlueX, CLAS12, ...
- Understand the dynamical mechanisms that form hadrons from quarks and gluons
  - Demonstrated efficacy of finite-volume methods
  - S-matrix formalism increasingly important going forward

e.g.,

SCALARS	HYBRID MESONS	BARYONS
Evolution of poles with varying quark mass	Decays (initially at heavy masses) → $(b_1\pi)_S, (\rho\pi)_P, (\eta\pi)_P, (\eta'\pi)_P$ → Photo decays	Simpler(?) decays (also at heavy masses) $\Lambda(1405) \rightarrow (\Sigma\pi / N\bar{K})_S$

# Hadron Spectrum Collaboration

## JEFFERSON LAB

Raul Briceno  
Jozef Dudek  
Robert Edwards  
Balint Joo  
David Richards  
Frank Winter

## MESON SPECTRUM

*PRL* 103 262001 (2009)  $I = 1$   
*PRD* 82 034508 (2010)  $I = 1, K^*$   
*PRD* 83 111502 (2011)  $I = 0$   
*JHEP* 07 126 (2011)  $c\bar{c}$   
*PRD* 88 094505 (2013)  $I = 0$   
*JHEP* 05 021 (2013)  $D, D_s$

## HADRON SCATTERING

*PRD* 83 071504 (2011)  $\pi\pi I = 2$   
*PRD* 86 034031 (2012)  $\pi\pi I = 2$   
*PRD* 87 034505 (2013)  $\pi\pi I = 1, \rho$   
*PRL* 113 182001 (2014)  $\pi K, \eta K$   
*PRD* 91 054008 (2015)  $\pi K, \eta K$   
*PRD* 92 094502 (2015)  
*PRD* 93 094506 (2016)  
*ARRIVE*: 1607.05900  
*ARRIVE*: 1607.07093

## TRINITY, DUBLIN

Mike Peardon  
Sinead Ryan  
David Wilson

## BARYON SPECTRUM

*PRD* 84 074508 (2011)  $(N, \Delta)^*$   
*PRD* 85 054016 (2012)  $(N, \Delta)_{\text{hyb}}$   
*PRD* 87 054506 (2013)  $(N \dots \Xi)^*$   
*PRD* 90 074504 (2014)  $\Omega_{ccc}^*$   
*PRD* 91 094502 (2015)  $\Xi_{cc}^*$

## MATRIX ELEMENTS

*PRD* 90 014511 (2014)  $f_{\pi^*}$   
*PRD* 91 114501 (2015)  $M' \rightarrow \gamma M$   
*PRL* 115 242001 (2015)  
*PRD* 93 114508 (2016)

## TATA, MUMBAI

Nilmani Mathur

## “TECHNOLOGY”

*PRD* 79 034502 (2009) lattices  
*PRD* 80 054506 (2009) distillation  
*PRD* 85 014507 (2012)  $\vec{p} > 0$

## CAMBRIDGE

Graham Moir  
Christopher Thomas

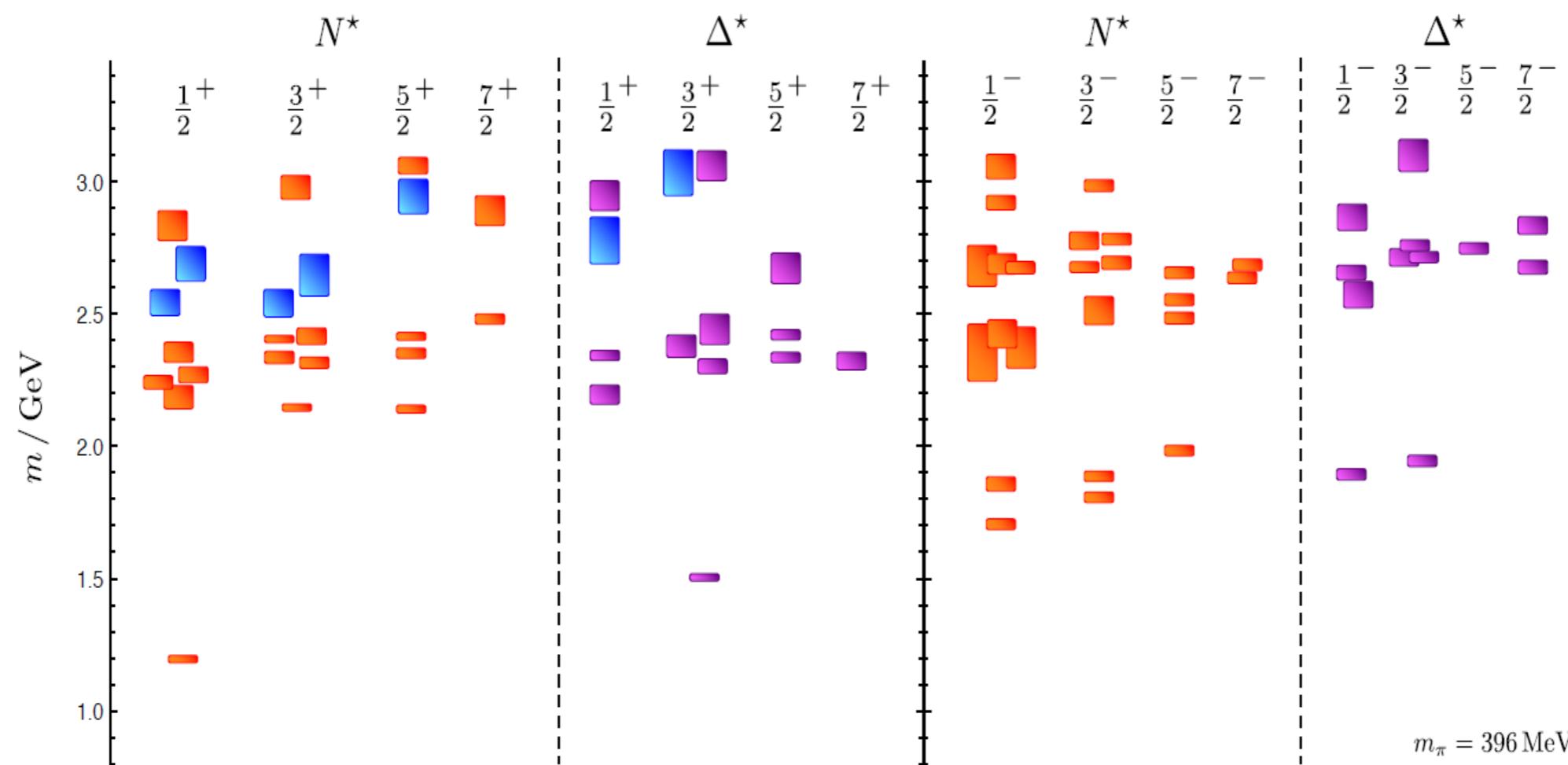
# Backup

---

# Are there missing baryons?

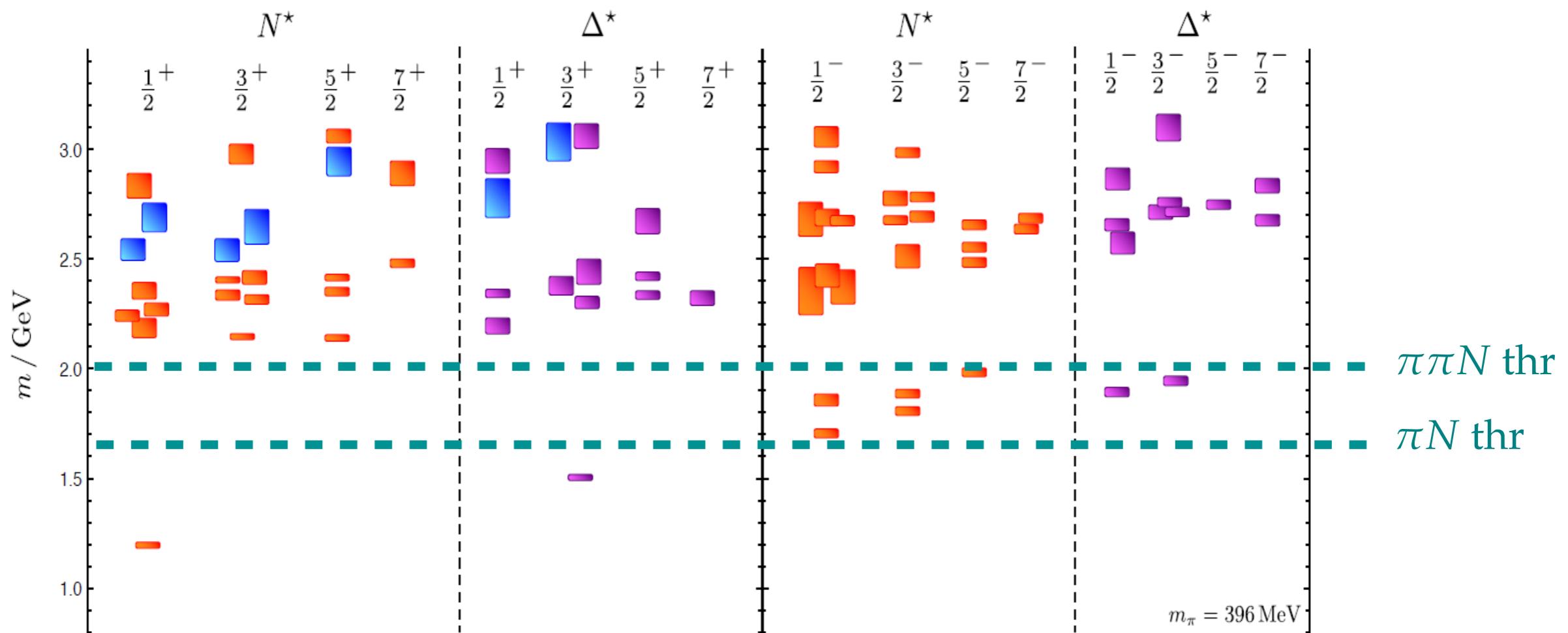
- Initial determination of spectrum with only  $qqq$  style operators
  - See rich spectrum, including hybrid-like states

PRD 84 & 85



# Are there missing baryons?

- Initial determination of spectrum with only  $qqq$  style operators *PRD 84 & 85*
- See rich spectrum, including hybrid-like states
- However, no operators that look like  $\pi N$  or  $\pi\pi N$  - missing scattering states

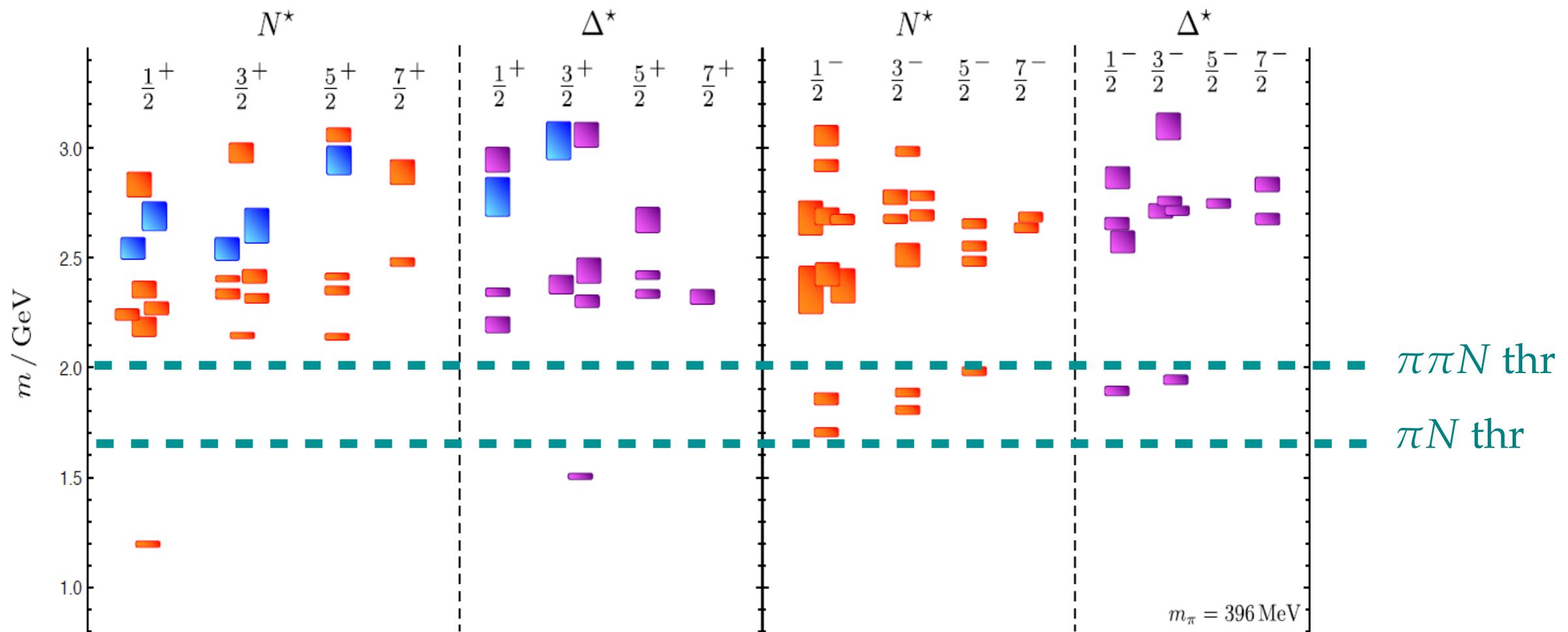


# Are there missing baryons?

- Initial determination of spectrum with only  $qqq$  style operators
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  - Some initial results in S11 & P13 have appeared

PRD 84 & 85

GRAZ GROUP



# Are there missing baryons?

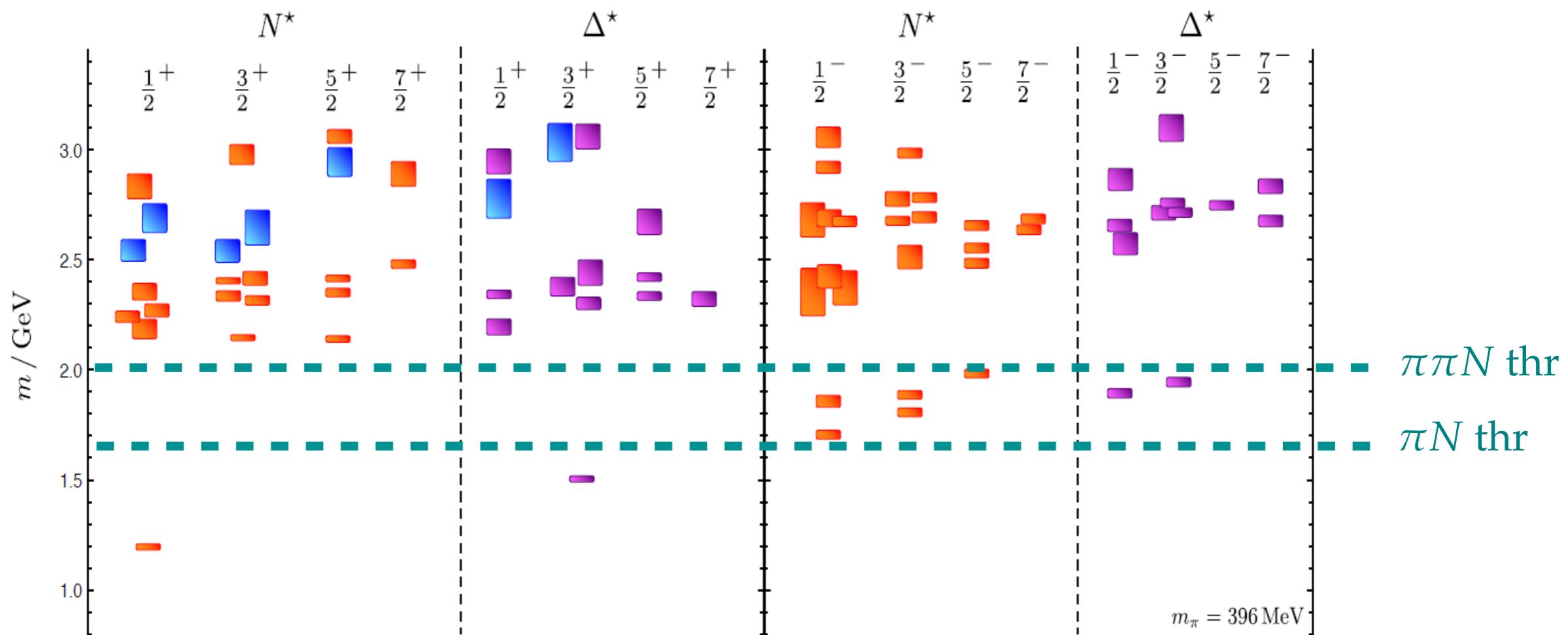
- Initial determination of spectrum with only  $qqq$  style operators
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  - However, no operators that look like  $\pi N$  or  $\pi\pi N$  - missing scattering states
  - Some initial results in S11 & P13 have appeared

PRD 84 & 85

GRAZ GROUP

- Development of three-body formalism required

HANSEN & SHARPE - MUCH PROGRESS



# $\pi K/\eta K$ scattering & kaon resonances

- Example of coupled-channel scattering



- Compute finite-volume spectrum

$$\bar{u} \Gamma s$$

$$\sum_{\hat{k}_1, \hat{k}_2} C(\Lambda, \vec{P}; \vec{k}_1, \vec{k}_2) \pi^\dagger(\vec{k}_1) K^\dagger(\vec{k}_2)$$

$$\sum_{\hat{k}_1, \hat{k}_2} C(\Lambda, \vec{P}; \vec{k}_1, \vec{k}_2) \eta^\dagger(\vec{k}_1) K^\dagger(\vec{k}_2)$$

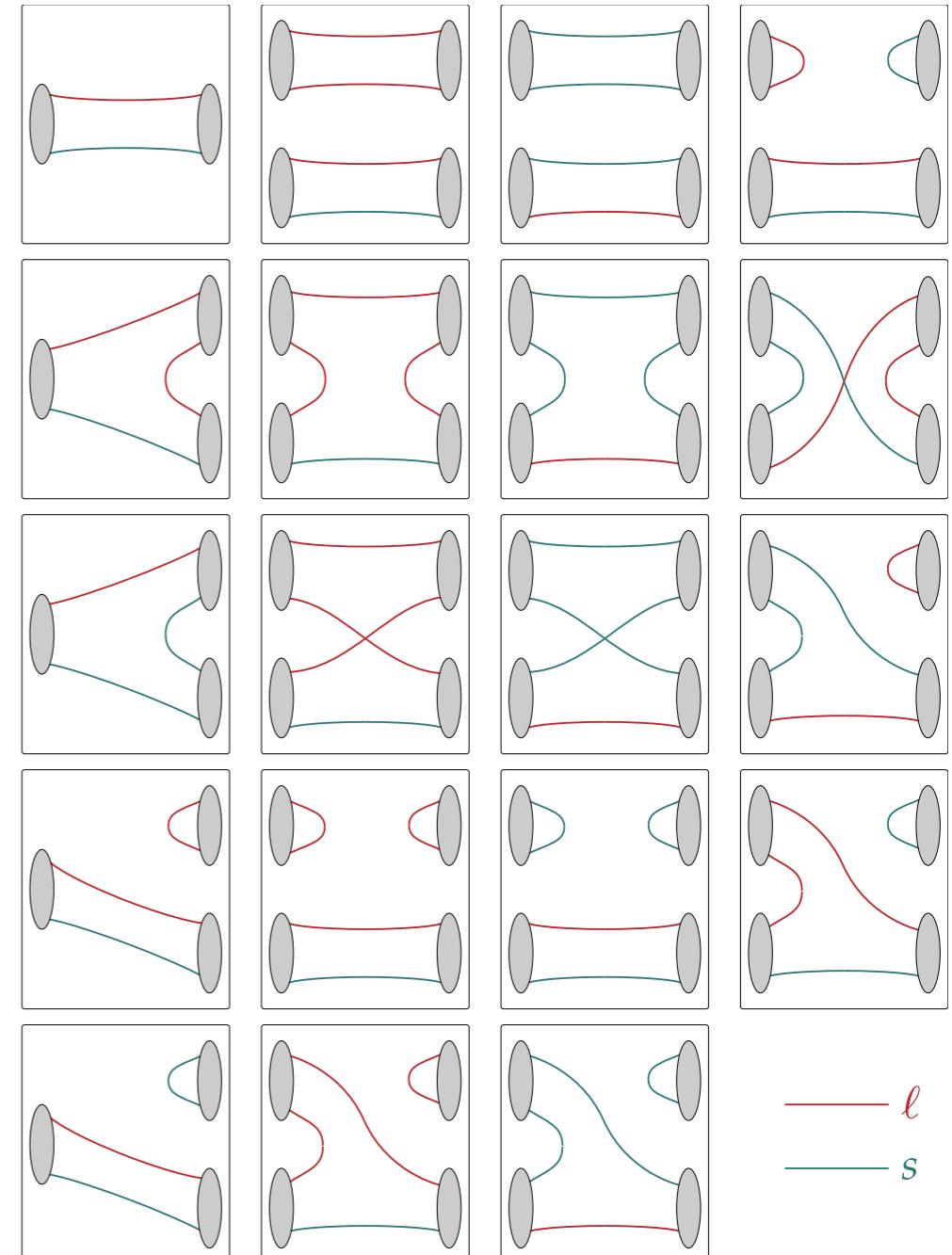
PRL 113 182001  
PRD 91 054008

# $\pi K/\eta K$ scattering & kaon resonances

- Example of coupled-channel scattering



WICK CONTRACTIONS



- Compute finite-volume spectrum

$$\bar{u} \Gamma s$$

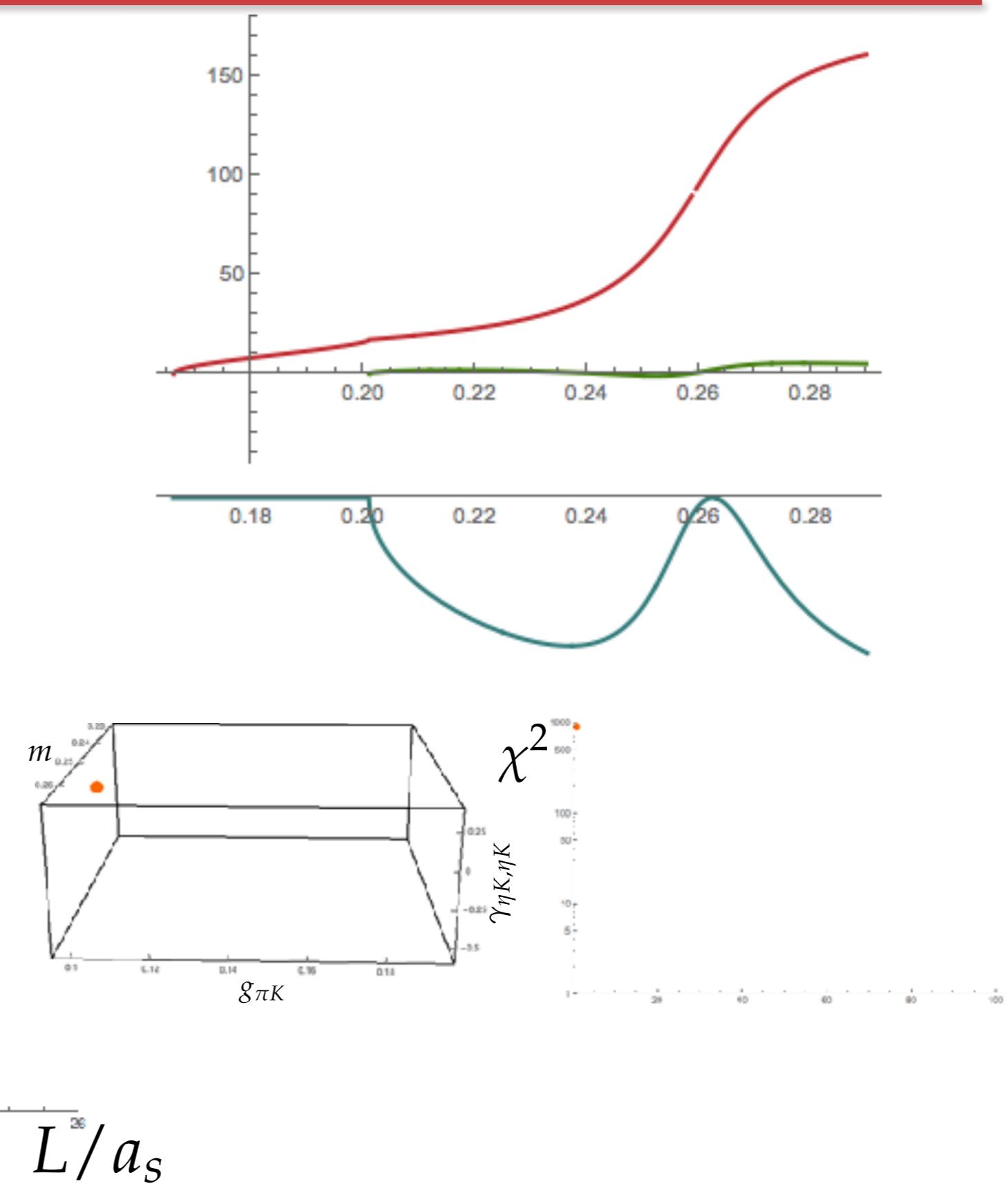
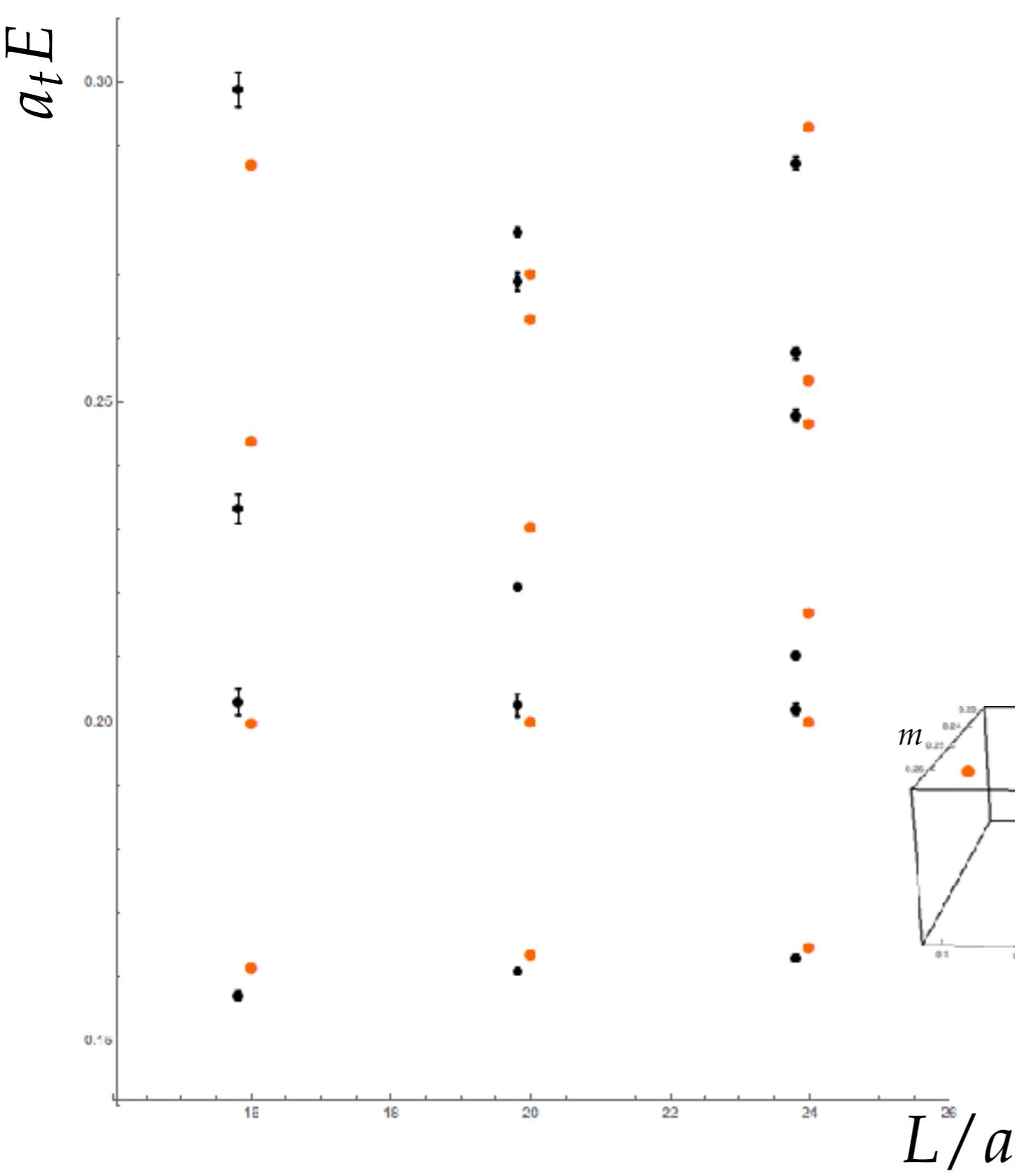
$$\sum_{\hat{k}_1, \hat{k}_2} C(\Lambda, \vec{P}; \vec{k}_1, \vec{k}_2) \pi^\dagger(\vec{k}_1) K^\dagger(\vec{k}_2)$$

$$\sum_{\hat{k}_1, \hat{k}_2} C(\Lambda, \vec{P}; \vec{k}_1, \vec{k}_2) \eta^\dagger(\vec{k}_1) K^\dagger(\vec{k}_2)$$

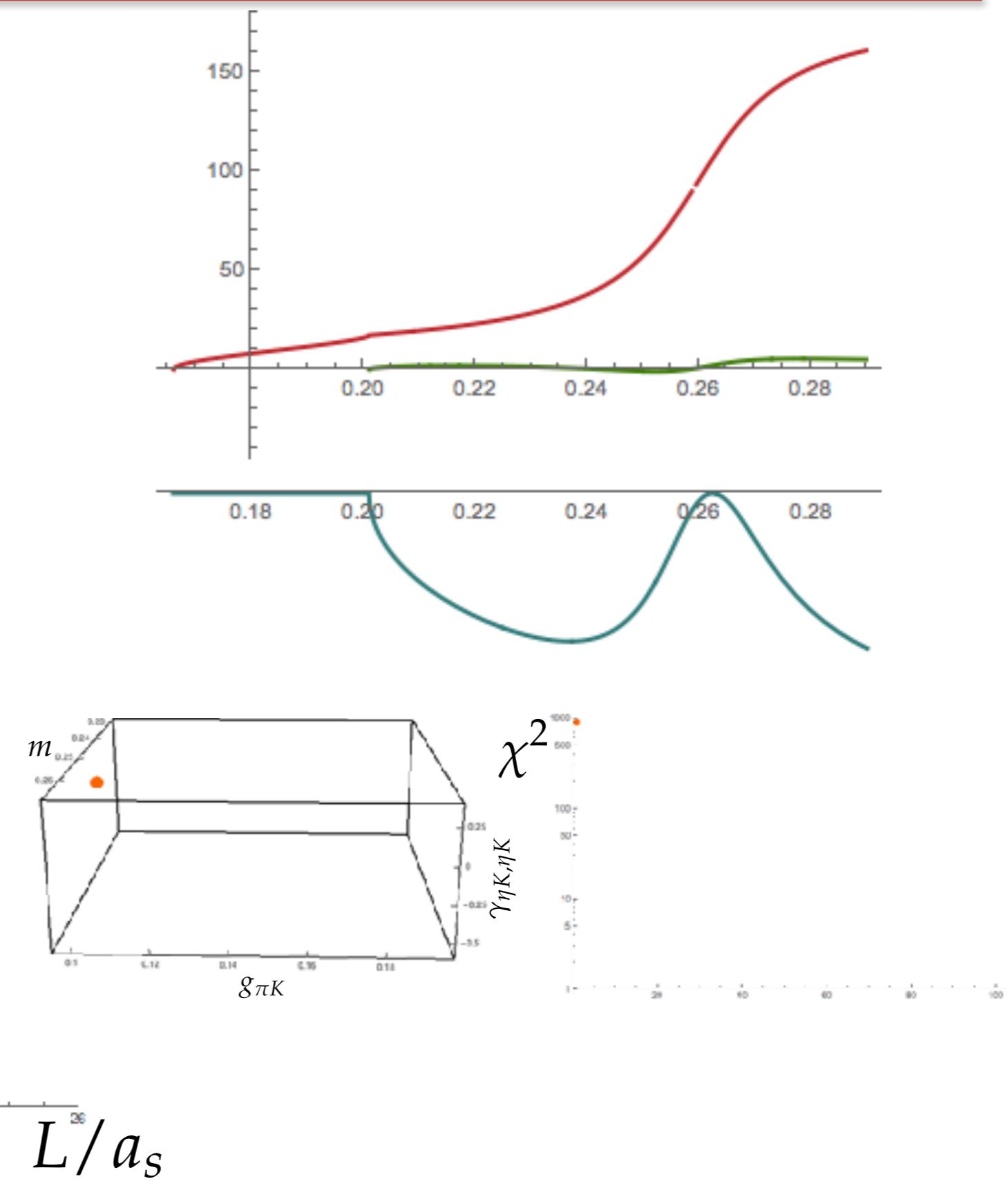
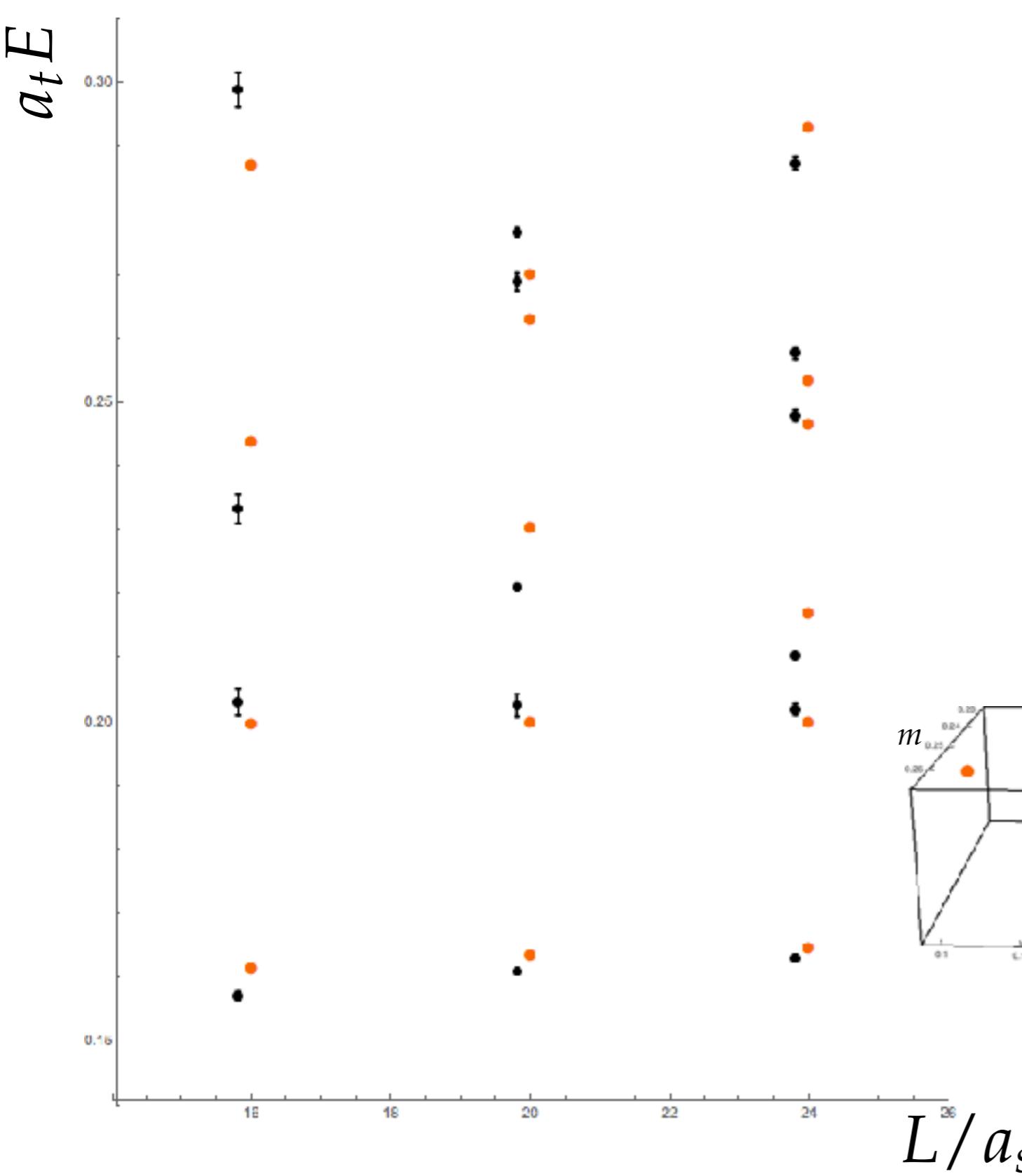
PRL 113 182001  
PRD 91 054008

—  $\ell$   
—  $s$

# $\pi K/\eta K$ scattering



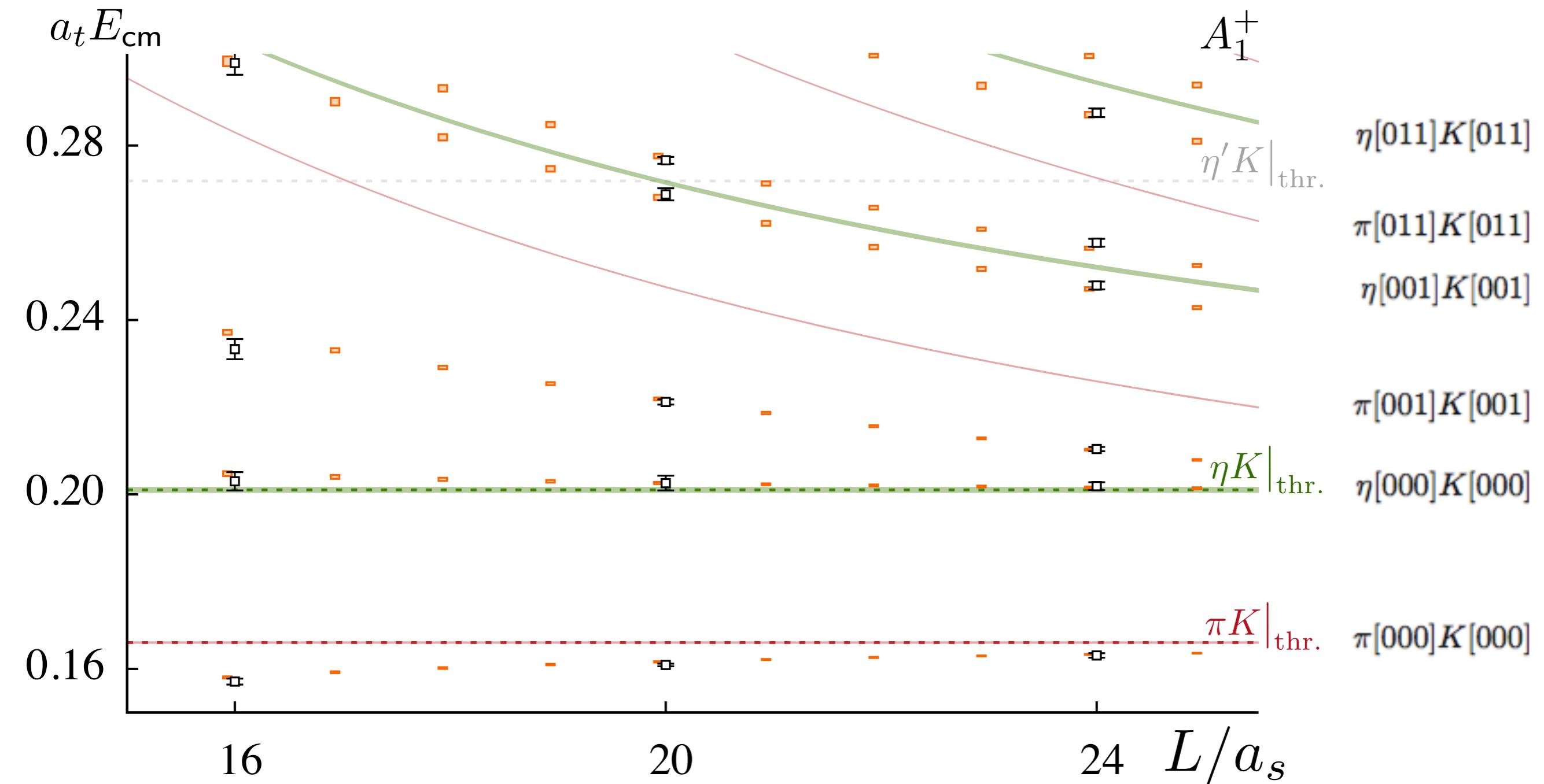
# $\pi K/\eta K$ scattering



# $\pi K/\eta K$ scattering

$$\chi^2/N_{\text{dof}} = \frac{6.40}{15 - 6} = 0.71$$

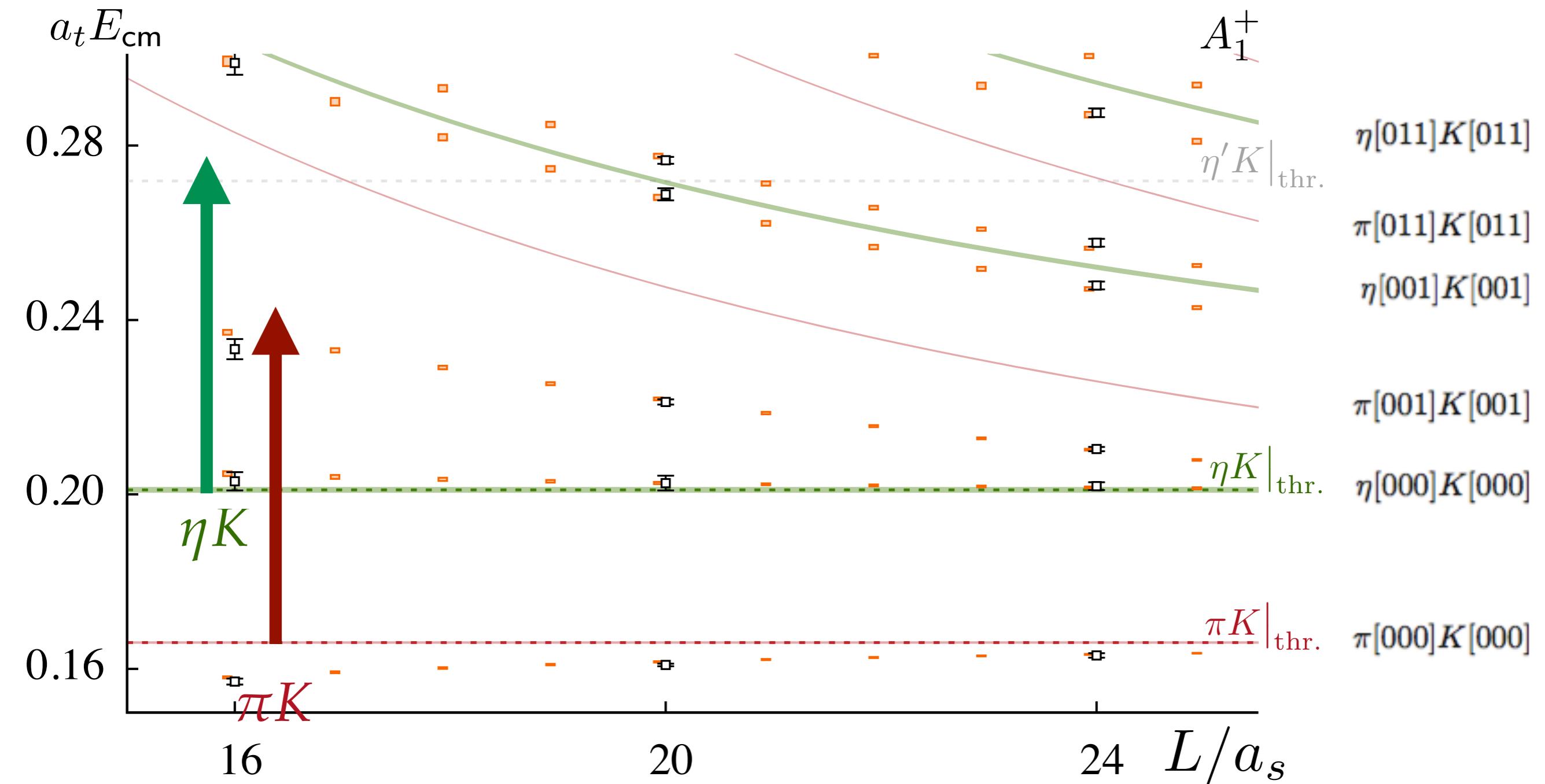
$m_\pi \sim 391 \text{ MeV}$



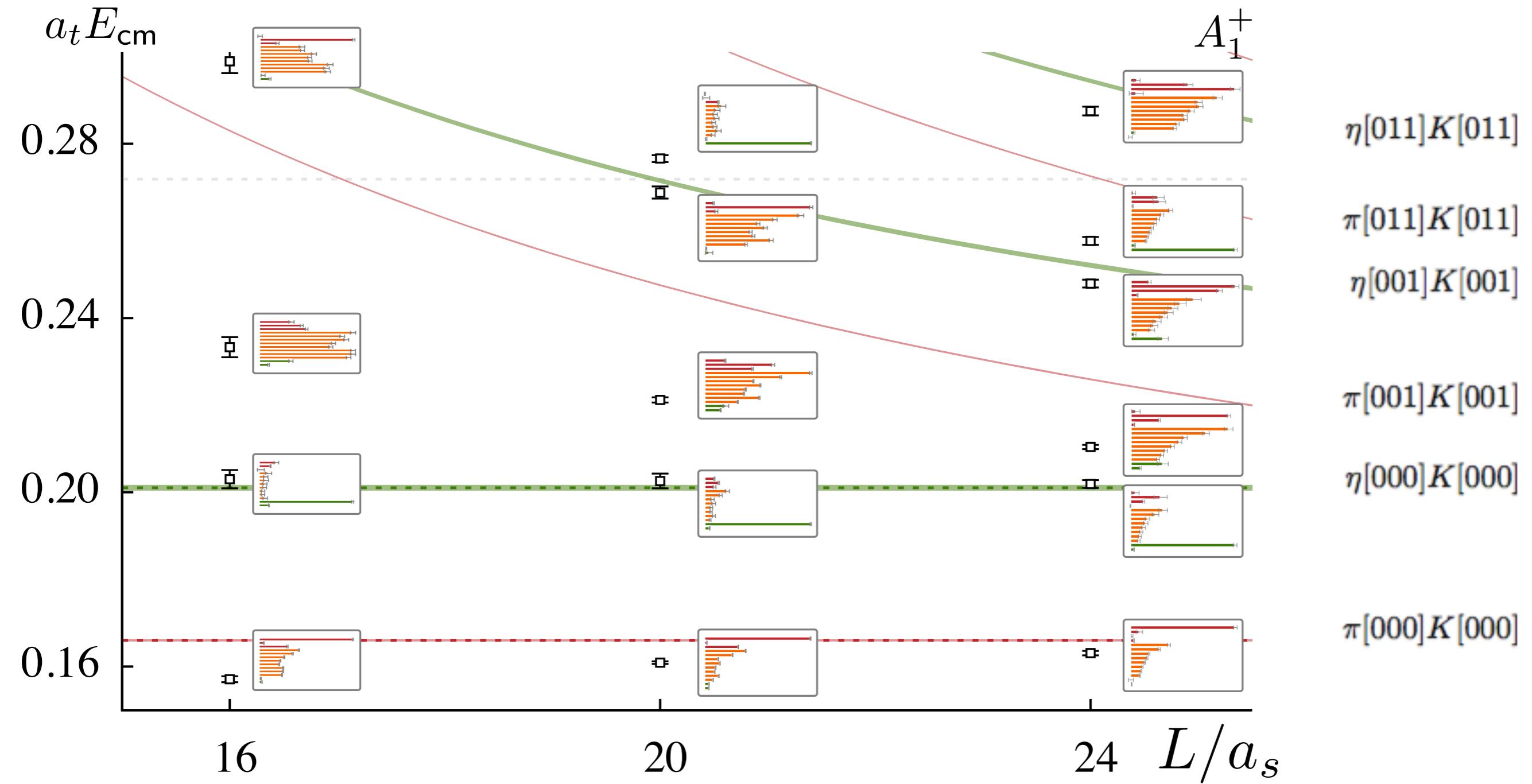
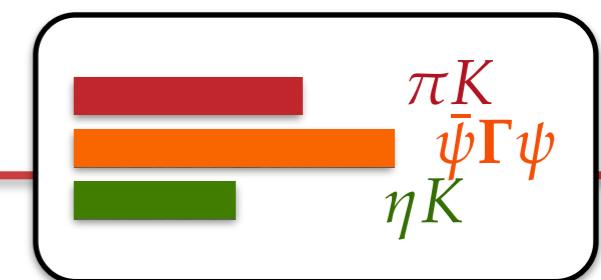
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# $\pi K/\eta K$ scattering



# $\pi K/\eta K$ scattering

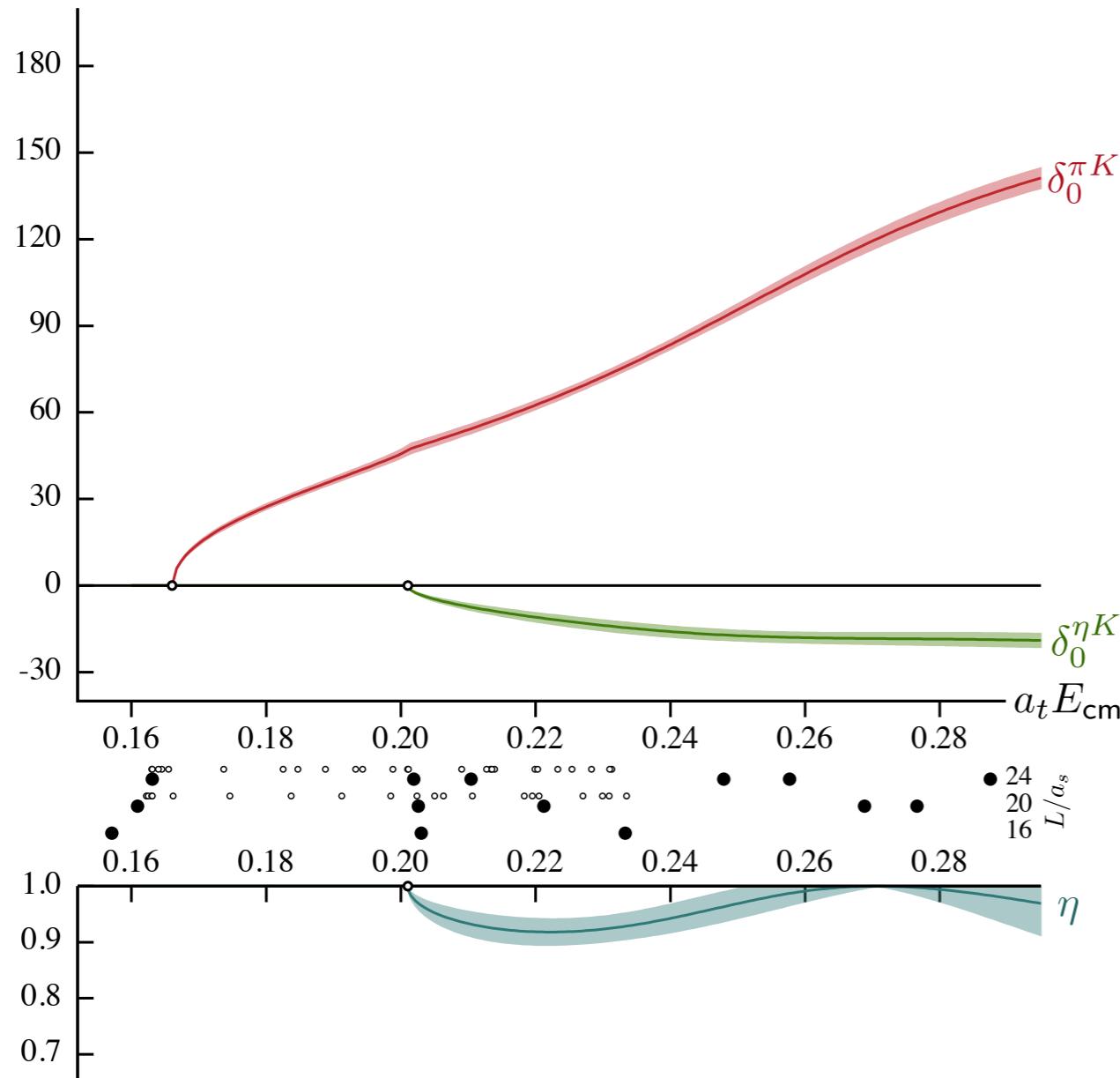
$m_\pi \sim 391$  MeV

- Describe all the finite-volume spectra

$$\chi^2/N_{\text{dof}} = \frac{49.1}{61 - 6} = 0.89$$

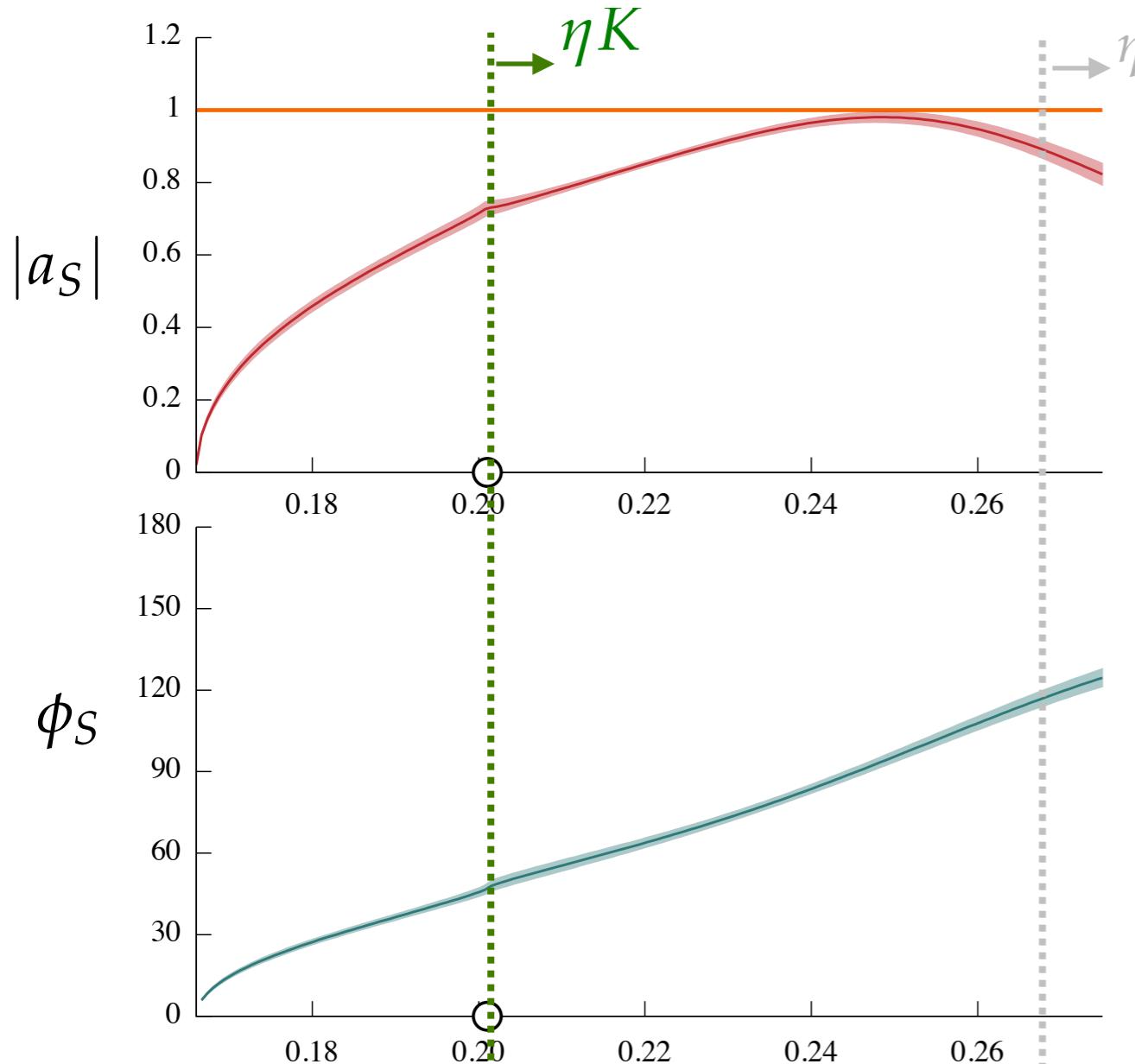
$$S_{\pi K, \pi K} = \eta e^{2i\delta^{\pi K}}$$
$$S_{\eta K, \eta K} = \eta e^{2i\delta^{\eta K}}$$

## S-WAVE $\pi K/\eta K$ SCATTERING

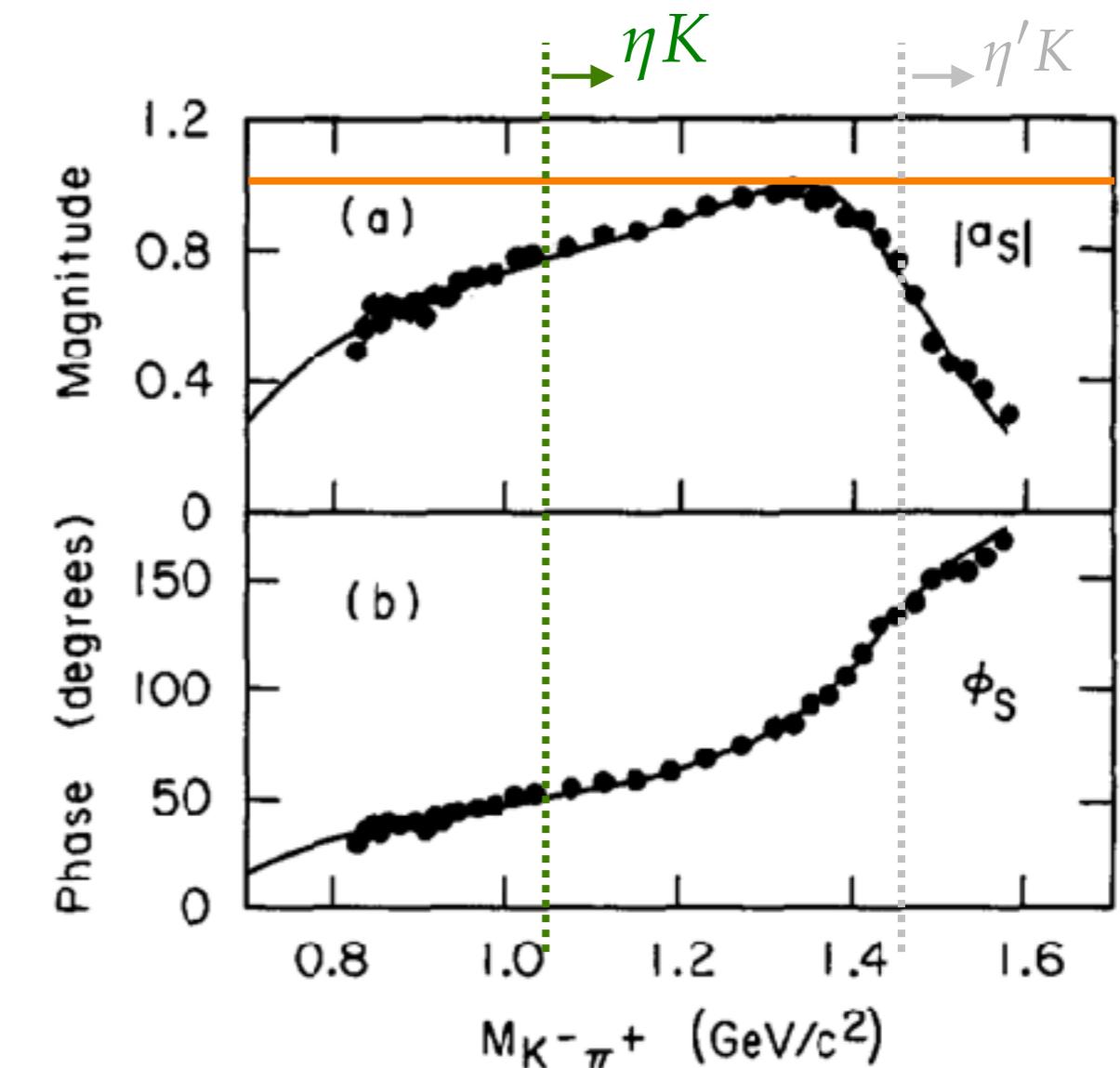


# Versus experimental scattering

## S-WAVE $\pi K \rightarrow \pi K$ AMPLITUDE



$m_\pi \sim 391 \text{ MeV}$

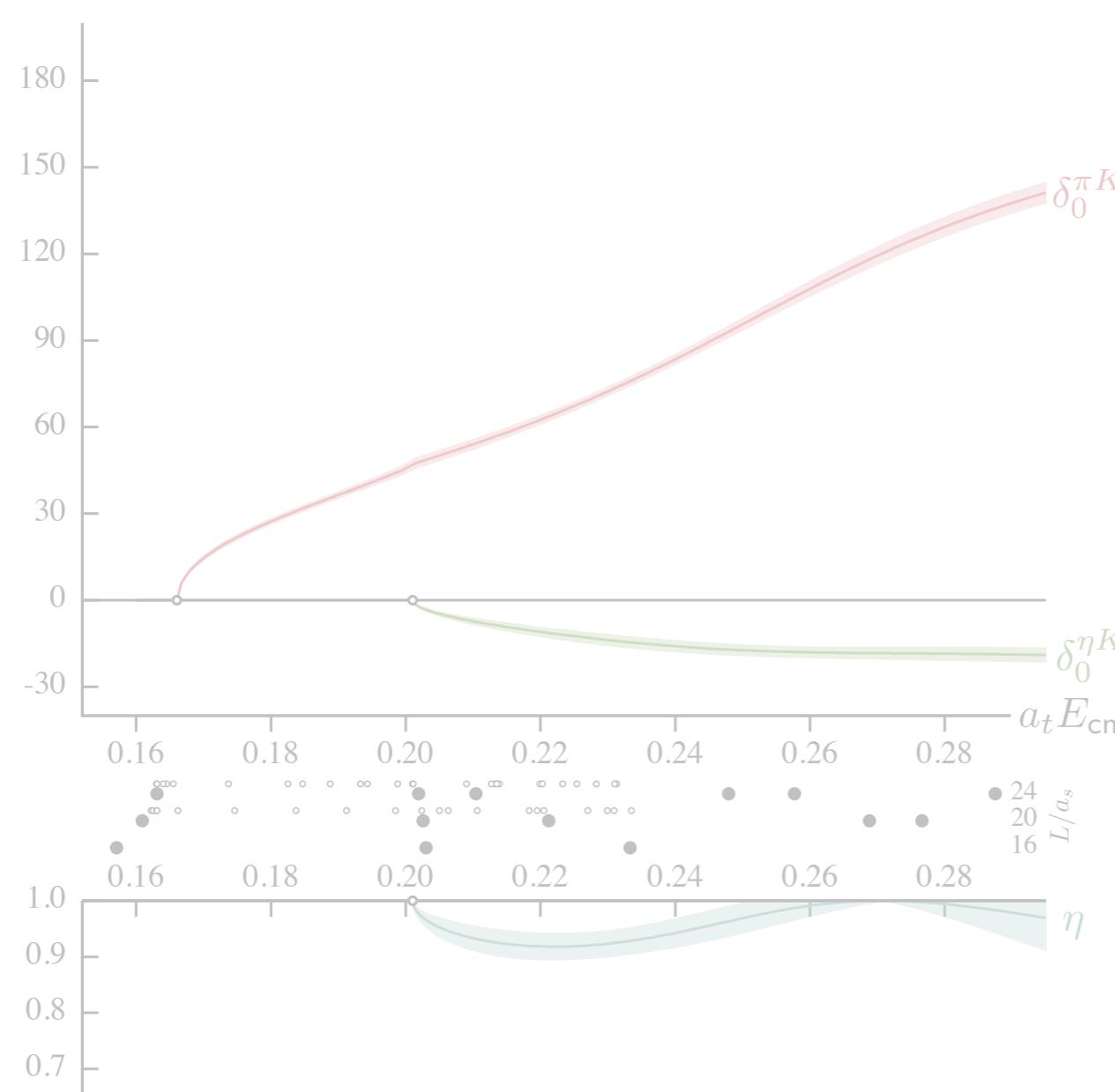


LASS, NPB296 493

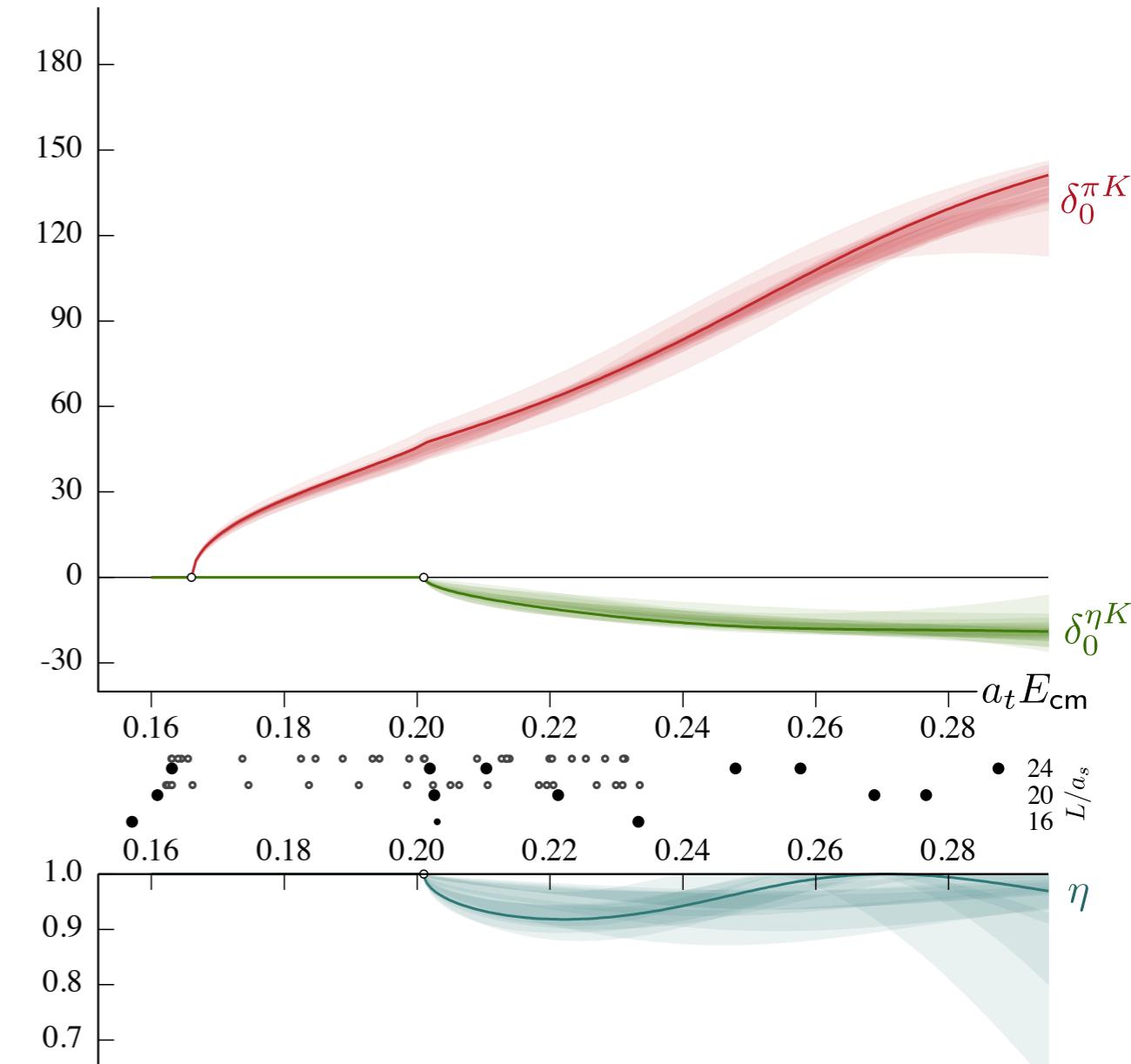
# $\pi K/\eta K$ scattering

$m_\pi \sim 391$  MeV

- Are the result parameterization dependent ?
  - Try a range of parameterizations ...



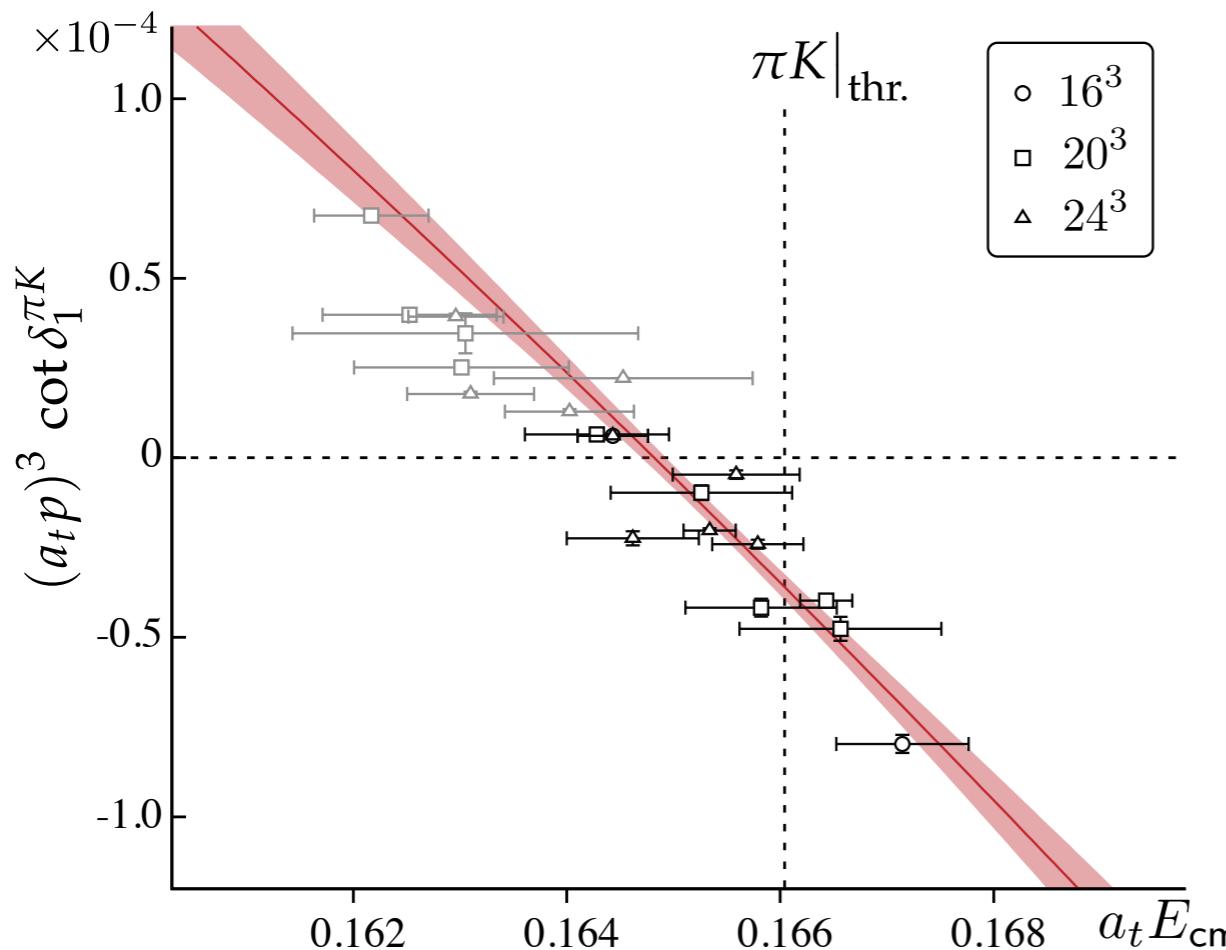
S-WAVE  $\pi K/\eta K$  SCATTERING



– gross features are robust

# $\pi K/\eta K$ scattering amplitudes

## P-WAVE $\pi K$ SCATTERING



Use a Breit-Wigner with  
a subthreshold mass

$$k^3 \cot \delta_1 = (m_R^2 - s) \frac{6\pi\sqrt{s}}{g_R^2}$$

$$\rightarrow g = 5.93(30)$$

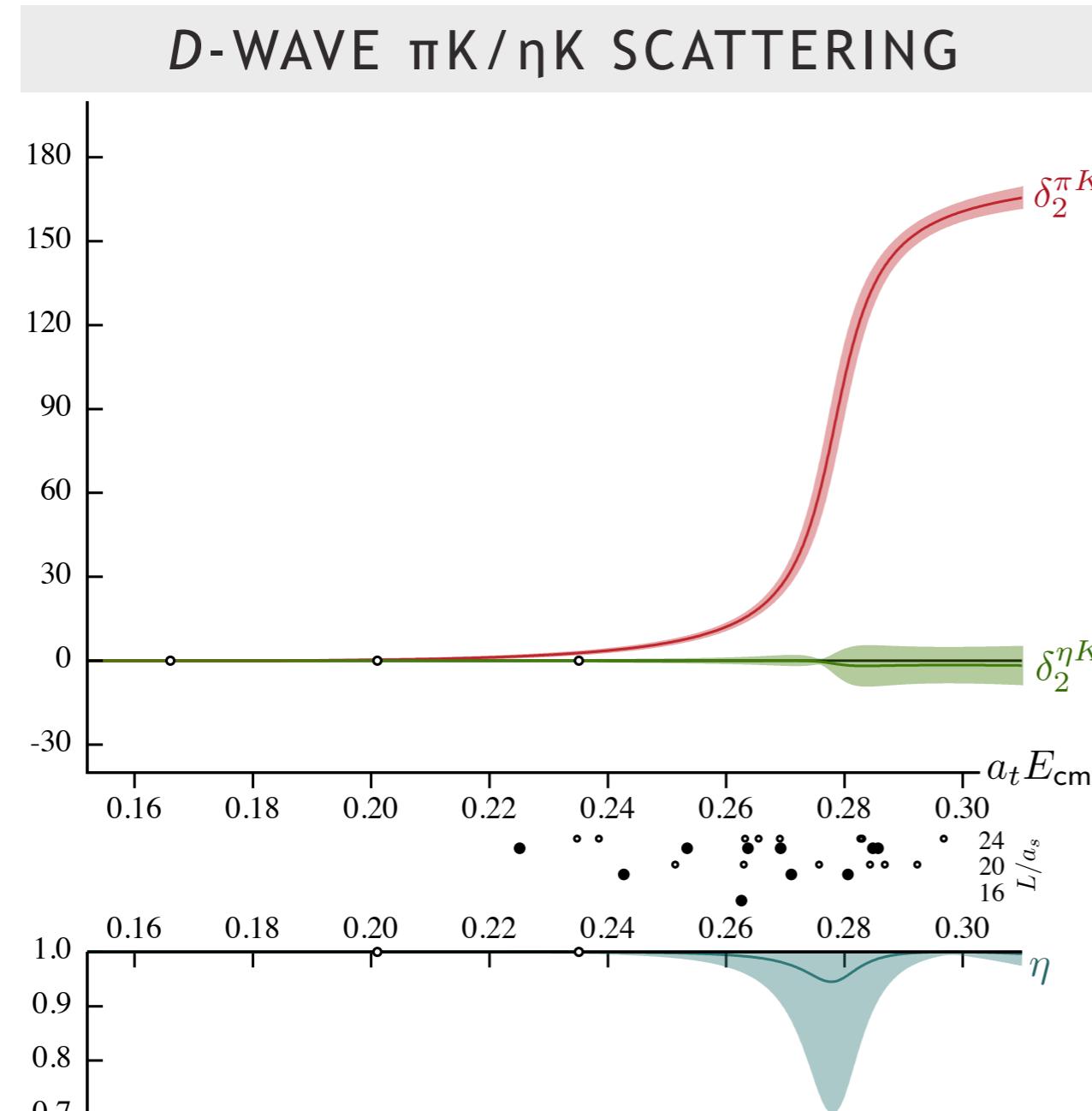
Vector (shallow) bound-state

Quark mass accident that it lies  
so close to threshold ...

$g_{\text{phys.}} = 5.5(2)$  PDG

# $\pi K/\eta K$ scattering

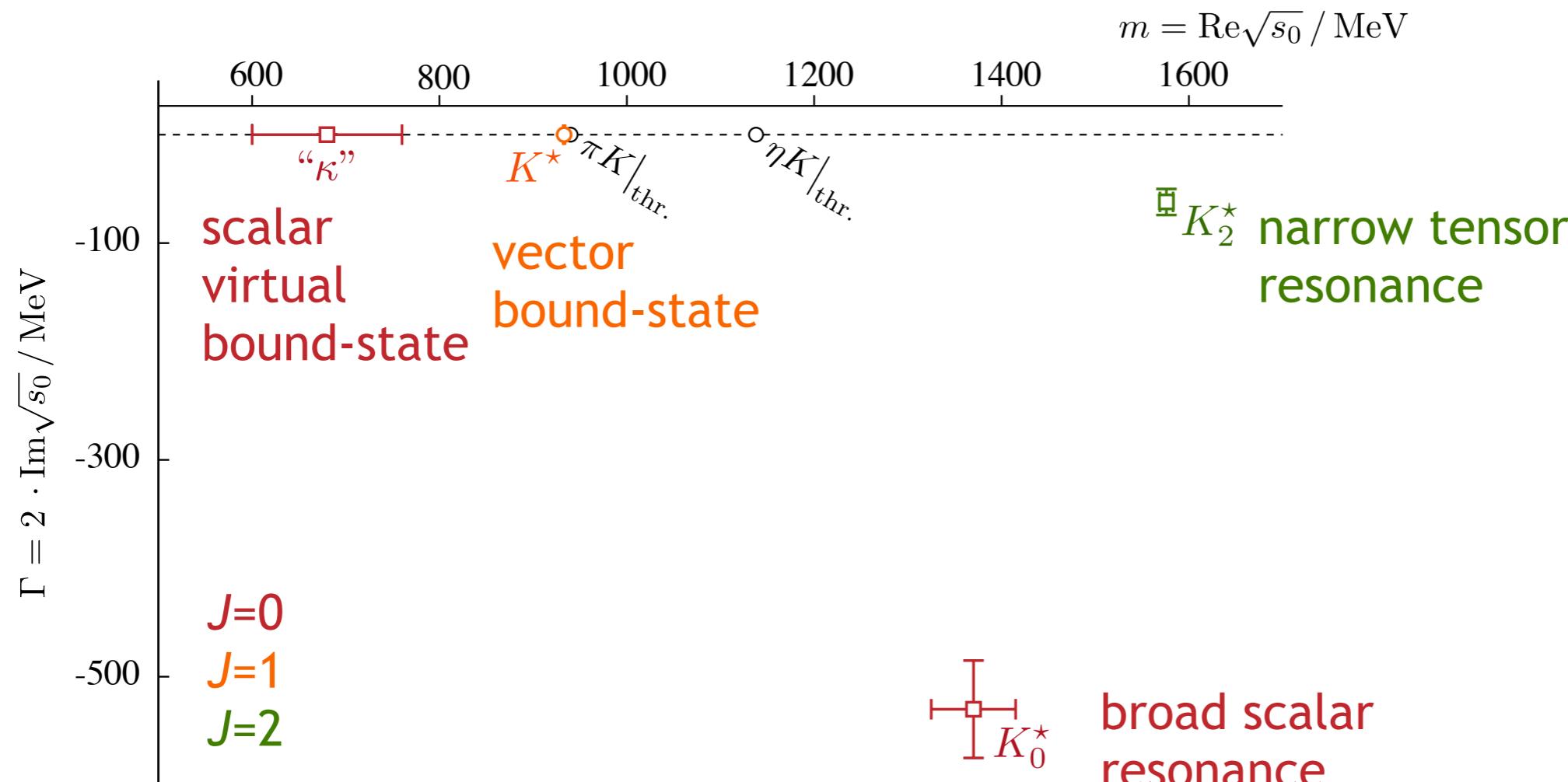
- Clear narrow resonance in  $D$ -wave scattering



$m_\pi \sim 391 \text{ MeV}$

# Singularity content

- extract  $t$ -matrix poles from partial waves

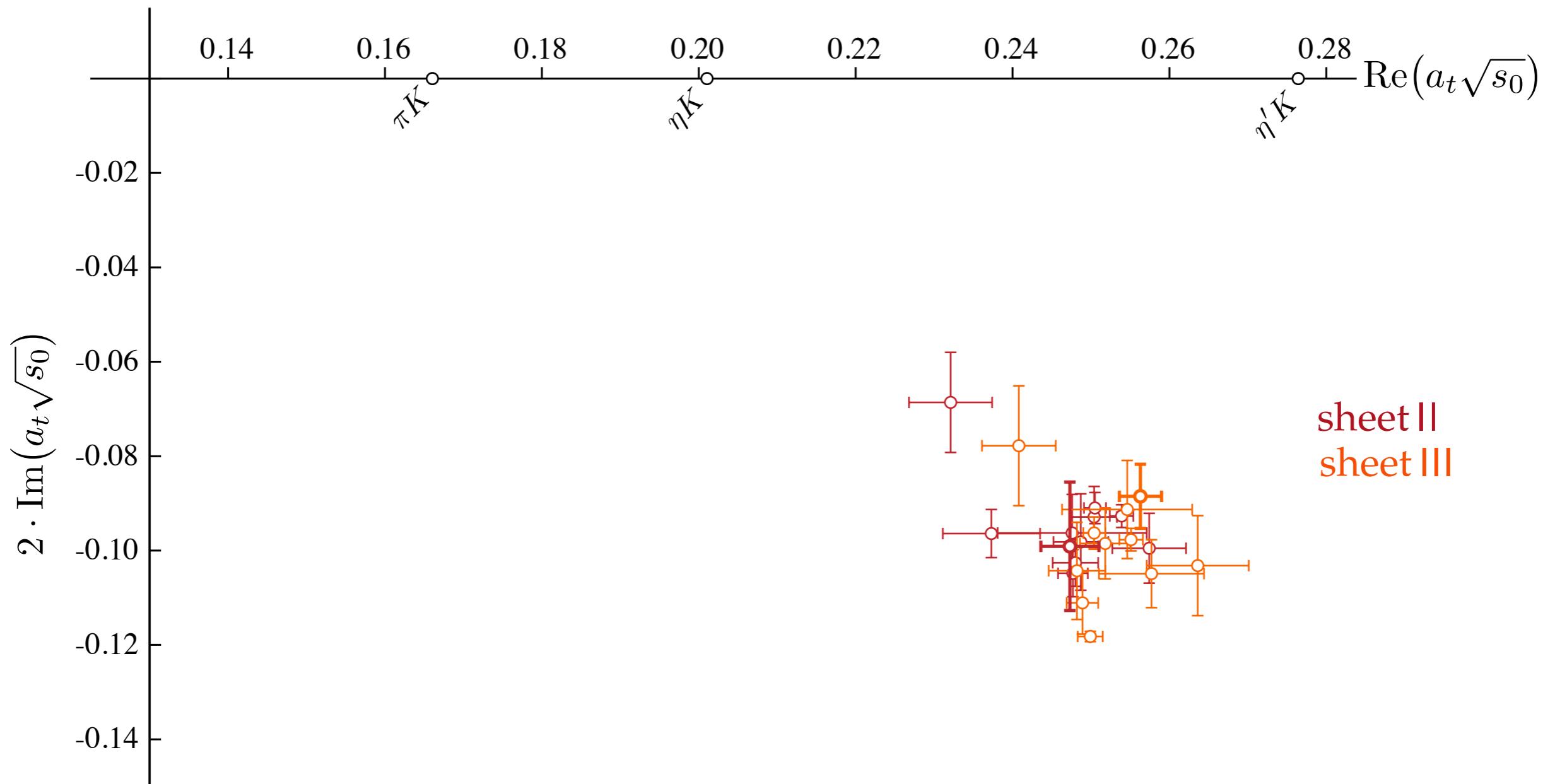


$m_\pi \sim 391 \text{ MeV}$

PRL 113 182001  
PRD 91 054008

# $\pi K/\eta K$ S-wave resonance

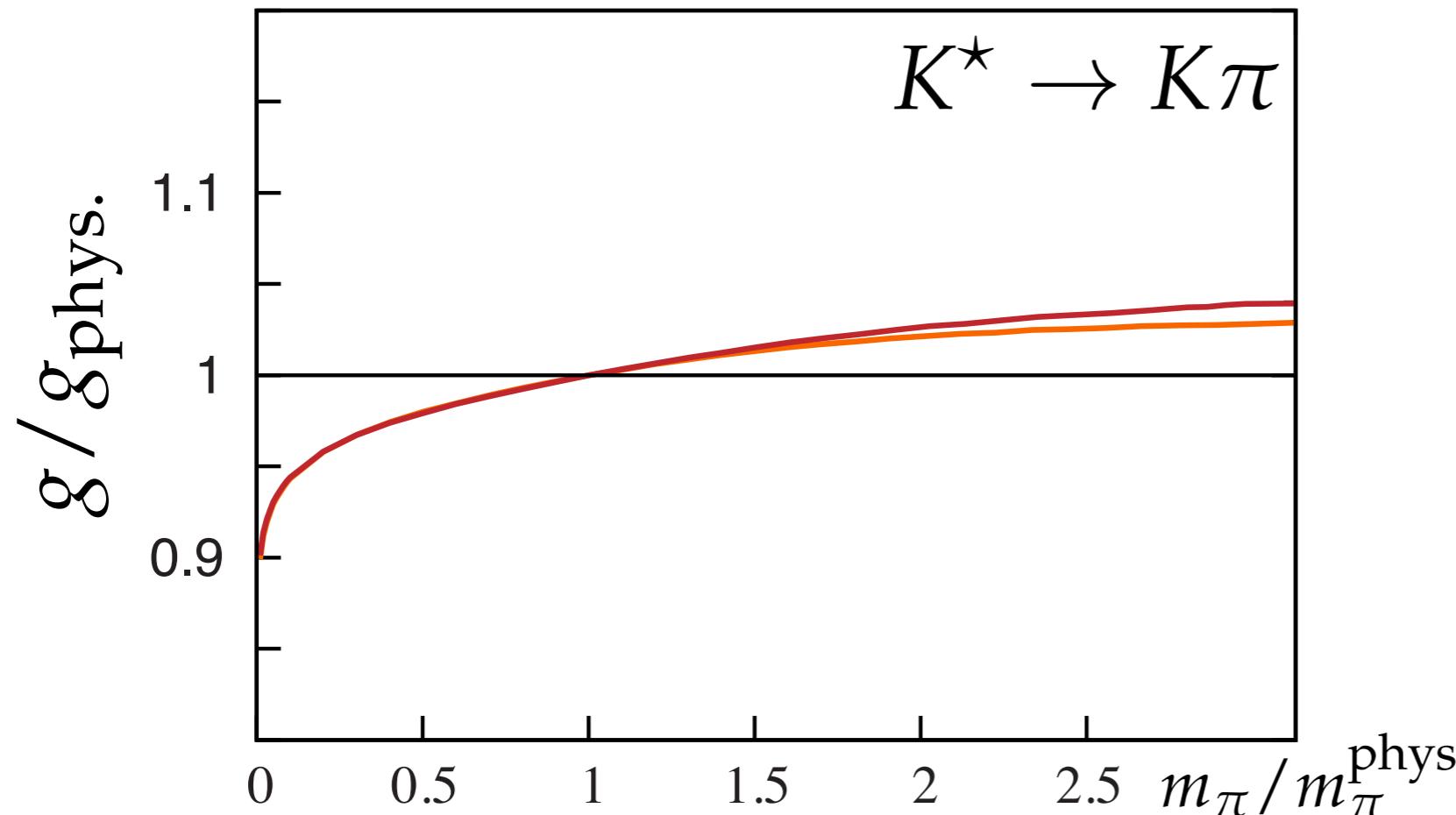
- $t$ -matrix pole position under variation of parameterization



# $K^*\pi K$ coupling with changing quark mass

- Unitarized  $SU(3)_F$  chiral perturbation theory

NEBREDA & PELAEZ  
PRD81 054035 (2010)

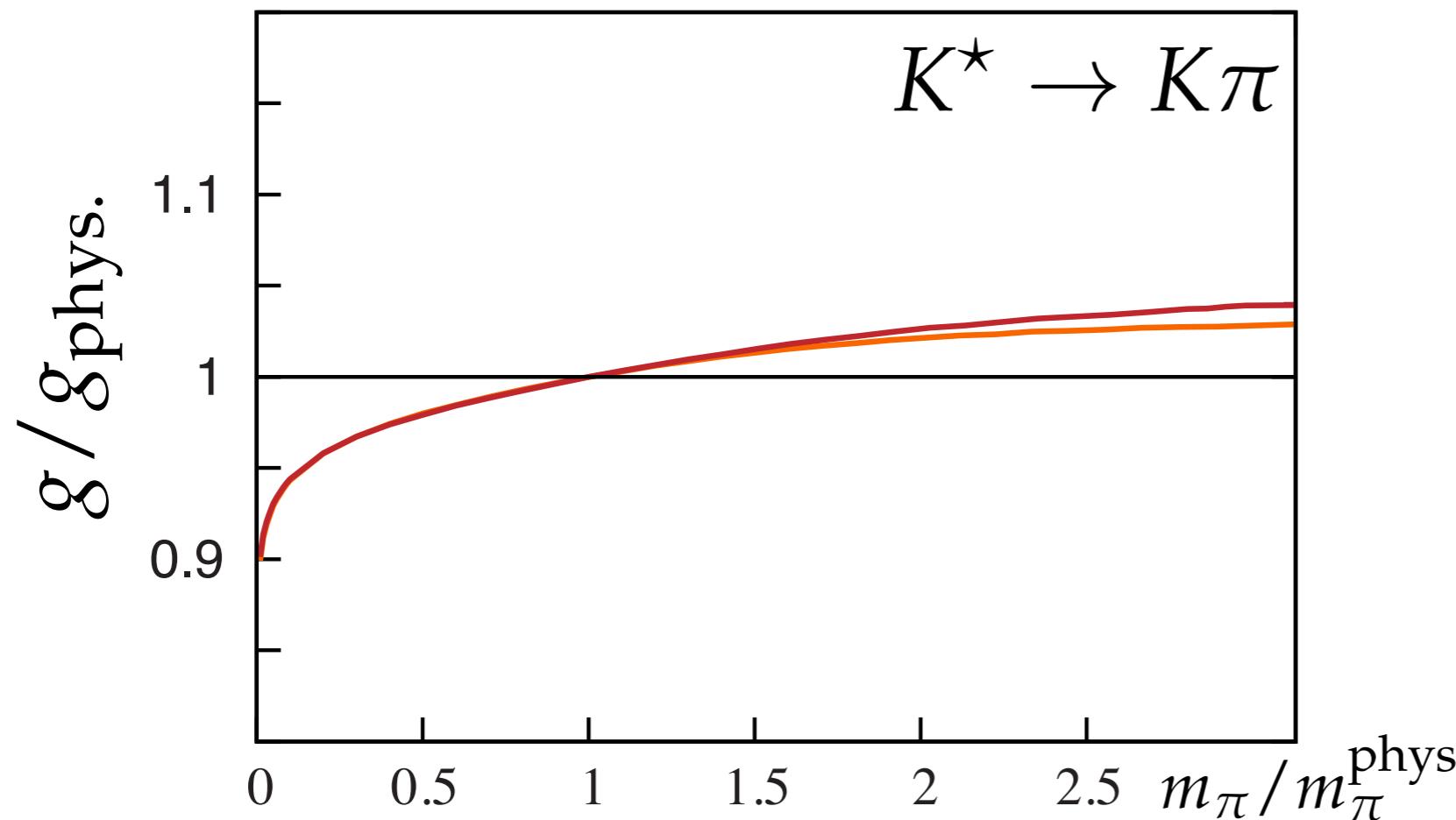


$g_{\text{phys.}} = 5.5(2)$  PDG

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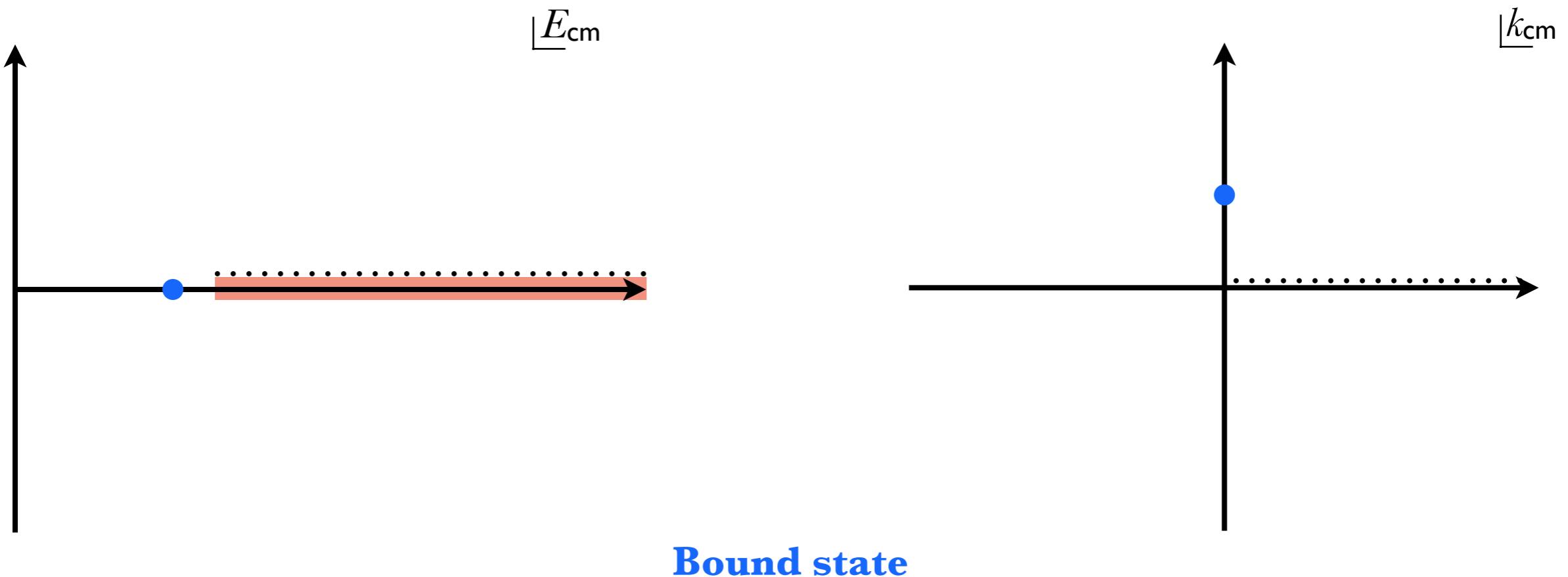
At  $m_\pi \sim 391$  MeV we have shallow bound state:  $g = 5.93 \pm 0.26$

That's the P-wave - what about S-wave?

# Varieties of poles

- Multi-sheet structure around a cut: single channel case

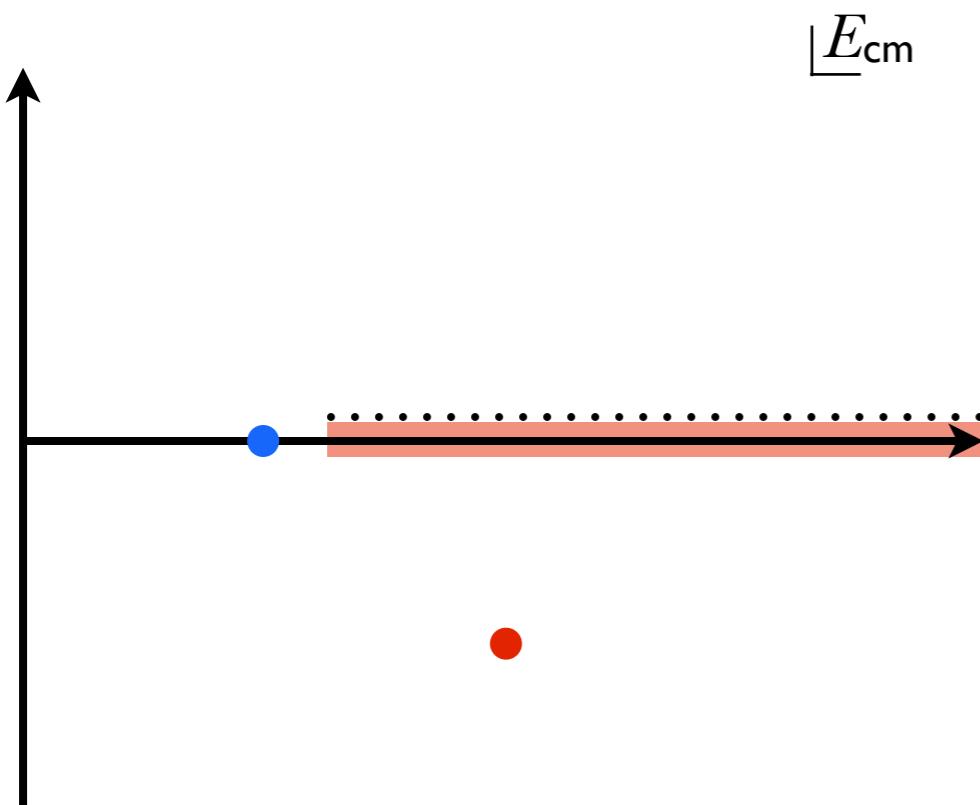
$$k_{cm} = \pm \frac{1}{2} \sqrt{E_{cm}^2 - 4m^2}$$



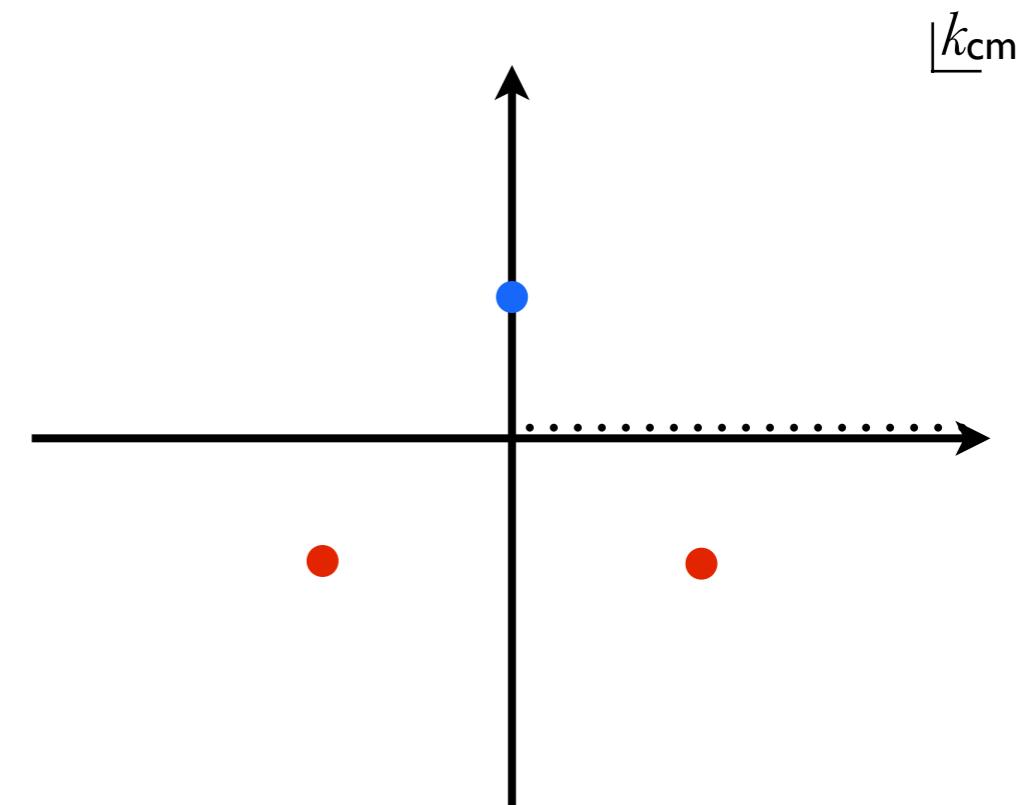
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$\underline{E}_{cm}$



$\underline{k}_{cm}$

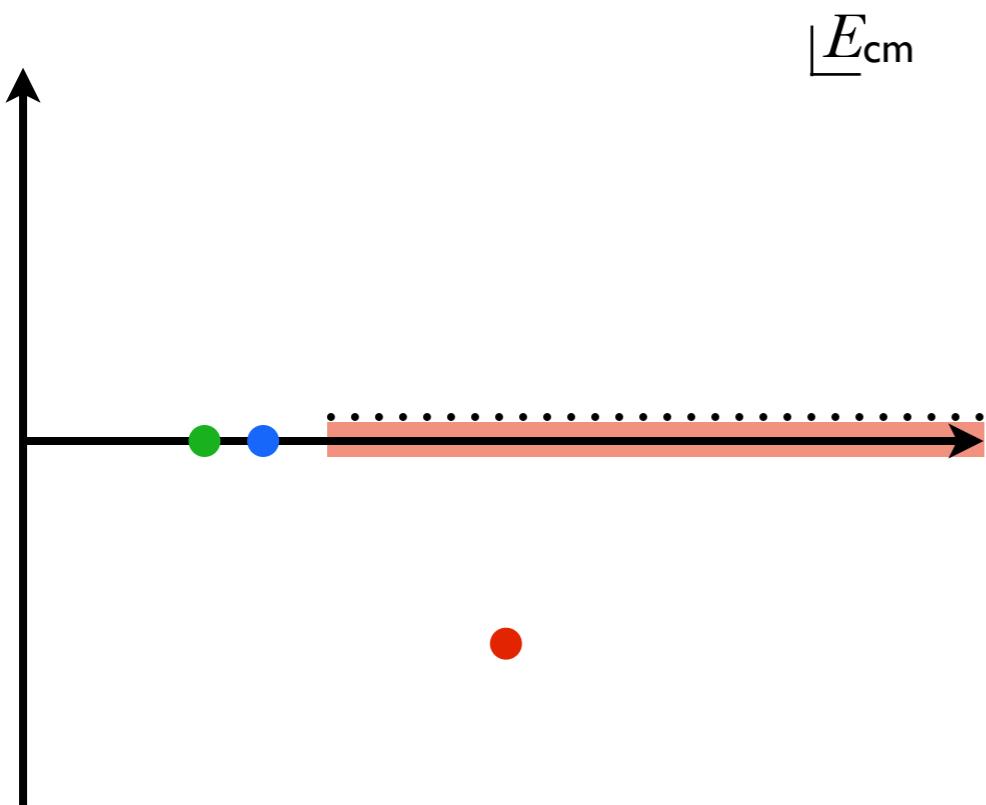
**Bound state**

**Resonance**

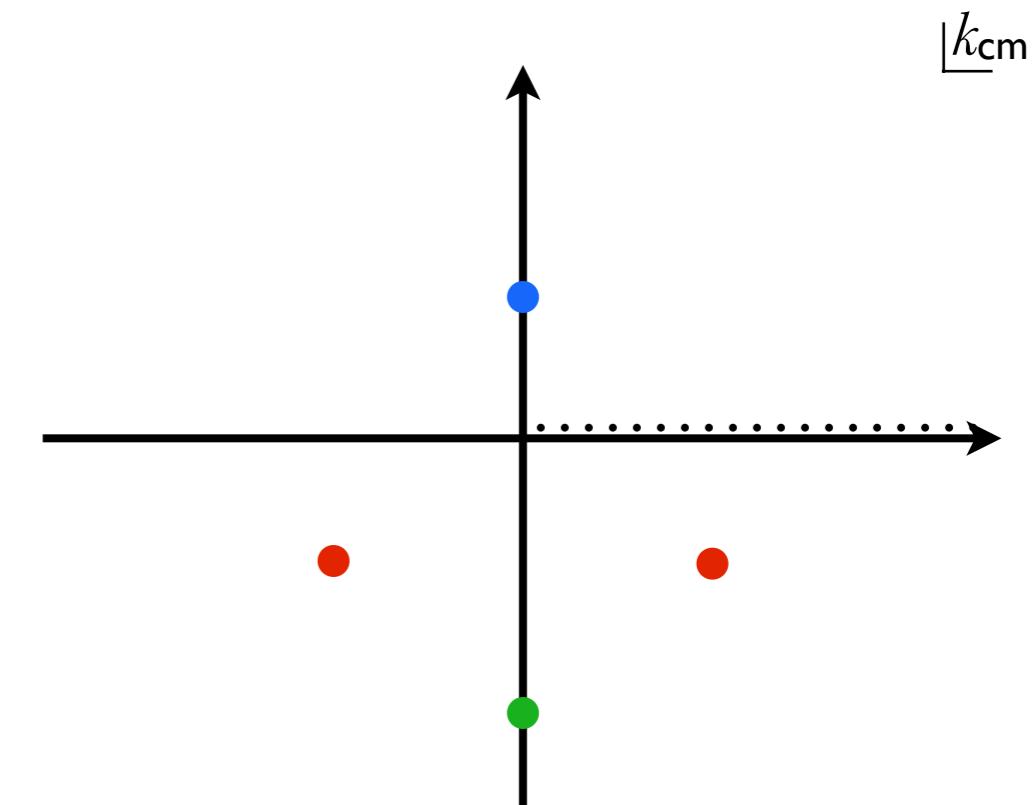
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**Bound state**  
**Resonance**  
**Virtual Bound state**



Familiar examples:  
N-N  ${}^3S_1$  deuteron  
 $\rho$   
N-N  ${}^1S_0$

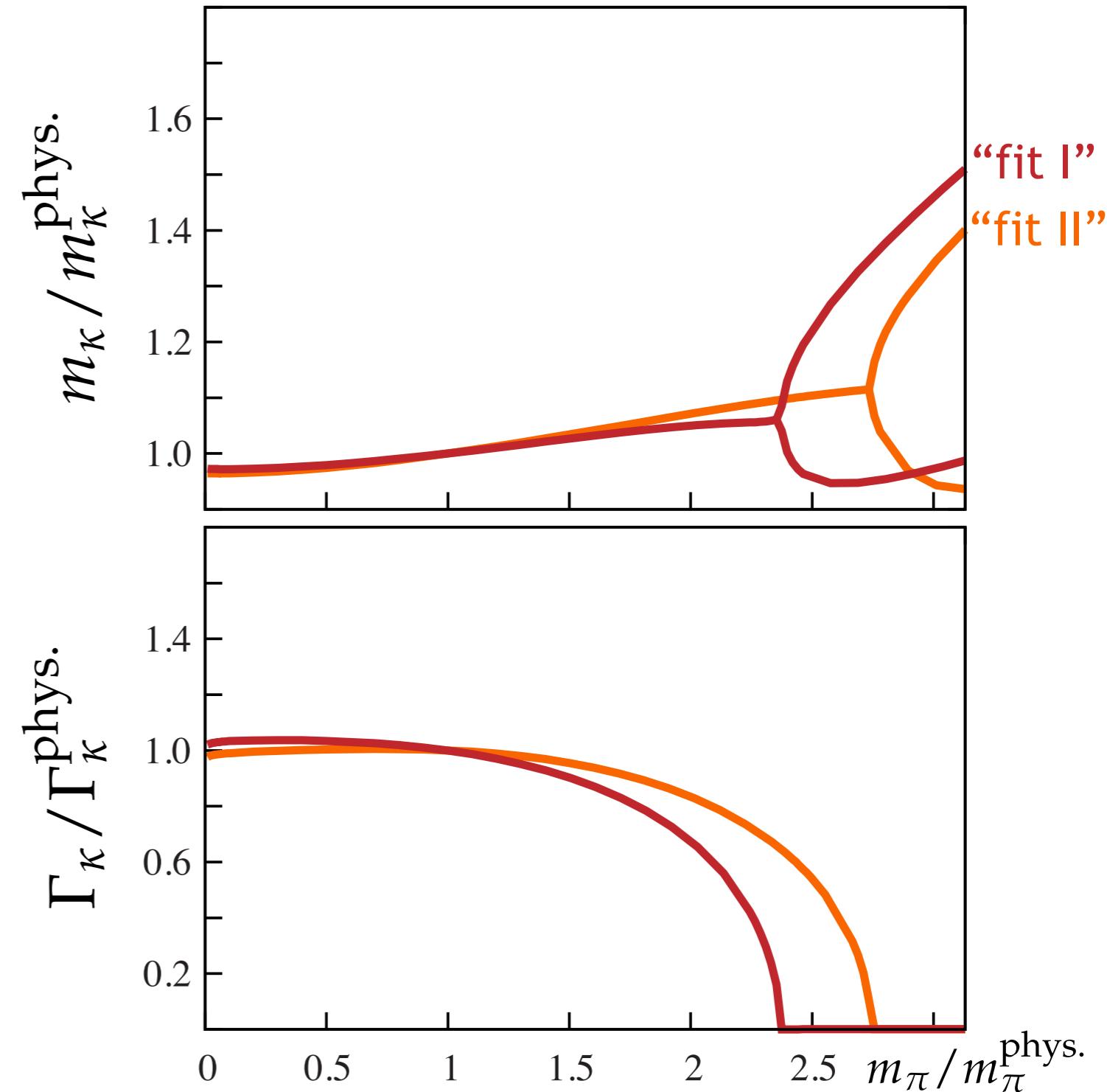
# $\kappa$ (kappa) pole with changing quark mass

- Unitarized SU(3)<sub>F</sub> chiral perturbation theory

- Resonance poles become virtual bound states somewhere near  $m_\pi \sim 2.5 m_\pi^{\text{phys}}$
- At higher pion mass virtual bound-state becomes bound

NEBREDA & PELAEZ  
PRD81 054035 (201)

$$\sqrt{s_0} = m + \frac{i}{2}\Gamma$$



DESCOTES-GENON

$$\sqrt{s_0} = 660(20) + \frac{i}{2} 550(25) \text{ MeV}$$

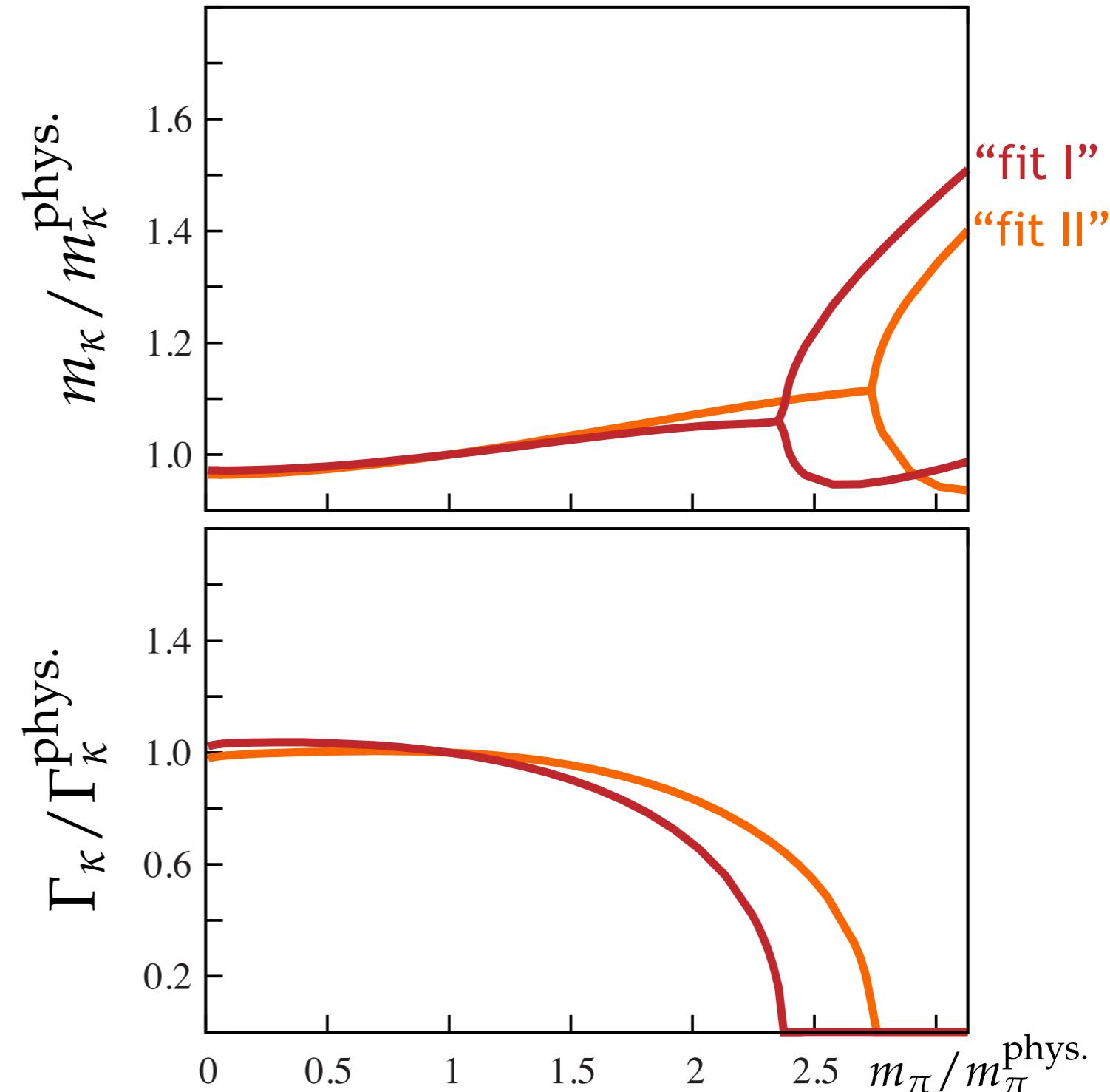
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At a lower pion mass, do we see a resonance?  
Stay tuned !