The quark model and the missing hyperons [1]

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[1] Talk at YSTAR 2016, JLab Nov. 16, 2016

In this talk we assume that the baryons are mainly composed of three quarks.

The focus will be on the <u>first</u> excited
Λ^* and Σ^* states.

To a large extent the theoretical observations to be presented are based on calculations within the cloudy (chiral) bag model.

We assume that quarks are confined.
To simulate confinement one can use the harmonic oscillator potential model or the MIT bag model.

In a bag model one can implement aspects of chiral symmetry.
Require the axial current to be continuous.
A consequence is that the three-quark bag (or baryon core) is surrounded by a pseudo-scalar meson cloud.

If one considers flavor SU(3), <u>the chiral (cloudy) model</u> will predict the decays of excited hyperons to a ground state baryon plus a meson (π, K and/or η).
(No new parameters added.)

The hadronic wave function is

$$\Psi = \Psi_{color}\ \Psi_{flavor}\ \Psi_{spin}\ \Psi_{space}$$

In this talk we assume that isospin is a good symmetry.
The masses $m_u = m_d = m_q$.
The s quark has mass $m_s > m_q$.
[In the (cloudy or MIT) bag model the u and d quarks are massless.]

The $\mathrm{SU}_F(3)$ is a broken symmetry.

Other assumptions:

(1) All hadrons are $SU(3)$-color singlets, i.e. Ψ_{color} is antisymmetric.

(2) Confinement is universal, i.e., the same condition for all quark flavors.

(3) Pauli says: Two identical quarks must have an anti-symmetric wave function.

(4) The quarks interact via an effective gluon exchange.

The non-relativistic one-gluon exchange (OGE) between quarks i and j, DeRujula et al. (1975):

$$H^{ij}_{hyp} \;=\; A_{ij}\left\{\frac{8\pi}{3}\vec{S}_i\cdot\vec{S}_j\delta^3(\vec{r}_{ij}) + \frac{1}{r^3_{ij}}\left(\frac{3(\vec{S}_i\cdot\vec{r}_{ij})(\vec{S}_j\cdot\vec{r}_{ij})}{r^2_{ij}} - \vec{S}_i\cdot\vec{S}_j\right)\right\}$$

The spin-spin and the tensor interaction are closely related.

Isgur and Karl argued that the spin-orbit force could be neglected.

In Bag models the spin-orbit from scalar confinement and OGE roughly cancel.

Effectively what remains are

the spin-spin and tensor interactions

due to OGE <u>and</u> the pseudo-meson cloud.

For the <u>ground state baryons</u> of three quarks,
the tensor force implies that they are a linear combination of
spatial S, S′ and D states
(similar to ^{3}H and ^{3}He wave functions).

This means, the <u>spatial wave function</u> of the baryons <u>ground state</u>
is <u>not</u> necessarily
in a pure <u>symmetric</u> state $|^2S_S\rangle$ as is commonly used.

For example, the nucleon state as evaluated by Isgur and Karl is:

$$|N\rangle \;\simeq\; 0.90|^2S_S\rangle - 0.34|^2S'_S\rangle - 0.27|^2S_M\rangle - 0.06|^2D_M\rangle$$

and the $\Lambda(1116)$ is:

$$|\Lambda\rangle \;\simeq\; 0.93|^2S_S\rangle - 0.30|^2S'_S\rangle - 0.20|^2S_M\rangle - 0.03|^4D_M\rangle - 0.05|\mathbf{1},^2S_M\rangle$$

The components beyond $|^2S_S\rangle$ will modify (sometimes strongly)
the excited baryon to ground states <u>decay widths</u>.

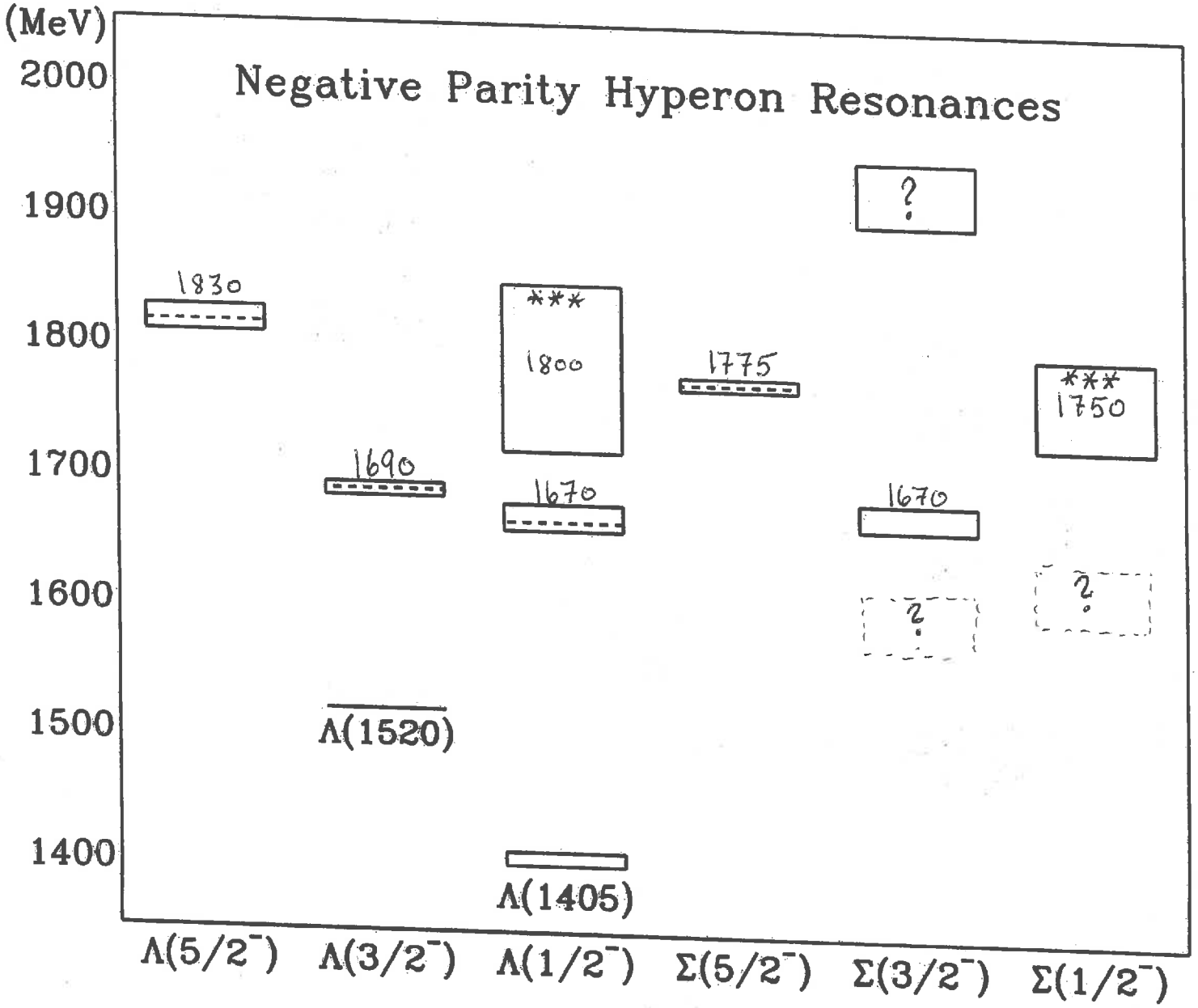

Negative Parity Hyperon Resonances
(MeV)
2000
1900
1800
1700
1600
1500
1400
1830
1690
***
1800
1670
Λ(1520)
Λ(1405)
1775
?
1670
?
***
1750
?
Λ(5/2⁻)
Λ(3/2⁻)
Λ(1/2⁻)
Σ(5/2⁻)
Σ(3/2⁻)
Σ(1/2⁻)

Table 1 : Hadronic relative decay branching ratios for Λ^* decays (PDG 2012).

$\Lambda^* - j^P$	decay product	%	comment	comment
$\Lambda(1405) - \frac{1}{2}^-$	$\pi\Sigma$	100	K^-p bound state?	–
$\Lambda(1520) - \frac{3}{2}^-$	$\bar{K}N$	45 ± 1	–	–
	$\pi\Sigma$	42 ± 1	–	–
	$\pi\pi\Lambda$	10 ± 1	via $\pi\Sigma(1385)$ mainly?	–
	$\pi\pi\Sigma$	0.9 ± 0.1	$\gamma\Lambda \sim 0.85\%$	–
$\Lambda(1670) - \frac{1}{2}^-$	$\bar{K}N$	$20 - 30$	–	–
	$\pi\Sigma$	$25 - 55$	–	–
	$\eta\Lambda$	$10 - 25$	–	–
$\Lambda(1690) - \frac{3}{2}^-$	$\bar{K}N$	$20 - 30$	–	–
	$\pi\Sigma$	$20 - 40$	–	–
	$\pi\pi\Lambda$	~ 25	–	–
	$\pi\pi\Sigma$	~ 20	–	–
	$\eta\Lambda$	?	–	–
$\Lambda(1800) - \frac{1}{2}^-$	$\bar{K}N$	$25 - 40$	$***$	–
	$\pi\Sigma$	–	seen	–
$\Lambda(1850) - \frac{5}{2}^-$	$\bar{K}N$	$3 - 10$	–	–
	$\pi\Sigma$	$35 - 75$	–	–
	$\eta\Lambda$	–	?	–

(i): In bag model evaluations we assume that one quark is in a P-state and the two others are in S-states.

(ii): Further we assume that the <u>symmetric</u> spatial wave function describes the three-quark c.o.m. motion of the excited hyperon.
(The c.o.m. boost operator is a symmetric operator.)

The excited Λ^* and Σ^* states therefore have spatially mixed symmetry states.

For a decay to the ground state it is the P-state quark which couples to the outgoing meson (π, $\bar{K}$ or η) (or the emitted photon).

The $\Lambda(1850)$ or the $\Sigma(1775)$ $\frac{5}{2}^-$ states are pure $|^4 8\rangle$ states.

The predicted quark model states for the three Λ^* $\frac{3}{2}^-$ and three Λ^* $\frac{1}{2}^-$ states (and similar for Σ^*) have the following structure

$$|\Lambda^*\rangle \;\simeq\; a|^2\mathbf{1}\rangle + b|^4\mathbf{8}\rangle + c|^2\mathbf{8}\rangle$$

Different quark models give different a, b and c coefficients, e.g., for the $|\Lambda(1520)\rangle$ the harmonic oscillator NRQM of Isgur and Karl gives $a = 0.92$, $b = -0.04$ and $c = 0.39$,

whereas the cloudy (chiral) bag model gives: $a = 0.95$, $b = -0.09$ and $c = 0.30$.

The radiative decay widths of the two models are:

NRQM: $\Gamma[\Lambda(1520) \to \Lambda\gamma] = 96$ **keV,** $\Gamma[\Lambda(1520) \to \Sigma^0\gamma] = 74$ **keV**

Cloudy bag: $\Gamma[\Lambda(1520) \to \Lambda\gamma] = 32$ **keV,** $\Gamma[\Lambda(1520) \to \Sigma^0\gamma] = 49$ **keV**

However

Table 1 : Radiative decay widths for $\Lambda(1405)$ and $\Lambda(1520)$ in keV in the chiral bag model from the different spin flavor multiplets of the Λ^*.

Transition	$\|^2 1\rangle$	$\|^4 8\rangle$	$\|^2 8\rangle$
$\Lambda(1520) \to \gamma\Lambda$	24	21	23
$\Lambda(1520) \to \gamma\Sigma^0$	91	58	93
$\Lambda(1405) \to \gamma\Lambda$	42	0.2	33
$\Lambda(1405) \to \gamma\Sigma^0$	98	0.03	91

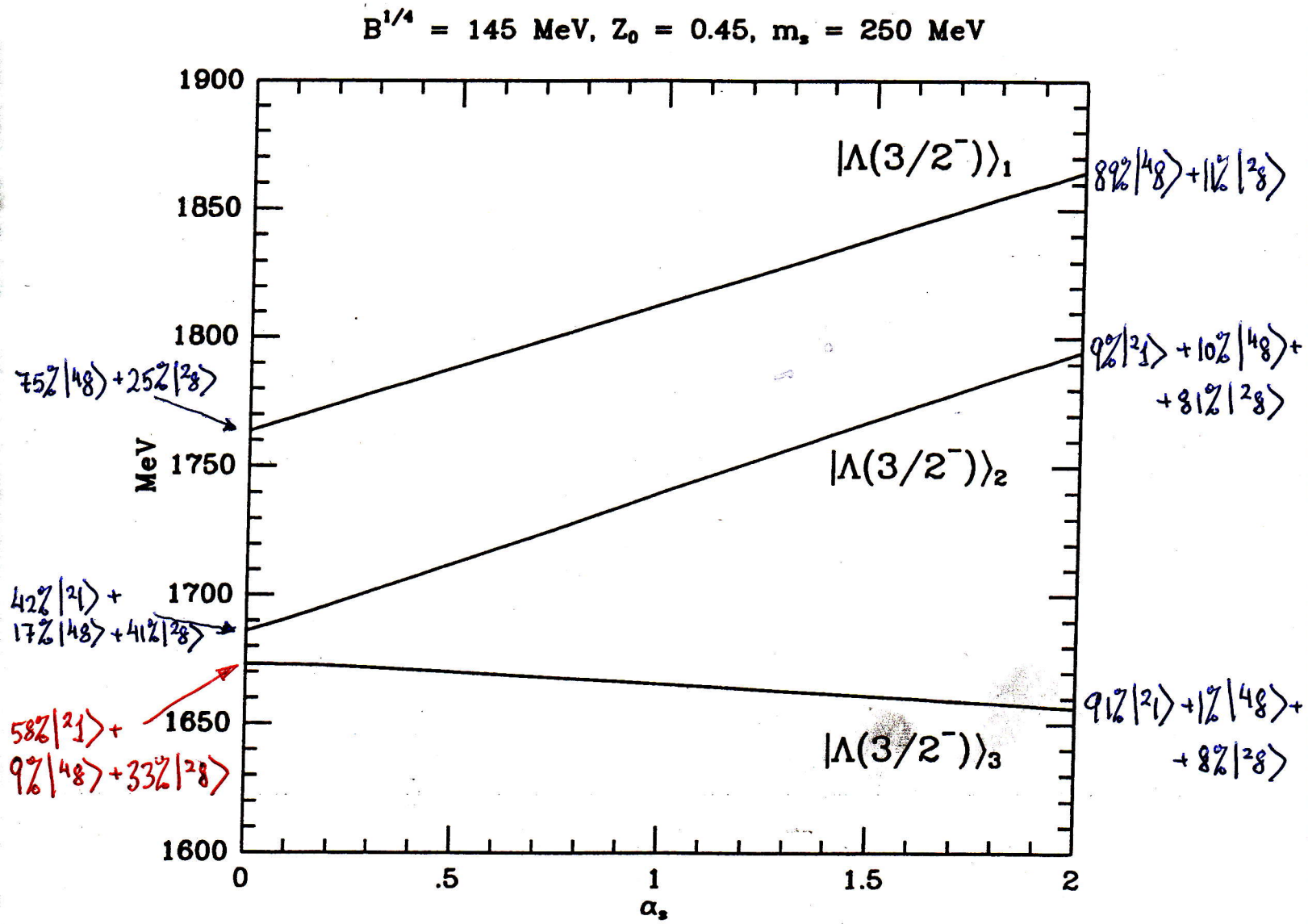

$B^{1/4}$ = 145 MeV, Z_0 = 0.45, m_s = 250 MeV
1900
1850
1800
1750
1700
1650
1600
MeV
0
.5
1
1.5
2
α_s
$|\Lambda(3/2^-)\rangle_1$
$|\Lambda(3/2^-)\rangle_2$
$|\Lambda(3/2^-)\rangle_3$
$8\%|^4 8\rangle + 11\%|^2 8\rangle$
$9\%|^2 1\rangle + 10\%|^4 8\rangle + 81\%|^2 8\rangle$
$91\%|^2 1\rangle + 1\%|^4 8\rangle + 8\%|^2 8\rangle$
$75\%|^4 8\rangle + 25\%|^2 8\rangle$
$42\%|^2 1\rangle + 17\%|^4 8\rangle + 41\%|^2 8\rangle$
$58\%|^2 1\rangle + 9\%|^4 8\rangle + 33\%|^2 8\rangle$

Figure 3b.

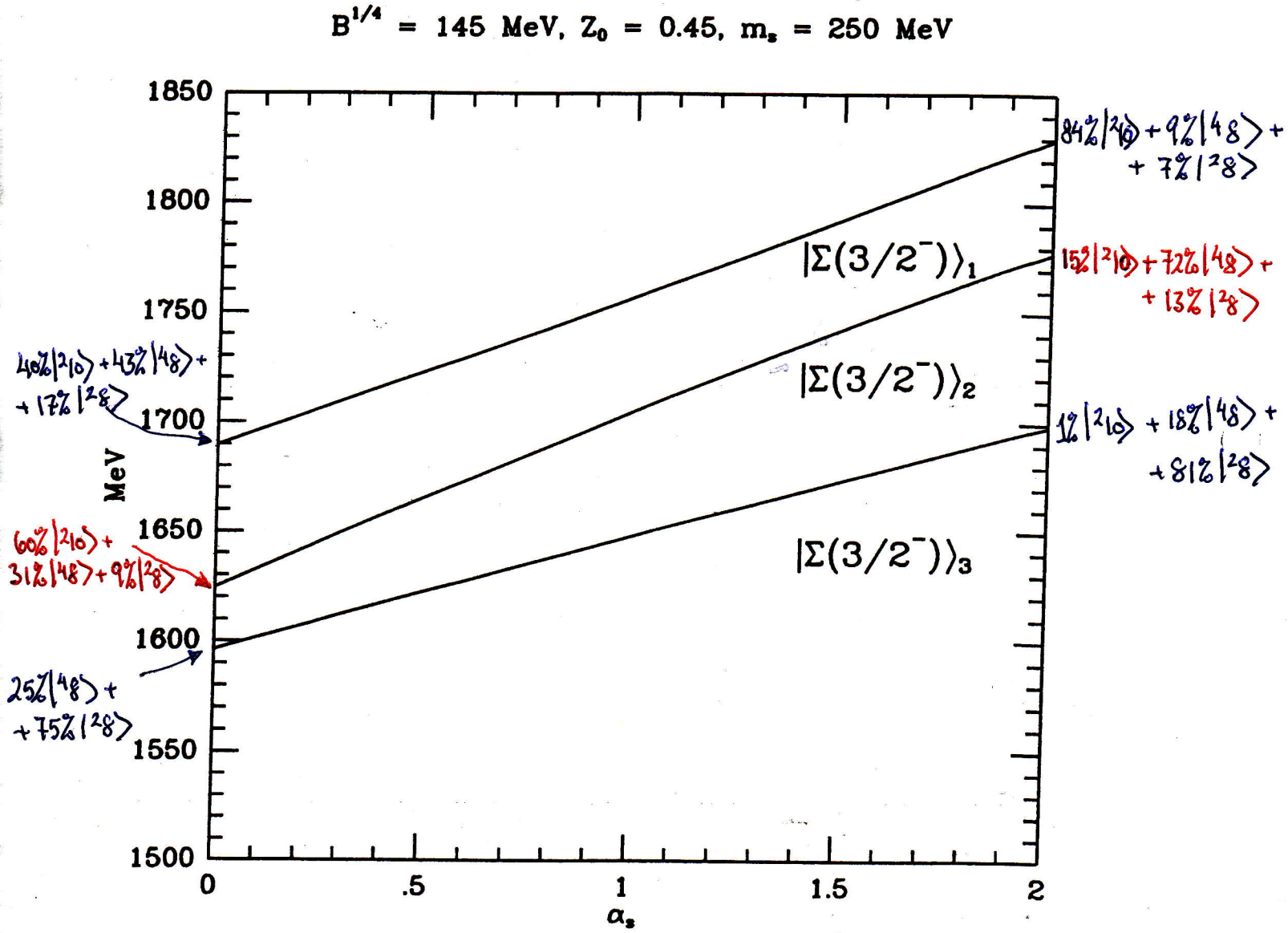

$B^{1/4}$ = 145 MeV, Z_0 = 0.45, m_s = 250 MeV
1850
1800
1750
1700
1650
1600
1550
1500
MeV
0
.5
1
1.5
2
α_s
$|\Sigma(3/2^-)\rangle_1$
$|\Sigma(3/2^-)\rangle_2$
$|\Sigma(3/2^-)\rangle_3$
84%|²10⟩ + 9%|⁴8⟩ + + 7%|²8⟩
15%|²10⟩ + 72%|⁴8⟩ + + 13%|²8⟩
1%|²10⟩ + 18%|⁴8⟩ + + 81%|²8⟩
40%|²10⟩ + 43%|⁴8⟩ + + 17%|²8⟩
60%|²10⟩ + 31%|⁴8⟩ + 9%|²8⟩
25%|⁴8⟩ + + 75%|²8⟩

Figure 3c.

$B^{1/4}$ = 145 MeV, Z_0 = 0.45, m_s = 250 MeV

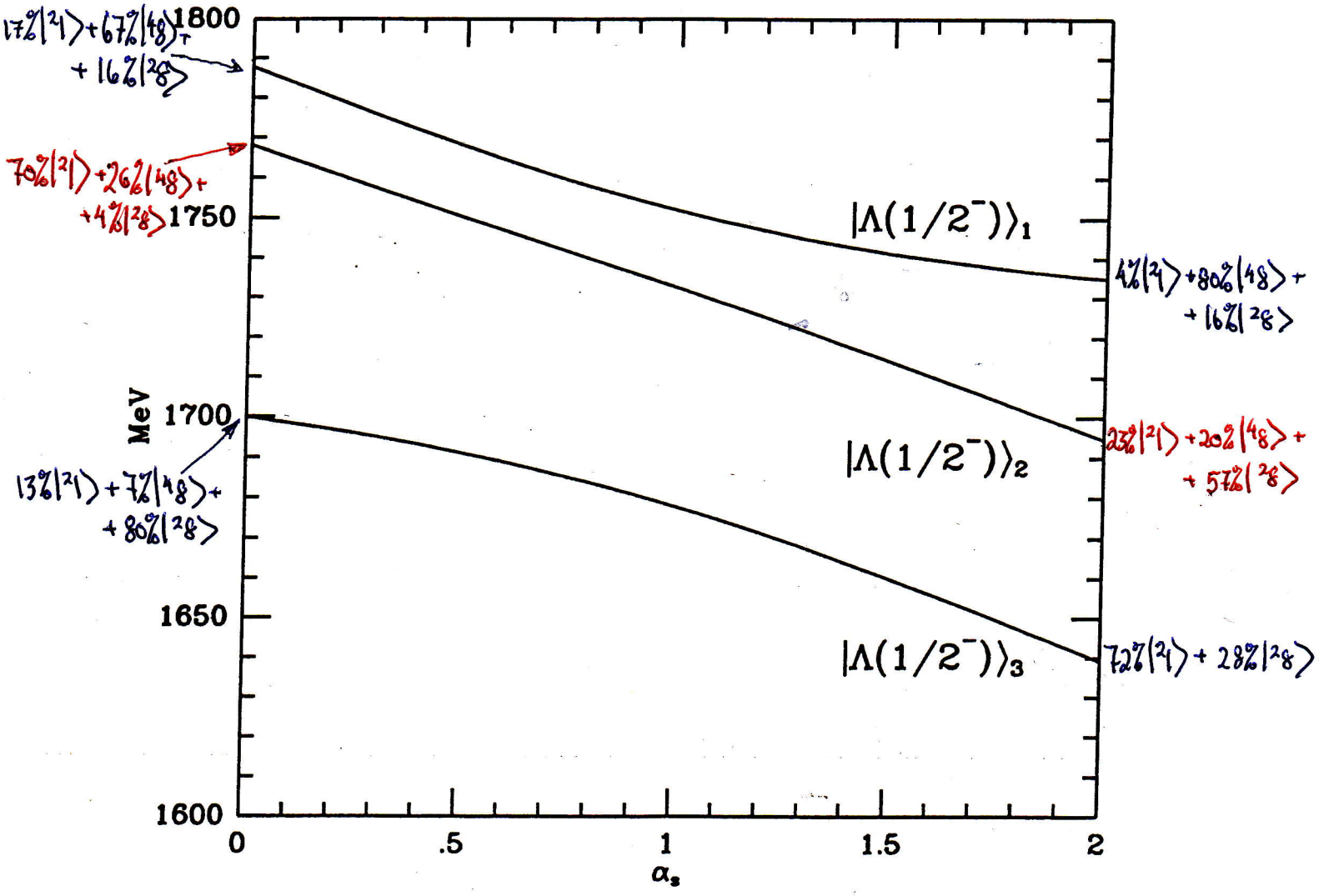


Figure 3d.

$B^{1/4}$ = 145 MeV, Z_0 = 0.45, m_s = 250 MeV

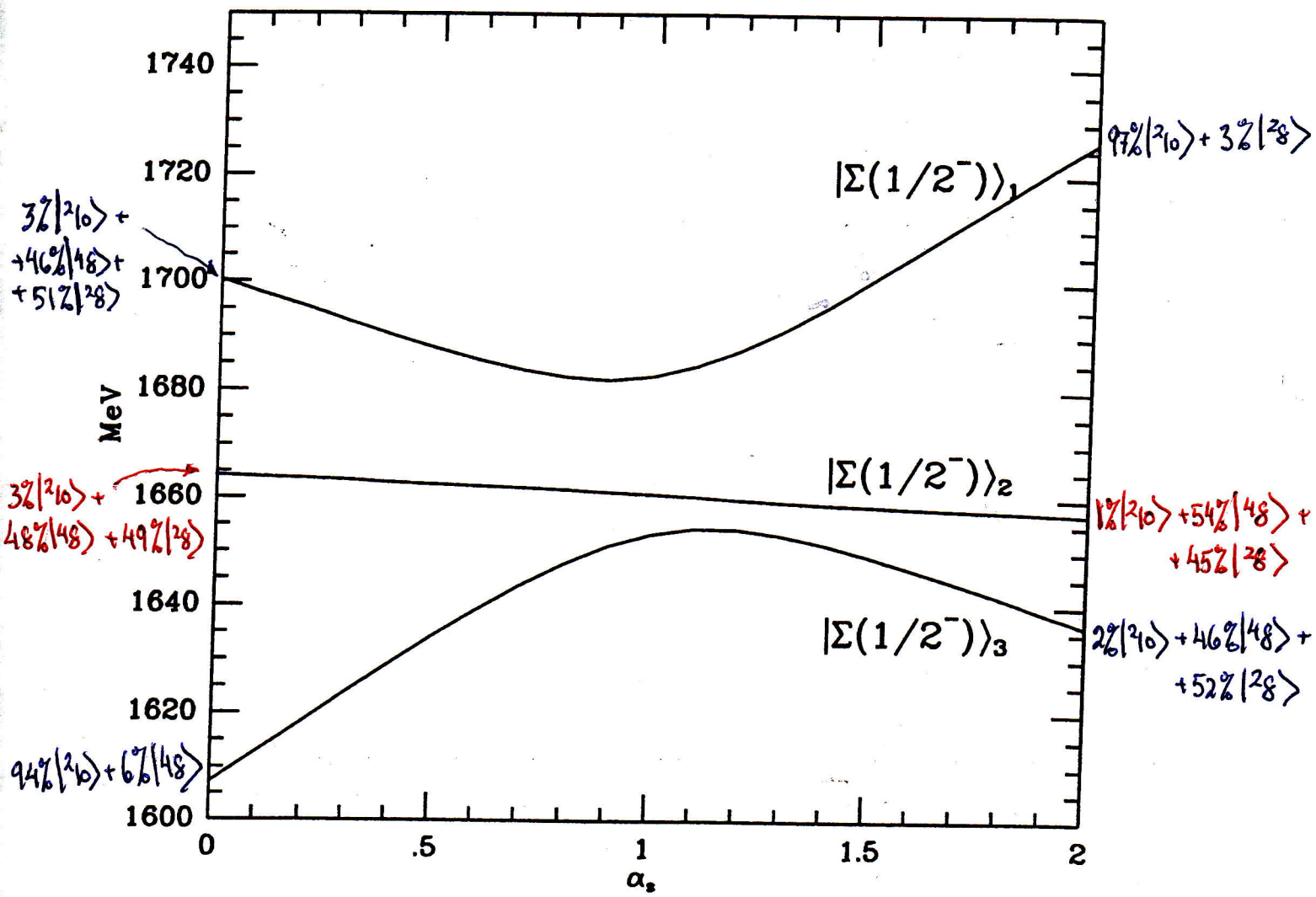


Figure 3e.

Hadronic decays of the two $(5/2)^-$ states

The three-quark spatial wave function has two relative coordinates

$$\vec{\rho} = (\vec{r}_1 - \vec{r}_2)/\sqrt{2} \text{ and } \vec{\lambda} = (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)/\sqrt{6}$$

Both $J^P = 5/2^-$ states have symmetric spin wave functions, $S = 3/2$.

(i): $\Lambda^*(1830)$, an iso-singlet, has a $\vec{\rho}$-dependent spatial wave function ($\vec{\rho}$ is anti-symmetric under $1 \leftrightarrow 2$).

(ii): $\Sigma^*(1775)$, an iso-triplet, has a $\vec{\lambda}$-dependent spatial wave function ($\vec{\lambda}$ is symmetric under $1 \leftrightarrow 2$).

Decay ratio consequences:

$\Lambda^*(1830)$ couples weakly to $\bar{K}N$ since the nucleon wave function is symmetric $1 \leftrightarrow 2$, whereas $\Sigma^*(1775)$ couples easily to $\bar{K}N$.

These findings are modified by H_{hyp}, and adding the mixed symmetric component $|^2S_M\rangle$ of the nucleon state gives the ratio of decay amplitudes (PRL 41, 1269):

$$\frac{\mathrm{A}(\Lambda(1830) \to \bar{K}N)}{\mathrm{A}(\Sigma(1775) \to \bar{K}N)} \simeq -0.28$$

The $\Lambda(1405) - \Lambda(1520)$ mass splitting

NRQM and bag models assume a pure qqq state for $\Lambda(1405)$.

Then $\Lambda(1405)$ and $\Lambda(1520)$ are almost mass degenerate.

(The NRQM of Isgur and Karl added an ad hoc spin-orbit force to explain the mass difference.
However, they argued no spin-orbit force in N^* and Δ^* systems.)

$\Lambda(1405)$ **is only 27 MeV below the K^-p threshold.**

Does $\Lambda(1405)$ contain large multi-quark component, e.g., $qqq\bar{q}q$?
or is it a K^-p bound state (Dalitz and Tuan 1960)? (a quark molecule?)

A bound state can be implemented in the cloudy (chiral) bag model if the meson cloud is treated non-perturbativly.

Apart from measurements of the K^-p atom by a KEK collaboration (1997) and by SIDDHARTA collaboration (2011), most data are old.

In the cloudy bag model ($m_q = 0$ MeV)

the quark P-state with j=3/2 has a lower energy than j=1/2.

The bag confinement condition introduces a spin-orbit splitting of the quark states.

Fortunately, the relativistic OGE introduces a spin-orbit force of opposite sign.

In cloudy bag model calculations the two spin-orbit contributions basically cancel.

Effectively what remains are

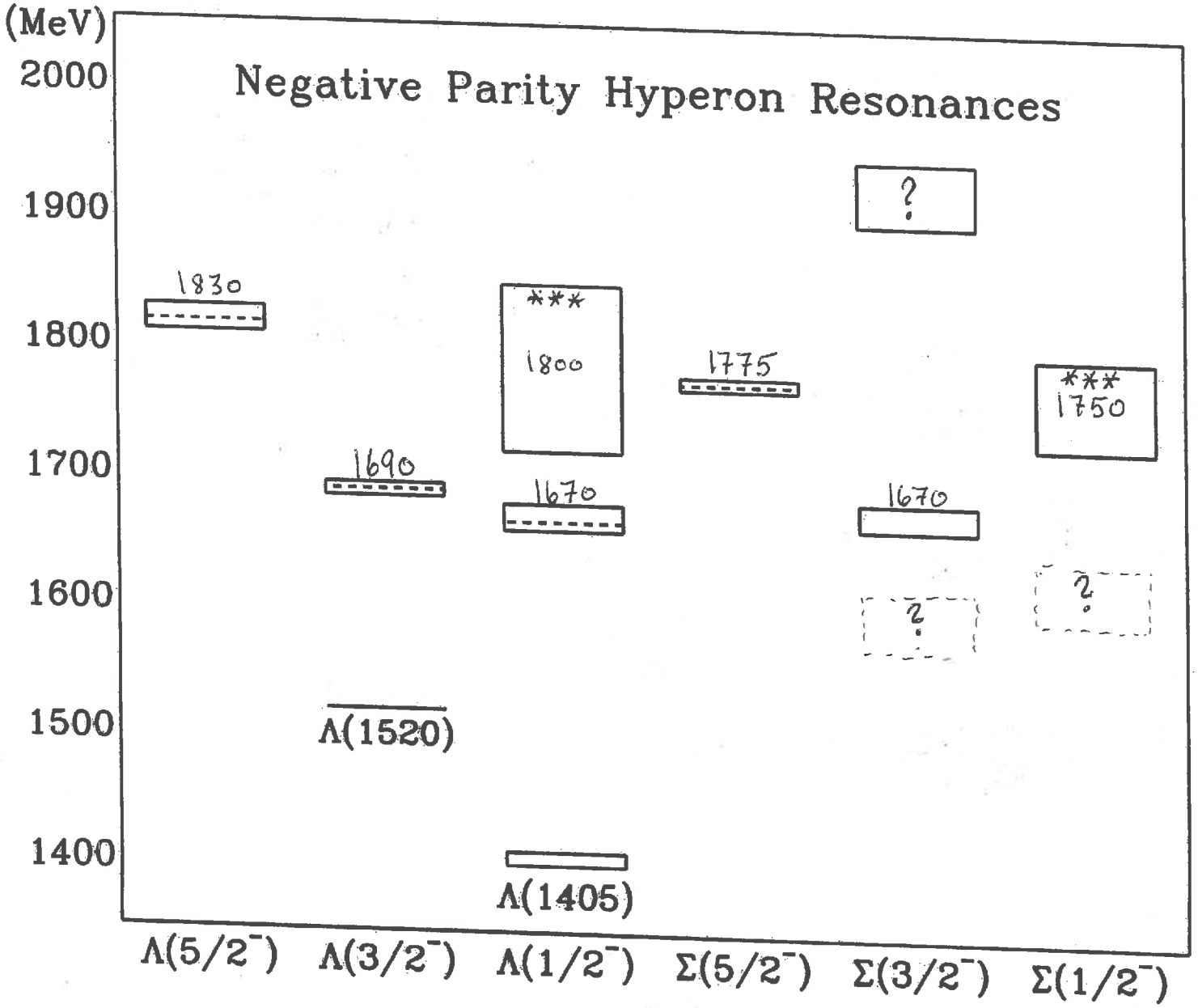
the spin-spin and tensor interactions

due to OGE and the pseudo-meson cloud.

These interactions strongly affects the decay rates of Λ^* and Σ^* states

to $\bar{K}N$ and $\pi\Sigma$.

We need <u>new</u> data to test the various quark models.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Λ(1405)
1775
?
1670
?
***
1750
?
Λ(5/2⁻)
Λ(3/2⁻)
Λ(1/2⁻)
Σ(5/2⁻)
Σ(3/2⁻)
Σ(1/2⁻)

SUMMARY

Do all the predicted three-quark lowest excited J^- Λ^* and Σ^* states exist?

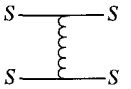
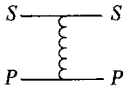
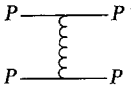
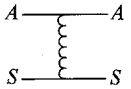
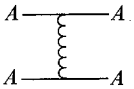
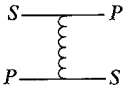
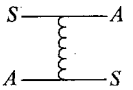
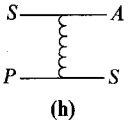
A K_L^0 beam on a hydrogen target can access the Σ^* states.

The reaction $\gamma^* + p \to K^+\Lambda^*$ could explore the Λ^* states.

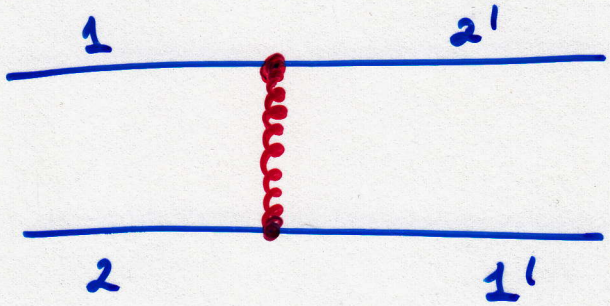
As shown, Λ^* and Σ^* decays to $\bar{K}N$, $\pi\Sigma$ etc.

Theory needs more accurately measured decay branching ratios in order to make progress.

Table 1. The numerical coefficients (column 2) and two-particle operators (column 3) for the gluon-exchange diagrams in column 1 are given. The coefficients are defined as in (5.4) and (5.5). Column 4 gives the number of the possible ways of reading the diagrams (and operators). See also the discussion following (5.6). The time dependence of the two last diagrams is suppressed in this table

Diagram	Coefficient	Operator	No. of operators
 (a)	$d^M_{SSSS} = -\frac{\alpha_s}{8}\frac{0.177}{R}$	$\lambda^a\sigma\otimes\lambda^a\sigma$	1
	$m^E_{SSSS} = \frac{\alpha_s}{8}\frac{0.278}{R}$	$\lambda^a\otimes\lambda^a$	1
 (b)	$d^M_{SSPP} = -\frac{\alpha_s}{8}\frac{0.112}{R}$	$\lambda^a\sigma\otimes\lambda^a\sigma$	2
	$m^E_{SSPP} = \frac{\alpha_s}{8}\frac{0.381}{R}$	$\lambda^a\otimes\lambda^a$	2
 (c)	$d^M_{PPPP} = -\frac{\alpha_s}{8}\frac{0.162}{R}$	$\lambda^a\sigma\otimes\lambda^a\sigma$	1
	$m^E_{PPPP} = \frac{\alpha_s}{8}\frac{0.555}{R}$	$\lambda^a\otimes\lambda^a$	1
 (d)	$d^M_{AASS} = -\frac{\alpha_s}{8}\frac{0.153}{R}$	$\lambda^a\mathbf{S}^{[\frac{3}{2}]}\otimes\lambda^a\sigma$	2
	$m^E_{AASS} = \frac{\alpha_s}{8}\frac{0.220}{R}$	$\lambda^a\otimes\lambda^a$	2
 (e)	$d^M_{AAAA} = -\frac{\alpha_s}{8}\frac{0.042}{R}$	$\lambda^a\mathbf{S}^{[\frac{3}{2}]}\otimes\lambda^a\mathbf{S}^{[\frac{3}{2}]}$	1
	$m^E_{AAAA} = \frac{\alpha_s}{8}\frac{0.183}{R}$	$\lambda^a\otimes\lambda^a$	1
	$o^M_{AAAA} = -\frac{\alpha_s}{8}\frac{0.006}{R}$	$\lambda^a\sigma^{[\frac{3}{2}]}_{nkt}\otimes\lambda^a\sigma^{[\frac{3}{2}]}_{nkt}$	1
	$k^E_{AAAA} = \frac{\alpha_s}{8}\frac{0.020}{R}$	$\lambda^a K^{[\frac{3}{2}]}_{nt}\otimes\lambda^a K^{[\frac{3}{2}]}_{nt}$	1
 (f)	$d^M_{SPPS} = -\frac{\alpha_s}{8}\frac{0.115}{R}$	$\lambda^a\sigma\otimes\lambda^a\sigma P(S\leftrightarrow P)$	2
	$d^E_{SPPS} = \frac{\alpha_s}{8}\frac{0.083}{R}$	$\lambda^a\sigma\otimes\lambda^a\sigma P(S\leftrightarrow P)$	2
 (g)	$d^M_{SAAS} = -\frac{\alpha_s}{8}\frac{0.011}{R}$	$\lambda^a\sigma^{[\frac{3}{2},\frac{1}{2}]}\otimes\lambda^a\sigma^{[\frac{1}{2},\frac{3}{2}]}P(S\leftrightarrow A)$	2
	$d^E_{SAAS} = \frac{\alpha_s}{8}\frac{0.647}{R}$	$\lambda^a\sigma^{[\frac{3}{2},\frac{1}{2}]}\otimes\lambda^a\sigma^{[\frac{1}{2},\frac{3}{2}]}P(S\leftrightarrow A)$	2
	$k^M_{SAAS} = -\frac{\alpha_s}{8}\frac{0.102}{R}$	$\lambda^a K^{[\frac{3}{2},\frac{1}{2}]}_{nt}\otimes\lambda^a K^{[\frac{1}{2},\frac{3}{2}]}_{nt}P(S\leftrightarrow A)$	2
	$k^E_{SAAS} = \frac{\alpha_s}{8}\frac{0.009}{R}$	$\lambda^a K^{[\frac{3}{2},\frac{1}{2}]}_{nt}\otimes\lambda^a K^{[\frac{1}{2},\frac{3}{2}]}_{nt}P(S\leftrightarrow A)$	2
(h)	$d^M_{SAPS} = -\frac{\alpha_s}{8}\frac{0.035}{R}$	$\lambda^a\sigma^{[\frac{3}{2},\frac{1}{2}]}\otimes\lambda^a\sigma P\begin{pmatrix}S\to A\\P\to S\end{pmatrix}$	4
	$d^E_{SAPS} = -\frac{\alpha_s}{8}\frac{0.202}{R}$	$\lambda^a\sigma^{[\frac{3}{2},\frac{1}{2}]}\otimes\lambda^a\sigma P\begin{pmatrix}S\to A\\P\to S\end{pmatrix}$	4
 (i)	$d^M_{SSAP} = \frac{\alpha_s}{8}\frac{0.028}{R}$	$\lambda^a\sigma\otimes\lambda^a\sigma^{[\frac{1}{2},\frac{1}{2}]}P\begin{pmatrix}S\to S\\A\to P\end{pmatrix}$	4
	$d^E_{SSAP} = 0$		

Quark Exchange Diagram



$$f^{\text{Symmetric}}_{\text{exchange}} = (\quad) \sum_{\sigma,\sigma'} \frac{M_q M_q}{\varepsilon_p \, \varepsilon_{p'}} \frac{\overbrace{\left(\bar{u}(p)\gamma^\mu u(p')\right)}^{j_\mu} \cdot \overbrace{\left(\bar{u}(p')\gamma_\mu u(p)\right)}^{j_\mu}}{(p'-p)^2 + \mu^2}$$

$$= \underset{\left(-\frac{g^2}{9}\right)}{(\quad)} \frac{1}{\varepsilon_p \, \varepsilon_{p'}} \frac{2M_q^2 + p\cdot p'}{(p'-p)^2 + \mu^2}$$

Limits:

(i) Non-Rel. $\quad -\dfrac{g^2}{9} \dfrac{1}{(\underline{p}'-\underline{p})^2 + \mu^2}$ $\leftarrow$ $\boxed{\text{attractive}}$

(ii) Relativistic

$$\xrightarrow[M_q \to 0]{} \quad + \frac{g^2}{18} \frac{1}{|\underline{p}|\,|\underline{p}'|}$$ $\leftarrow$ $\boxed{\text{repulsive}}$

FIRST

ELIMINATE SPURIOUS BAG STATES!

RECIPE

$\boxed{\text{Three quark Dirac spinors coupled in } jj \text{ basis}}$

$\parallel$

$\boxed{\text{Baryon wave function}}$

To eliminate spurious states transform to $\underline{L}\cdot\underline{S}$ basis.

to get rid of the spatially symmetric states (assumed spurious).

$$\underbrace{(\underline{P}_1+\underline{P}_2+\underline{P}_3)}_{\text{TOTAL MOMENTUM OF THREE QUARKS}}\ \overset{\text{symmetric}}{\boxed{S\,|\,S\,|\,S}} = \overset{\text{symmetric}}{\boxed{S\,|\,S\,|\,P}} = \boxed{\text{WAVE-FUNCTION SYMMETRIC UNDER INTERCHAN OF ANY TWO QUARKS.}}$$

EXAMPLE:

$\underline{j\text{-}j \text{ BASIS}}$ $\qquad$ $\boxed{L\text{-}S \text{ BASIS}}$

$$\begin{pmatrix} |\underline{10}, \tfrac{1}{2}, S_{1/2} S_{1/2} P_{3/2}\rangle \\ |\underline{10}, \tfrac{1}{2}, S_{1/2} S_{1/2} P_{1/2}\rangle \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & -\frac{\sqrt{8}}{3} \\ -\frac{\sqrt{8}}{3} & -\frac{1}{3} \end{pmatrix} \begin{pmatrix} |\underline{56}, {}^4 10_{1/2}\rangle \\ |\underline{70}, {}^2 10_{1/2}\rangle \end{pmatrix}$$

SU(3) decuplet

$\boxed{J = 1/2}$

SU(6) multiplets

$2S+1$

$10_{J=1/2}$

SU(3)

Comparing several different model calculations of the decay rate Γ_γ:

$$\Lambda(1520) \to \Lambda(1116) + \gamma$$

The configuration mixing in $\Lambda(1116)$ may change the rate by 50% or more.

A more precise determination of this decay rate is desirable.

Table 1 : The values of Γ_γ from various models.

Models	a	b	c	$\Lambda(1116)$	Γ_γ(keV)
NRQM	0.91	0.01	0.40	$^2S_S\rangle$	96
NRQM ($SU(6)-$basis)	0.91	0.01	0.40	$-$	98
χQM	0.91	0.01	-0.40	$^2S_S\rangle$	85
χQM	0.91	0.01	-0.40	mixed	134
NRQM ($uds-$basis)	$-$	$-$	$-$	mixed	154
MIT bag	0.86	0.34	-0.37	$-$	46
Chiral/Cloudy bag	0.95	0.09	-0.29	$^2S_S\rangle$	32
RCQM	0.91	0.01	0.40	mixed	215
Bonn $-$ CQM	$-$	$-$	$-$	$-$	258