Entropy shifts and Missing States

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Excited Hyperons in QCD Thermodynamics at Freeze-Out

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Works with Eugenio Megias, Lorenzo Salcedo, Wojciech Broniwoski, Pere Masjuan

Outline

- The Hadron Spectrum at zero temperature
- Quarks and gluons at finite temperature
- Quark Hadron duality at finite temperature
- Entropy shifts
- Anatomy of Hadron Resonance Gas
- Conclusions
- References

To count or not to count

- The best way to look for missing states is to count them

$$1, 2, 3 \ldots$$

- Thermodynamics is a way of counting
- The partition function of QCD counts

$$Z_{\rm QCD} = \sum_n e^{-E_n/T} \qquad H_{\rm QCD}\psi_n = E_n\psi_n$$

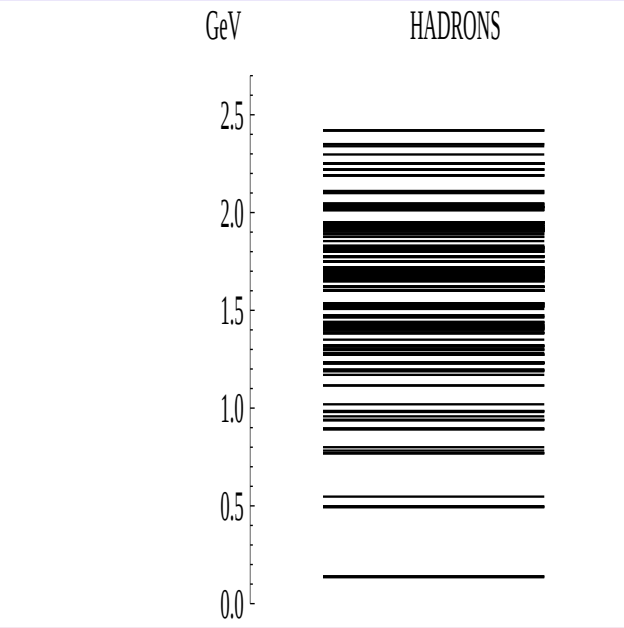
- Spectrum of QCD $\rightarrow$ Thermodynamics
- Colour singlet states (hadrons + ...???)
- Do we see quark-gluon substructure BELOW the "phase transition" ?
- Completeness relation in Hilbert space $\mathcal{H}_{\rm QCQ}$

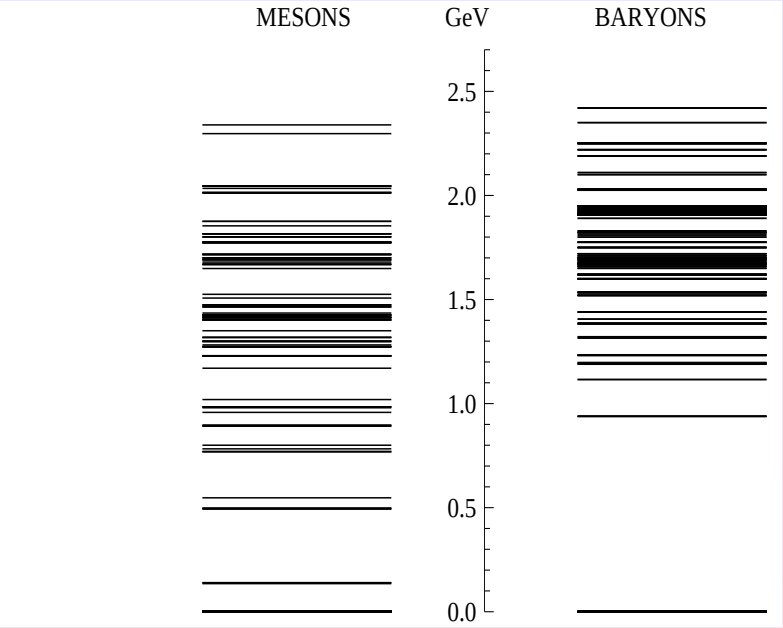
$$\mathbf{1} = \sum_n |\Psi_n\rangle\langle\Psi_n| \approx \underbrace{\sum_n |\bar{q}q; n\rangle\langle\bar{q}q; n|}_{\text{mesons}} + \underbrace{\sum_n |qqq; n\rangle\langle qqq; n|}_{\text{baryons}} + \underbrace{\sum_n |\bar{q}qg; n\rangle\langle\bar{q}qg; n|}_{\text{hybrids}} + \ldots$$

- Given H, is there a sum rule involving ALL resonances ?.

HADRONIC SPECTRUM AT ZERO TEMPERATURE

The states in the Particle Data Group (PDG) book

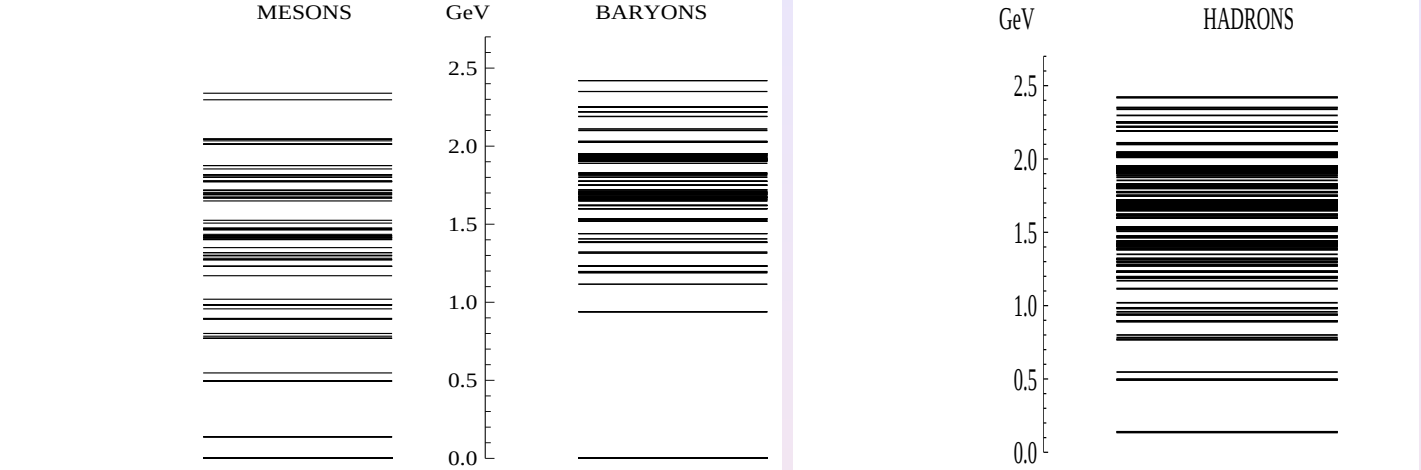




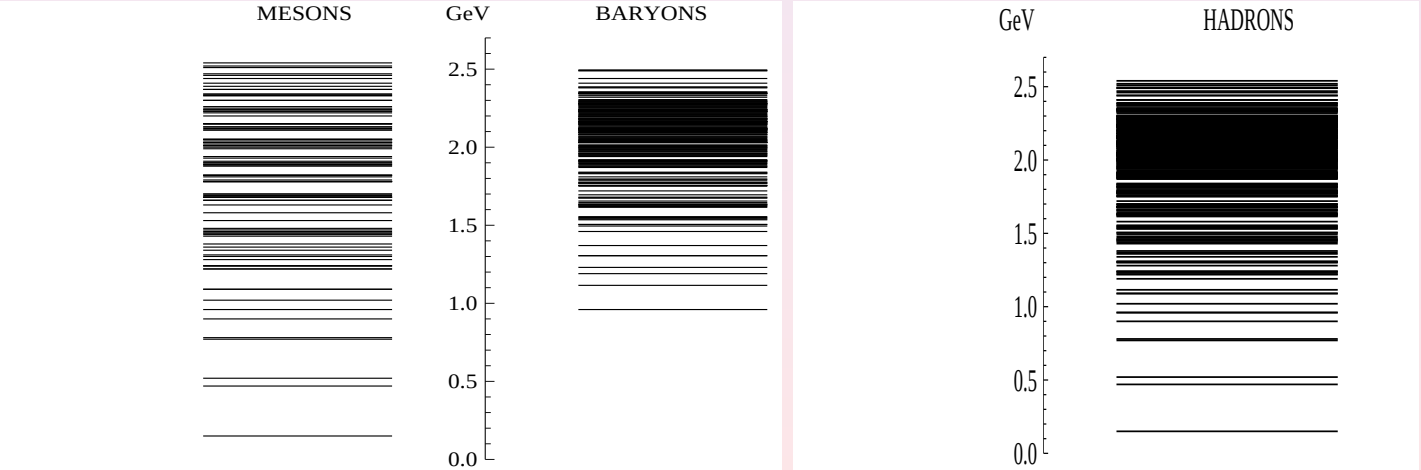
- No data (is a compilation)
- No particles (resonances)
- No book
- Which particles enter PDG ?

Hadron Spectrum (u,d,s)

- Particle Data Group (PDG) compilation 2016



- Relativized Quark Model (RQM) Isgur, Godfrey, Capstik, 1985

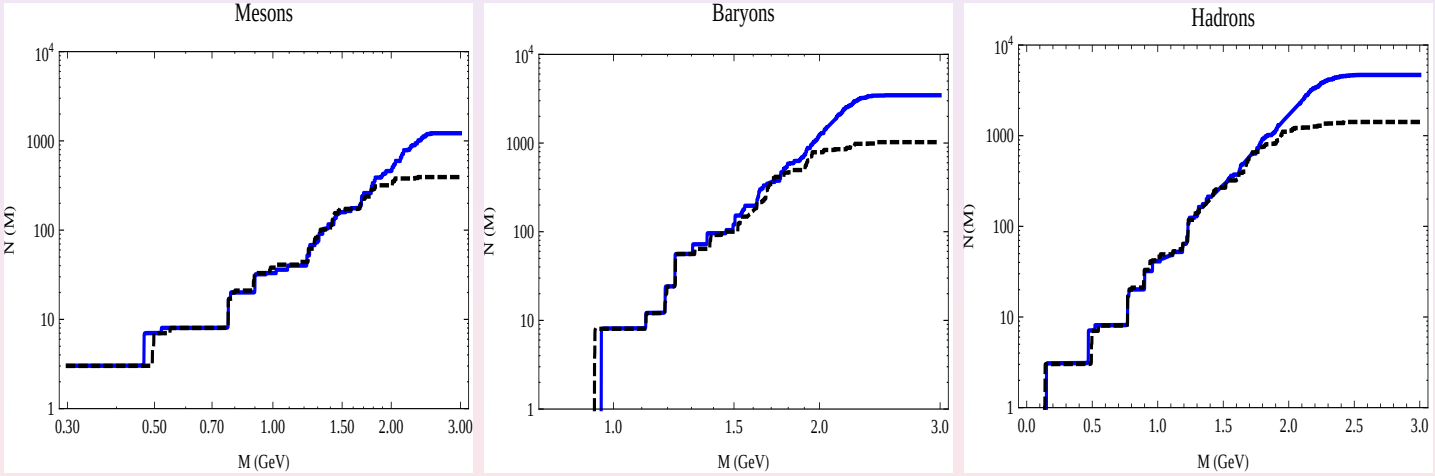


Cumulative number of states

- Compare $H_{\rm QCD}$, $H_{\rm PDG}$, $H_{\rm RQM}$ with staircase function

$$N(M) = \sum_n \theta(M - M_n)$$

- Which states count ?
- Is $N_{\rm QCD}(M)$ accessible ?



$N_{q\bar{q}} \sim M^6$ $\quad N_{qqq} \sim M^{12}$ $\quad N_{\bar{q}q\bar{q}q} \sim M^{18}$ $\quad N_{\rm hadrons} \sim e^{M/T_H}$

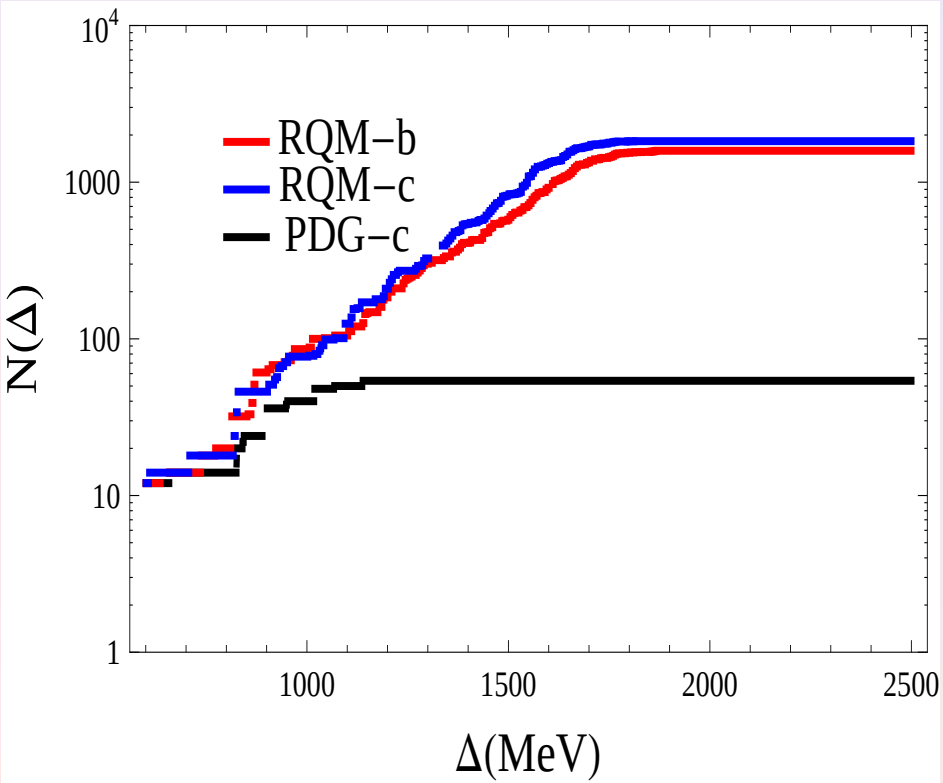
$T_H \sim 150\,\mathrm{MeV}$= Hagedorn temperature

Spectrum with one heavy quark

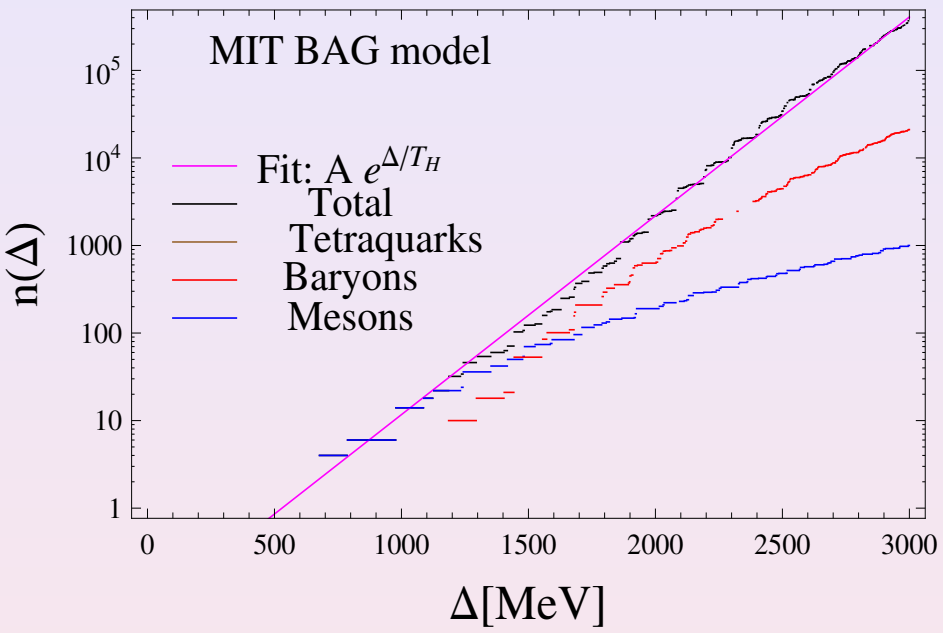
- T_H Hagedorn temperature for hadrons with ONE heavy quark

$$N(\Delta) = \sum_n \theta(\Delta - \Delta_n) \sim N_{\bar{q}Q}(\Delta) + N_{Qqq}(\Delta) + \cdots \sim e^{\Delta/T_H}$$

- Missing states !!.

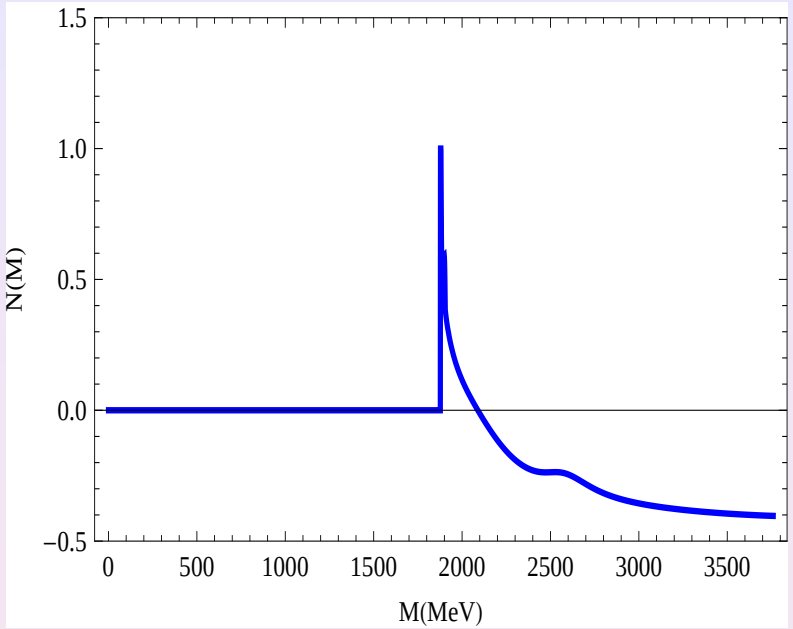


Hagedorn and The bootstrap



- Which are the complete set of states in the PDG ?
- Should X,Y,Z's or the deuteron or ^{208}Pb enter as multiquark states ?

Who counts ?



- The cumulative number in a given channel in the continuum with threshold $M_{\rm th}$

$$N(M) = \sum_n \theta(M - M_n) + [\delta(M) - \delta(M_{\rm th})]/\pi$$

- Levinson's theorem

$$N(\infty) = n_B + [\delta(\infty) - \delta(M_{\rm th})]/\pi = 0$$

- Deuteron doesn't count

Quark-Hadron duality at zero temperature

- In the confined phase we expect all observables to be represented by hadronic degrees of freedom.
  1. Gell-Mann–Oakes–Renner relation

$$2\,\underbrace{\langle \bar{q}q\rangle\, m_q}_{\text{quarks}} = -\,\underbrace{f_\pi^2 m_\pi^2}_{\text{hadrons}}\,, \tag{1}$$

  2. Transition form factor of the pion
  3. Effective Chiral lagrangians with resonances
  4. Deep inelastic scattering
- Are hadrons a complete set of states ?
- Is the PDG complete or overcomplete ?
- The “phase transition” is a smooth cross-over, so we expect to see departures from quark-hadron duality below T_c

QUARKS AND GLUONS AT FINITE TEMPERATURE

QCD at finite temperature

QCD Lagrangian:

$$\mathcal{L}_{\rm QCD} = \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_f \overline{q}^a_f (i\gamma_\mu D_\mu - m_f) q^a_f \; ;$$

Partition function

$$\begin{aligned} Z_{\rm QCD} &= {\rm Tr} e^{-H/T} = \sum_n e^{-E_n/T} \\ &= \int \mathcal{D}A_{\mu,a} \exp\left[-\frac{1}{4}\int d^4x (G^a_{\mu\nu})^2\right] {\rm Det}(i\gamma_\mu D_\mu - m_f) \end{aligned}$$

Boundary conditions and Matsubara frequencies

$$q(\vec{x},\beta) = -q(\vec{x},0) \qquad A_\mu(\vec{x},\beta) = A_\mu(\vec{x},0) \qquad \beta = 1/T$$

$$\int \frac{dp_0}{2\pi} f(p_0) \to T \sum_n f(w_n)$$

$$w_n = (2n+1)\pi T \qquad w_n = 2n\pi T$$

Thermodynamic relations

- Statistical mechanics of non-interacting particles

$$\log Z = V\eta g_i \int \frac{d^3p}{(2\pi)^3} \log\left[1 + \eta e^{-E_p/T}\right] \qquad E_p = \sqrt{p^2 + m^2}$$

$\eta = -1$ for bosons ; $\eta = -1$ for fermions ; g_i-number of species

$$\begin{aligned} F &= -T \log Z \qquad P = -T\frac{\partial F}{\partial V} \\ S &= -\frac{\partial(TF)}{\partial T} \qquad E = F + TS \end{aligned}$$

- High temperature limit $\rightarrow$ Free gas of gluons and quarks

$$P = \left[2(N_c^2 - 1) + 4N_c N_f \frac{7}{8}\right] \frac{\pi^2}{90} T^4$$

Interaction measure (trace anomaly)

$$\Delta \equiv \frac{\epsilon - 3p}{T^4} \rightarrow 0 \qquad (T \rightarrow \infty)$$

Symmetries in QCD

Colour gauge invariance

$$q(x) \to e^{i\sum_a (\lambda_a)^c \alpha_a(x)} q(x) \equiv g(x)q(x)$$
$$A^g_\mu(x) = g^{-1}(x)\partial_\mu g(x) + g^{-1}(x)A_\mu(x)g(x)$$

Only periodic gauge transformations are allowed:

$$g(\vec{x}, x_0 + \beta) = g(\vec{x}, x_0)\,, \qquad \beta = 1/T\,.$$

In the static gauge $\partial_0 A_0 = 0$

$$g(x_0) = e^{i2\pi x_0 \lambda/\beta}\,, \quad \text{where} \quad \lambda = \mathrm{diag}(n_1, \cdots, n_{N_c})\,, \quad \mathrm{Tr}\lambda = 0\,.$$

Large Gauge Invariance: $\Rightarrow$ periodicity in A_0 with period $2\pi/\beta$

$$A_0 \to A_0 + 2\pi T \mathrm{diag}(n_j) \qquad \text{Gribov copies}$$

Explicitly Broken in perturbation theory (non-perturbative finite temperature gluons)

Symmetries in QCD

In the limit of massless quarks ($m_f = 0$),

- Invariant under scale

$$(\mathbf{x} \longrightarrow \lambda \mathbf{x})$$

  Broken by quantum corrections regularization (Trace anomaly)

$$\epsilon - 3p = \frac{\beta(g)}{2g} \langle (G^a_{\mu\nu})^2 \rangle \neq 0 \,,$$

- Chiral **Left** $\leftrightarrow$ **Right** transformations.

$$q(x) \to e^{i \sum_a (\lambda_a)^f \alpha_a} q(x) \qquad q(x) \to e^{i \sum_a (\lambda_a)^f \alpha_a \gamma_5} q(x)$$

  Broken by chiral condensate in the vacuum

$$\langle \bar{q} q \rangle \neq 0$$

Symmetries in QCD

Gluodynamics: In the limit of heavy quarks ($m_f \to \infty$)

$$Z \to \int \mathcal{D}A_{\mu,a} \exp\left[-\frac{1}{4}\int d^4x (G^a_{\mu\nu})^2\right] \mathrm{Det}(-m_f)$$

Larger symmetry ('t Hooft) Center Symmetry $\mathbb{Z}(N_c)$

$$g(\vec{x}, x_0 + \beta) = z\, g(\vec{x}, x_0)\,, \qquad z^{N_c} = 1, \quad (z \in \mathbb{Z}(N_c))\,.$$

$$g(x_0) = e^{i2\pi x_0 \lambda/(N_c\beta)}\,, \qquad A_0 \to A_0 + \frac{2\pi T}{N_c}\mathrm{diag}(n_j)$$

The Poyakov loop

$$L_T = \frac{1}{N_c}\langle \mathrm{tr}_c e^{iA_0/T}\rangle = e^{-F_q/T} = e^{i2\pi/N_c} L_T = 0$$

$F_q = \infty$ means CONFINEMENT
At high temperatures $A_0/T << 1$

$$L_T = 1 - \frac{\langle \mathrm{tr}_c A_0^2\rangle}{2N_c T^2} + \cdots = e^{-\frac{\langle \mathrm{tr}_c A_0^2\rangle}{2N_c T^2} + \cdots}$$

In full QCD $L_T = \mathcal{O}(e^{-m_q/T}) \neq 0$ Large Violation of center sym.

Lattice results in full QCD

The chiral-deconfinement cross over is a unique prediction of lattice QCD

- Order parameter of chiral symmetry breaking ($m_q = 0$)
  Quark condensate $SU(N_f) \otimes SU(N_f) \to SU_V(N_f)$

$$\langle \bar{q}q \rangle \neq 0 \quad T < T_c \qquad \langle \bar{q}q \rangle = 0 \quad T > T_c$$

- Order parameter of deconfinement ($m_q = \infty$)
  Polyakov loop: Center symmetry $Z(N_c)$ broken

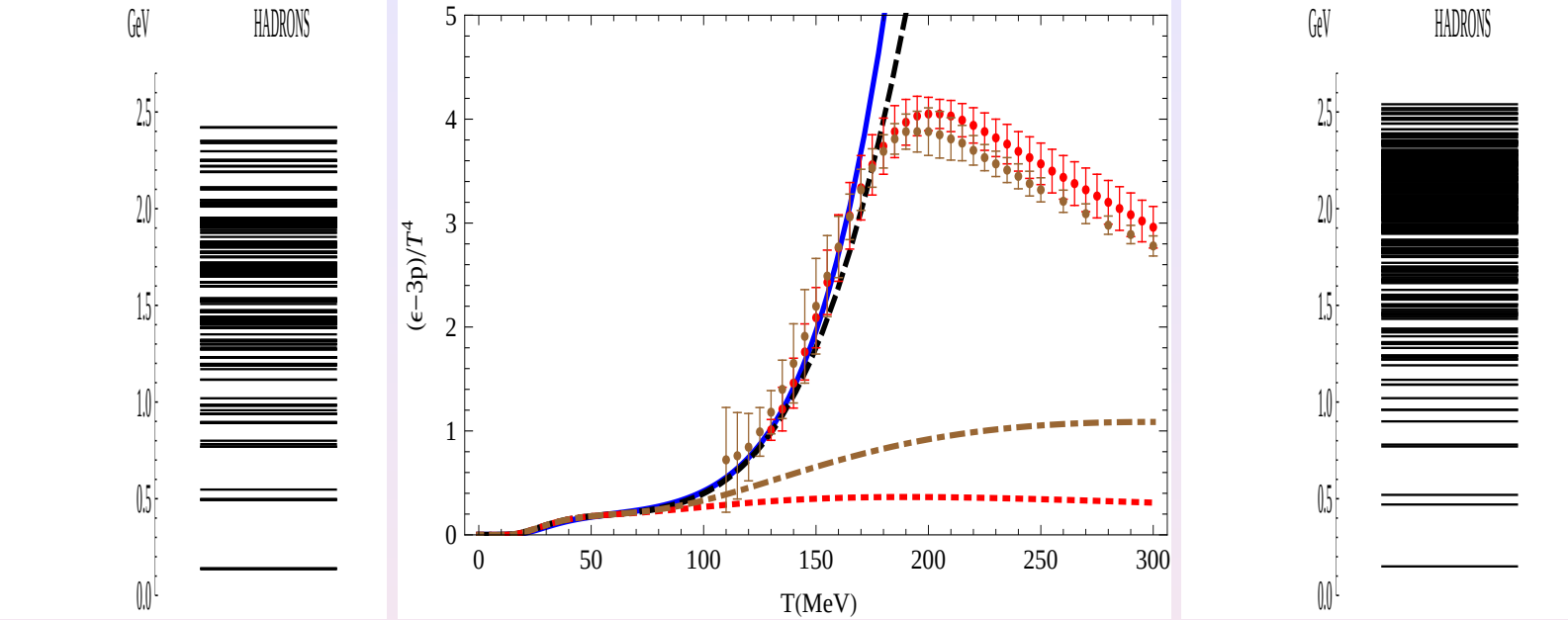
$$L_T = \frac{1}{N_c} \langle \mathrm{tr}_c e^{iA_0/T} \rangle = 0 \quad T < T_c \qquad L_T = \frac{1}{N_c} \langle \mathrm{tr}_c e^{iA_0/T} \rangle = 1 \quad T > T_c$$

- In the real world m_q is finite. The chiral-deconfinement crossover (connected) crossed correlator (never computed on lattice),

$$\langle \bar{q}q \, \mathrm{tr}_c \, e^{igA_0/T} \rangle - \langle \bar{q}q \rangle \langle \mathrm{tr}_c \, e^{igA_0/T} \rangle = \frac{\partial L_T}{\partial m_q} \, , \tag{2}$$

QUARK-HADRON DUALITY AT FINITE TEMPERATURE

QCD Spectrum and Trace anomaly



- PDG is thermodynamically equivalent to RQM !!

$$\mathcal{A}_{\rm HRG}(T) \equiv \frac{\epsilon - 3P}{T^4} = \frac{1}{T^4}\sum_n \int \frac{d^3p}{(2\pi)^3}\frac{E_n(p) - \vec{p}\cdot\vec{\nabla}_p E_n(p)}{e^{E_n(p)/T} + \eta_n} \,,$$

$$E_n(p) = \sqrt{p^2 + M_n^2} \qquad \eta_n = \pm 1$$

- Non-interacting Hadron-Resonance Gas works for $T < 0.8\,T_c$
- Spectrum $\rightarrow$ Thermodynamics

Fluctuations of conserved charges

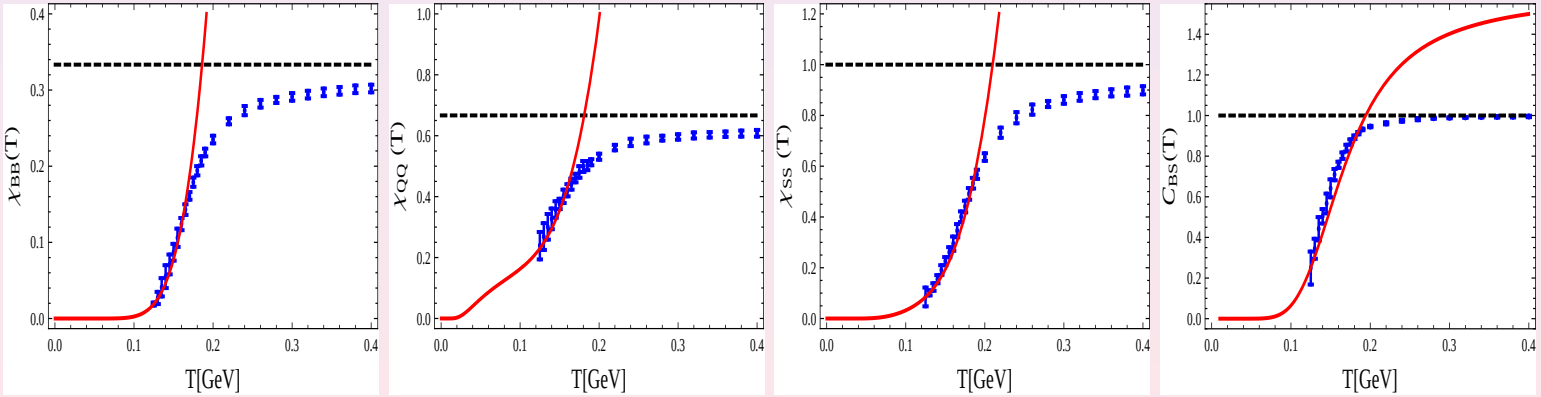
- Conserved charges

$$\langle N_B \rangle_T = 0 \qquad \langle N_Q \rangle_T = 0 \qquad \langle N_S \rangle_T = 0$$

- Vacuum Fluctuations

$$\chi_{BB}(T) = \langle N_B^2 \rangle_T \to \frac{1}{N_c} \qquad \chi_{QQ}(T) = \langle N_Q^2 \rangle_T \to \sum_{i=1}^{N_f} q_i^2$$

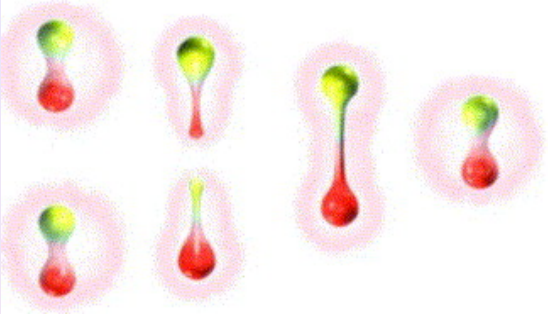
$$\chi_{SS}(T) = \langle N_S^2 \rangle_T \to 1 \qquad C_{BS}(T) = -3\frac{\langle N_S N_B \rangle_T}{\langle N_S^2 \rangle_T} \to 1$$

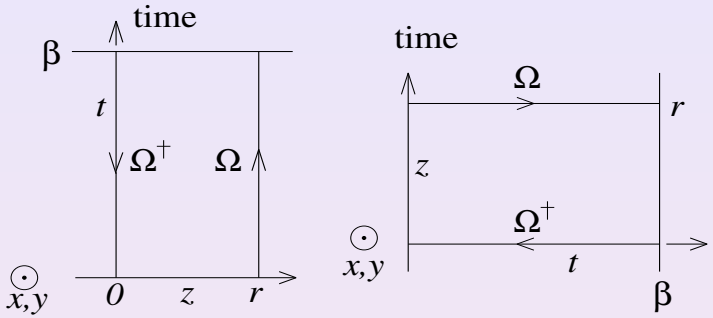


- $C_{BS}^{\rm PDG} < C_{BS}^{\rm QCD}$ (Missing states ?, Significant ?)

Spectral representation

- Two conjugate sources are placed in the medium at temperature T and a separation distance r generate a Free energy shift





$$e^{-\Delta F(r,T)} = \langle \mathrm{tr}_R \Omega(r) \mathrm{tr}_{\bar{R}} \Omega(0) \rangle_T$$

- Standard representation (ratio of partition functions)

$$e^{-\Delta F(r,T)} = \frac{Z_{R\otimes\bar{R}}(r,T)}{Z_0(T)} = \frac{\sum_n e^{-E_n^{R\otimes\bar{R}}(r)/T}}{\sum_n e^{-E_n^0(r)/T}}$$

- Spectral r-representation

$$e^{-\Delta F(r,T)} = \sum_n |\langle n, T|\mathrm{Tr}_R \Omega^\dagger|0, T\rangle|^2 \, e^{-r w_n(T)}$$

- Inequalities

$$\partial_r \Delta F(r,T) \geq 0 \qquad \partial_r^2 \Delta F(r,T) \leq 0$$

Double heavy hadron spectrum and correlators

- String breaking for the $\bar{Q}Q \to \bar{B}B$ (level crossing)

$$M_{\bar{Q}Q}(r_c) = V_{Q\bar{Q}}(r_c) + m_{\bar{Q}} + m_Q = M_{\bar{B}} + M_B \,,$$

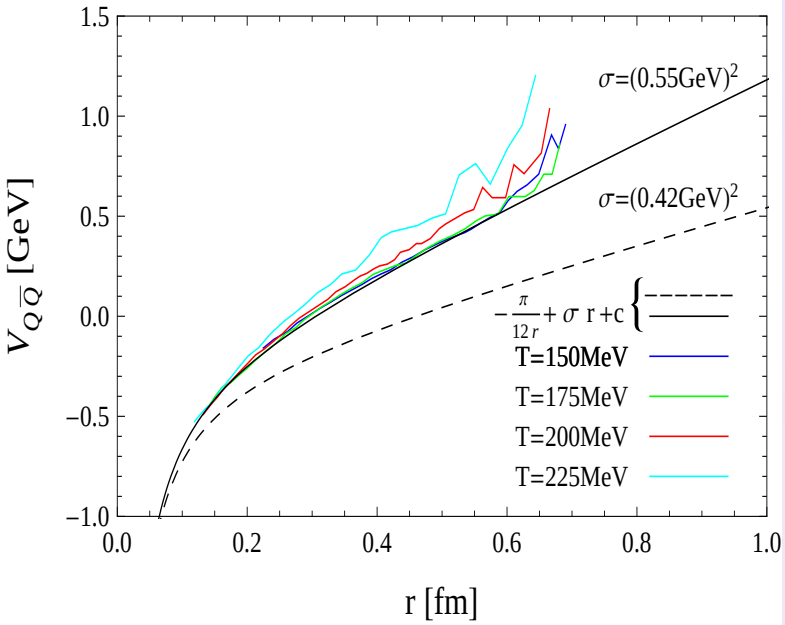
- No mixing

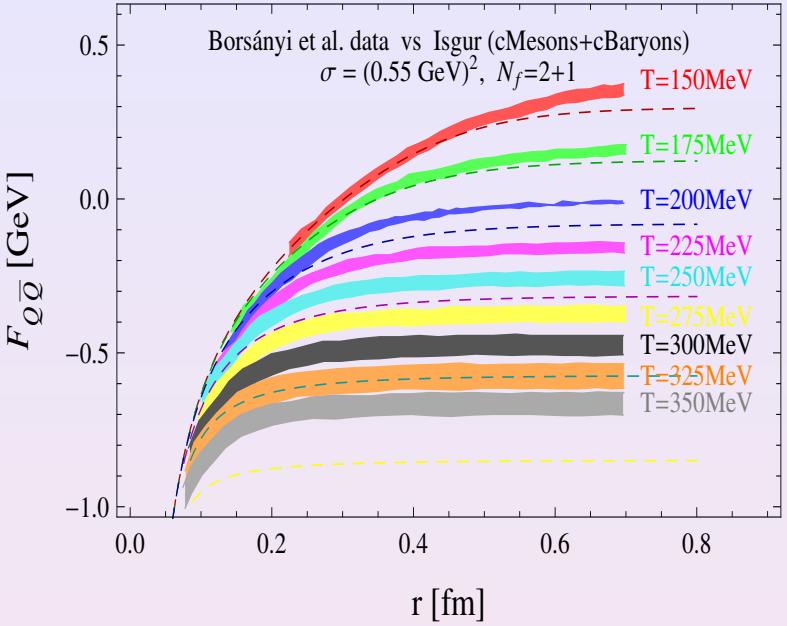
$$e^{-\Delta F(r,T)/T} = e^{-V_Q(r)/T} + \left(\sum_n e^{-\Delta_H^{(n)}/T} \right)^2$$

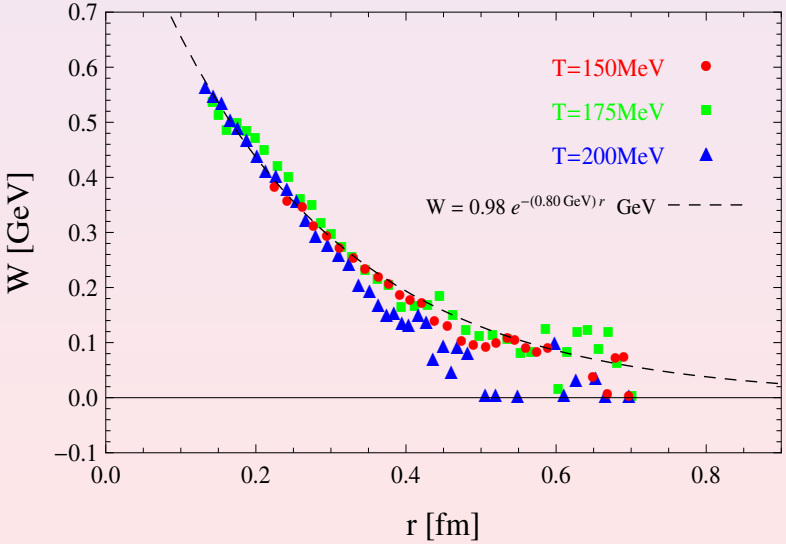
- Two modes model

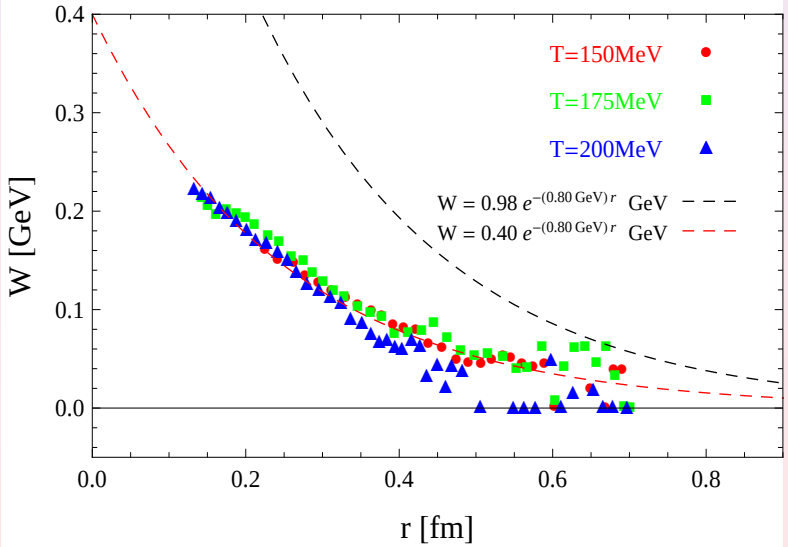
$$V(r) = \begin{pmatrix} -\frac{4\alpha}{3r} + \sigma r & W(r) & \\ W(r) & 2\Delta & \\ & & \ddots \end{pmatrix} , \qquad W(r) = g e^{-mr}$$

Free energies

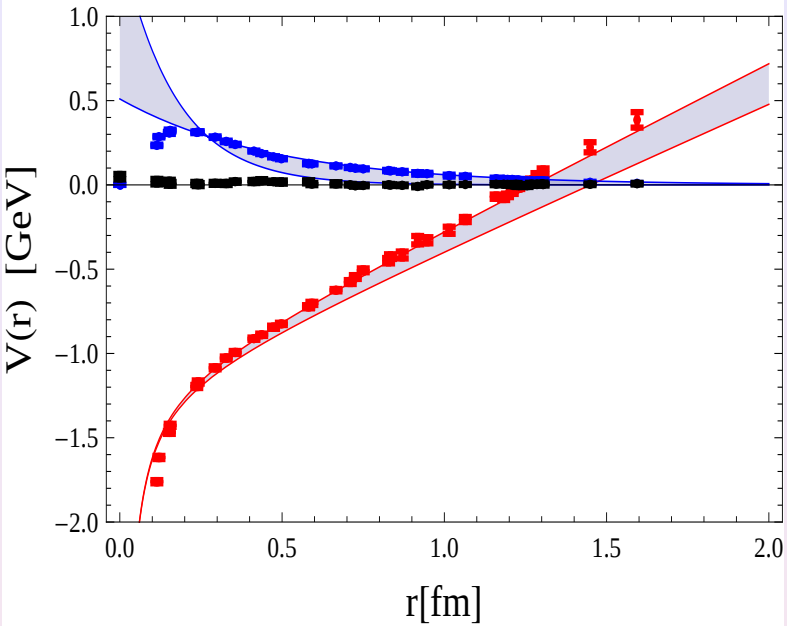


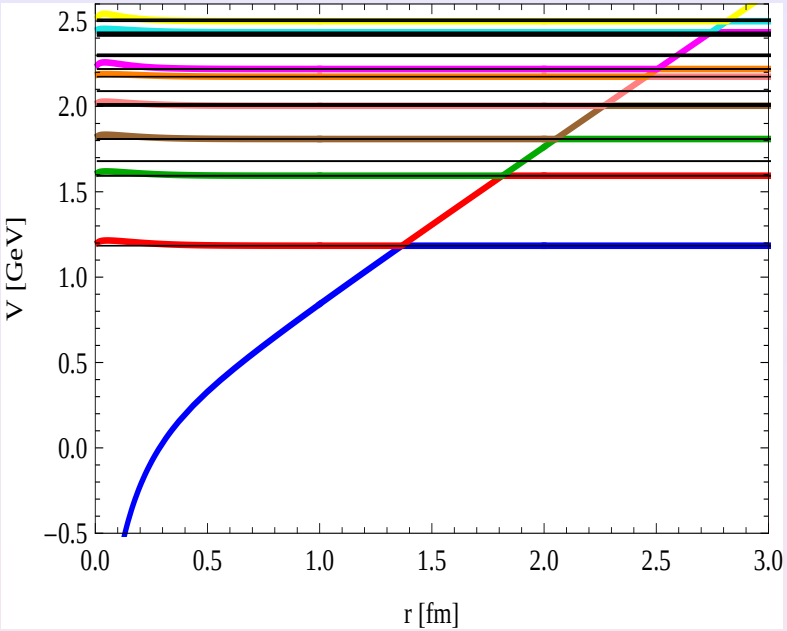






Avoided crossing from thermodynamics





- String tension is only defined in Quenched Approximation
- String breaks. How determine string tension ?
- Using thermodynamics WITH mixing

$$\sqrt{\sigma} = 0.424(14)\mathrm{GeV}, \quad g = 0.98(47)\mathrm{GeV} m = 0.80(38)$$

- Spectral representation $\rightarrow$ Quantum phase transition AT ZERO TEMPERATURE and complex string tension as a pole in the second Riemann sheet

ENTROPY SHIFTS

Thermodynamic shifts

- Add one extra heavy charge belonging to rep R to the vacuum
- Energy of the states changes under the presence of the charge

$$E_n \to E_n^R \to \Delta_n^R + m_R + \dots$$

- In the static gauge $\partial_0 A_0 = 0$ the Ployakov loop operator

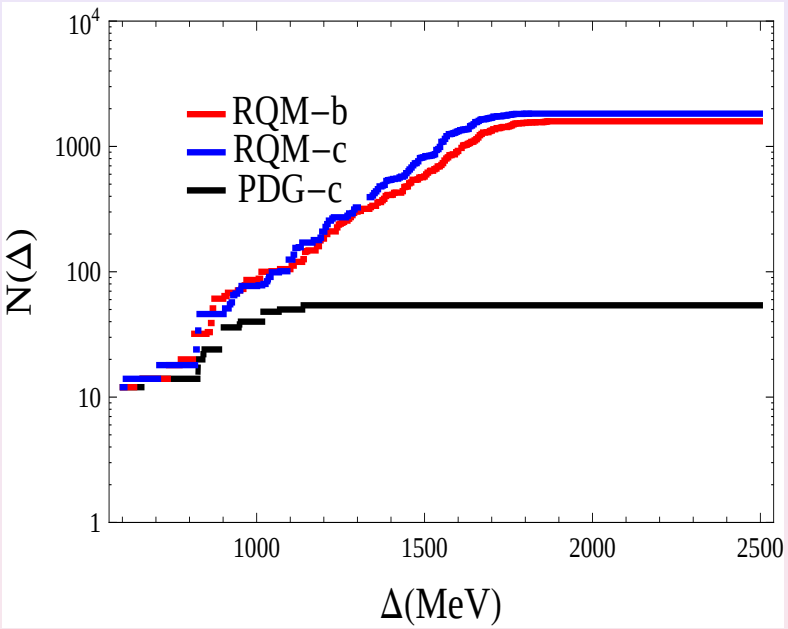
$$tr\Omega(\vec{r}) = \mathrm{tr} e^{iA_0(\vec{r})/T}$$

- The ratio of partition functions $\to$ Free energy shift

$$\langle tre^{iA_0/T} \rangle = \frac{Z_R}{Z_0} = e^{-\Delta F_R/T} = \frac{\sum_n e^{-\Delta_n/T}}{1 + \dots}$$

Counting states (Hagedorn-Polyakov temperature)

$$N(\Delta) = \sum_n \theta(\Delta - \Delta_n) \sim e^{\Delta/T_{H,L}} \rightarrow L(T) \sim \int N'(\Delta) e^{-\Delta/T}\, d\Delta$$



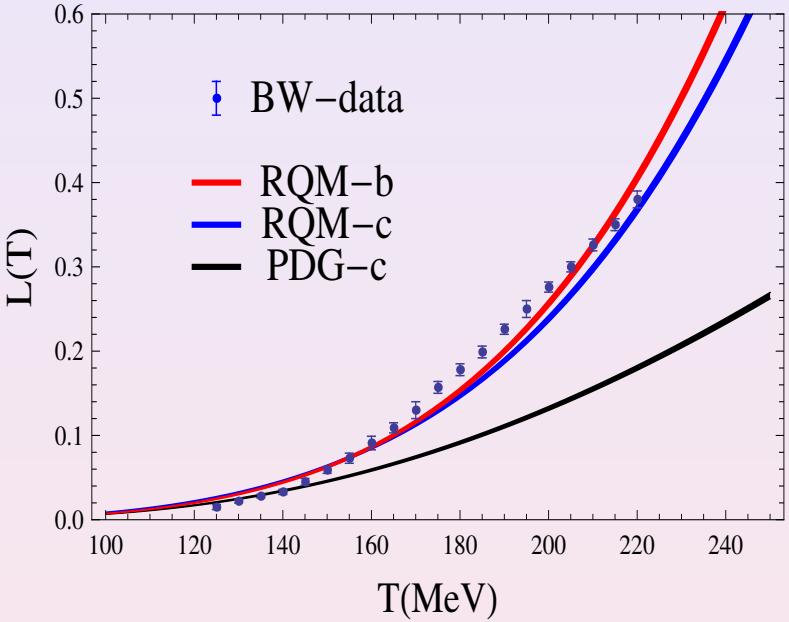


Figure: Left: $N(\Delta)$ as a function of the c-quark and b-quark mass subtracted hadron mass $\Delta = M - m_Q$ (in) with u, d and s quarks, computed in the RQM vs PDG. Right: Polyakov loop as a function of temperature (in MeV).

- Polyakov loop ambiguity removed by entropy shift

$$\langle \mathrm{tr}\Omega(0)\rangle_T = e^{-F_Q(T)/T} \to \Delta S_Q(T) = -\partial_T F_Q(T)$$

- Third principle of thermodynamics for degenerate states

$$\Delta S_Q(0) = \log(2N_f)\,, \qquad \Delta S_Q(\infty) = \log N_c$$

- RGE equation for specific heat

$$\Delta c_Q = T\frac{\partial S_Q}{\partial T} = \frac{\partial}{\partial T}\left\{T\int d^4x\left[\frac{\langle \mathrm{tr}\Omega\,\Theta(x)\rangle}{\langle \mathrm{tr}\Omega\rangle} - \langle\Theta(x)\rangle\right]\right\} \equiv \frac{\partial U_Q}{\partial T}$$

- Energy momentum tensor

$$0 = \mu\frac{dS_Q}{d\mu} = \beta(g)\frac{\partial S_Q}{\partial g} - \sum_q m_q(1+\gamma_q)\frac{\partial S_Q}{\partial m_q} - T\frac{\partial S_Q}{\partial T}$$

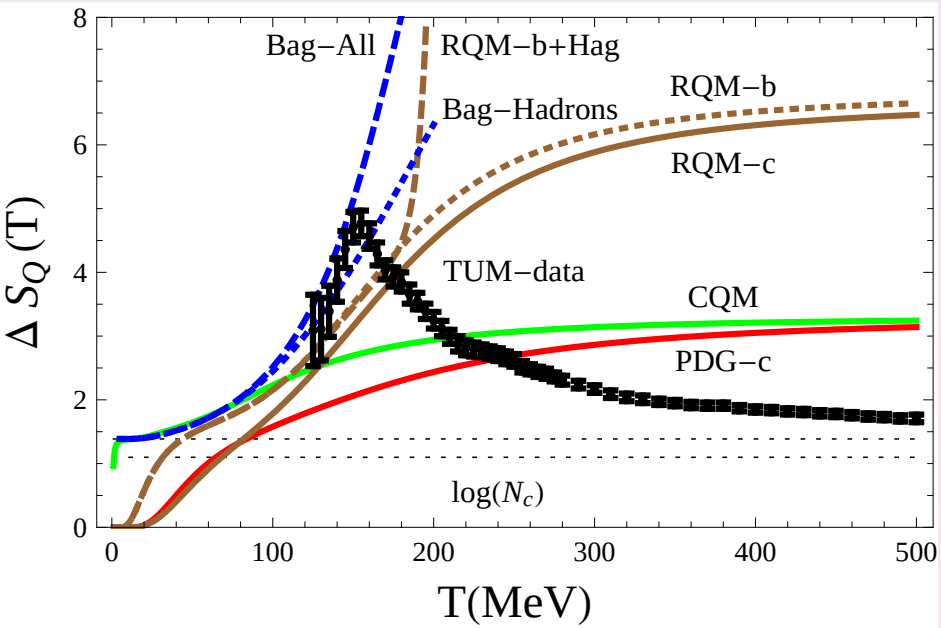
- Entropy shift IS NOT a true entropy $c = T\partial_T S = (\Delta H)^2/T^2 > 0$

From Hadron resonance gas ...

- TUM collaboration $125 < T 6000\mathrm{MeV}$
- Constituent Quark Model $M = 300\mathrm{MeV}$, $m_u = 2.5\mathrm{MeV}$,$m_d = 5\mathrm{MeV}$, $m_s = 95\mathrm{MeV}$.

$$L = \sum_{q=u,d,s} g_q e^{-M_{\bar{Q}q}/T} + \sum_{q,q'=u,d,s} g_{q,q'} e^{-M_{\bar{Q}qq'}/T} + \ldots$$

- All=($Q\bar{q}$, Qqq and $Q\bar{q}g$) Hadron spectrum (missing states !!)

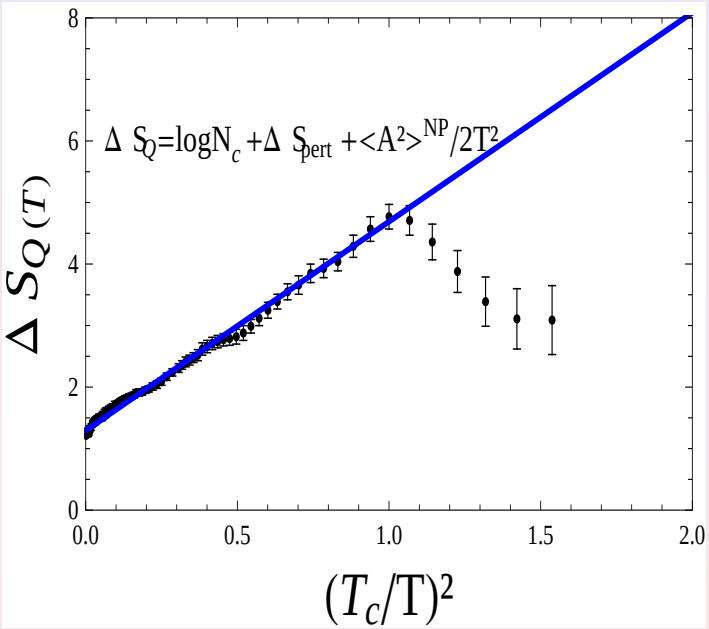


... to Power corrections

- Dim-2 condensates

$$\langle tr(e^{iA_0 T})\rangle \sim N_c \exp\left[-g^2 \frac{\langle (A_0^a)^2\rangle}{4N_c T^2}\right] \rightarrow$$

$$S_Q(T) = \frac{\langle \mathrm{tr}(\bar{A}_0^2)\rangle^{\mathrm{NP}}}{2N_c T^2} + S_{\mathrm{pert}}(T) + \log(N_c)$$



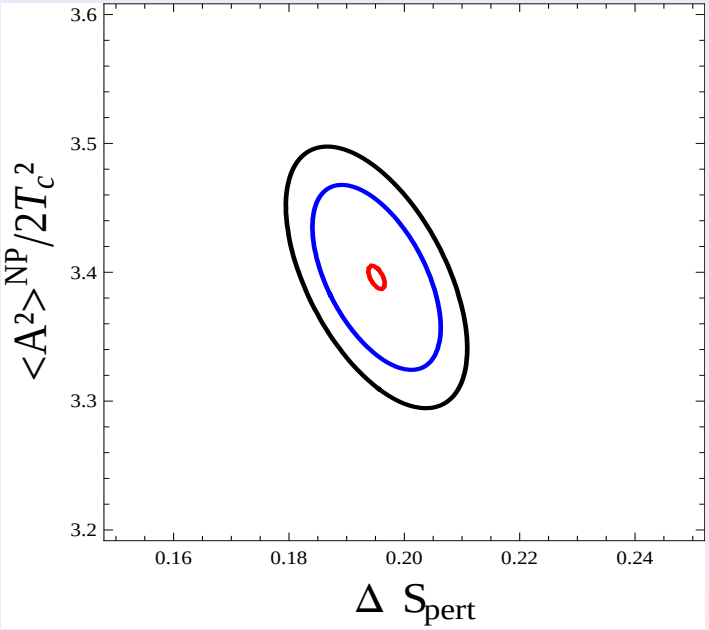


Figure: Left panel: TUM lattice data for the entropy as a function of the inverse squared temperature in unites of the critical temperature.

ANATOMY OF HADRON RESONANCE GAS

Limitations of the Hadron resonance gas

In the HRG Hadrons are taken STABLE, POINT-LIKE AND ELEMENTARY

- Hadrons are composite
- Hadrons have a size
- Hadron Resonances have a finite width

From chiral quark models to hadron resonance gas

- Use local Wilson line $\Omega(x) = e^{igA_0(x)}$ as quantum variable
- Quantization of Multiquark states: Create/Anhiquilate a quark at point $\vec{x}$ and momentum p

$$\Omega(x)e^{-E_P/T} \qquad \Omega(x)^+ e^{-E_P/T}$$

- At low temperatures quark Boltzmann factor small $e^{-E_p/T} < 1$.
  The action becomes small

$$S_q[\Omega] = 2N_f \int \frac{d^3x d^3p}{(2\pi)^3} \left[\mathrm{tr}_c\Omega(x) + \mathrm{tr}_c\Omega(x)^+\right] e^{-E_p/T} + \dots$$

$$Z = \int D\Omega\, e^{-S[\Omega]} = \int D\Omega \left(1 - S[\Omega] + \frac{1}{2}S[\Omega]^2 + \dots\right)$$

- $\bar{q}q$ contribution

$$Z_{\bar{q}q} = (2N_f)^2 \int \frac{d^3x_1 d^3p_1}{(2\pi)^3} \int \frac{d^3x_2 d^3p_2}{(2\pi)^3} e^{-E_1/T} e^{-E_2/T} \underbrace{\langle \mathrm{tr}_c \Omega(\vec{x}_1) \mathrm{tr}_c \Omega^\dagger(\vec{x}_2) \rangle}_{e^{-\sigma|\vec{x}_1 - \vec{x}_2|/T}}$$

$$= (2N_f)^2 \int \frac{d^3x_1 d^3p_1}{(2\pi)^3} \frac{d^3x_2 d^3p_2}{(2\pi)^3} e^{-H(x_1,p_1;x_2,p_2)/T}$$

$\bar{q}q$ Hamiltonian

$$H(x_1, p_1; x_2, p_2) = E_1 + E_2 + V_{12} \, .$$

- Quantization in the CM frame $p_1 = -p_2 \equiv p$

$$\left(2\sqrt{p^2 + M^2} + V_{q\bar{q}}(r) \right) \psi_n = M_n \psi_n \, .$$

- Boosting the CM to any frame with momentum P

$$Z_{\bar{q}q} \to \sum_n \int \frac{d^3R d^3P}{(2\pi)^3} e^{-E_n(P)/T}$$

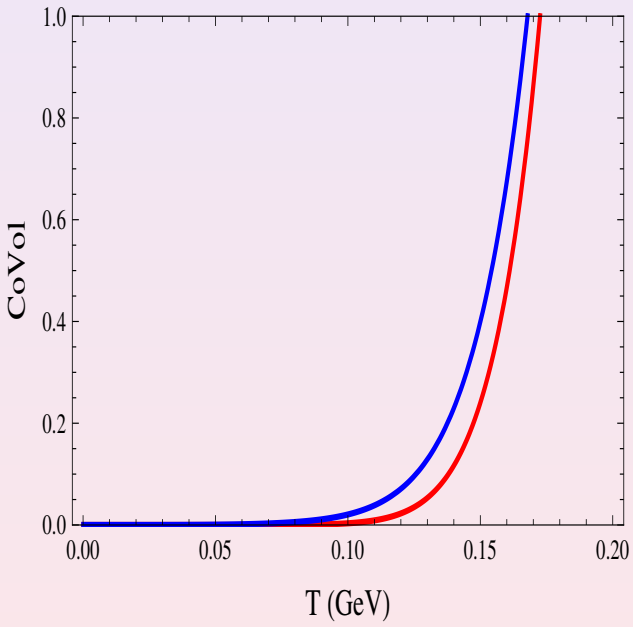
A GAS OF NON INTERACTING MESONS ! (valid to $\bar{q}q\bar{q}q$)

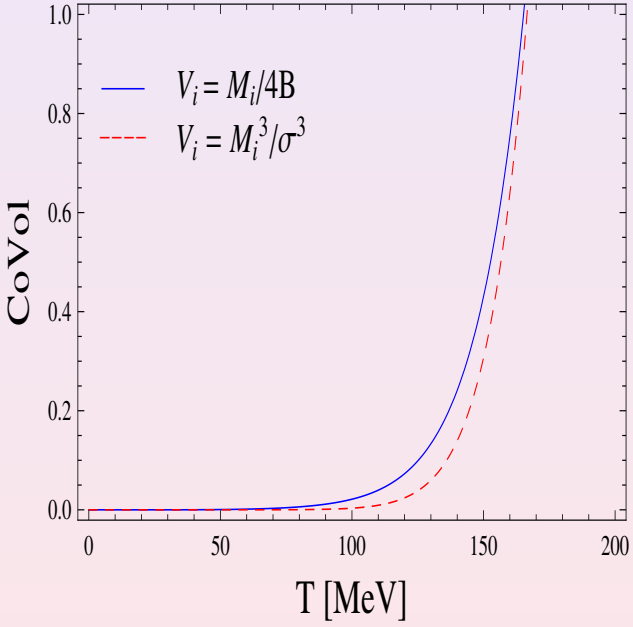
Excluded Volume constraint

$$\sum V_i N_i \leq V \qquad \sum_i V_i \int \frac{d^3p}{(2\pi)^3} \frac{g_i}{e^{E_i(p)/T} \pm 1} \leq 1$$

MIT bag model

$$V_i = M_i/(4B) \qquad B = (0.166 GeV)^4$$

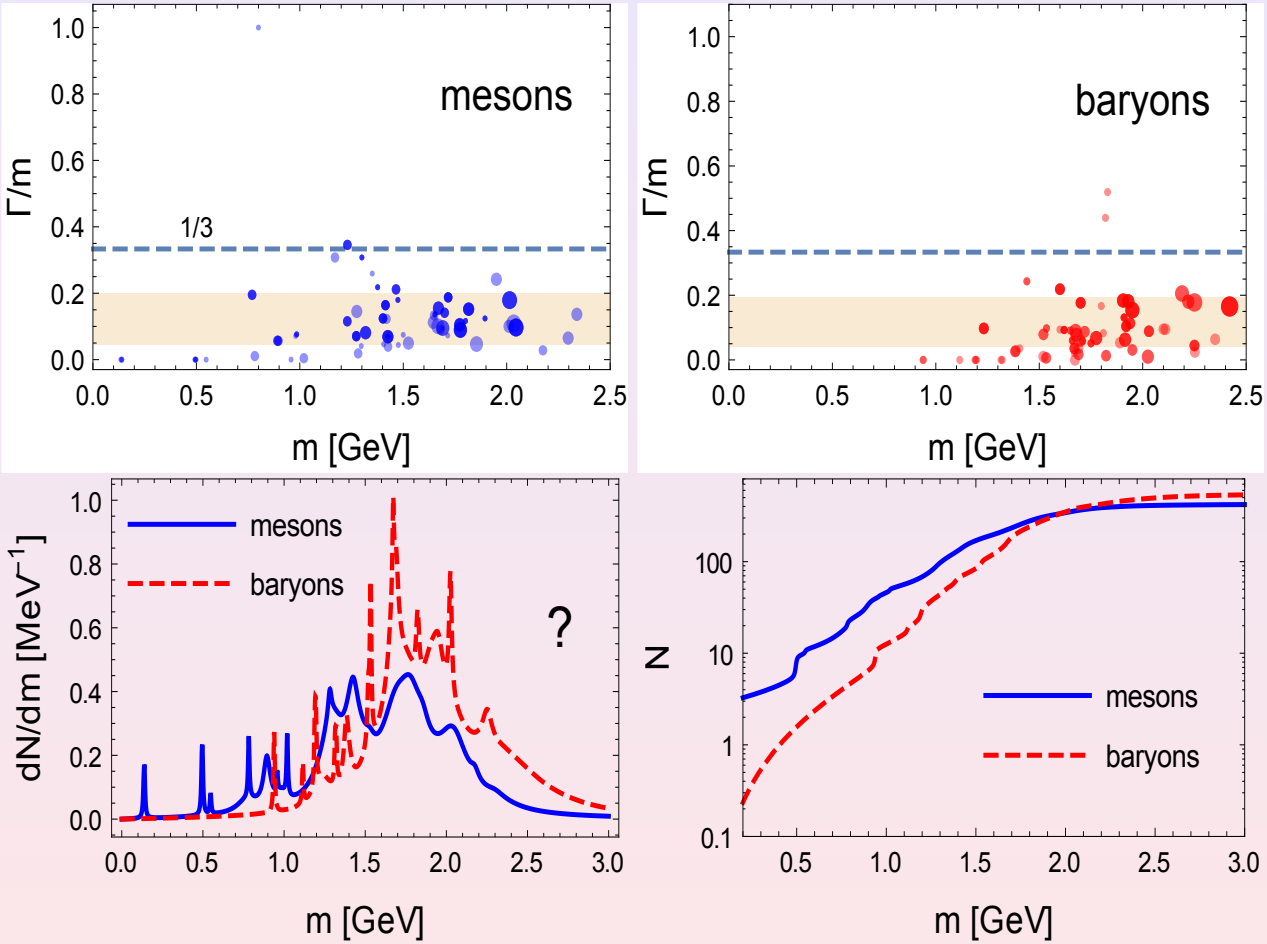




When hadrons overlapp, excluded volume corrections are important → percolation

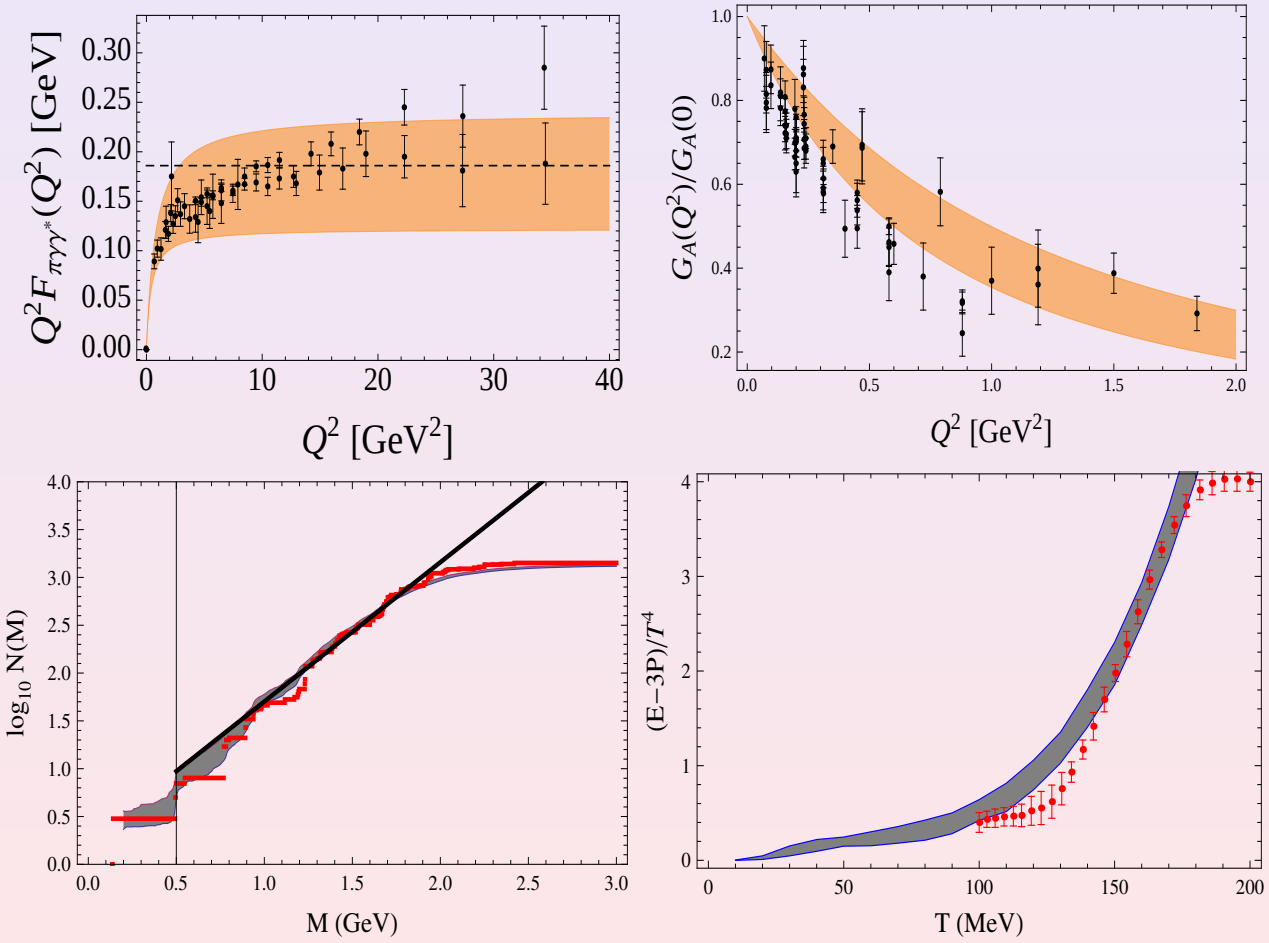
Resonance spectrum and width

- Resonances have a *mass spectrum* (what is the mass?)



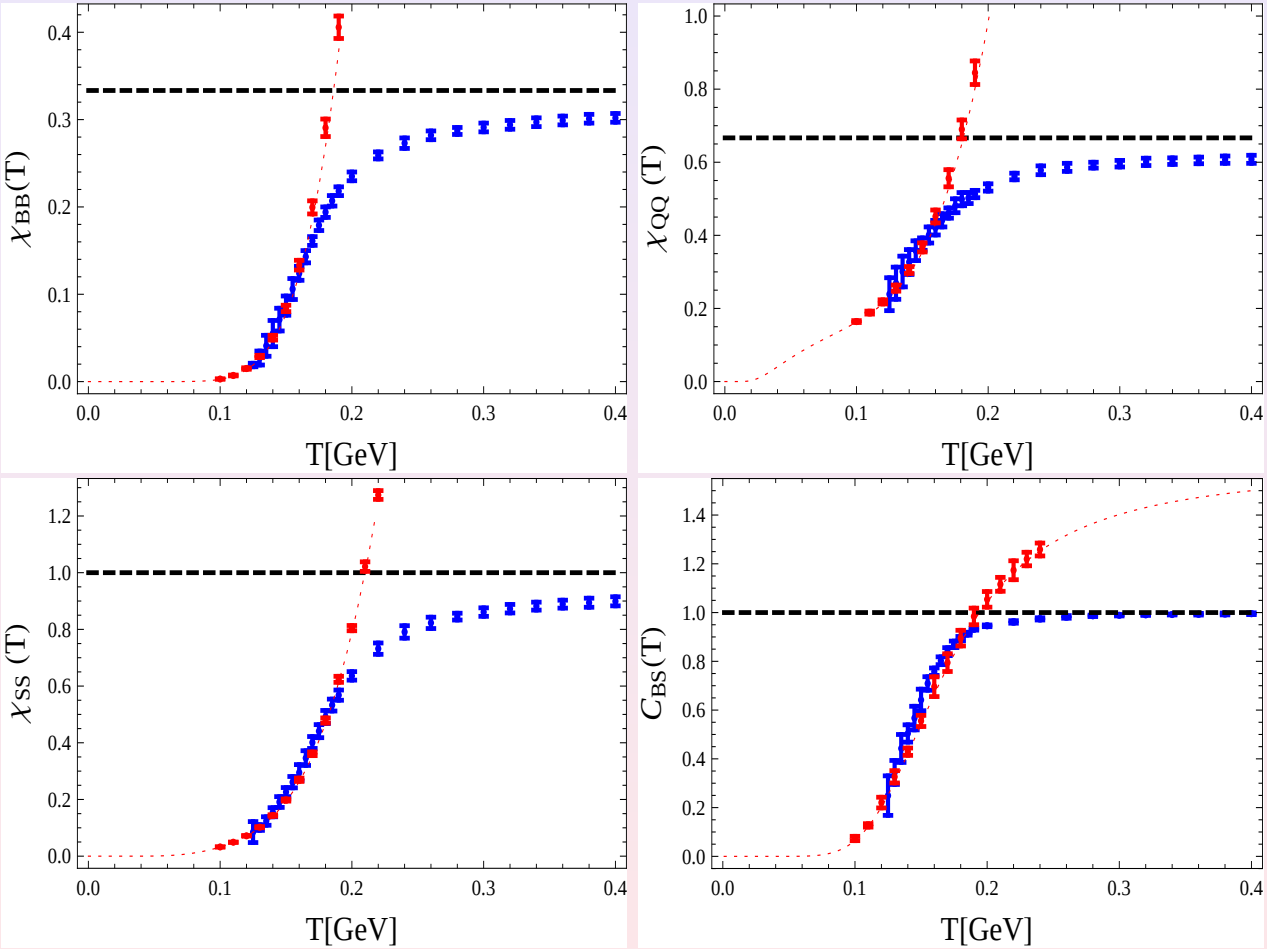
Resonance spectrum and the half-width rule

- The **half-width rule**: $\Delta M_R = \Gamma_R/2$ or $\Delta M_R^2 = M_R \Gamma_R$
- Then take a RANDOM mass (error estimate)
- Hadronic Meson dominated form factors, ...



Fluctuations of conserved charges

$$\chi_{AB}^{\mathrm{PDG}} \pm \Delta\chi_{AB}^{\mathrm{PDG}} \sim \chi_{AB}^{\mathrm{QCD}} \pm \Delta\chi_{AB}^{\mathrm{QCD}}$$



CONCLUSIONS

Conclusions:

- Quark Hadron Duality suggests that at low temperatures Hadrons can be considered as a complete basis of states in terms of a hadron resonance gas. The HRG works up to relatively large temperatures.
- PDG states incorporate currently just $q\bar{q}$ or qqq states which fit into the quark model. What states are needed when approaching the crossover from below ?
- Saturating at subcritical temperatures requires many hadronic states, so the excited spectrum involves relativistic effects even for heavy quarks.
- Uncertainty estimates from the HRG may change our perception of what we understand by a missing states.
- Polyakov loops in fundamental and higher representations allow to deduce multiquark quark states, gluelumps etc. containing one or several heavy quark states. Clear hints for missing states.

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