the excited spectrum of QCD

the spectrum of excited hadrons

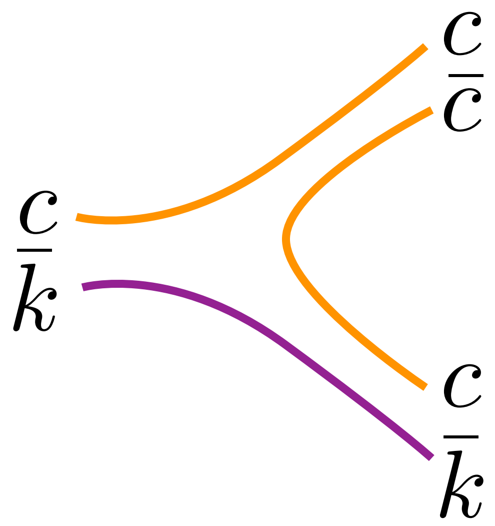
let's begin with a **convenient fiction** :

imagine that QCD were such that there was a spectrum of **stable** excited hadrons

e.g. suppose we set up QCD with just two degenerate flavours of quark with mass roughly that of the charm quark

$$m_c = m_k \sim 1.5\,\mathrm{GeV}$$

then we'd expect a spectrum of $c\bar{k}$ hadron states starting at about 3 GeV that are stable up to about 6 GeV

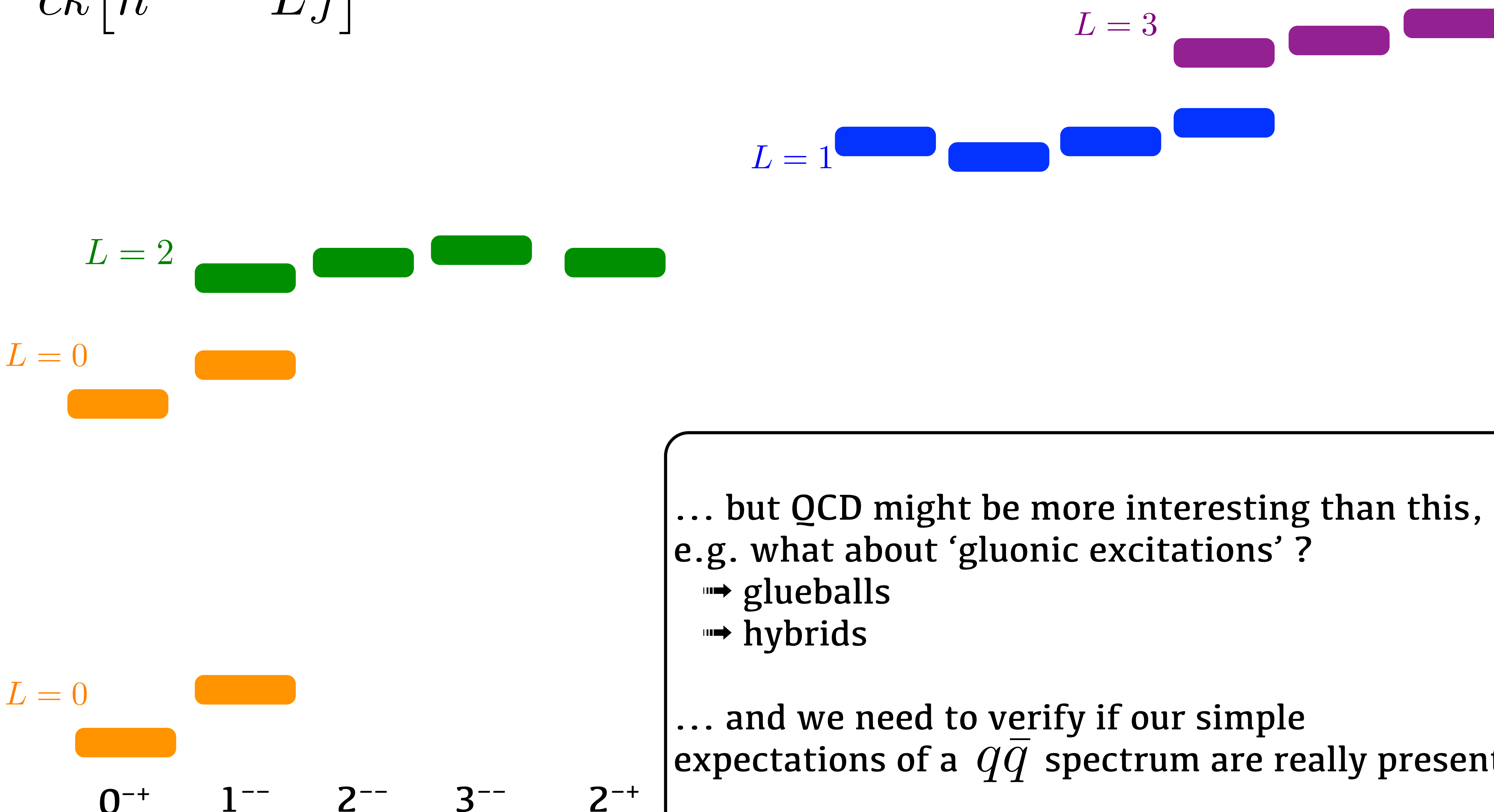


except perhaps if glueballs are important ?

the spectrum of excited hadrons

might expect something like the non-relativistic quark model

$$c\bar{k}\left[n\,^{2S+1}L_J\right]$$



... but QCD might be more interesting than this, e.g. what about 'gluonic excitations' ?
- ➟ glueballs
- ➟ hybrids

... and we need to verify if our simple expectations of a $q\bar{q}$ spectrum are really present

the spectrum of excited hadrons

we'd like to map out the spectrum of states in each J^{PC}

need interpolating fields that transform like the desired J^{PC}

e.g. local fermion bilinears

$$\begin{aligned}
\bar{\psi}\gamma_5\psi &\sim 0^{-+} \\
\bar{\psi}\psi &\sim 0^{++} \\
\bar{\psi}\gamma_i\psi &\sim 1^{--} \\
\bar{\psi}\gamma_5\gamma_i\psi &\sim 1^{++} \\
\epsilon_{ijk}\bar{\psi}\gamma_j\gamma_k\psi &\sim 1^{+-}
\end{aligned}$$

… very limited in J^{PC} coverage

one possible extension: include gauge-covariant derivatives

$$\overleftrightarrow{D}_i = \overleftarrow{D}_i - \overrightarrow{D}_i = \overleftarrow{\partial}_i - \overrightarrow{\partial}_i - 2igA_i$$

e.g. $\bar{\psi}\overleftrightarrow{D}_i\psi \sim 1^{--}$

$\bar{\psi}\gamma_i\overleftrightarrow{D}_j\psi \sim ?$ $\quad i = 1\ldots3$, $j = 1\ldots3$

9 elements

operator is reducible

the spectrum of excited hadrons

$\bar{\psi}\gamma_i \overleftrightarrow{D}_j \psi \sim ?$ $\quad i = 1\ldots 3$, $j = 1\ldots 3$ — 9 elements — operator is reducible

very easy to build a scheme where the operators are irreducible:

$$\gamma_m \equiv \sum_i \epsilon_i(m)\gamma_i \qquad \vec{\epsilon}(m=\pm) = \mp\tfrac{1}{\sqrt{2}}\big[1, \pm i, 0\big]$$

$$\overleftrightarrow{D}_m \equiv \sum_i \epsilon_i(m)\overleftrightarrow{D}_i \qquad \vec{\epsilon}(m=0) = \big[0, 0, 1\big]$$

spin-1
circular basis

$$\Longrightarrow \big\langle 1 m_1; 1 m_2 \big| J M \big\rangle \, \bar{\psi}\gamma_{m_1} \overleftrightarrow{D}_{m_2} \psi \sim J^{++}$$

with J=0,1,2

Hadron Spectrum Collaboration
has used up to three derivatives:

$$\big\langle 1 m_1; j_{234} m_{234} \big| J M \big\rangle$$
$$\big\langle 1 m_3; j_{24} m_{24} \big| j_{234} m_{234} \big\rangle$$
$$\big\langle 1 m_2; 1 m_4 \big| j_{24} m_{24} \big\rangle$$
$$\bar{\psi}\gamma_{m_1} \overleftrightarrow{D}_{m_2} \overleftrightarrow{D}_{m_3} \overleftrightarrow{D}_{m_4} \psi$$

can build a big basis this way covering all J ≤ 4

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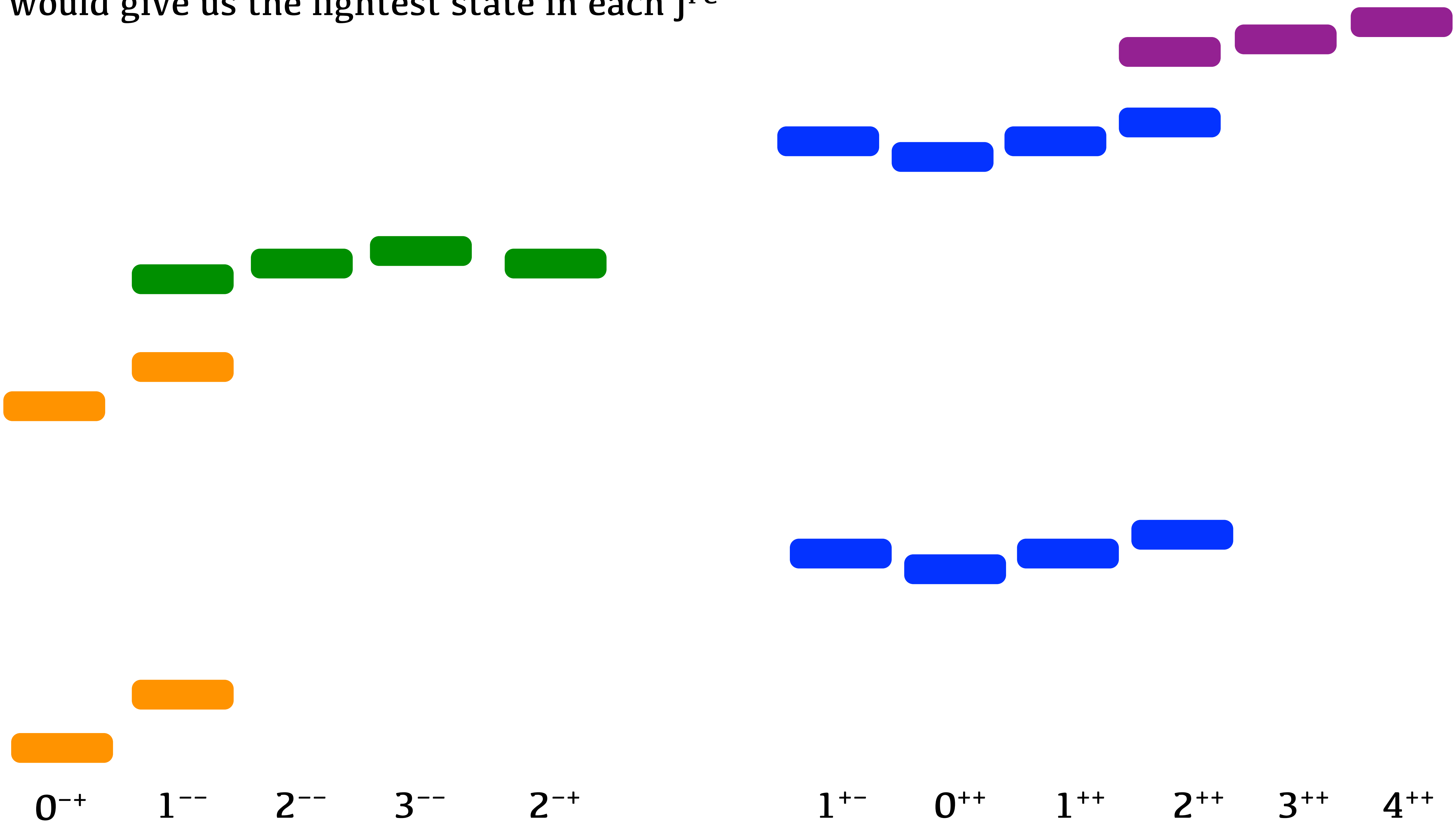
the spectrum of excited hadrons

so we could compute correlators for each J^{PC} and look at effective masses at large t

would give us the lightest state in each J^{PC}



we want more than this . . .

the spectrum of excited hadrons

we need to be able to extract **excited** states

$$C(t) = \sum_{\mathfrak{n}} A_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t}$$

a weighted sum of exponentials
- just do a fit to the time-dependence ?

(fit variables : A_0, A_1 ... , E_0, E_1 ...)

this is a very bad way to approach this problem

➟ suppose two states are (nearly) degenerate
- fit won't be able to tell if there are two states or one !

➟ how do we determine how many states to include in the fit
- if we decrease t_{min} to use more of the data, need more states ?

fortunately there is a very powerful method available ...

variational approach

suppose we have multiple operators for a given J^{PC} $\quad \mathcal{O}_1,\ \mathcal{O}_2,\ \mathcal{O}_3 \ldots$

e.g. $J^{PC} = 1^{--}$

$$\bar{\psi}\gamma_m\psi$$
$$\bar{\psi}\overleftrightarrow{D}_m\psi$$
$$\langle 1m_1; 1m_2|1m\rangle \bar{\psi}\gamma_5 \overleftrightarrow{D}_{m_1}\overleftrightarrow{D}_{m_2}\psi$$
$$\langle 1m_1; 2m_D|1m\rangle\langle 1m_2; 1m_3|2m_D\rangle \bar{\psi}\gamma_{m_1}\overleftrightarrow{D}_{m_2}\overleftrightarrow{D}_{m_3}\psi$$
$$\vdots$$

compute a **matrix of correlation functions**

$$C_{ij}(t) = \big\langle 0\big|\mathcal{O}_i(t)\mathcal{O}_j(0)\big|0\big\rangle$$
$$= \sum_{\mathfrak{n}} Z_i^{(\mathfrak{n})} Z_j^{(\mathfrak{n})} e^{-E_{\mathfrak{n}} t}$$

$$Z_i^{(\mathfrak{n})} = \big\langle \mathfrak{n}\big|\mathcal{O}_i(0)\big|0\big\rangle$$

solve the 'generalised eigenvalue problem' : $\quad C(t)v^{(\mathfrak{n})} = \lambda_{\mathfrak{n}}(t) C(t_0) v^{(\mathfrak{n})}$

eigenvalues, 'principal correlators' $\quad \lambda_{\mathfrak{n}}(t) \sim e^{-E_{\mathfrak{n}}(t-t_0)}$

eigenvectors are 'orthogonal' $\quad v^{(\mathfrak{m})\dagger} C(t_0) v^{(\mathfrak{n})} = \delta_{\mathfrak{m},\mathfrak{n}}$

variational approach

the interpretation is relatively simple

the eigenvectors indicate the optimal linear combination of $\mathcal{O}_i$ to interpolate $|\mathfrak{n}\rangle$

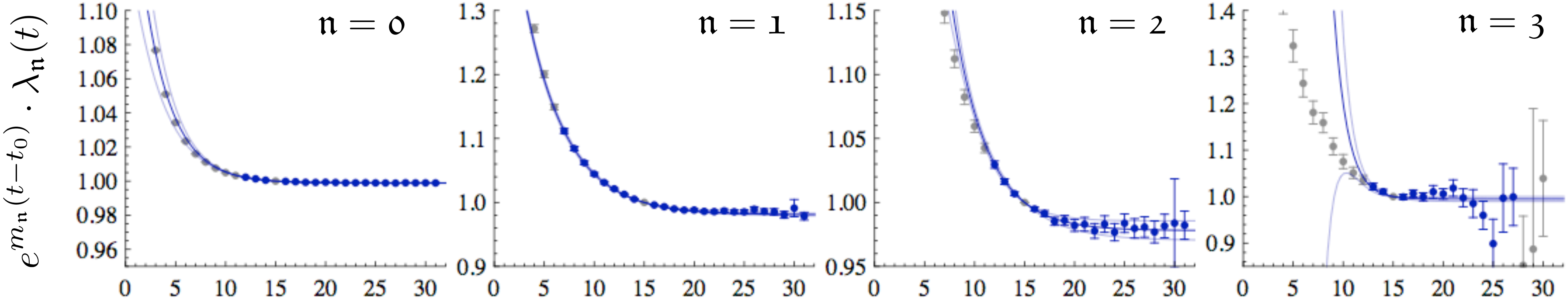
$$\Omega_{\mathfrak{n}} = \sum_i v_i^{(\mathfrak{n})} \mathcal{O}_i \qquad \langle \mathfrak{m} | \Omega_{\mathfrak{n}} | 0 \rangle \approx \delta_{\mathfrak{m},\mathfrak{n}}$$

degenerate states are easy to deal with - they might have $E_{\mathfrak{m}} = E_{\mathfrak{n}}$

- but they have orthogonal $v^{(\mathfrak{m})}$, $v^{(\mathfrak{n})}$

variational approach

principal correlators $t_0 = 15$



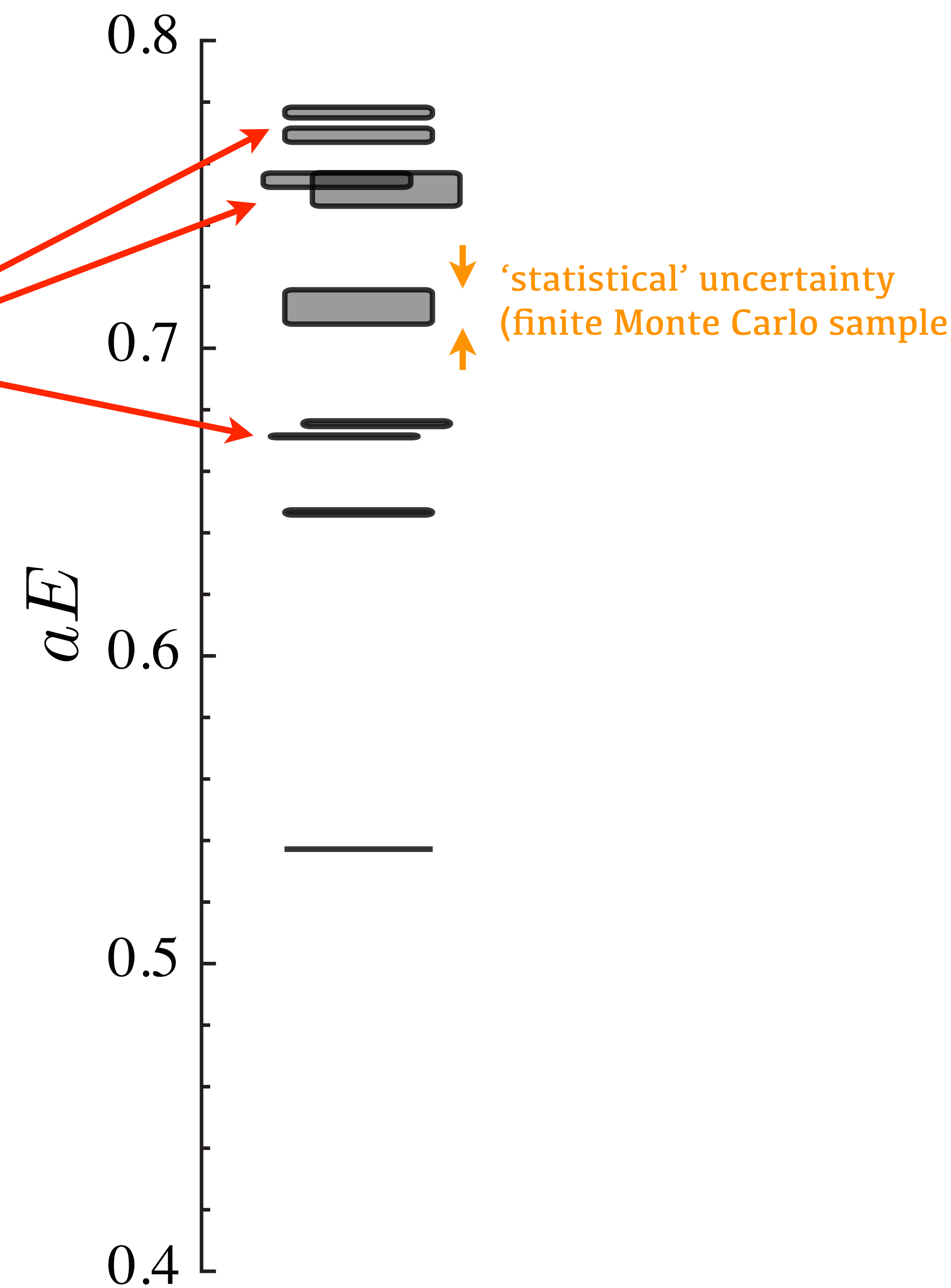
a real example - T_1^{--} in charmonium

superimposed J=1,3,4 spectra

26 operators

variational analysis of
26×26 matrix of correlators

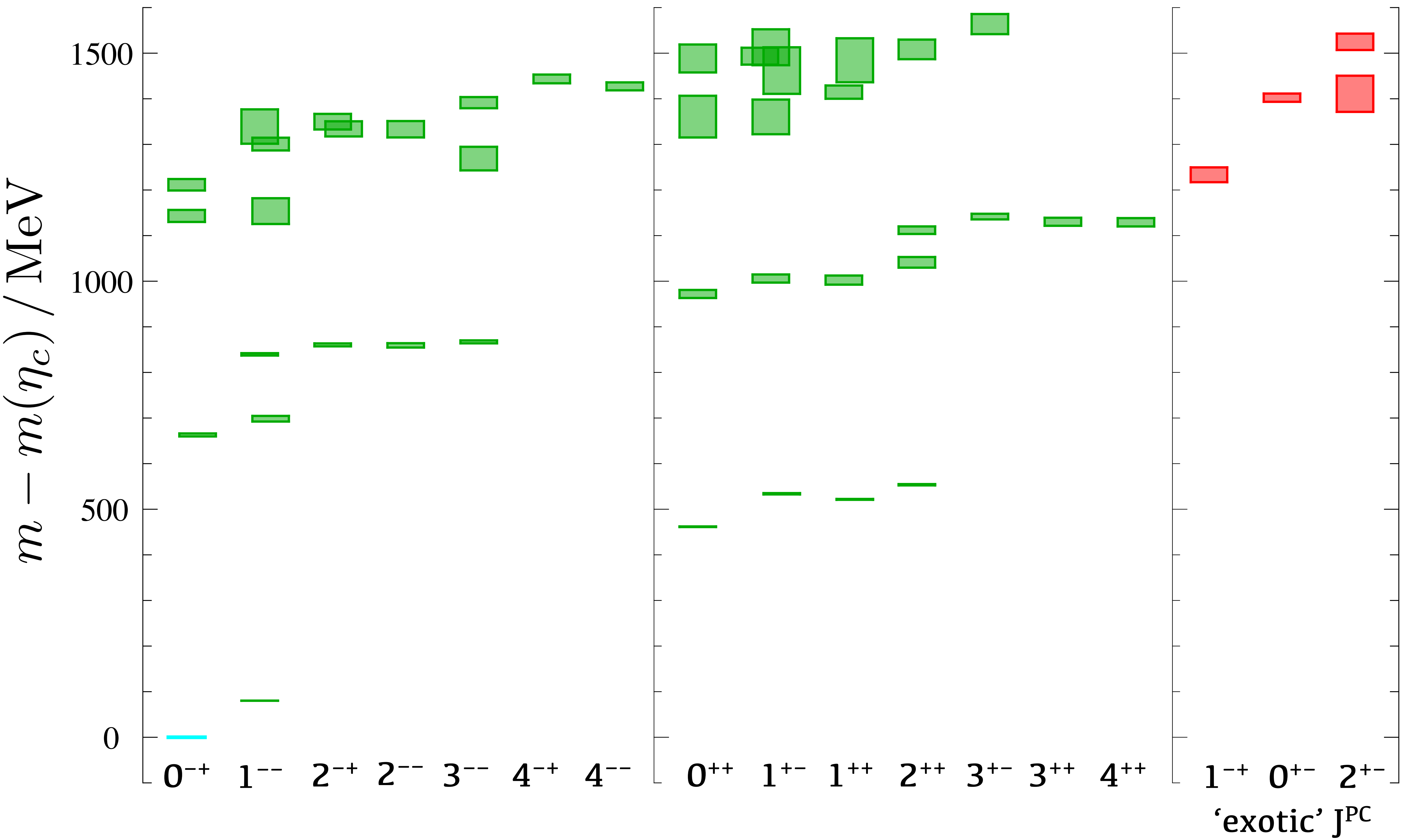
multiple approximate degeneracies



the charmonium spectrum from a lattice QCD calc

perform variational analysis in each quantum number

Hadron Spectrum Collaboration
arXiv:1204.5425



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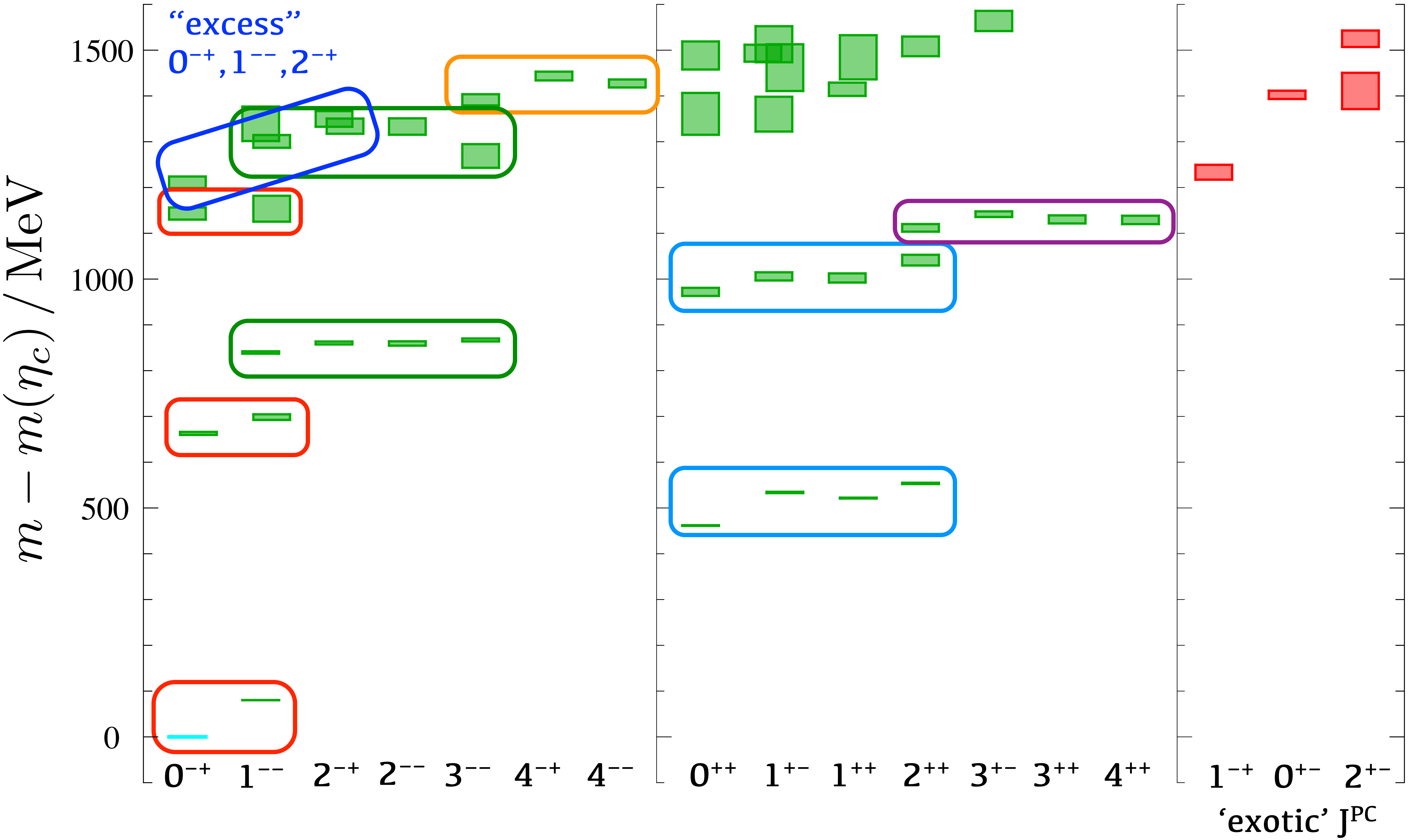

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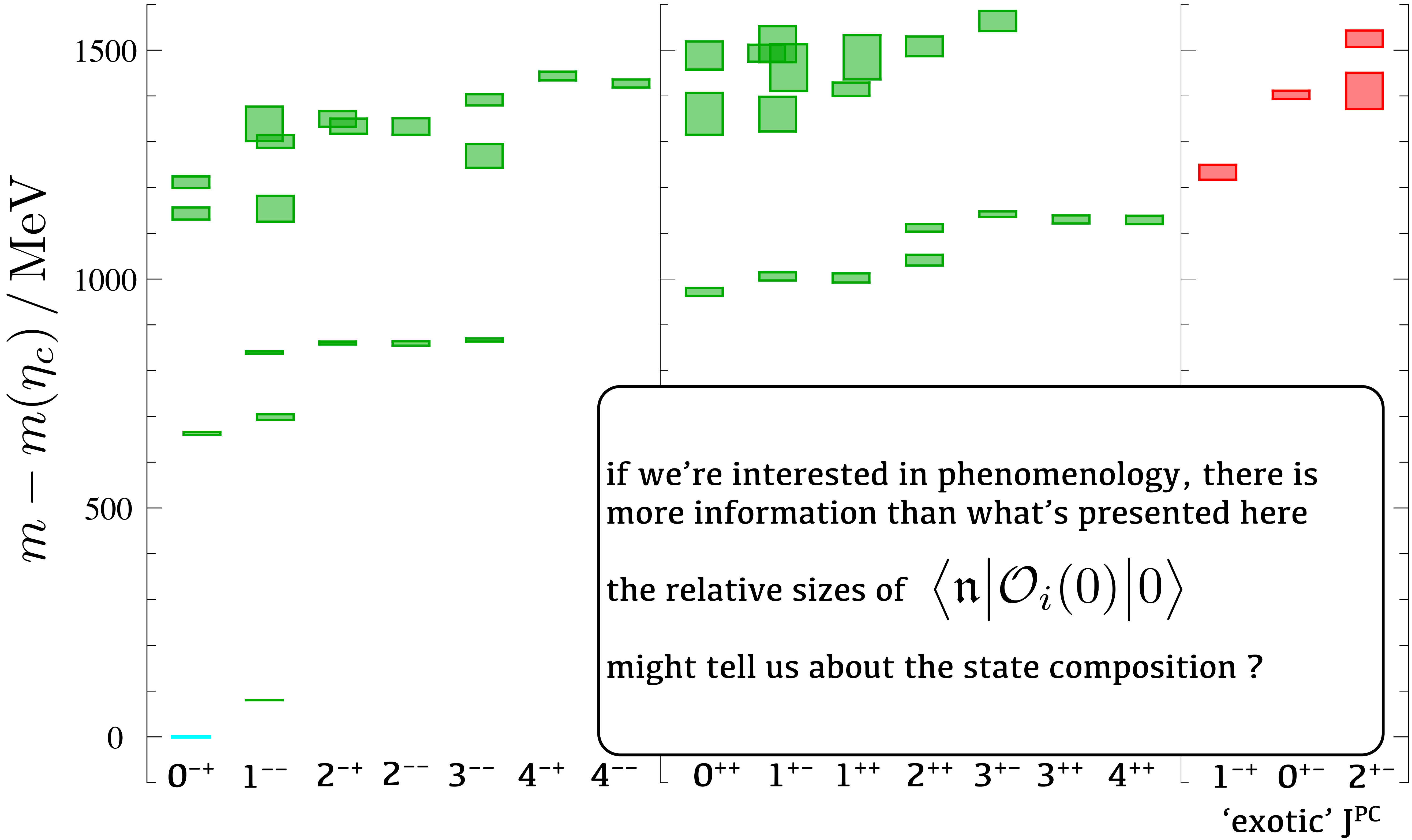


if we're interested in phenomenology, there is more information than what's presented here

the relative sizes of $\langle \mathfrak{n} | \mathcal{O}_i(0) | 0 \rangle$

might tell us about the state composition ?

back to the operators ...

e.g. $J^{PC}=1^{--}$ consider a model-interpretation

$$\bar{\psi}\gamma_m \tfrac{1}{2}(1-\gamma_0)\psi$$

spin-structure: $$\psi \sim \begin{bmatrix} 1 \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m} \end{bmatrix} \chi$$

$$\tfrac{1}{2}(1-\gamma_0)\psi \sim \begin{bmatrix} 1 \\ 0 \end{bmatrix} \chi$$

upper component projector

$$\bar{\psi}\gamma_m \tfrac{1}{2}(1-\gamma_0)\psi \sim \phi^\dagger \sigma_m \chi$$

$^3S_1$

back to the operators . . .

e.g. $J^{PC}=1^{--}$ consider a model-interpretation

$$\langle 1 m_1; 2 m_2 | 1 m \rangle \; \bar{\psi} \gamma_{m_1} D^{[2]}_{J=2,m_2} \tfrac{1}{2}(1-\gamma_0)\psi$$

$$D^{[2]}_{J,m} \equiv \langle 1 m_1; 1 m_2 | J m \rangle \overleftrightarrow{D}_{m_1} \overleftrightarrow{D}_{m_2}$$

without gauge-fields: $D^{[2]}_{J=2,m} \to Y_2^m(\overleftrightarrow{\partial})$

$$\sim \langle 1 m_1; 2 m_2 | 1 m \rangle \cdot \phi^\dagger \sigma_{m_1} \chi \cdot Y_2^{m_2}(\vec{q})$$

$^3D_1$

$q\bar{q}$ relative momentum

back to the operators ...

e.g. $J^{PC}=1^{--}$

$$\bar{\psi}\gamma_5 D^{[2]}_{J=1,m}\tfrac{1}{2}(1-\gamma_0)\psi$$

$$D^{[2]}_{J=1,m} \equiv \langle 1m_1; 1m_2 | 1m\rangle \overleftrightarrow{D}_{m_1}\overleftrightarrow{D}_{m_2}$$

antisymmetric CGC

without gauge-fields: $D^{[2]}_{J=1,m} \to 0$

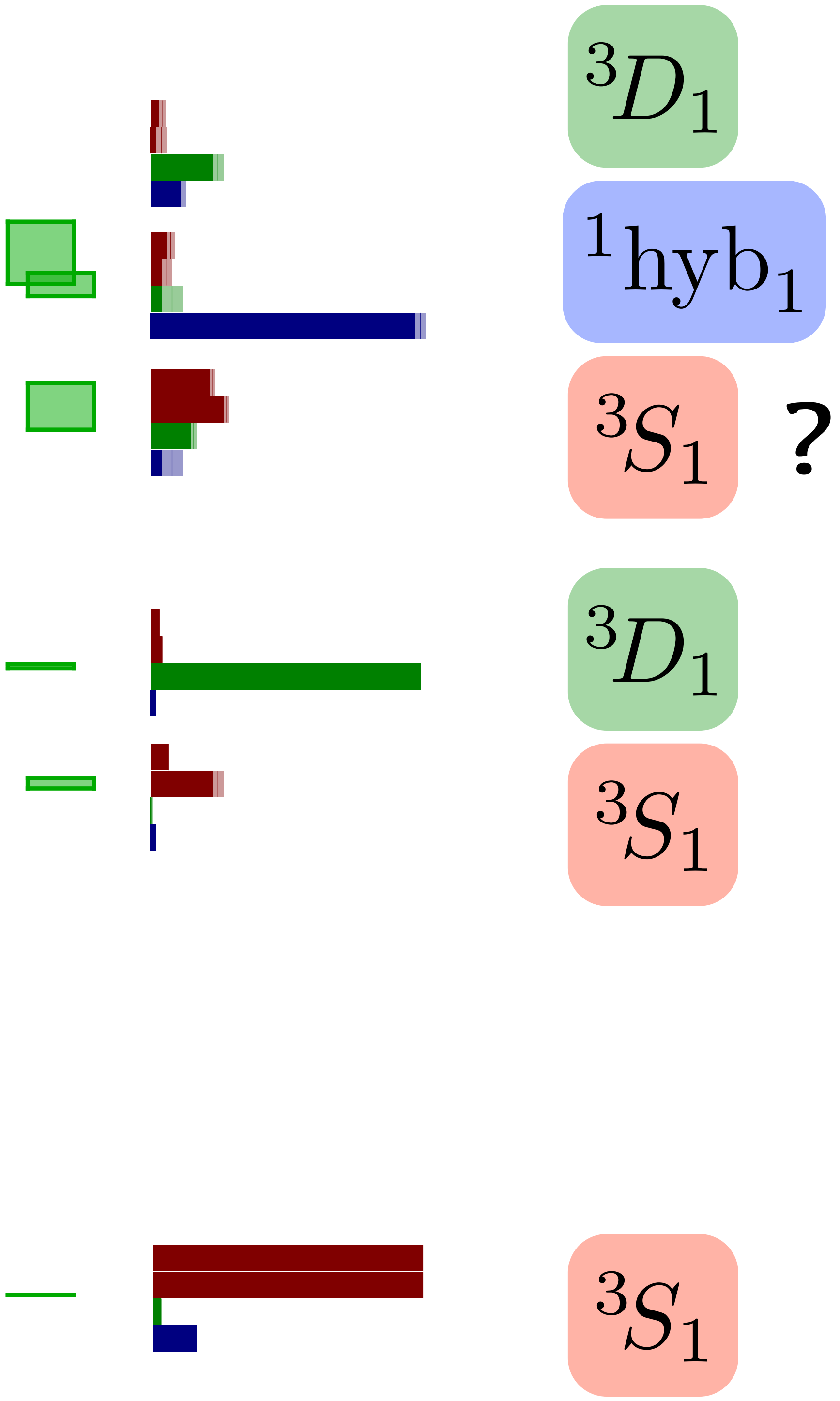
with gauge-fields $D^{[2]}_{J=1,m} \propto [D_i, D_j] \propto F_{ij}$ chromomagnetic part of field-strength tensor

$$\underbrace{\bar{\psi}\gamma_5 t^a \psi}_{q\bar{q}_{\mathbf{8}}(^1S_0)}\, B^a_m$$

$^1\mathrm{hyb}_1$

operator overlaps

e.g. $J^{PC}=1^{--}$



the charmonium spectrum from a lattice QCD calc

perform variational analysis in each quantum number

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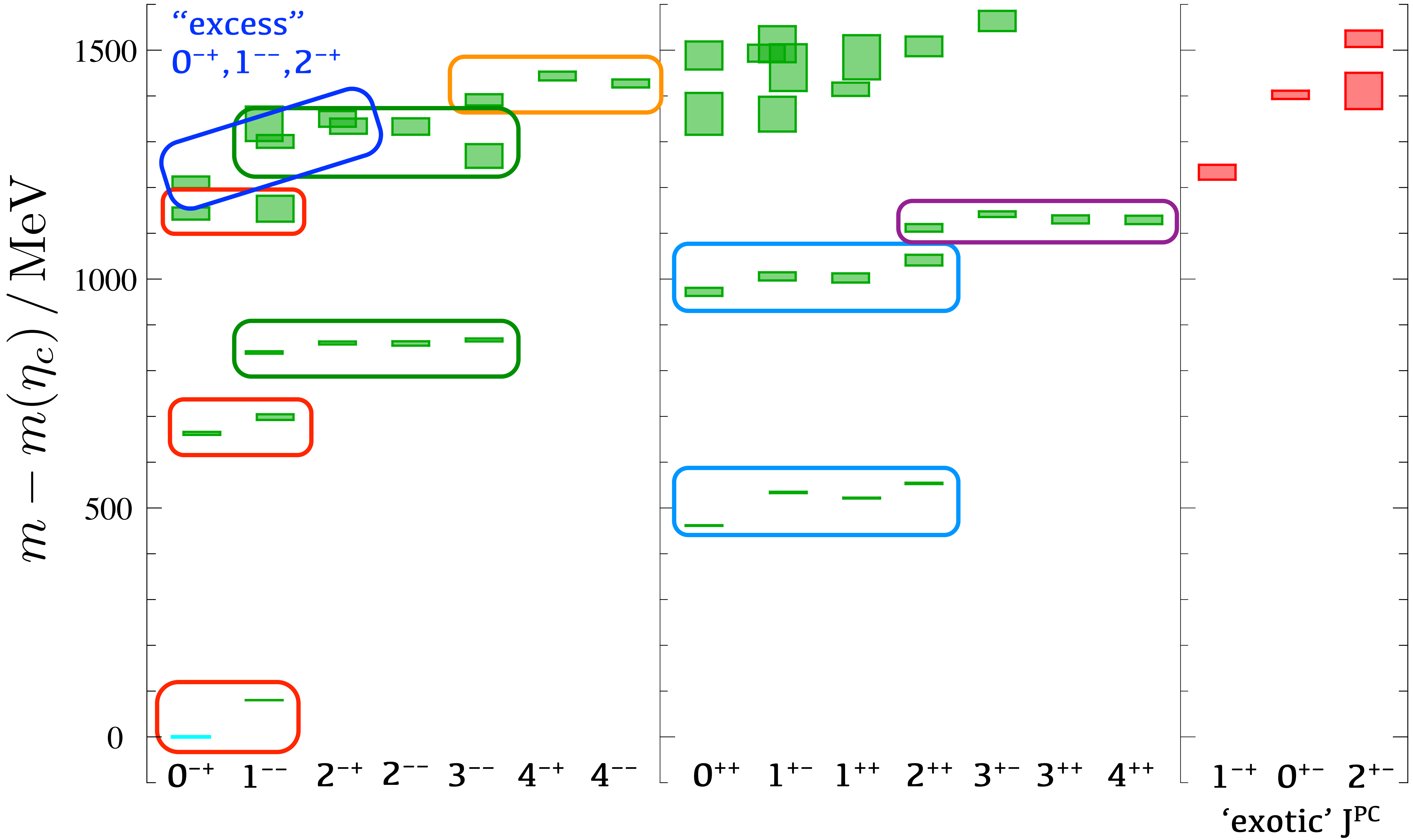
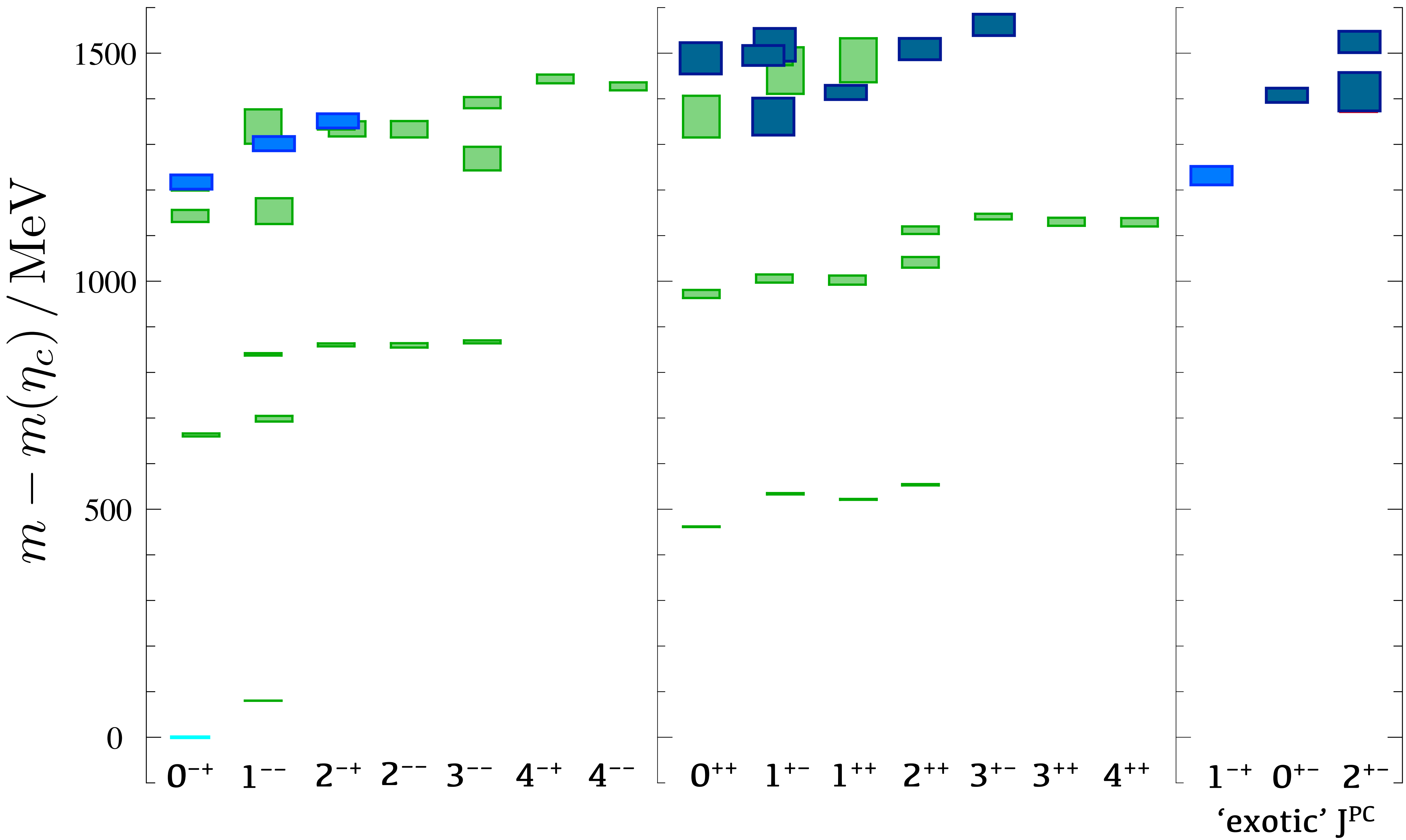


the charmonium spectrum from a lattice QCD calc

can isolate dominant hybrid character across the spectrum



hybrid mesons

a phenomenology of hybrid mesons based upon QCD calculations

a chromomagnetic field configuration is lowest excitation

$$q\bar{q}_{\mathbf{8}}(^1S_0)B_{\mathbf{8}} \sim 0^{-+} \otimes 1^{+-} = 1^{--}$$

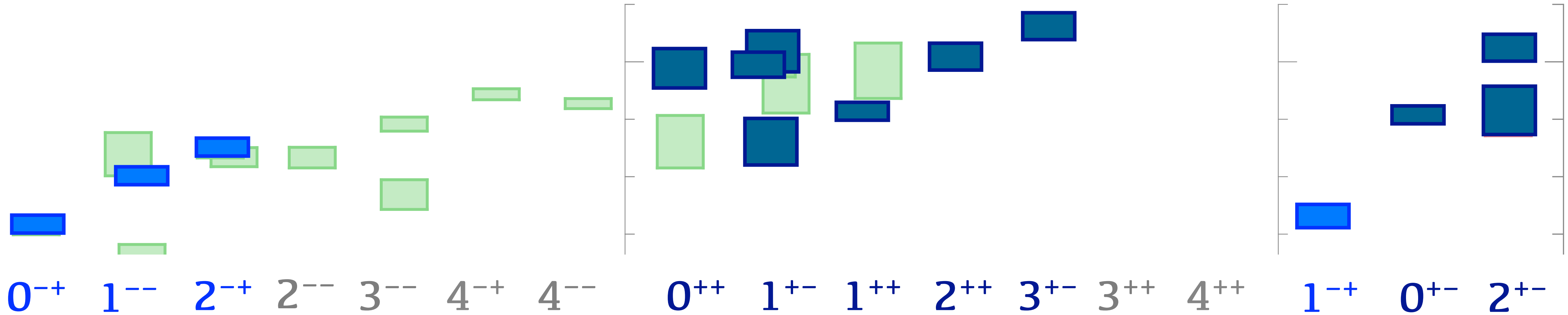
$$q\bar{q}_{\mathbf{8}}(^3S_1)B_{\mathbf{8}} \sim 1^{--} \otimes 1^{+-} = (0,1,2)^{-+}$$

$$q\bar{q}_{\mathbf{8}}(^1P_1)B_{\mathbf{8}} \sim 1^{+-} \otimes 1^{+-} = (0,1,2)^{++}$$

$$q\bar{q}_{\mathbf{8}}(^3P_0)B_{\mathbf{8}} \sim 0^{++} \otimes 1^{+-} = 1^{+-}$$

$$q\bar{q}_{\mathbf{8}}(^3P_1)B_{\mathbf{8}} \sim 1^{++} \otimes 1^{+-} = (0,1,2)^{+-}$$

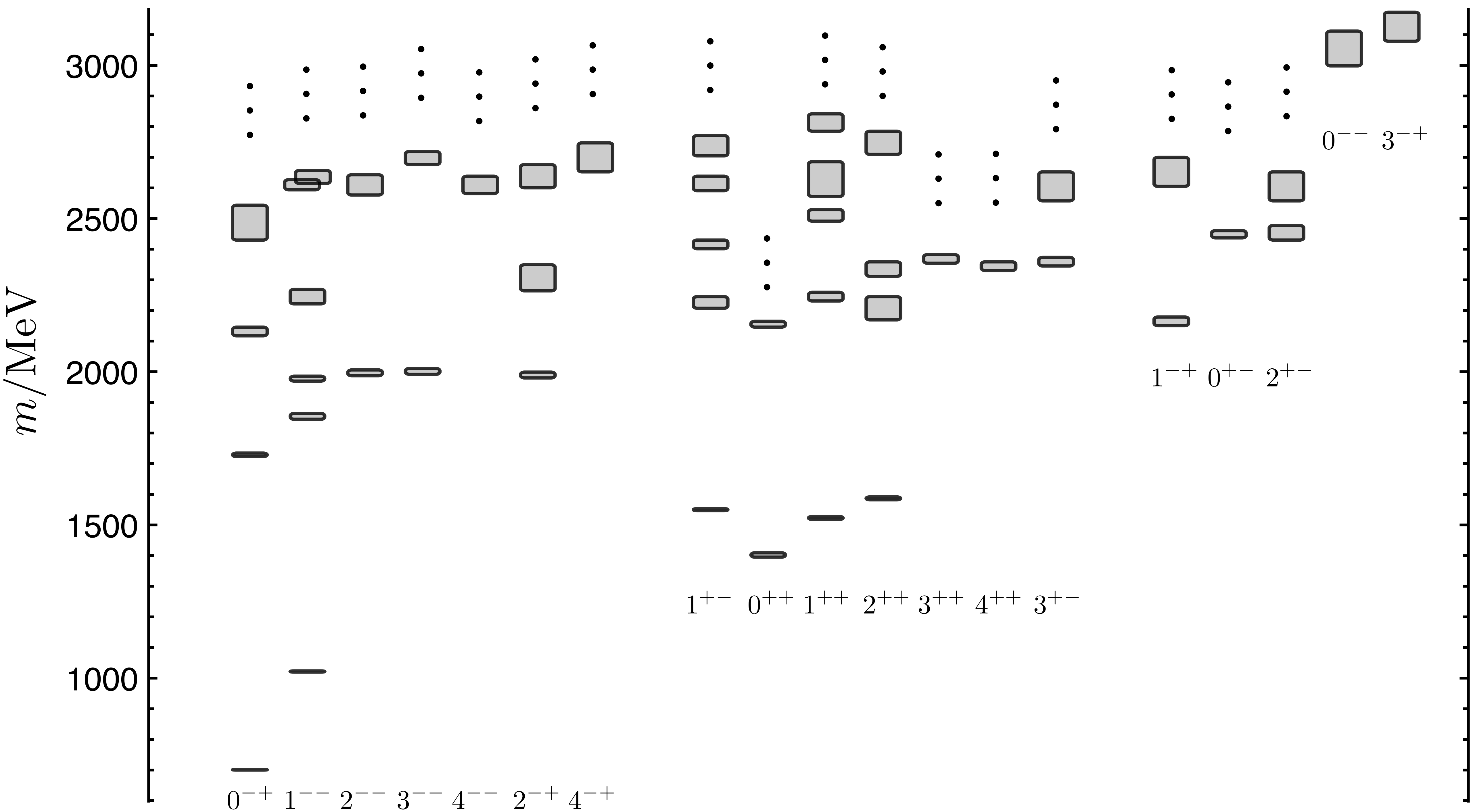
$$q\bar{q}_{\mathbf{8}}(^3P_2)B_{\mathbf{8}} \sim 2^{++} \otimes 1^{+-} = (1,2,3)^{+-}$$



lighter quarks - isovector mesons

three flavours of quark - all at the strange quark mass

m(“π”) ~ 700 MeV

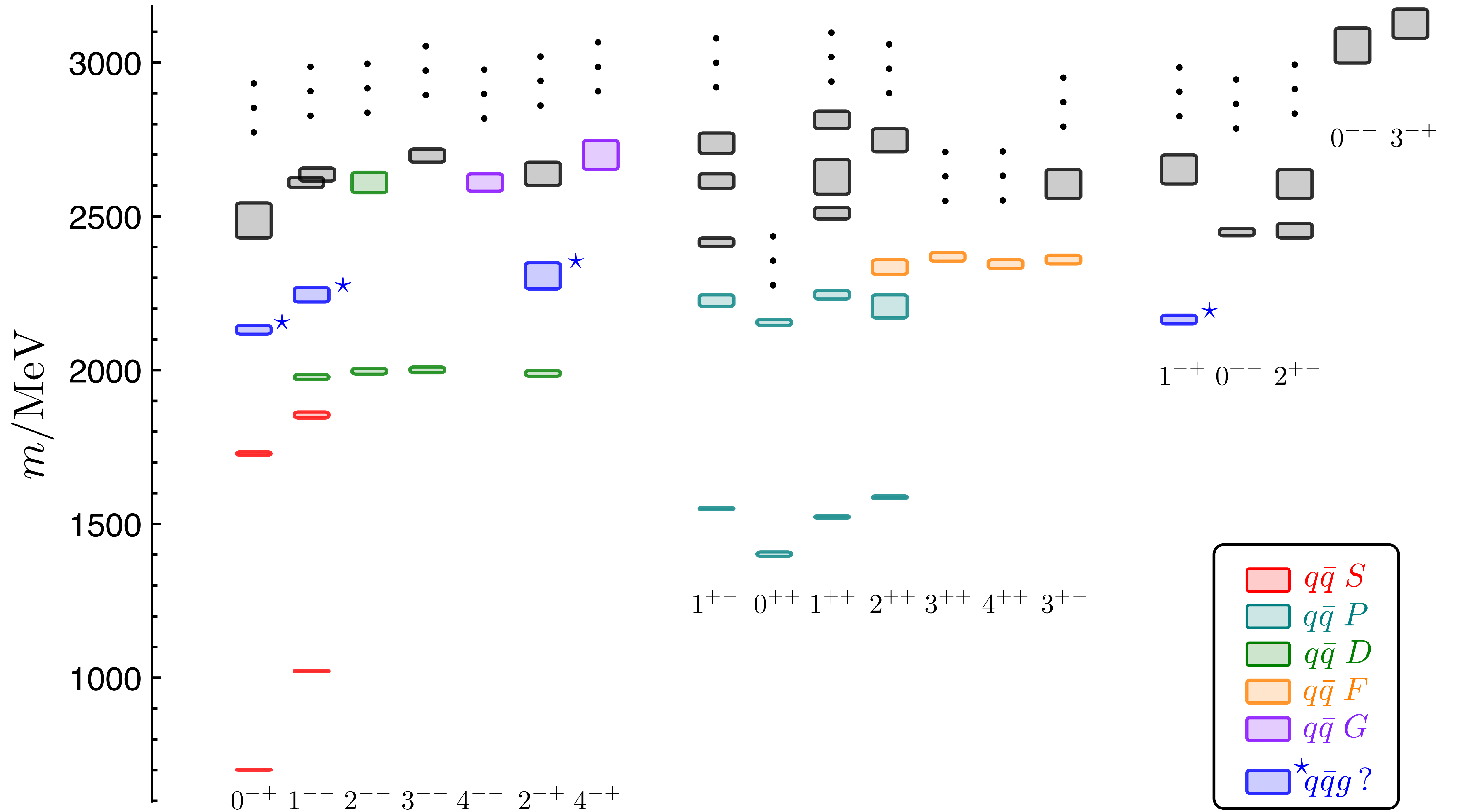


lighter quarks - isovector mesons

three flavours of quark - all at the strange quark mass

m(“π”) ~ 700 MeV

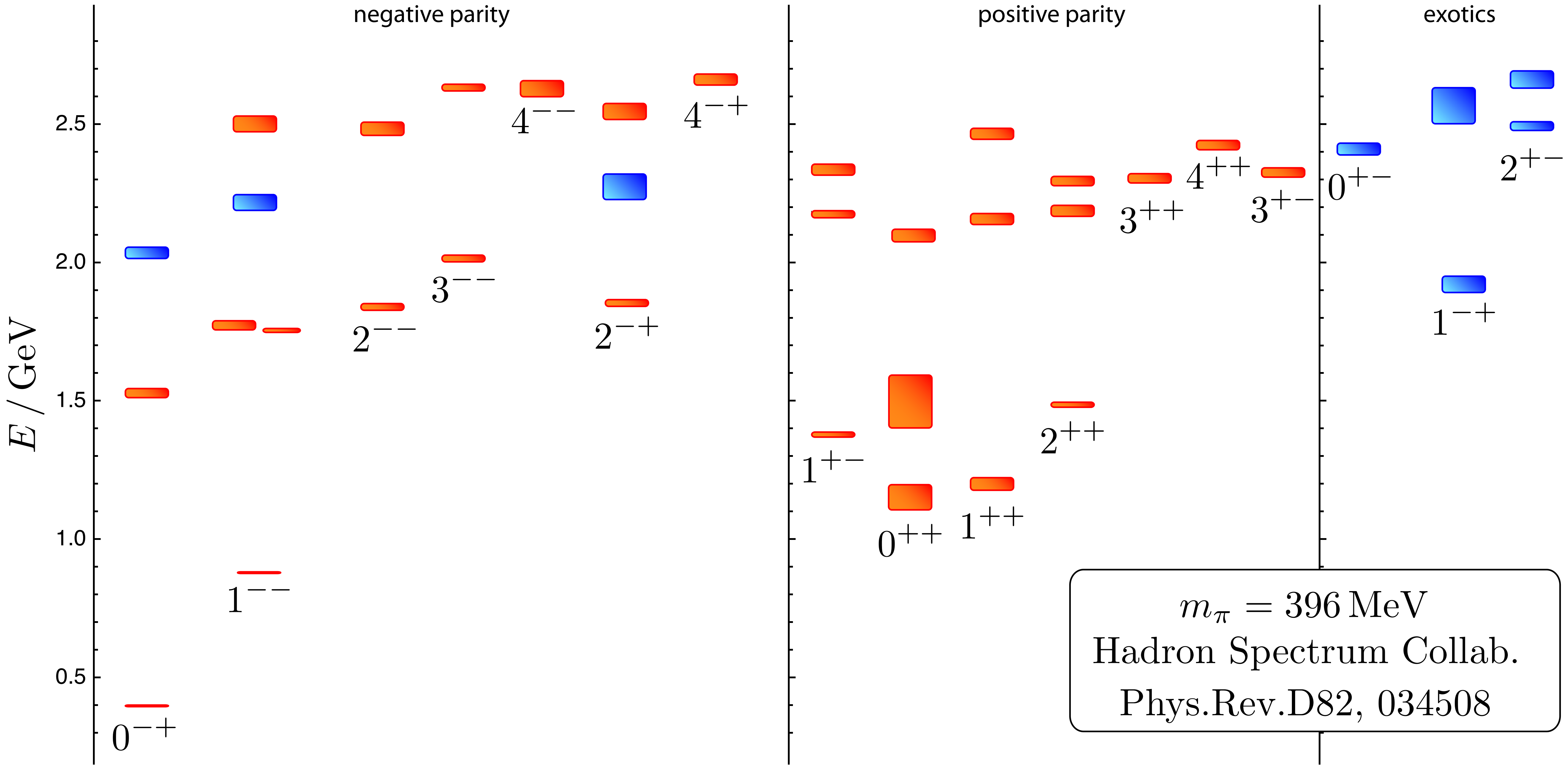
interpretations based on operator overlaps



lighter quarks - isovector mesons

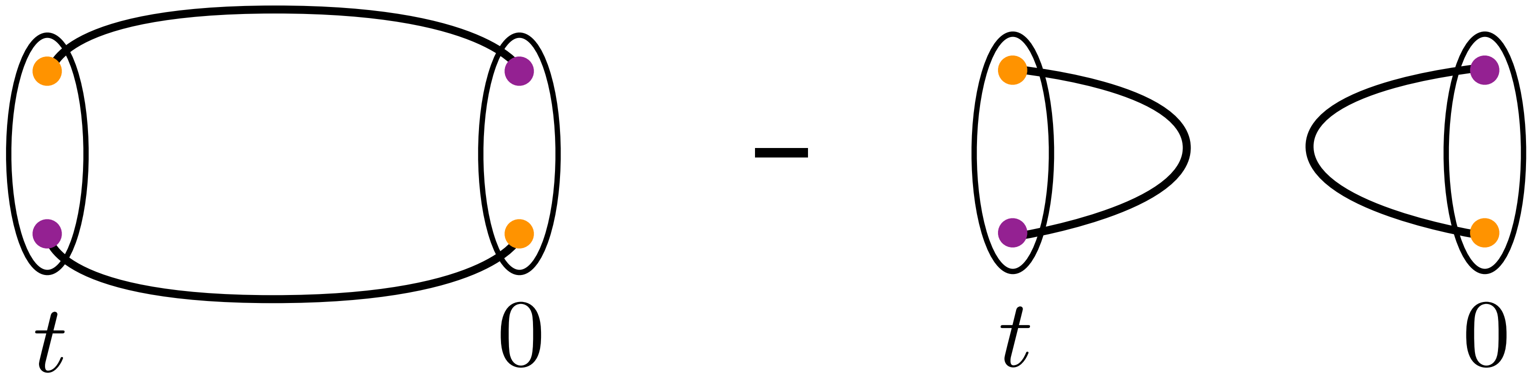
three flavours of quark
- degenerate up/down quarks
- correct strange quark mass

m(π) ~ 400 MeV



isoscalar mesons

difference w.r.t. isovector mesons is addition of ‘disconnected’ diagrams



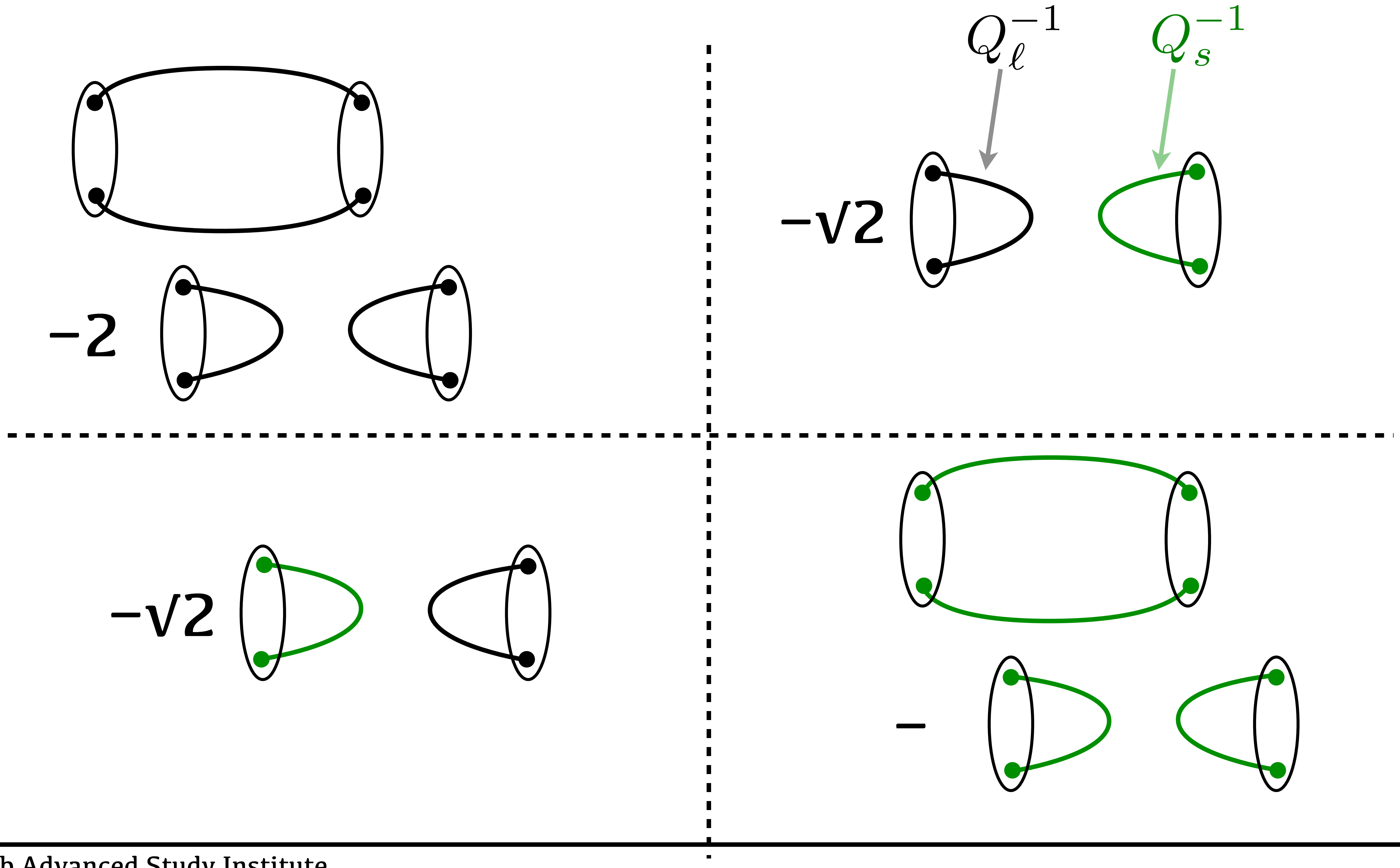
$$\bar{\psi}\mathbf{\Gamma}'\psi(t)\cdot\bar{\psi}\mathbf{\Gamma}\psi(0) \quad - \quad \bar{\psi}\mathbf{\Gamma}'\psi(t)\cdot\bar{\psi}\mathbf{\Gamma}\psi(0)$$

$$\mathrm{tr}\left[Q^{-1}_{t,0}\mathbf{\Gamma}Q^{-1}_{0,t}\mathbf{\Gamma}'\right] \qquad \mathrm{tr}\left[Q^{-1}_{t,t}\mathbf{\Gamma}'\right]\,\mathrm{tr}\left[Q^{-1}_{0,0}\mathbf{\Gamma}\right]$$

challenging using
‘traditional’ methods

isoscalar mesons

hidden 'light' and hidden 'strange' can mix $\frac{1}{\sqrt{2}}\left(u\bar{u}+d\bar{d}\right)$ $s\bar{s}$



isoscalar mesons

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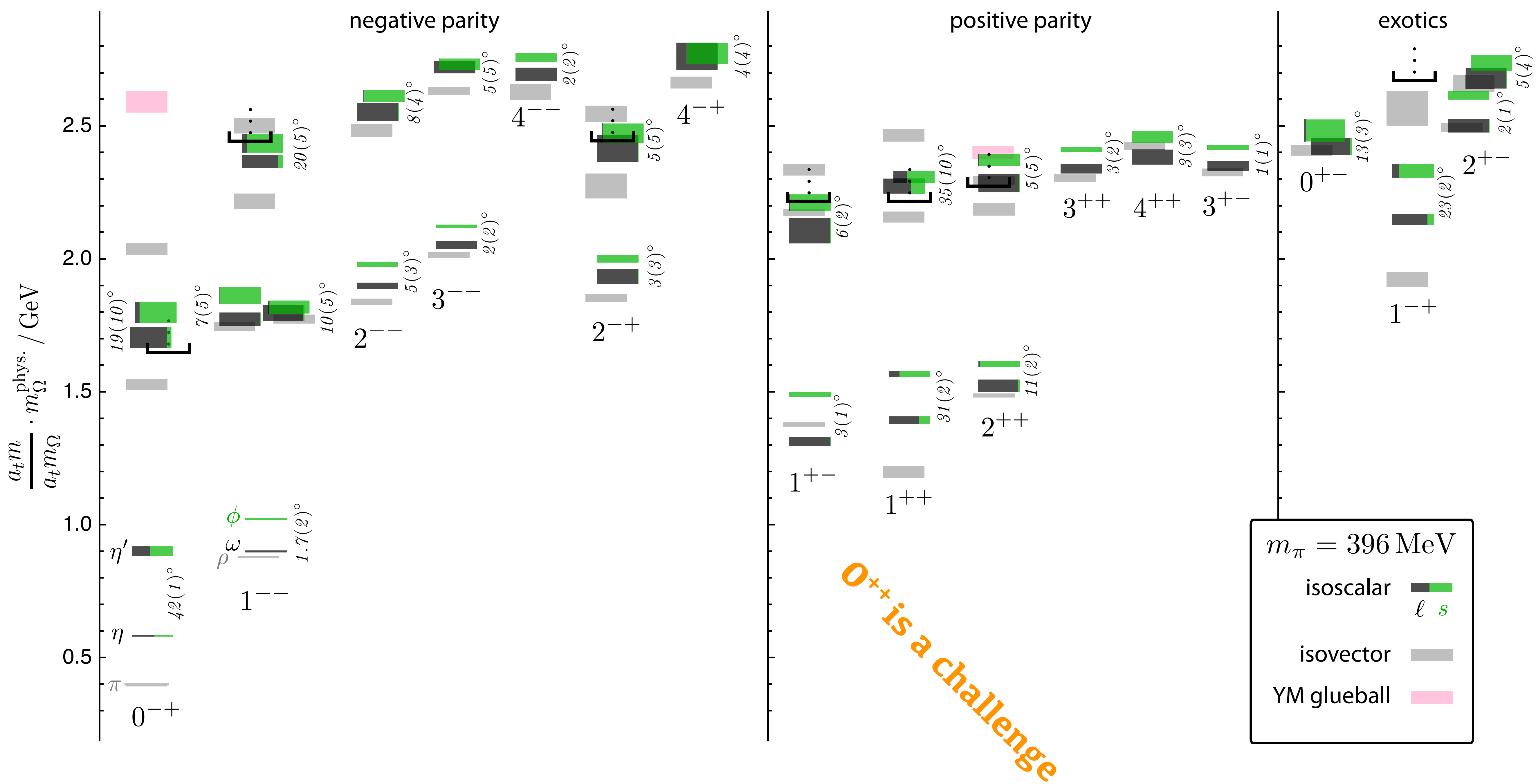
PRD83 111502 (2011)



baryons

analogous large basis of operators for baryons - three quark fields respecting permutation (anti-)symmetry

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PRD85 054016 (2012)