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Lattice QCD

- Best first principle-tool to extract predictions for the theory of strong interactions in the non-perturbative regime

- Uncertainties:
  - Statistical: finite sample, error $\sim \frac{1}{\sqrt{\text{sample size}}}$
  - Systematic: finite box size, unphysical quark masses

- Given enough computer power, uncertainties can be kept under control

- Unprecedented level of accuracy in lattice data

- Results can be compared to experimental measurements
QCD phase diagram
Interacting hadronic matter in the ground state can be well approximated by a non-interacting resonance gas.

The pressure can be written as:

\[ p^{HRG}/T^4 = \frac{1}{VT^3} \sum_{i \in \text{mesons}} \ln Z^M_{m_i}(T, V, \mu X^a) + \frac{1}{VT^3} \sum_{i \in \text{baryons}} \ln Z^B_{m_i}(T, V, \mu X^a) \]

where

\[ \ln Z^M_{m_i}/B = \pm \frac{V d_i}{2\pi^2} \int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\varepsilon_i/T}) , \]

with energies \( \varepsilon_i = \sqrt{k^2 + m_i^2} \), degeneracy factors \( d_i \) and fugacities

\[ z_i = \exp \left( \frac{\sum_a X_i^a \mu X^a}{T} \right) . \]

\( X^a \): all possible conserved charges, including the baryon number \( B \), electric charge \( Q \), strangeness \( S \).

Input: hadronic spectrum
QCD Equation of state at $\mu_B=0$

- EoS available in the continuum limit, with realistic quark masses
- Agreement with HRG model at low temperatures

WB: S. Borsanyi et al., 1309.5258
HotQCD: A. Bazavov et al., 1407.6387, PRD (2014)
Evolution of a Heavy Ion Collision

- **Chemical freeze-out**: inelastic reactions cease: the chemical composition of the system is fixed (particle yields and fluctuations)
- **Kinetic freeze-out**: elastic reactions cease: spectra and correlations are frozen (free streaming of hadrons)
- Hadrons reach the detector
Hadron yields

- E=mc²: lots of particles are created
- Particle counting (average over many events)
- HRG model: test hypothesis of hadron abundancies in equilibrium

\[
N_i = -T \frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i V}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T]} \pm 1
\]

Resonances contribute to the effective number of stable species through their branching ratios \( d_{r \rightarrow i} \).

\[
\tilde{N}_i = N_i + \sum_r d_{r \rightarrow i} N_r
\]
The thermal fits

- Fit is performed minimizing the $X^2$
- Fit to yields: parameters $T$, $\mu_B$, $V$
- Fit to ratios: the volume $V$ cancels out

- Changing the collision energy, it is possible to draw the freeze-out line in the $T$, $\mu_B$ plane

- The results depend dramatically on the hadronic spectrum: the freeze-out temperature was $\sim 175$ MeV in 2005 and is now $\sim 156$ MeV
The thermal fits

- Fit is performed minimizing the $\chi^2$

- Fit to yields: parameters $T$, $\mu_B$, $V$

- Fit to ratios: the volume $V$ cancels out

- Changing the collision energy, it is possible to draw the freeze-out line in the $T$, $\mu_B$ plane

- The results depend dramatically on the hadronic spectrum: the freeze-out temperature was $\sim 175$ MeV in 2005 and is now $\sim 156$ MeV

Notice that the particle yields also depend on $\mu_S$ and $\mu_Q$

However, $\mu_S$ and $\mu_Q$ are not free parameters

They are fixed imposing the following conditions:

$$<n_S> = 0 \quad <n_Q> = 0.4<n_B>$$

$\mu_S$ and $\mu_Q$ become functions of $T$ and $\mu_B$
Do we have a common freeze-out line?

- Yield fits seem to hint at a higher temperature for strange particles

R. Preghenella for ALICE, SQM 2012  
M. Floris, QM 2014.
Missing strange states?

- Quark Model predicts not-yet-detected (multi-)strange hadrons

\[ \left( \frac{\mu_S}{\mu_B} \right)_{LO} = -\chi_1^{BS}/\chi_2^S + \chi_1^{QS} \mu_Q/\mu_B \]

- QM-HRG improves the agreement with lattice results for the baryon-strangeness correlator:

- The effect is only relevant at finite \( \mu_B \)

- Feed-down from resonance decays not included

A. Bazavov et al., PRL (2014); Quark Model states from Capstick and Isgur, PRD (1986) and Ebert, Faustov and Galkin, PRD (2009).
Missing strange states?

- Quark Model predicts not-yet-detected (multi-)strange hadrons

- The agreement with the HRG model improves with some observables but gets worse with others

- $\chi_4^S/\chi_2^S$ is proportional to $\langle S^2 \rangle$ in the system

- It seems to indicate that the quark model predicts too many multi-strange states

A. Bazavov et al., PRL (2014); Quark Model states from Capstick and Isgur, PRD (1986) and Ebert, Faustov and Galkin, PRD (2009).
Missing strange states?

- We took every state listed in the 2016 PDG (including * and ** states)

- We took two versions of the Quark Model: hypercentral Quark Model (hQM) and quark-diquark (qD)

- We constructed different spectra by merging the observed particles with the not-yet-detected ones

- We invented new observables which allow to separate the contributions of hadrons to the thermodynamics according to their quantum numbers

- We tested the predictions of the three different HRG models thus obtained by comparing them to lattice QCD results
Fluctuations of conserved charges

- Definition:

\[ \chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p/T^4}{\partial (\mu_B/T)^l \partial (\mu_S/T)^m \partial (\mu_Q/T)^n}. \]

- Relationship between chemical potentials:

\[ \mu_u = \frac{1}{3} \mu_B + \frac{2}{3} \mu_Q; \]
\[ \mu_d = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q; \]
\[ \mu_s = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q - \mu_S. \]

- They can be calculated on the lattice and compared to experiment
Missing strange states?

- Idea: define linear combinations of fluctuations which receive contributions only from particles with a given quantum number

- They allow to compare PDG and QM prediction for each sector separately

\[
P_S(\hat{\mu}_B, \hat{\mu}_S) = P_{0|1|} \cosh(\hat{\mu}_S) + P_{1|1|} \cosh(\hat{\mu}_B - \hat{\mu}_S) + P_{1|2|} \cosh(\hat{\mu}_B - 2\hat{\mu}_S) + P_{1|3|} \cosh(\hat{\mu}_B - 3\hat{\mu}_S)
\]

\[
P_{0|1|} = \chi_2^S - \chi_{22}^{BS}
\]

\[
P_{1|1|} = \frac{1}{2} \left( \chi_4^S - \chi_2^S + 5\chi_{13}^{BS} + 7\chi_{22}^{BS} \right)
\]

\[
P_{1|2|} = -\frac{1}{4} \left( \chi_4^S - \chi_2^S + 4\chi_{13}^{BS} + 4\chi_{22}^{BS} \right)
\]

\[
P_{1|3|} = \frac{1}{18} \left( \chi_4^S - \chi_2^S + 3\chi_{13}^{BS} + 3\chi_{22}^{BS} \right)
\]

A. Bazavov et al., PRL (2013)
How many states? $|S|=0$


The columns hQM and qD contain additional states, not included in the PDG2016.
How many states? \(|S| = 1\)


The columns hQM and qD contain additional states, not included in the PDG2016.
How many states? \(|S|=2\) and \(|S|=3\)


The columns hQM and qD contain additional states, not included in the PDG2016.
How many states?

In total we have (isospin multiplicity and particle-antiparticle):

- PDG 2005: 144
- PDG 2016: 608 (**, *** and **** states)
- PDG 2016+: 738 (including also * states)
- hQM: 247 additional states (not included in PDG 2016+)
- qD: 79 additional states (not included in PDG 2016+)
Missing strange states?

- First observable: \((\mu_s/\mu_B)_{LO}\)

- Adding states to the spectrum allows to reproduce the lattice data

- Very small difference for this observable between PDG, PDG+hQM and PDG+qD

WB collaboration, in preparation
Missing strange states?

- Second observable: $\chi_4^S/\chi_2^S$

- This observable is extremely sensitive to the strangeness content of the system

- It looks like we need more $|S|=1$ states or less multi-strange ones

WB collaboration, in preparation
Missing strange states?

- The HRG model based on the PDG 2016 spectrum yields a good description of the data.
- There is room to add more states beyond those predicted by the hQM or qD.
Missing strange states?

- The qD model adds $|S|=2$ states in the intermediate mass range.
Missing strange states?

- It looks like we need more $|S|=3$ states
Not enough strange mesons

- Both Quark Model and PDG 2016 underestimate the partial pressure due to strange mesons
- This explains why the QM overestimates $\frac{\chi_4^S}{\chi_2^S}$: more strange mesons would bring the curve down

Relevance of * states

- Adding states to the spectrum allows to reproduce the lattice data
- We need the * states from the PDG 2016: the spectrum with only **, *** and **** states is not enough
Effect of resonance decays

- To extract the freeze-out parameters, we need to know how these additional states decay.
- The decays have a big effect on the freeze-out parameters.
Unprecedented precision in lattice QCD thermodynamics allows to extract information on the hadronic spectrum.

The most up-to-date PDG 2016 list with **, *** and **** states is not enough to reproduce the lattice results.

Good agreement is found by including * states and/or Quark Model states.

Large effect of decays on freeze-out parameters from yields.

We need to know how these states decay, to use them in heavy ion physics.
Effect of resonance decays

- We used the PDG2014 to estimate the effect of resonance decays on the fit to proton and charge fluctuations.
- The results agree with the ones obtained with the PDG2012 within errorbars.

![Graph showing effect of resonance decays](image)

**WB collaboration, in preparation**