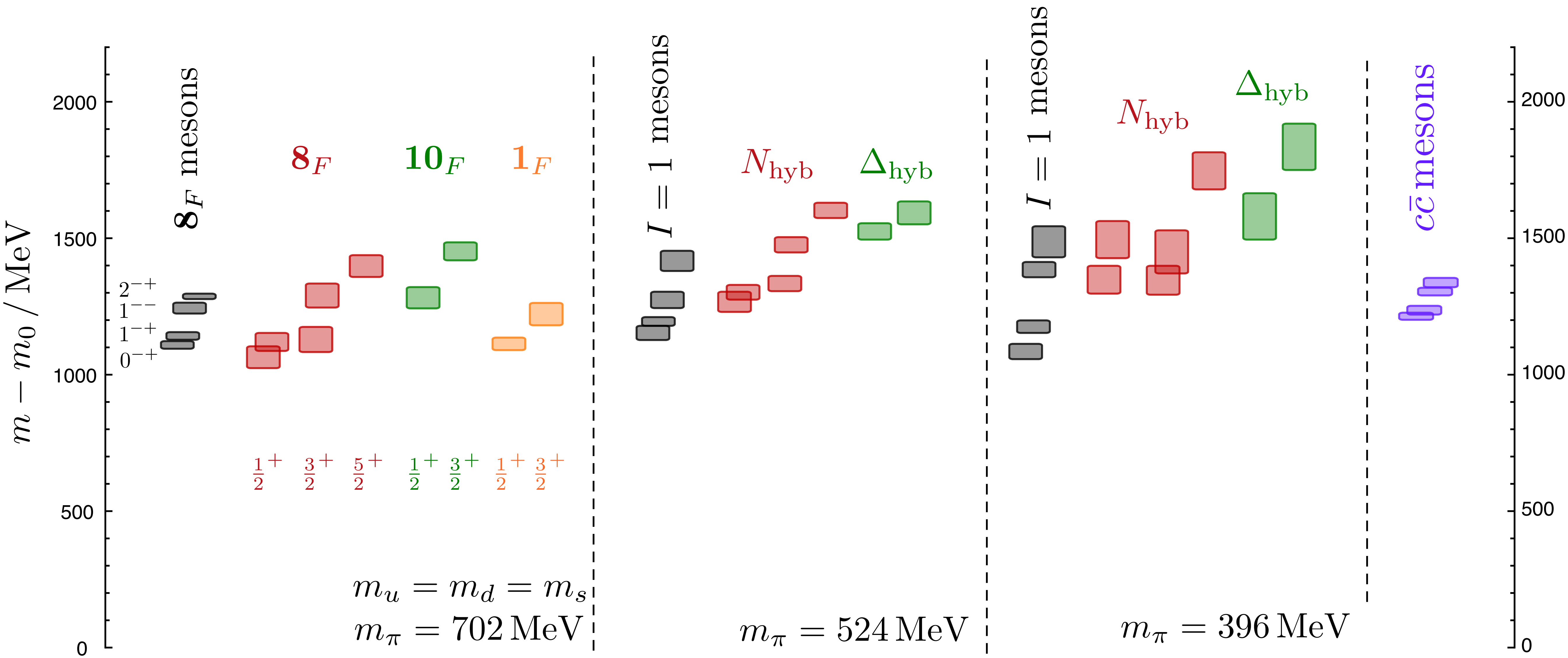
hybrids (mesons and baryons)



the resonance spectrum of QCD

or,

“where are you hiding the scattering amplitudes?”

real QCD

real QCD has very few stable particles : $\pi, \; K, \; N, \; \Sigma, \; \Lambda, \; \Xi, \; \Omega$

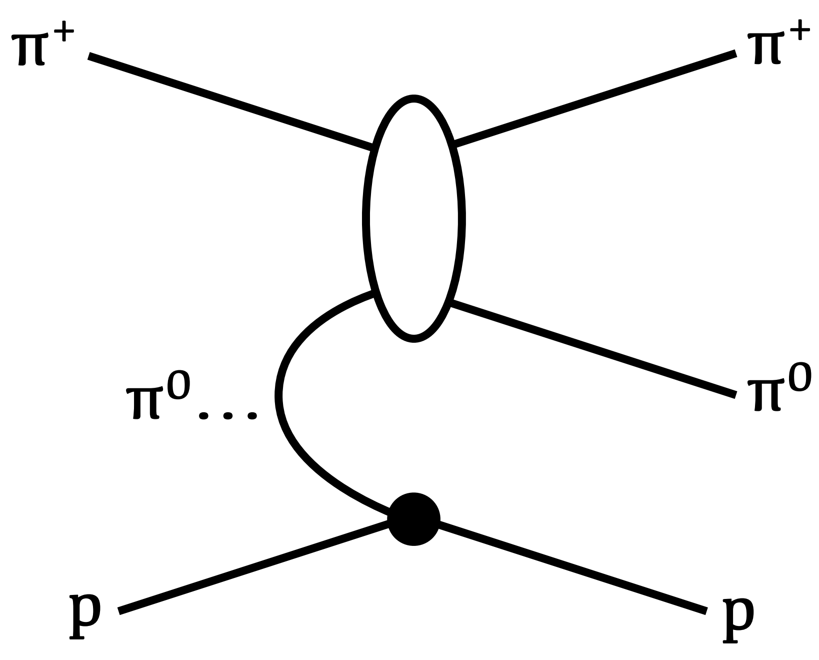
'states' like ρ , Δ ... are **resonances**

asymptotic states of the theory include multi-pion states

e.g. in the CM frame $\int d\hat{p}\; Y_L^m(\hat{p})\, \big|\pi(\vec{p})\pi(-\vec{p})\big\rangle$

with $\left|\vec{p}\right|$ varying continuously up from zero

e.g. ππ scattering in isospin-1



real QCD

real QCD has very few stable particles : $\pi,\ K,\ N,\ \Sigma,\ \Lambda,\ \Xi,\ \Omega$

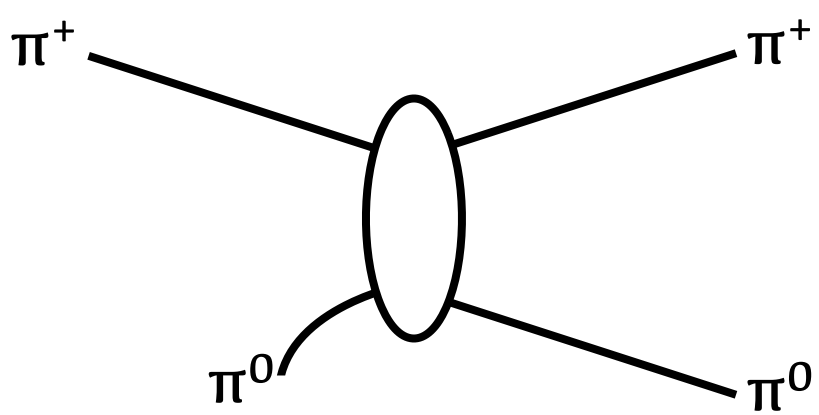
'states' like ρ , Δ ... are **resonances**

asymptotic states of the theory include multi-pion states

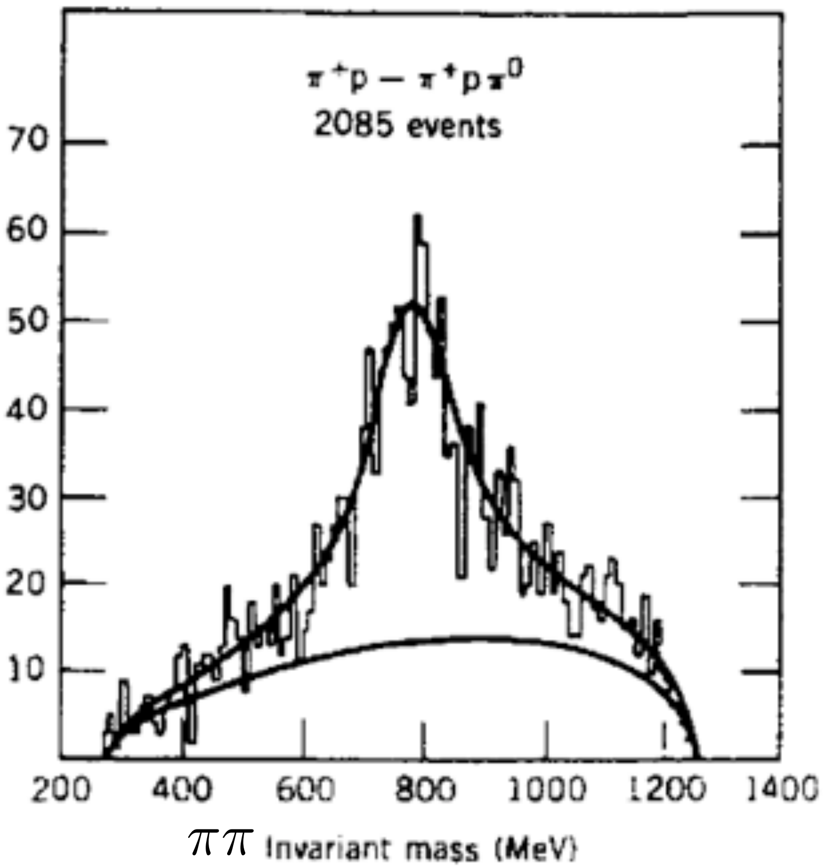
e.g. in the CM frame $\int d\hat{p}\; Y_L^m(\hat{p})\, \big|\pi(\vec{p})\pi(-\vec{p})\big\rangle$

with $\left|\vec{p}\right|$ varying continuously up from zero

e.g. ππ scattering in isospin-1



for small t



real QCD

real QCD has very few stable particles : $\pi, \; K, \; N, \; \Sigma, \; \Lambda, \; \Xi, \; \Omega$

'states' like ρ , Δ ... are **resonances**

asymptotic states of the theory include multi-pion states

e.g. in the CM frame $\int d\hat{p}\; Y_L^m(\hat{p})\, \big|\pi(\vec{p})\pi(-\vec{p})\big\rangle$

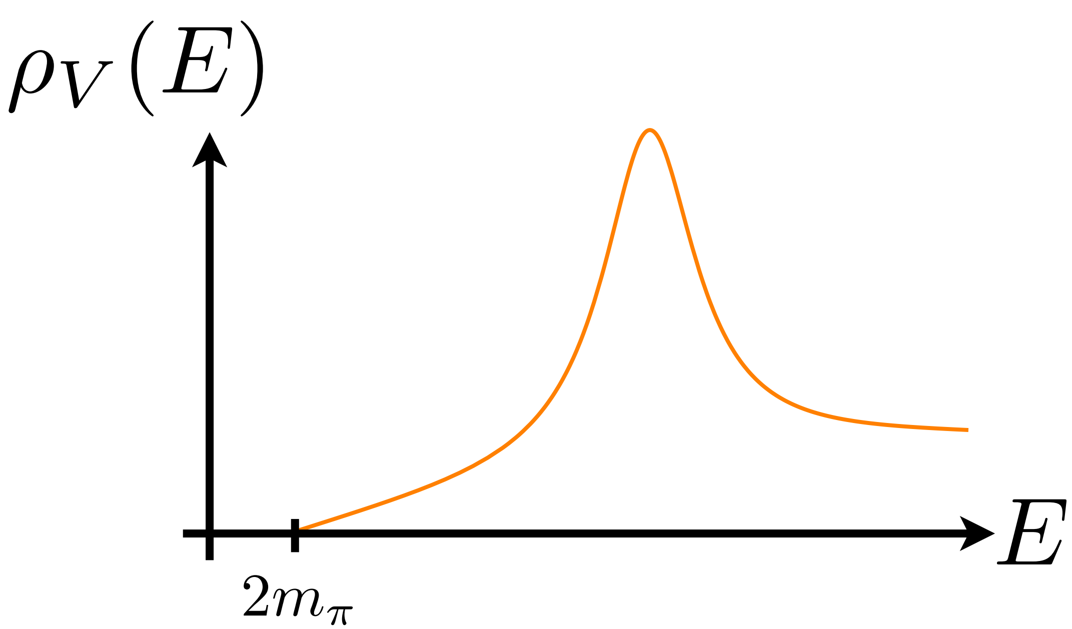
with $\big|\vec{p}\big|$ varying continuously up from zero

within the field-theory we have correlators

e.g. the (Euclidean) vector correlator

$$C_V(t) = \int dE\; e^{-Et}\, \rho_V(E)$$

spectral function



the spectrum is continuous !

real QCD

real QC… : $\pi,\ K,\ N,\ \Sigma,\ \Lambda,\ \Xi,\ \Omega$

'states'

asy… …clude multi-pion states

$Y_L^m(\hat{p})\,\big|\pi(\vec{p})\pi(-\vec{p})\big\rangle$

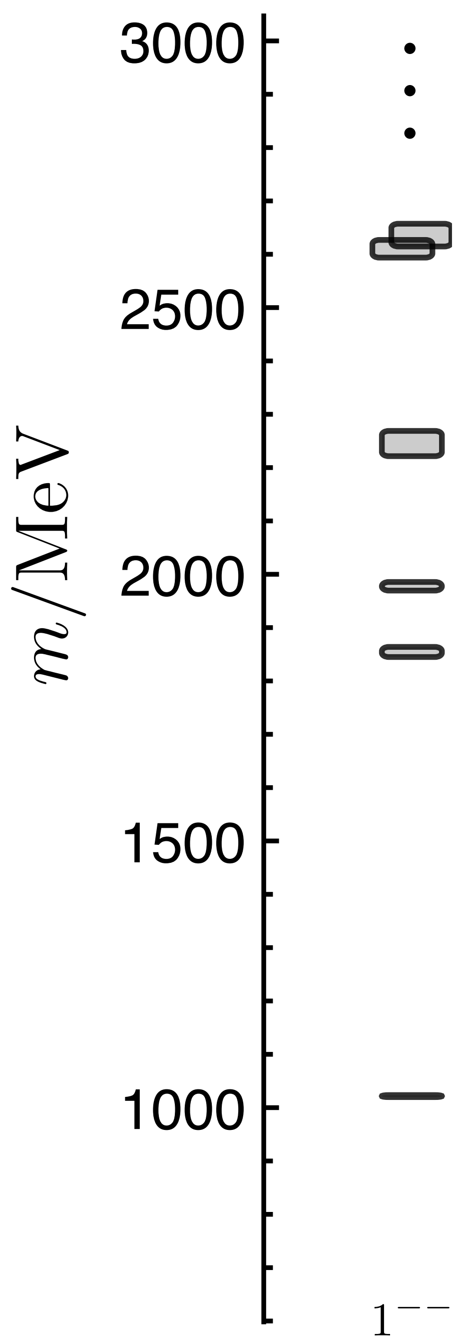
with $|\vec{p}|$ varying continuously up from zero

withi… …relators

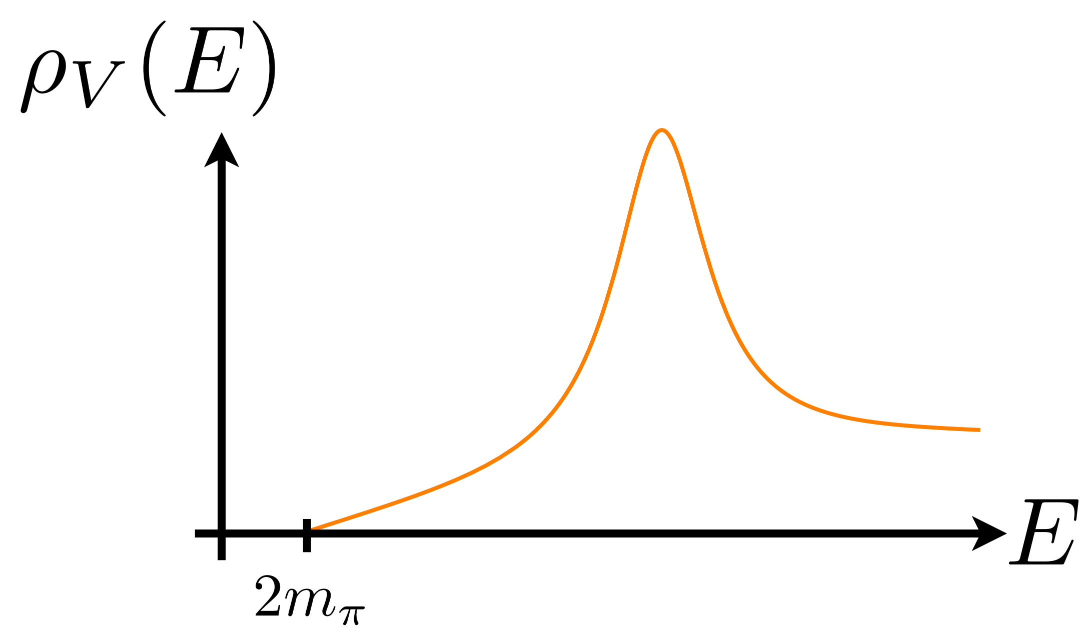
e… …relator

…tral
…tion

a lattice calc.



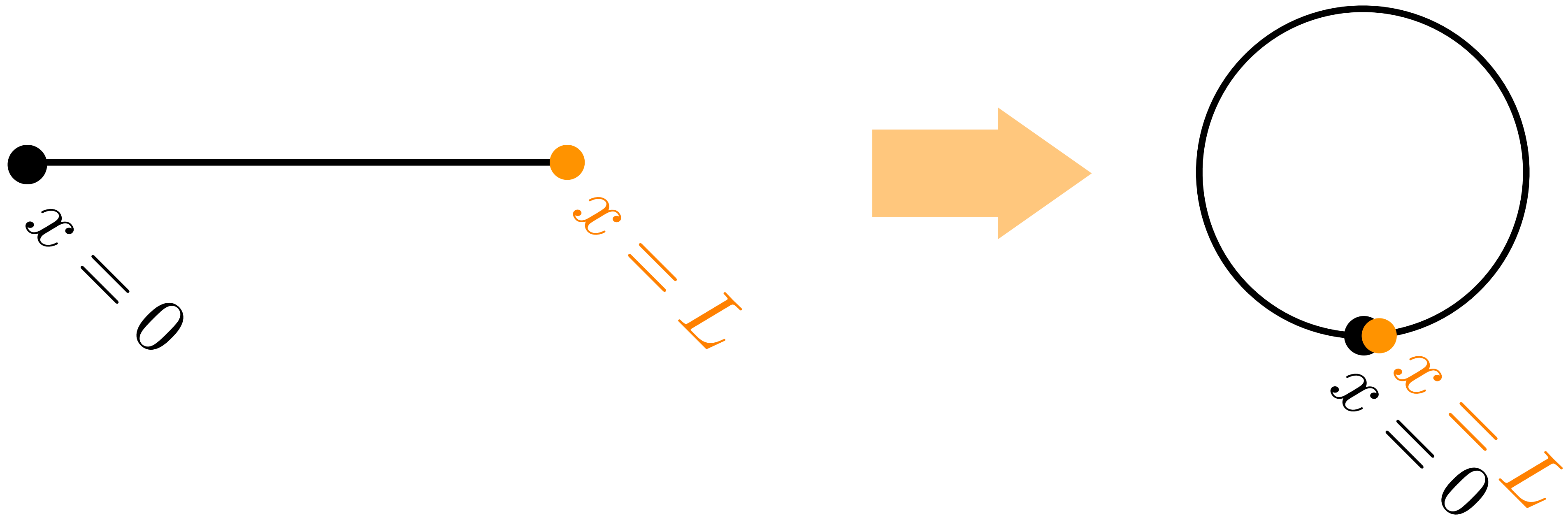
a discrete spectrum ! ?



the spectrum is continuous !

field theory in a finite volume

consider the case of one space dimension with a periodic boundary condition



this will make the allowed momenta of a free particle discrete :

$$\psi_p(x) = e^{ipx}$$ free particle

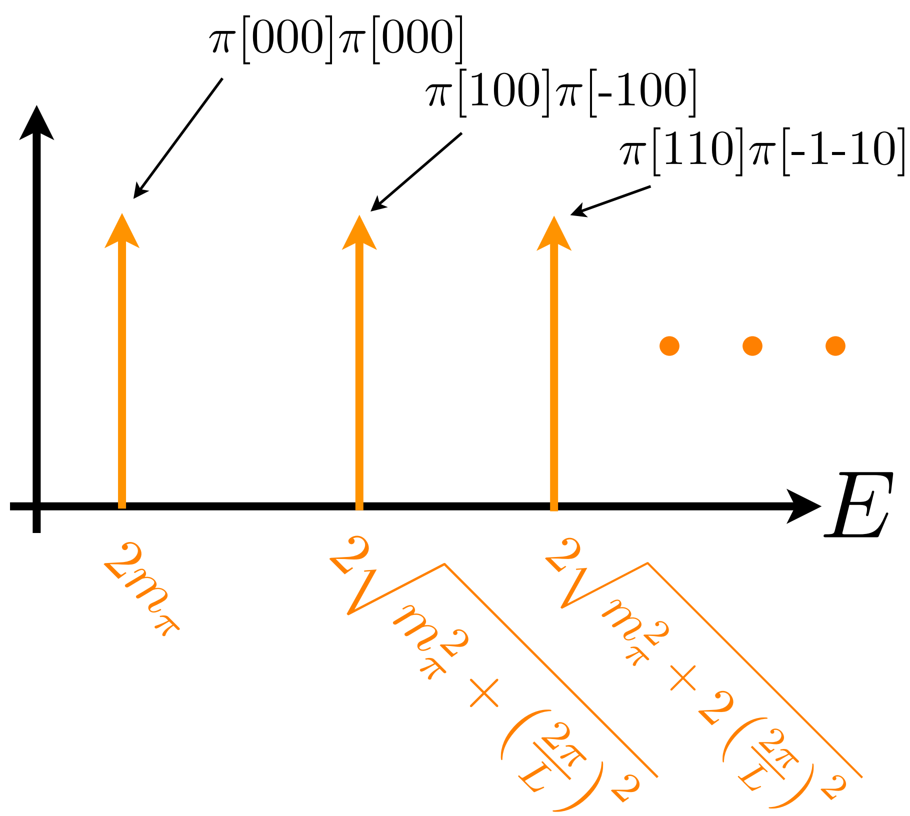
$$\psi_p(x) = \psi_p(x+L)$$ periodic boundary condition

$$e^{ipL} = 1 \implies p = \frac{2\pi}{L} n$$ for integer n

non-interacting two-particle states in a finite volume

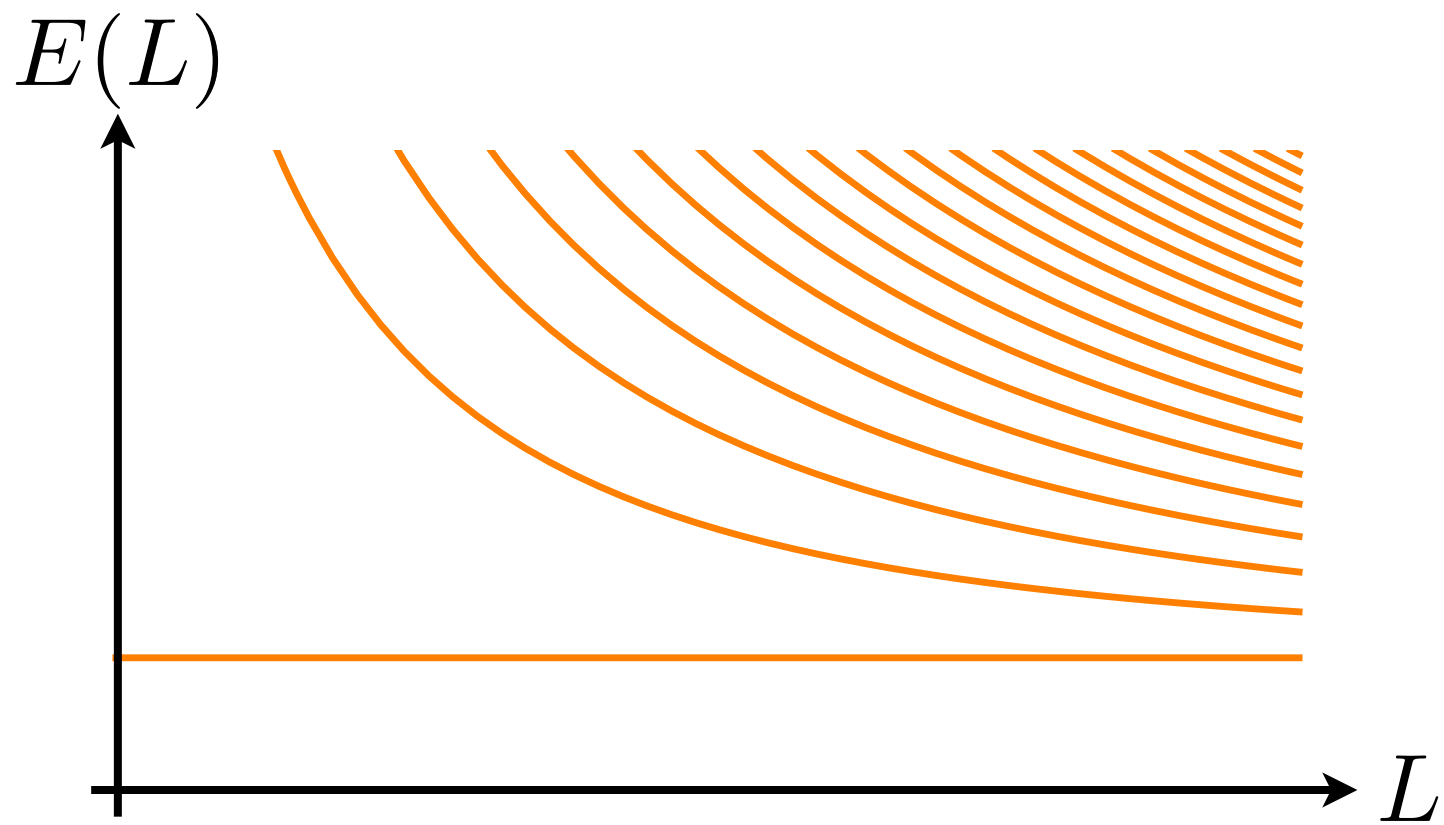
in a three-dimensional cubic box

$$\vec{p} = \frac{2\pi}{L}\left[n_x, n_y, n_z\right]$$



non-interacting
spectral function

the spectrum is discrete and volume-dependent



two-particle states in a finite volume

but hadrons do interact, and sometimes strongly

e.g. $$\pi\pi \to \rho \to \pi\pi$$
$$\pi N \to \Delta \to \pi N$$

what is the impact of these interactions on the finite-volume spectrum ?

two-particle states in a finite volume

e.g. non-rel quantum mechanics in one-dimension

two spinless bosons separated by x

interacting through a potential $V(x)$

in center-of-momentum frame

scattering solutions of

$$-\frac{1}{m}\frac{d\psi}{dx^2} + V(x)\psi = E\psi$$

$$E = \frac{k^2}{m} > 0$$

$$\psi(x \to \infty) = N \cos\left[k|x| + \delta(k)\right]$$

finite length world $-L/2 < x < L/2$ with periodic b.c.

wavefunction and derivative continuous at boundary

$$\frac{1}{\psi}\frac{d\psi}{dx}\bigg|_{x=-L/2} = \frac{1}{\psi}\frac{d\psi}{dx}\bigg|_{x=+L/2}$$

$$-k\tan\left[\tfrac{kL}{2} + \delta(k)\right] = k\tan\left[\tfrac{kL}{2} + \delta(k)\right]$$

$$\tan\left[\tfrac{kL}{2} + \delta(k)\right] = 0$$

$$kL + 2\delta(k) = 0 \mod 2\pi$$

two-particle states in a finite volume

e.g. quantum mechanics in one-dimension

$$kL + 2\delta(k) = 0 \mod 2\pi$$

discrete & **volume-dependent** spectrum of scattering states

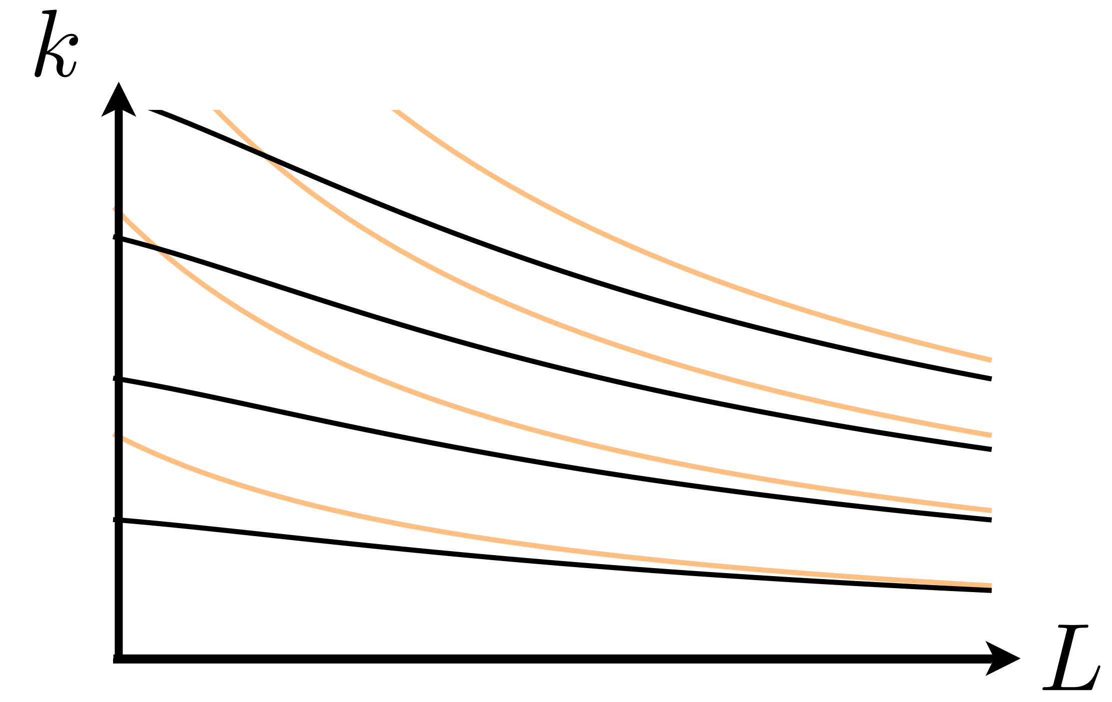
$$E = \frac{k^2}{m} \qquad\qquad k = \frac{2\pi}{L}\left(n - \frac{\delta(k)}{\pi L}\right)$$

non-interacting momentum

shift due to interaction

e.g. a weak attraction

$$\delta(k) = a\,k$$



two-particle states in a finite volume

the analogous expression for two-particle elastic scattering
in a finite cubic volume has been derived by Lüscher

somewhat complicated by the lack of full rotational symmetry (a cube)

for our purposes, we'll pretend that it's as simple as

$$\delta_\ell(E) = f_\ell(E, L)$$

phase-shift in partial wave ℓ
at scattering energy E

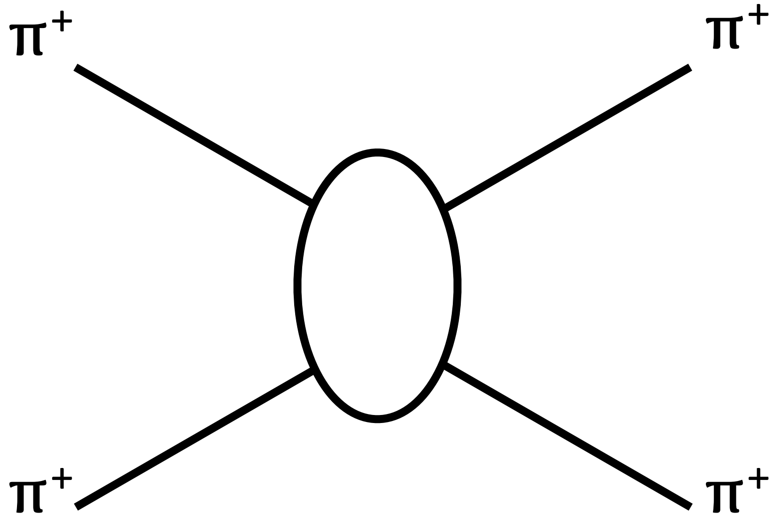
known function of
energy and box length

so we 'measure' $E_\mathfrak{n}$ on one or more volumes

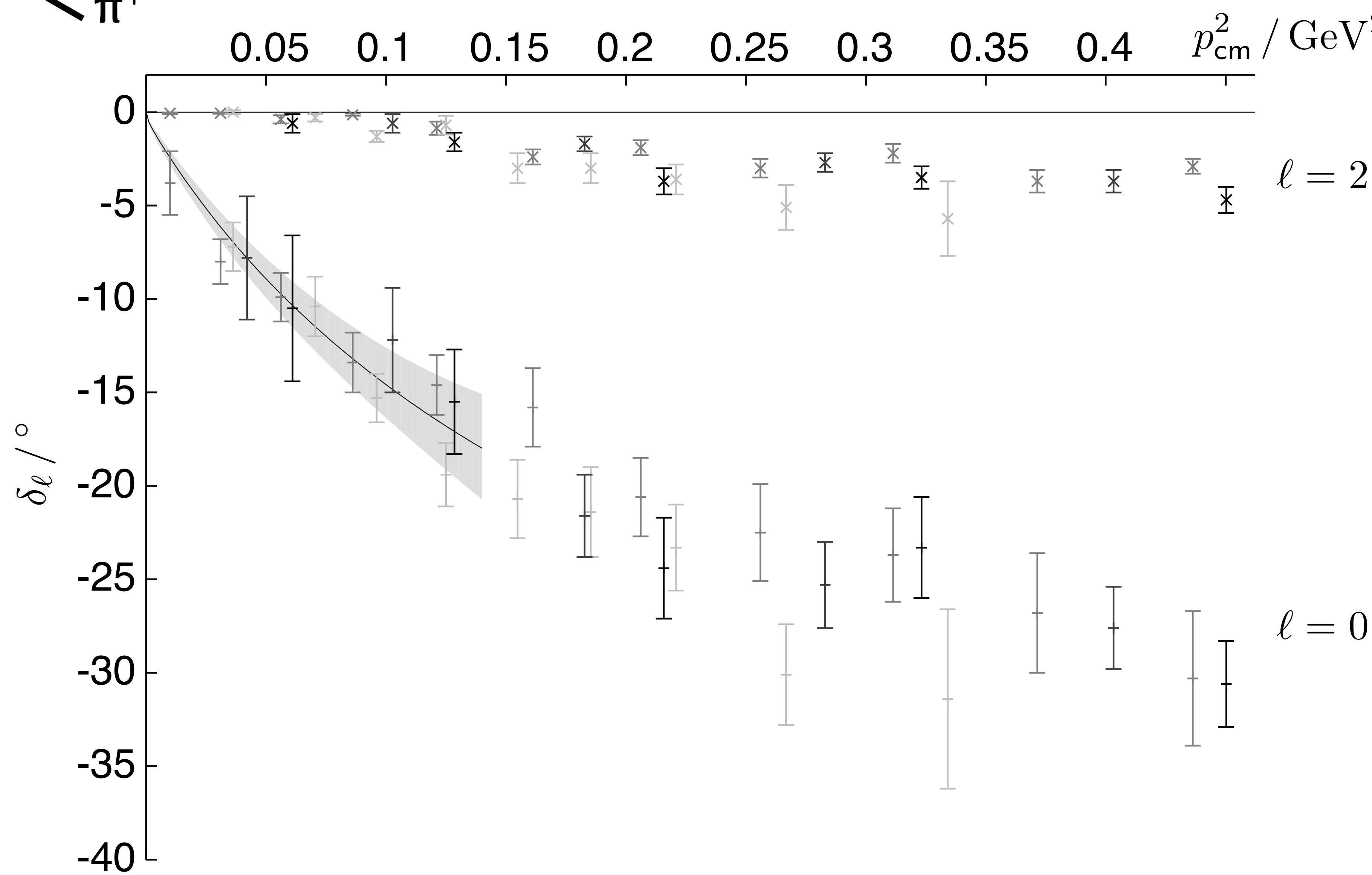
and plug into the formula to determine δ at discrete energies

ππ isospin-2 scattering

e.g.



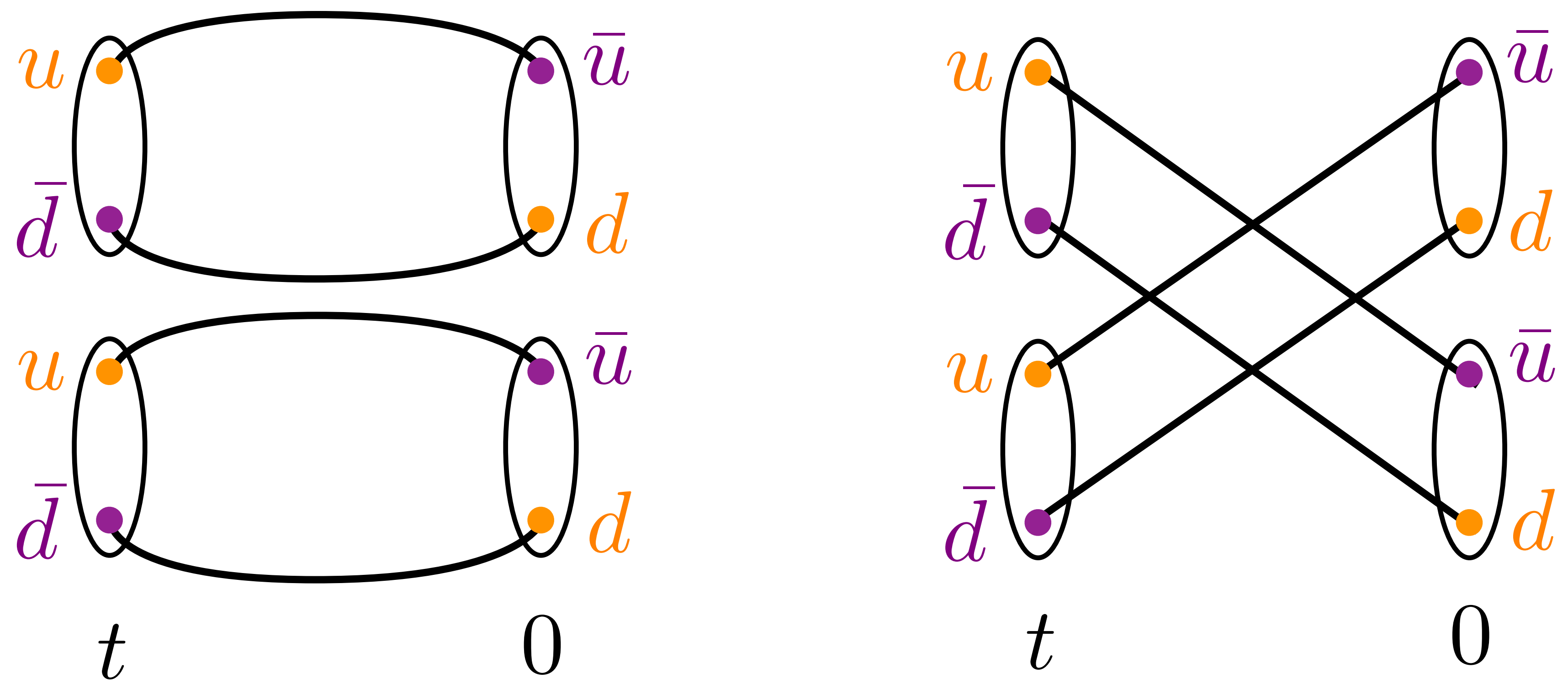
empirically: weak & repulsive scattering



ππ isospin-2 scattering - field theory calculation

$$C(t) = \langle 0 \big| \mathcal{O}^\dagger_{\pi^+}(t)\mathcal{O}^\dagger_{\pi^+}(t) \cdot \mathcal{O}_{\pi^+}(0)\mathcal{O}_{\pi^+}(0) \big| 0 \rangle$$

no quark-annihilation
in these correlators



variational in a basis
of operators :

"ππ" of various
relative momenta

e.g. $\vec{P} = [000]$

$\pi_{[000]}\pi_{[000]}$

$\pi_{[100]}\pi_{[-100]}$

$\pi_{[110]}\pi_{[-1-10]}$

$\pi_{[111]}\pi_{[-1-1-1]}$

$\vdots$

ππ isospin-2 scattering

Hadron Spectrum Collaboration
arXiv:1203.6041

variational in a basis of operators :

"ππ" of various relative momenta

e.g. $\vec{P} = [000]$

$\pi_{[000]}\pi_{[000]}$

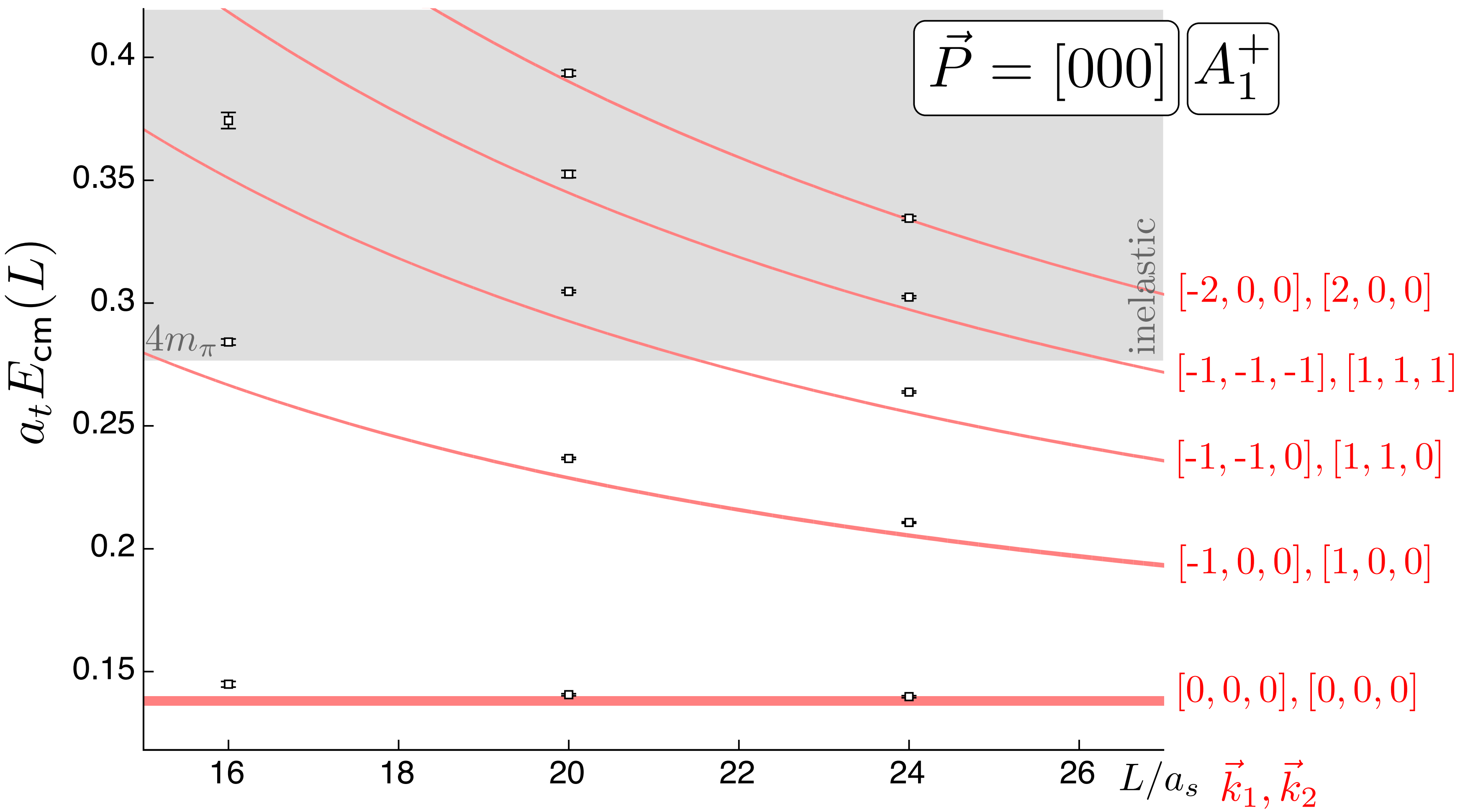
$\pi_{[100]}\pi_{[-100]}$

$\pi_{[110]}\pi_{[-1-10]}$

$\pi_{[111]}\pi_{[-1-1-1]}$

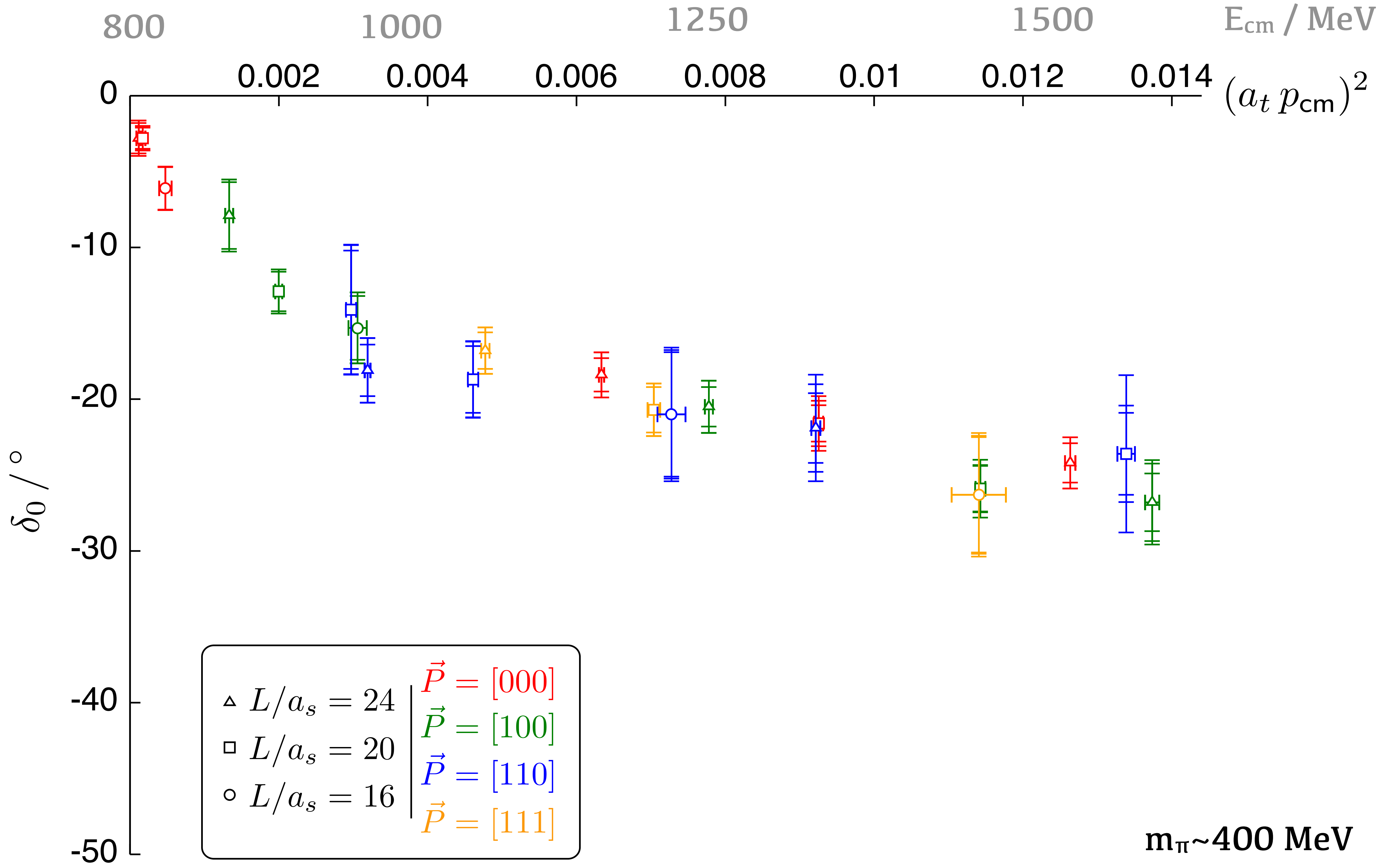
$\vdots$

extracted energies shifted upward slightly from non-interacting pion pairs



ππ isospin-2 scattering

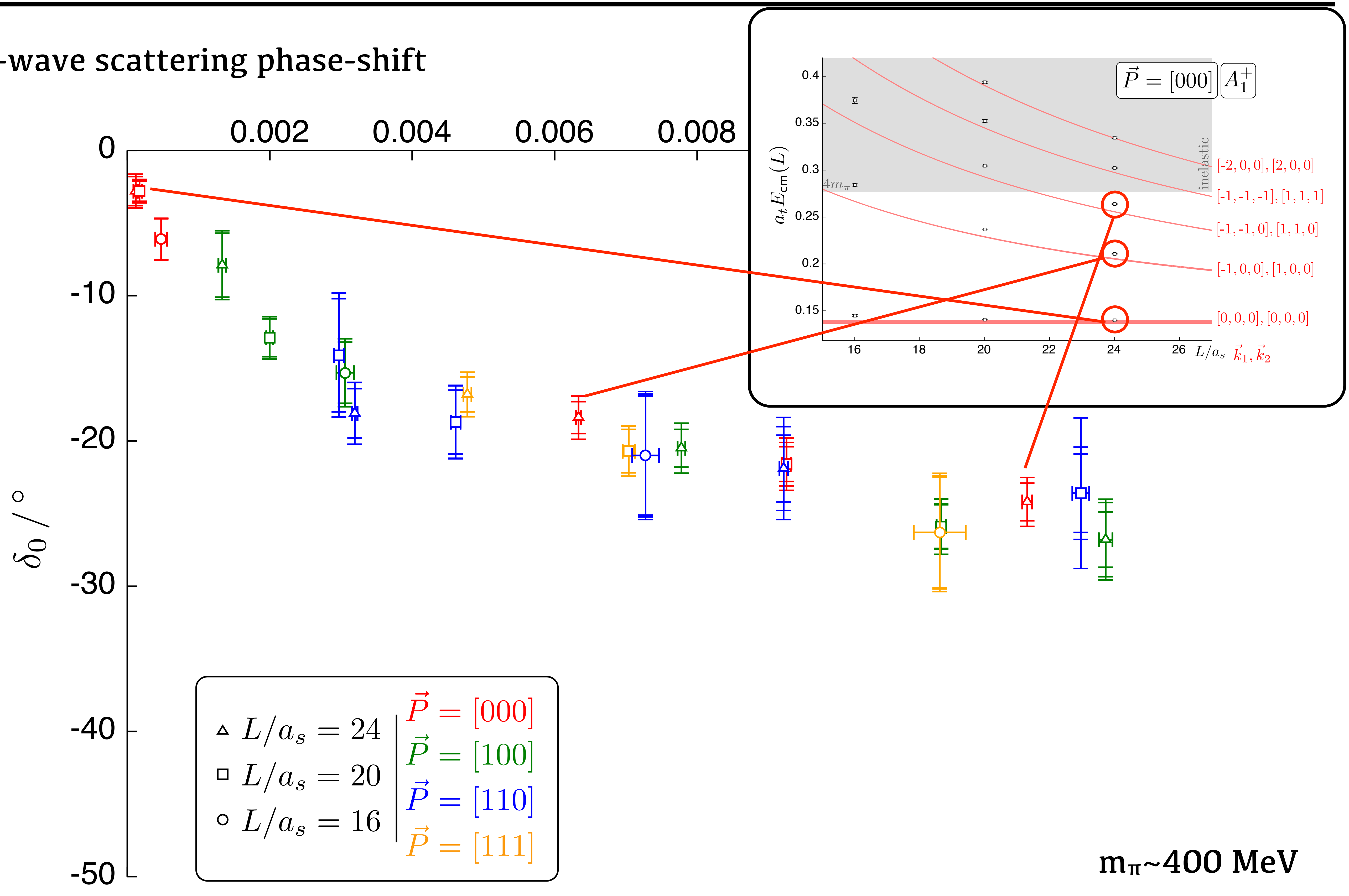
S-wave scattering phase-shift



m_π~400 MeV

ππ isospin-2 scattering

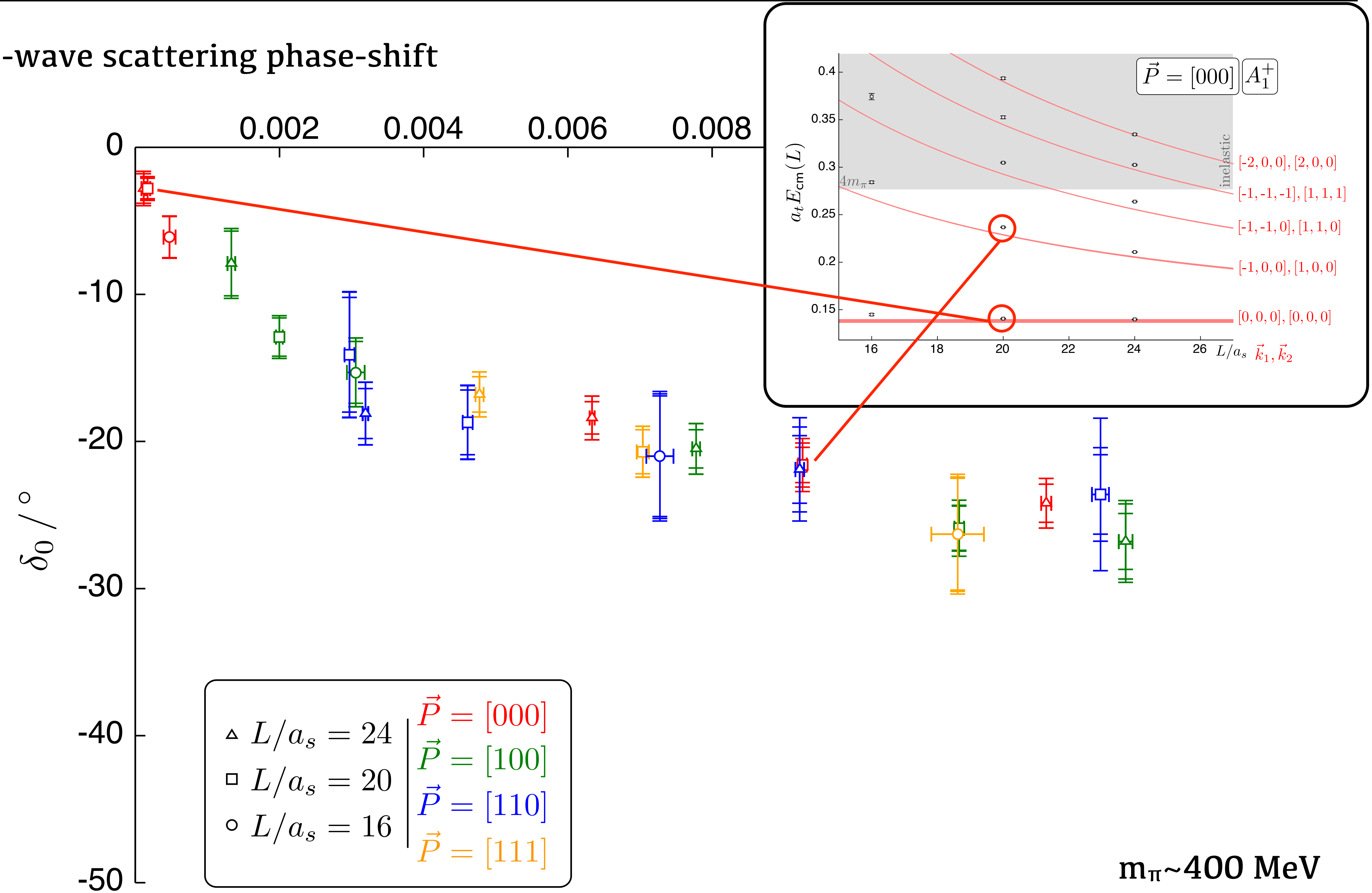
S-wave scattering phase-shift



m_π~400 MeV

ππ isospin-2 scattering

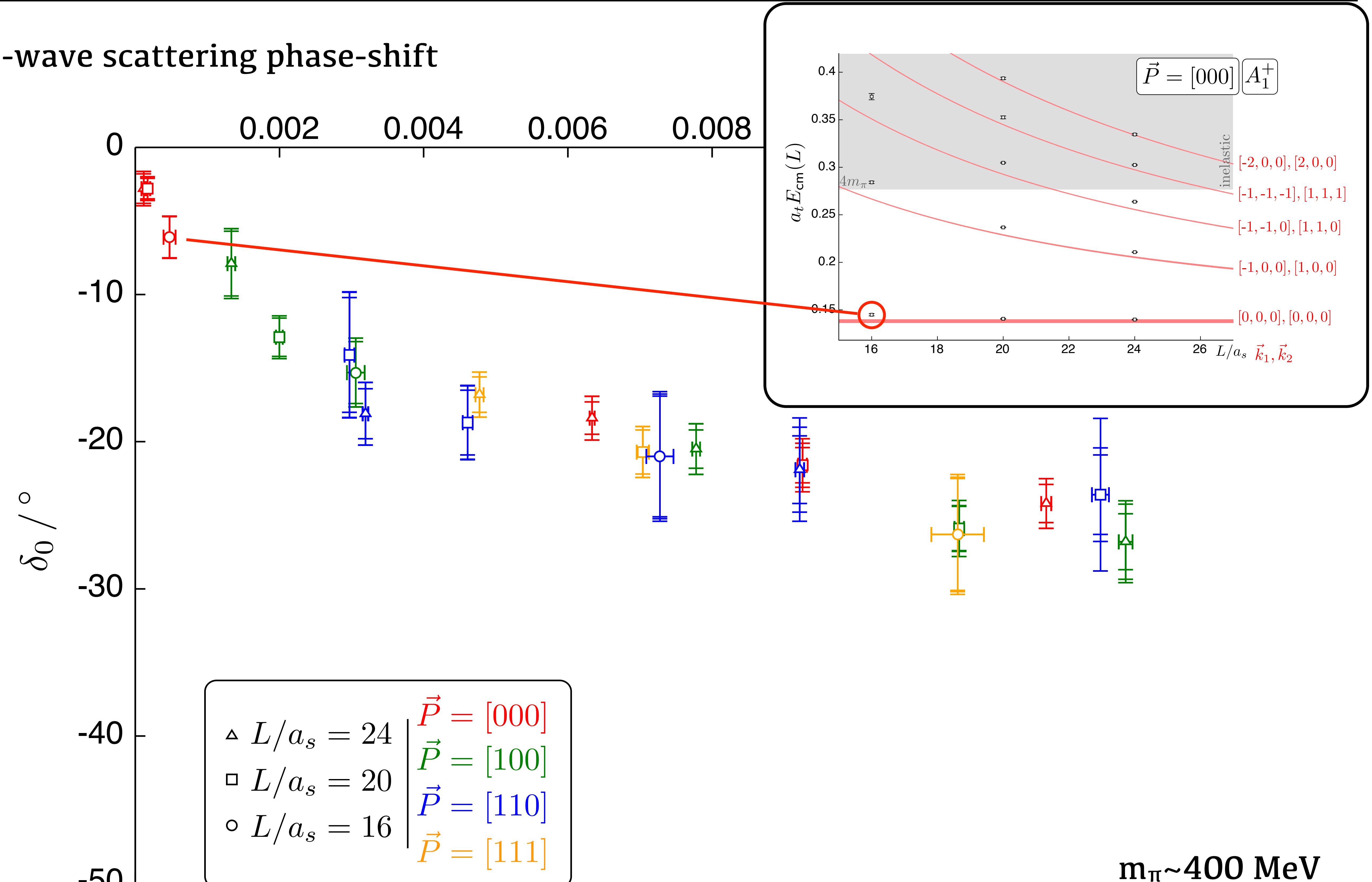
S-wave scattering phase-shift



m_π~400 MeV

ππ isospin-2 scattering

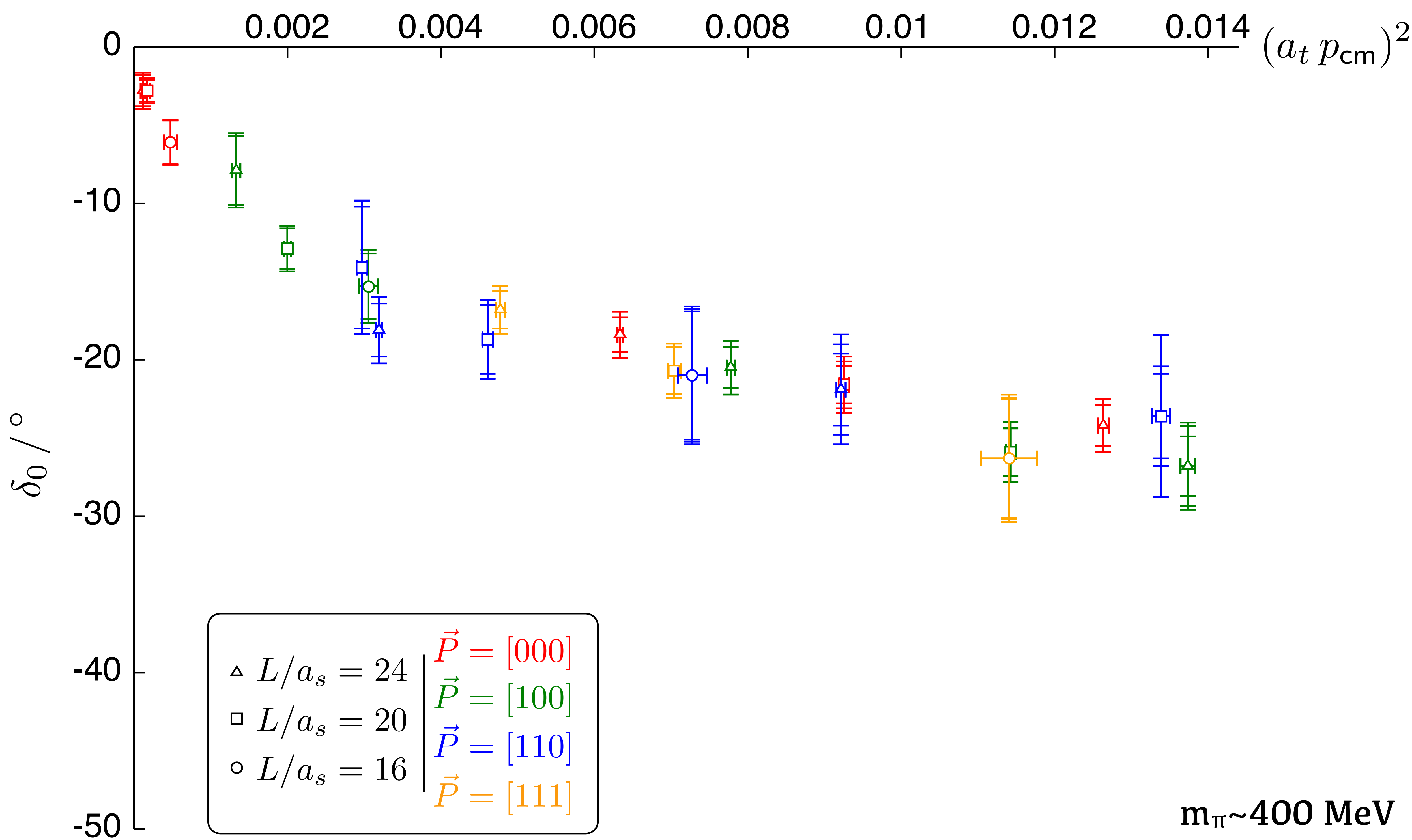
S-wave scattering phase-shift



m_π~400 MeV

ππ isospin-2 scattering

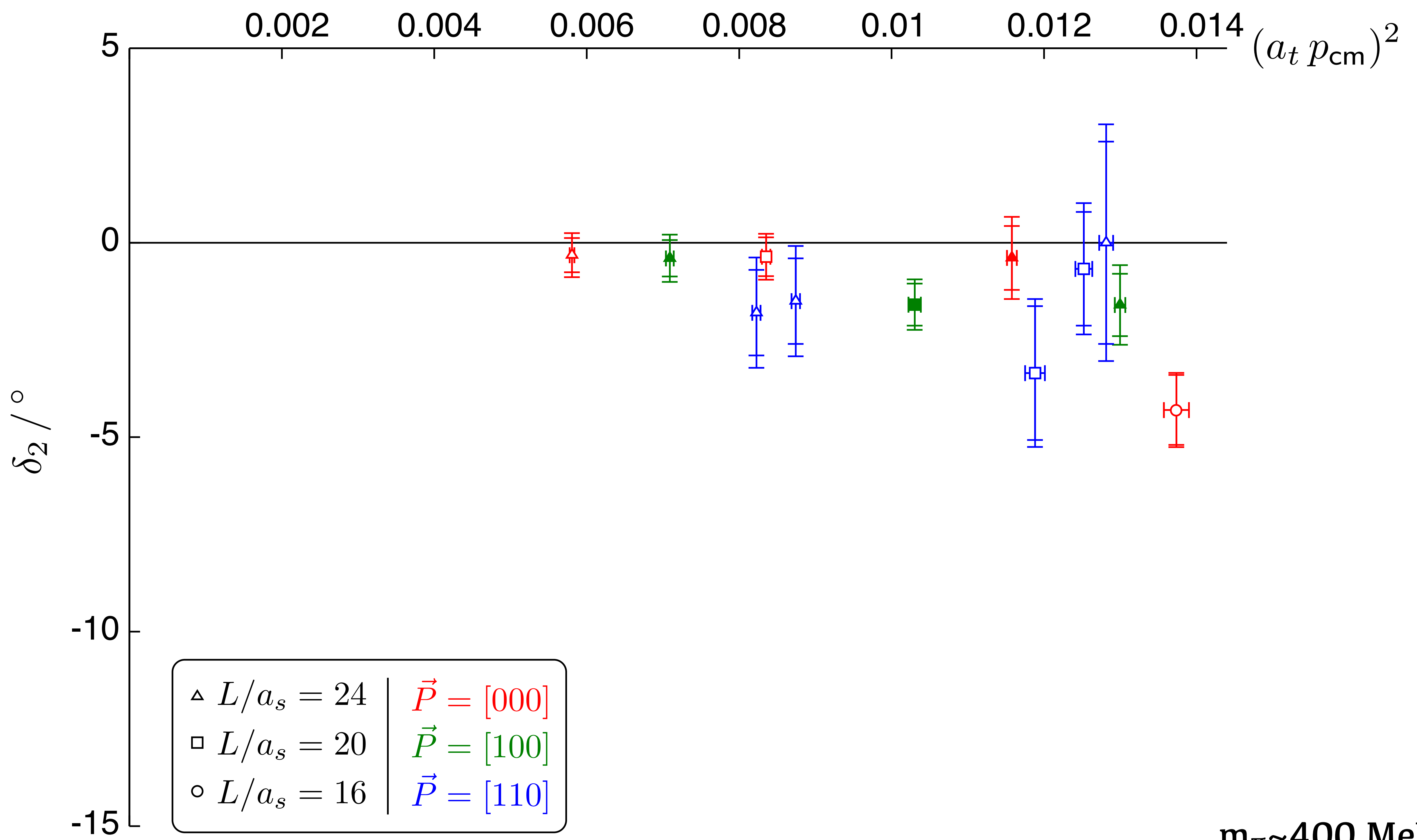
S-wave scattering phase-shift



m_π~400 MeV

ππ isospin-2 scattering

D-wave scattering phase-shift

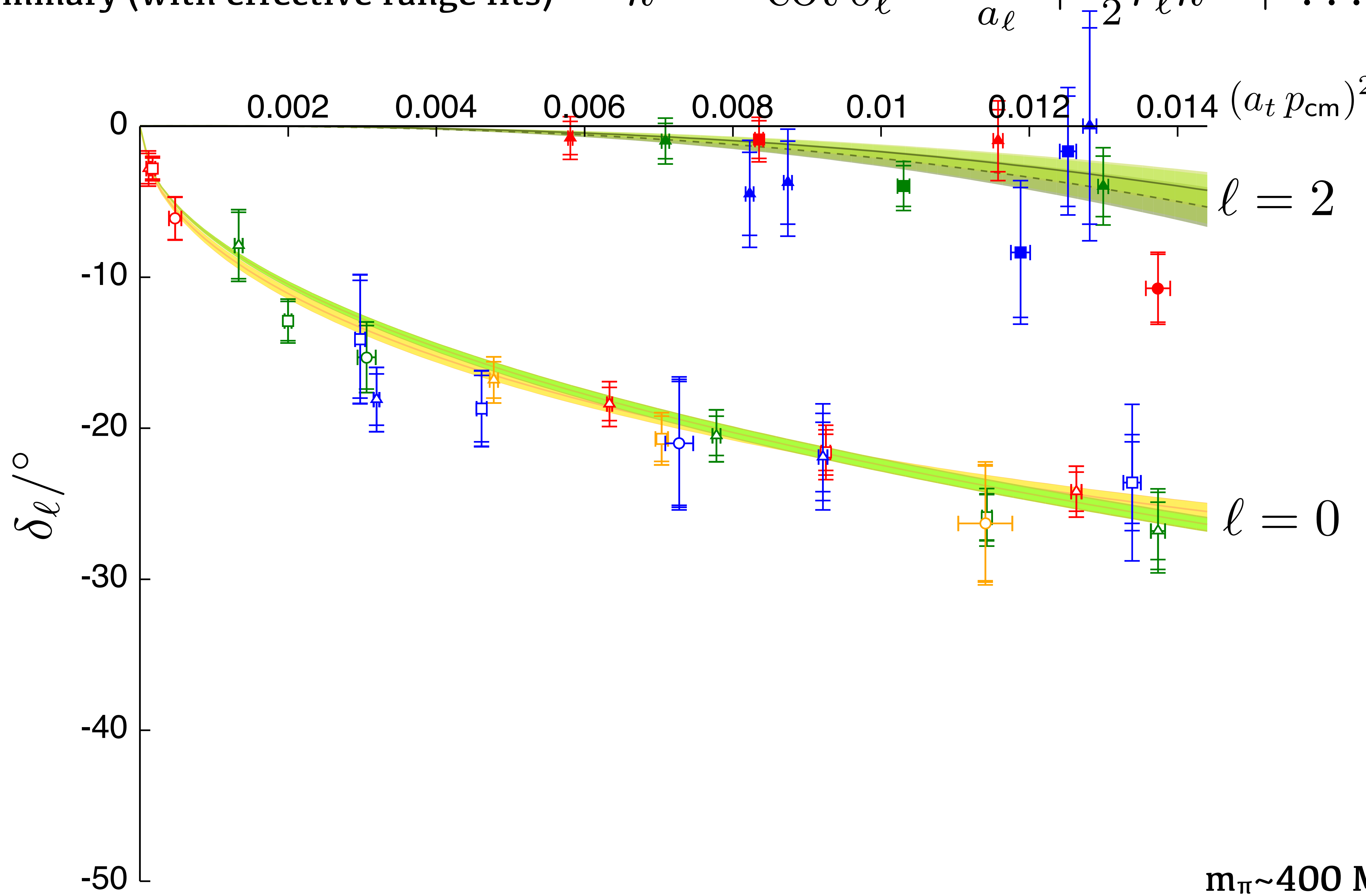


m_π~400 MeV

ππ isospin-2 scattering

summary (with effective range fits)

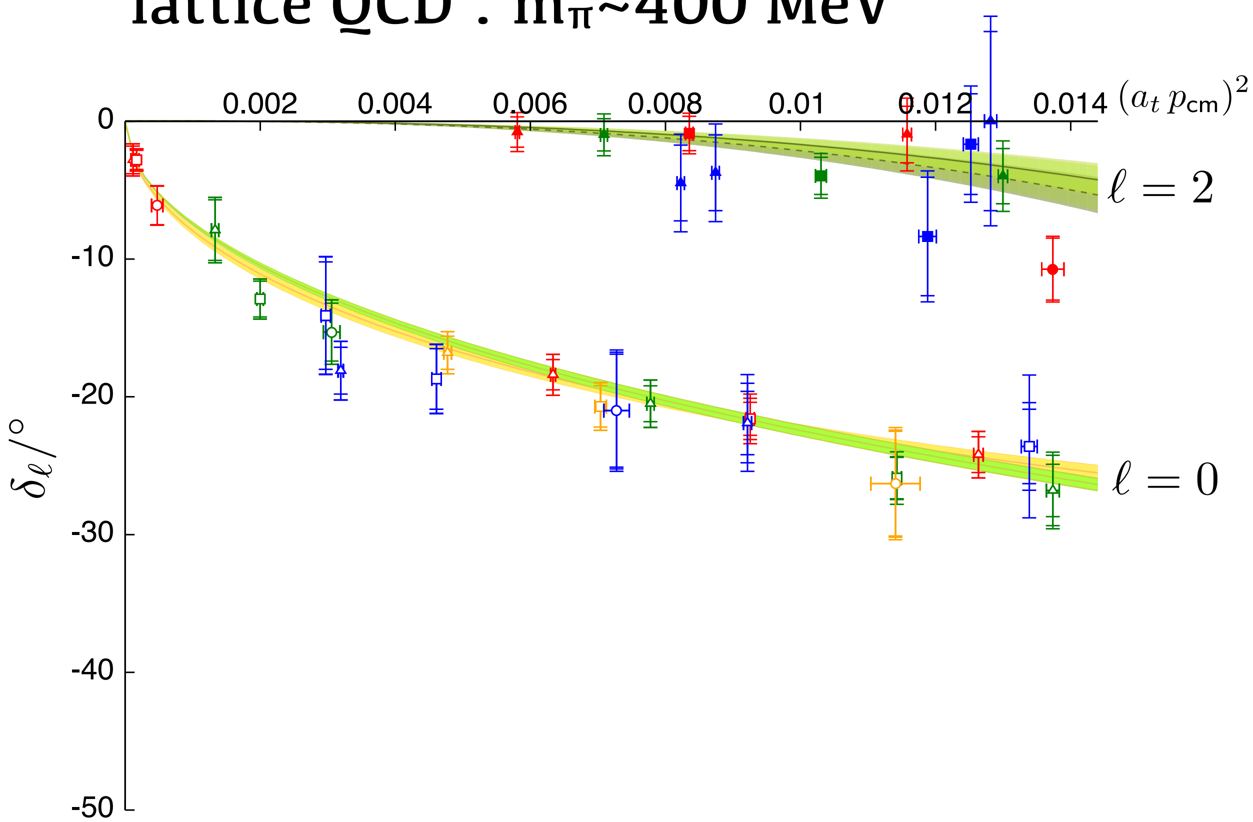
$$k^{2\ell+1}\cot\delta_\ell = \frac{1}{a_\ell} + \frac{1}{2}r_\ell k^2 + \ldots$$



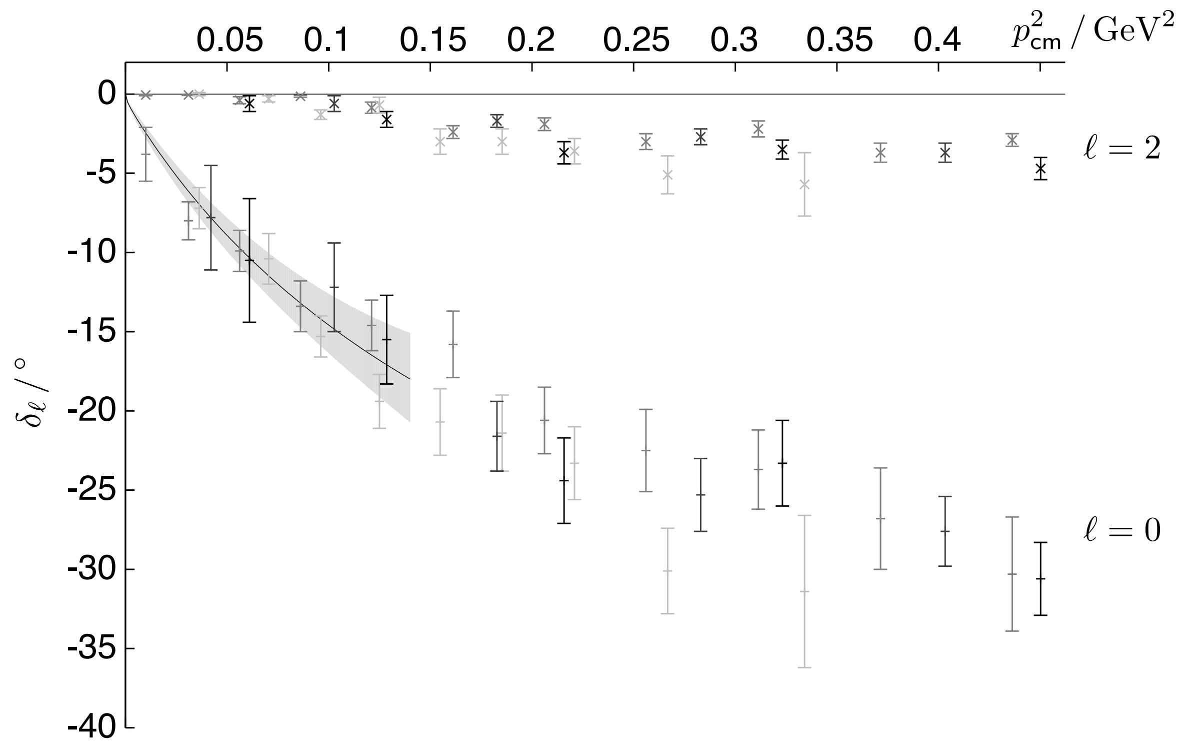
m_π~400 MeV

ππ isospin-2 scattering

lattice QCD : mπ~400 MeV



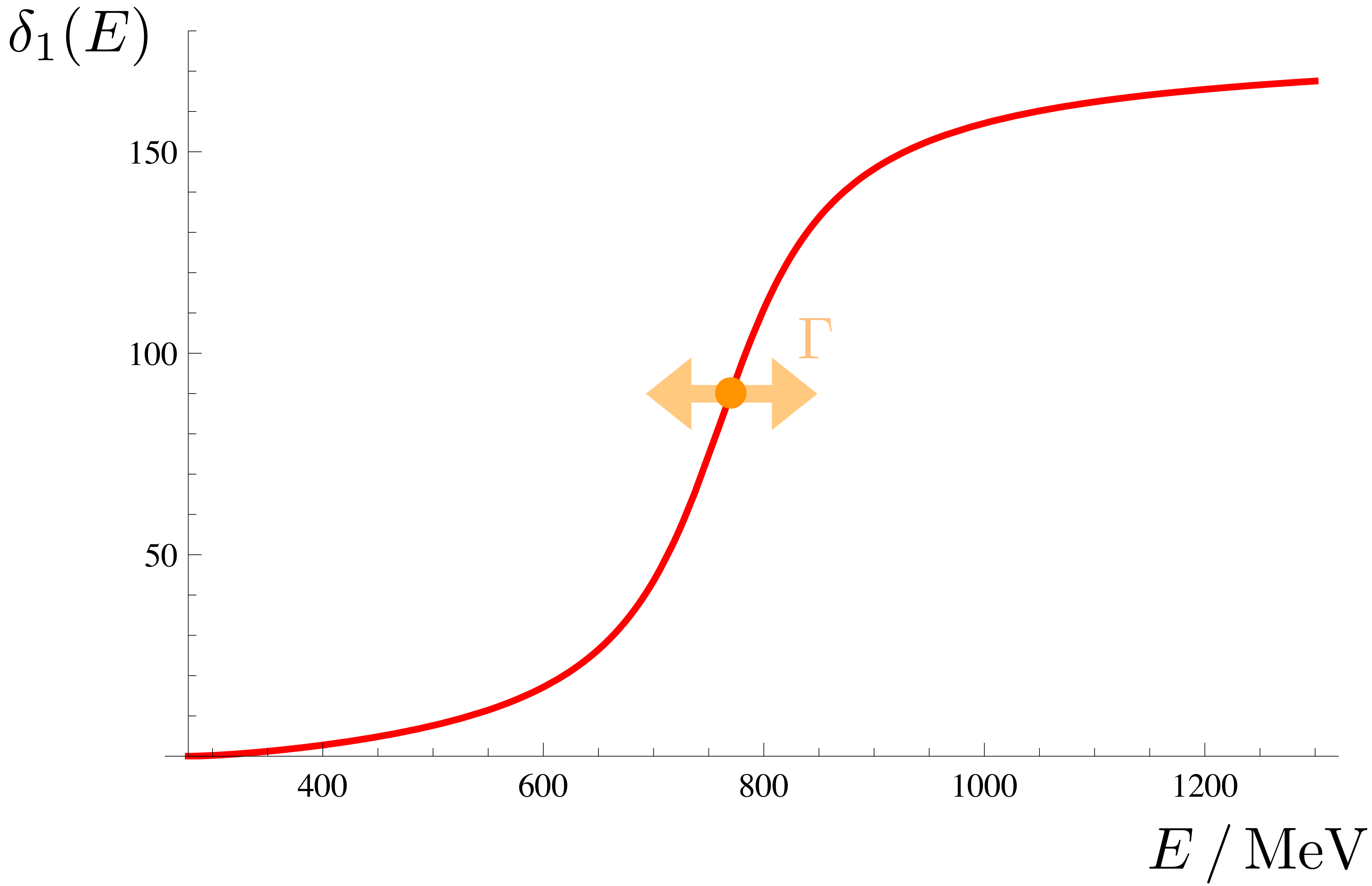
experimental compendium



ππ isospin-1 scattering

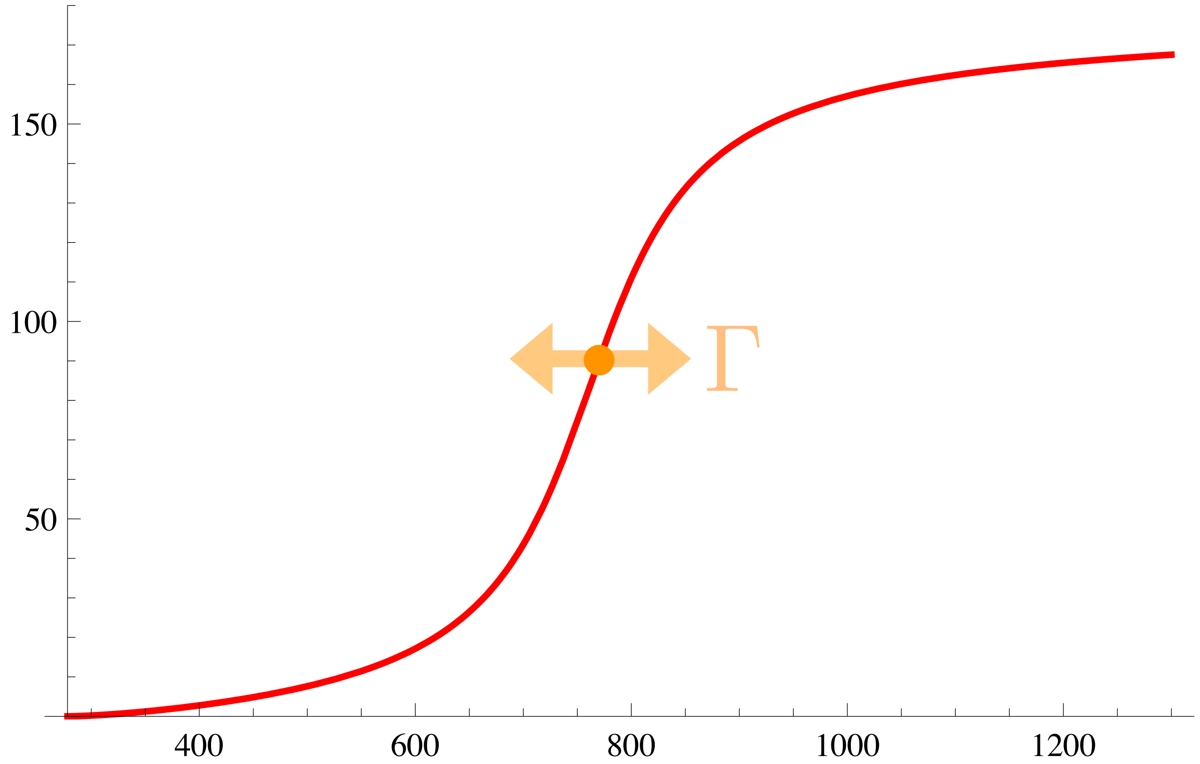
more interesting scattering channels are those featuring resonances

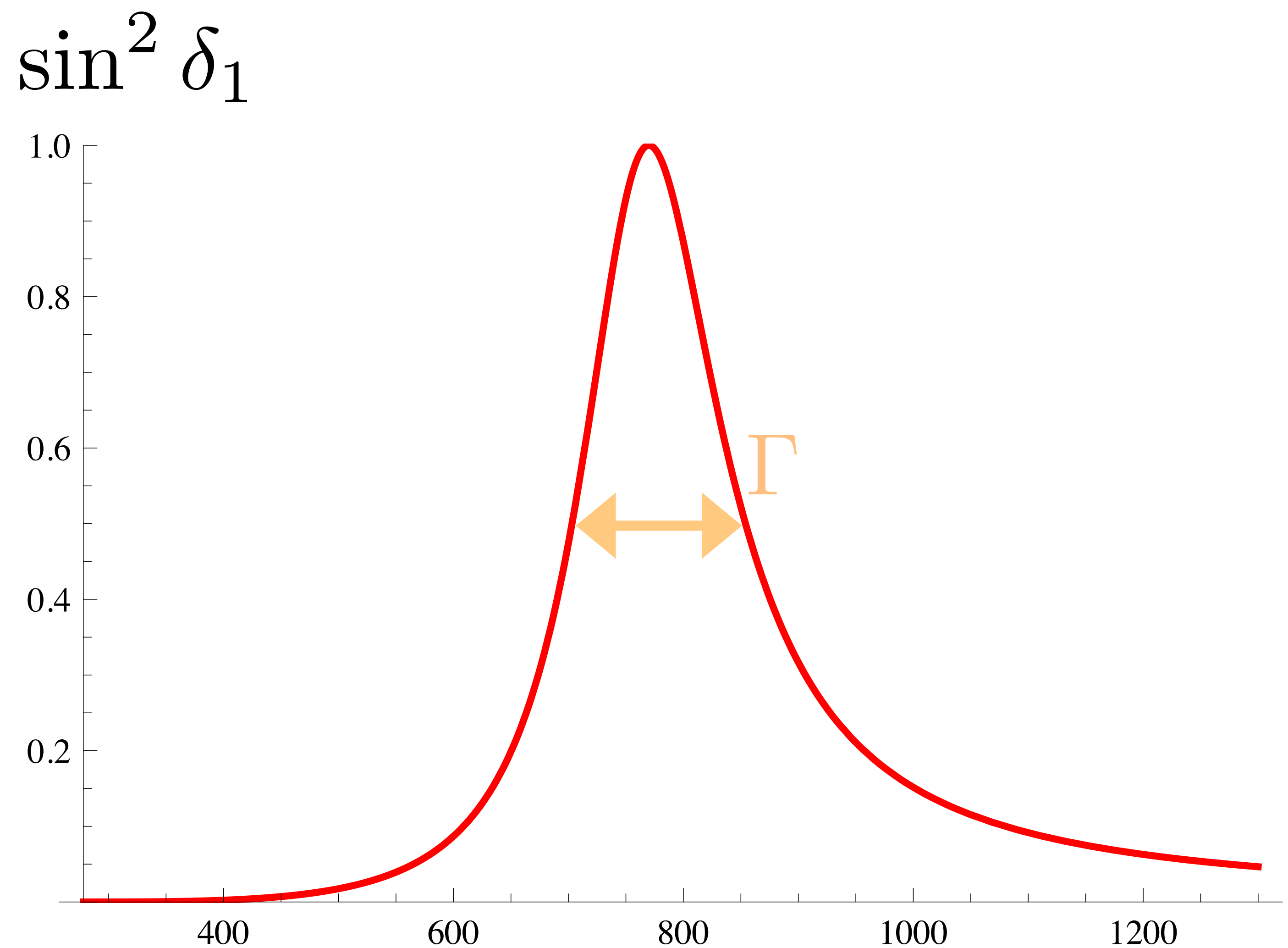
e.g. the **ρ** in ππ



ππ isospin-1 scattering

more interesting scattering channels are those featuring resonances

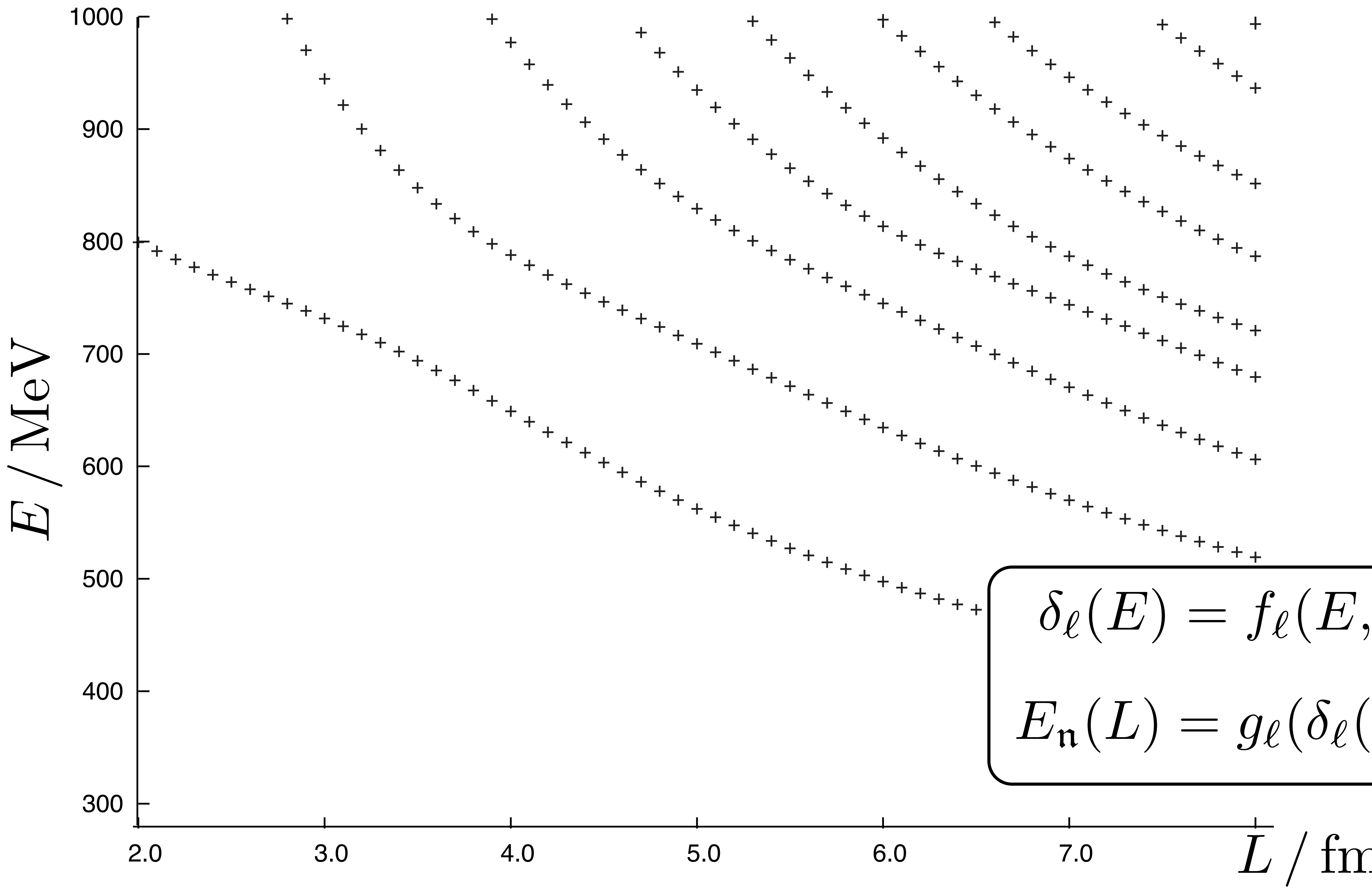




ππ isospin-1 scattering

expected finite-volume spectrum given a ρ resonance

m_π=139 MeV
m_ρ=770 MeV
Γ_ρ=150 MeV

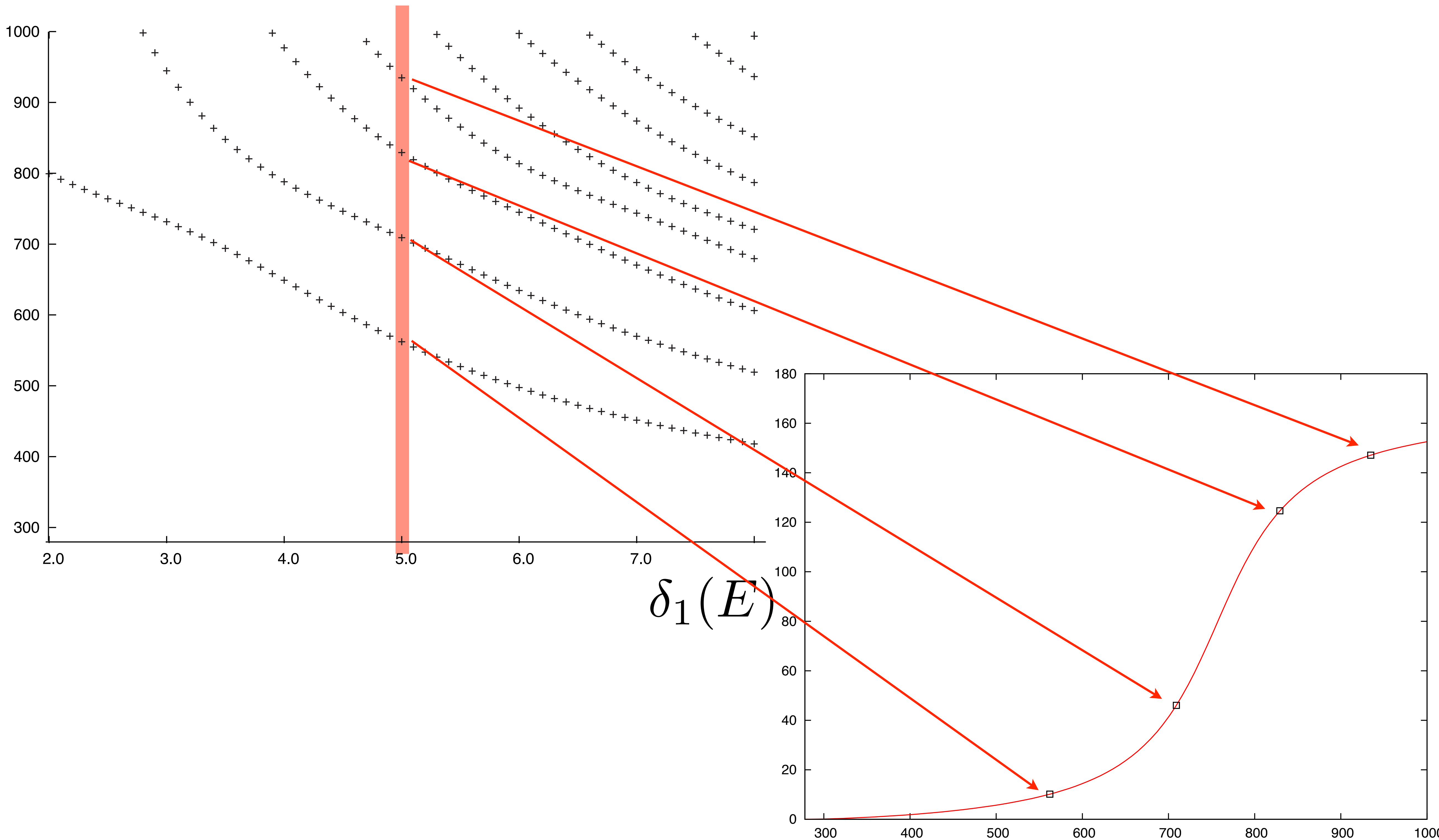


$$\delta_\ell(E) = f_\ell(E, L)$$

$$E_{\mathfrak{n}}(L) = g_\ell(\delta_\ell(E), L)$$

ππ isospin-1 scattering

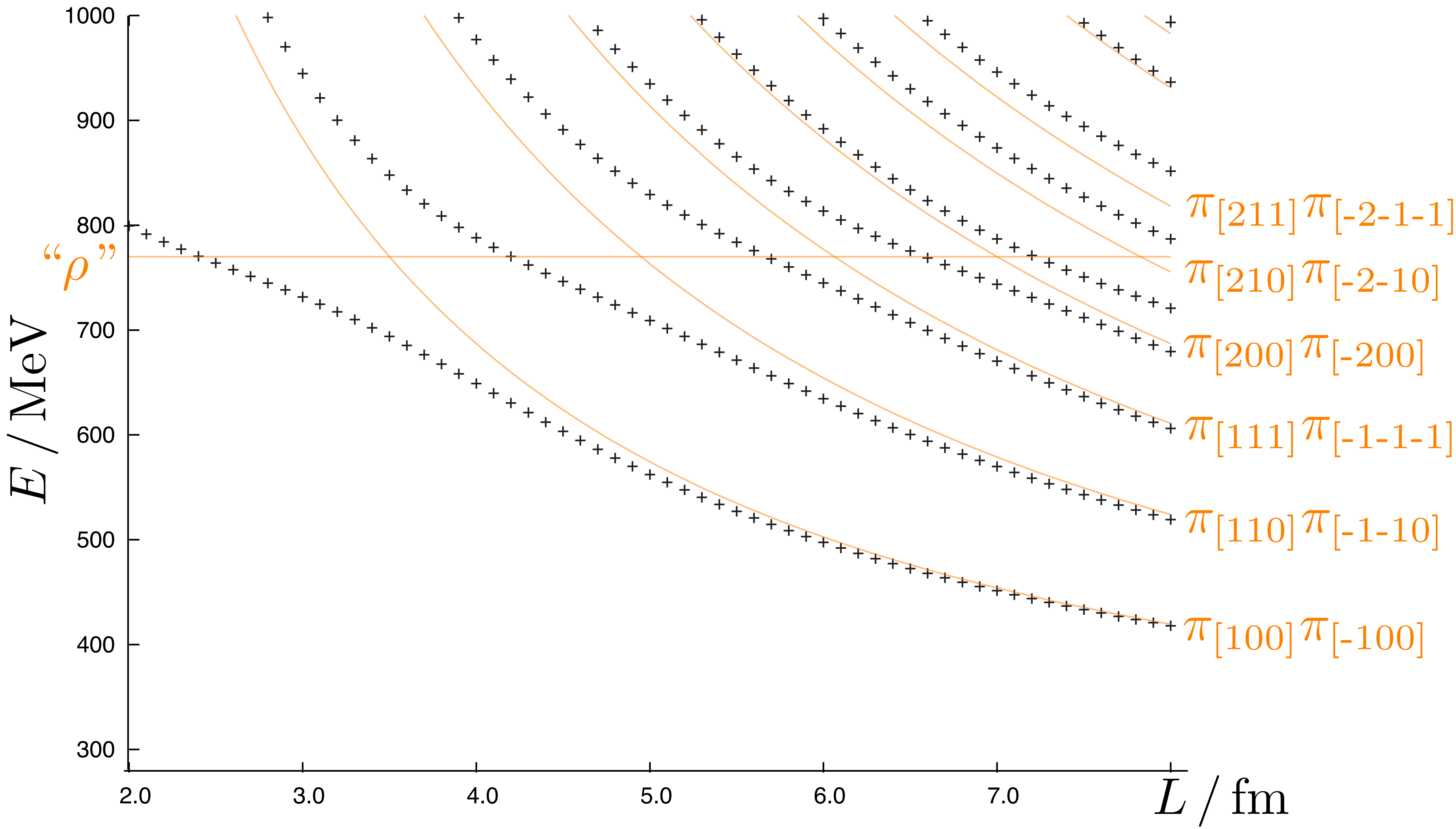
a strongly volume-dependent spectrum



ππ isospin-1 scattering

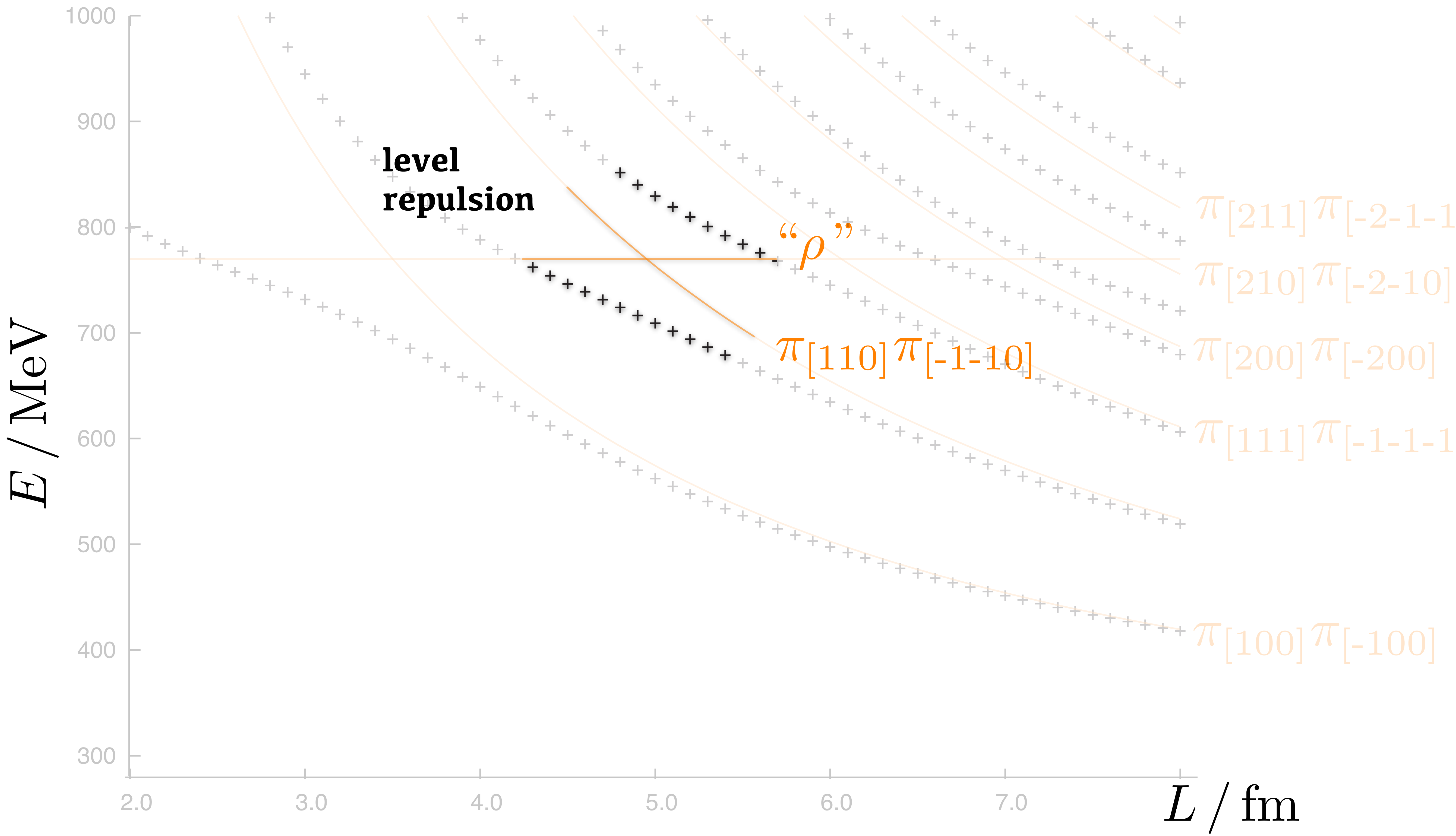
expected finite-volume spectrum given a ρ resonance

m_π=139 MeV
m_ρ=770 MeV
Γ_ρ=150 MeV



ππ isospin-1 scattering

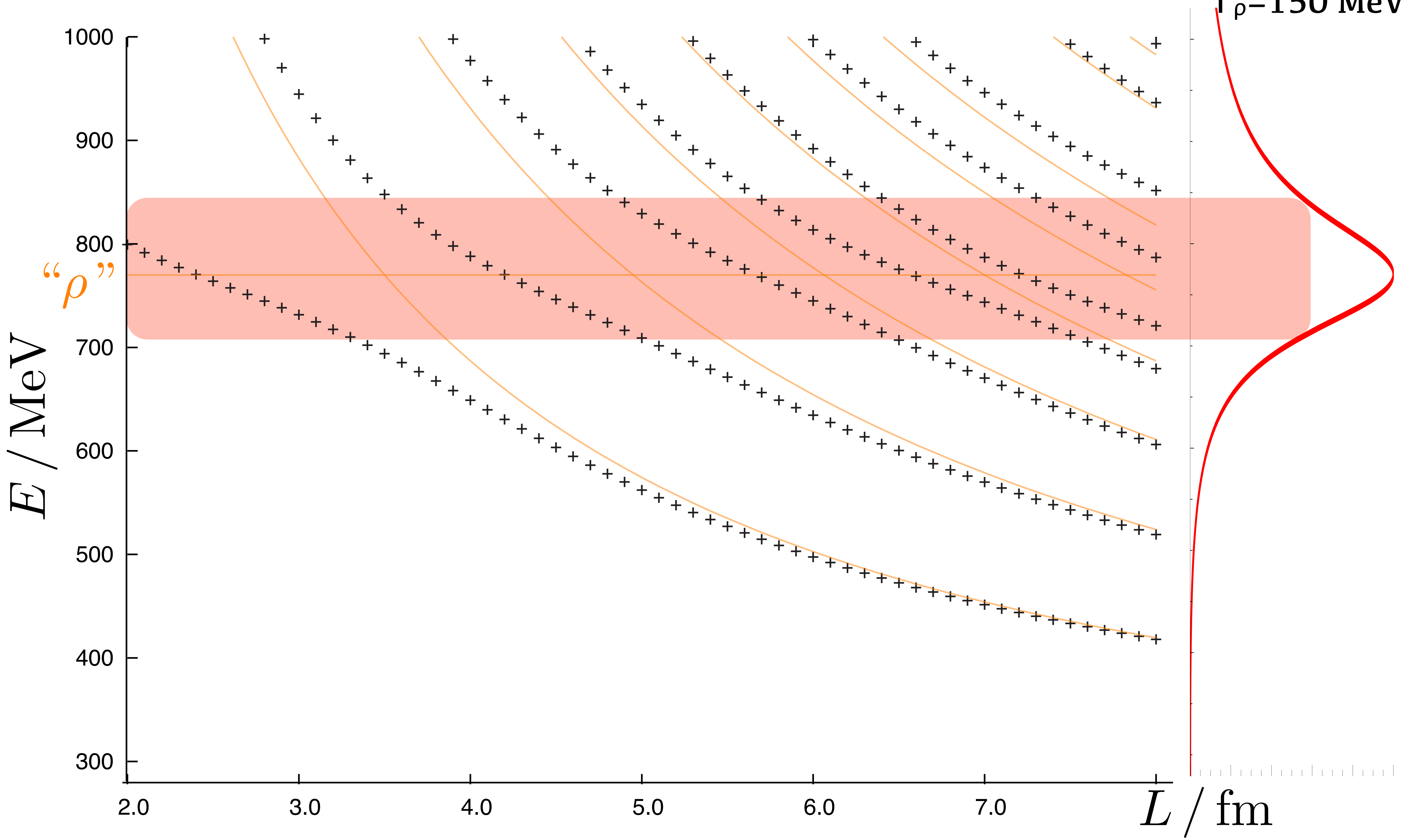
expected finite-volume spectrum given a ρ resonance



ππ isospin-1 scattering

expected finite-volume spectrum given a ρ resonance

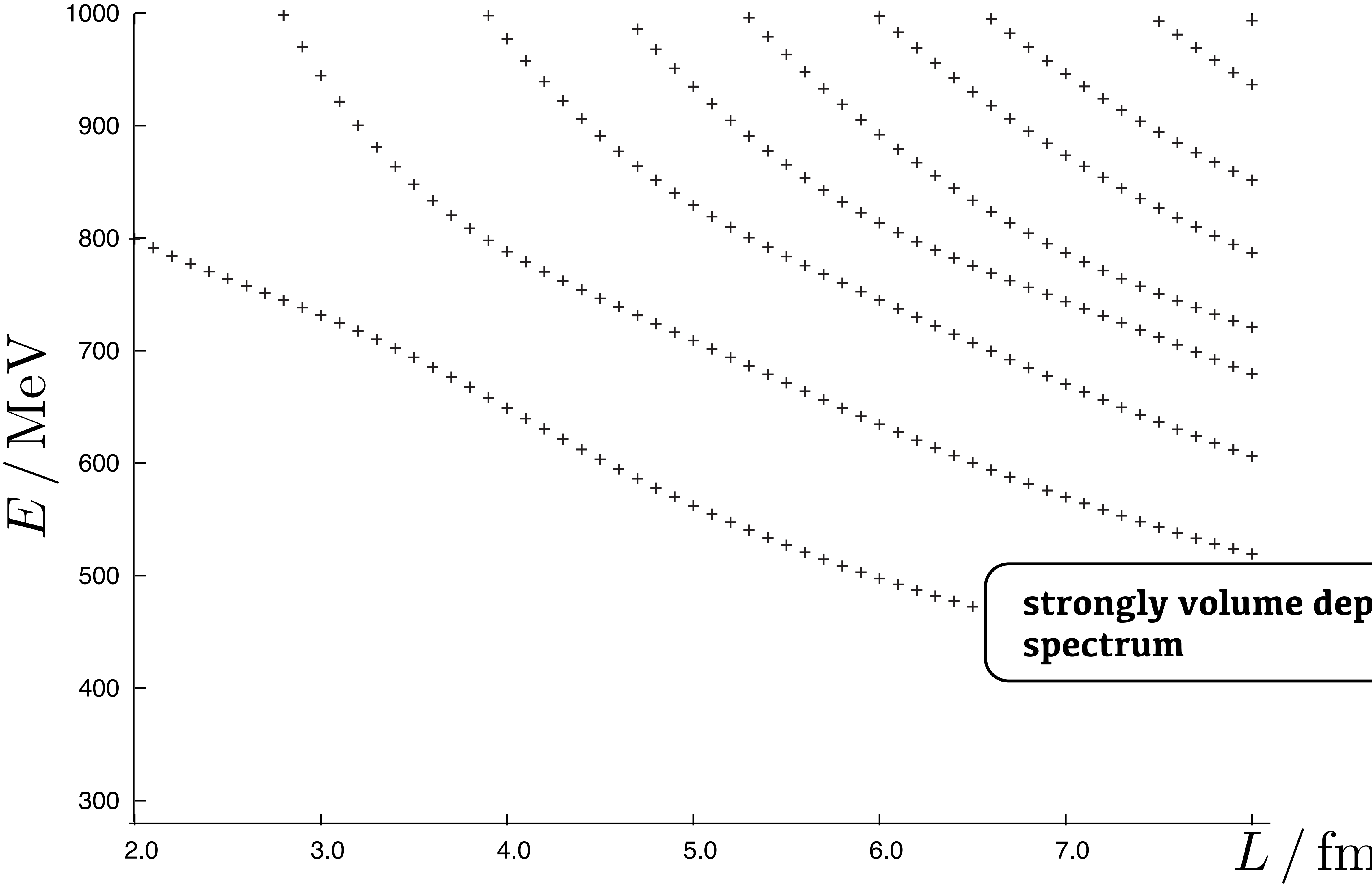
m_π=139 MeV
m_ρ=770 MeV
Γ_ρ=150 MeV



ππ isospin-1 scattering

expected finite-volume spectrum given a ρ resonance

m_π=139 MeV
m_ρ=770 MeV
Γ_ρ=150 MeV

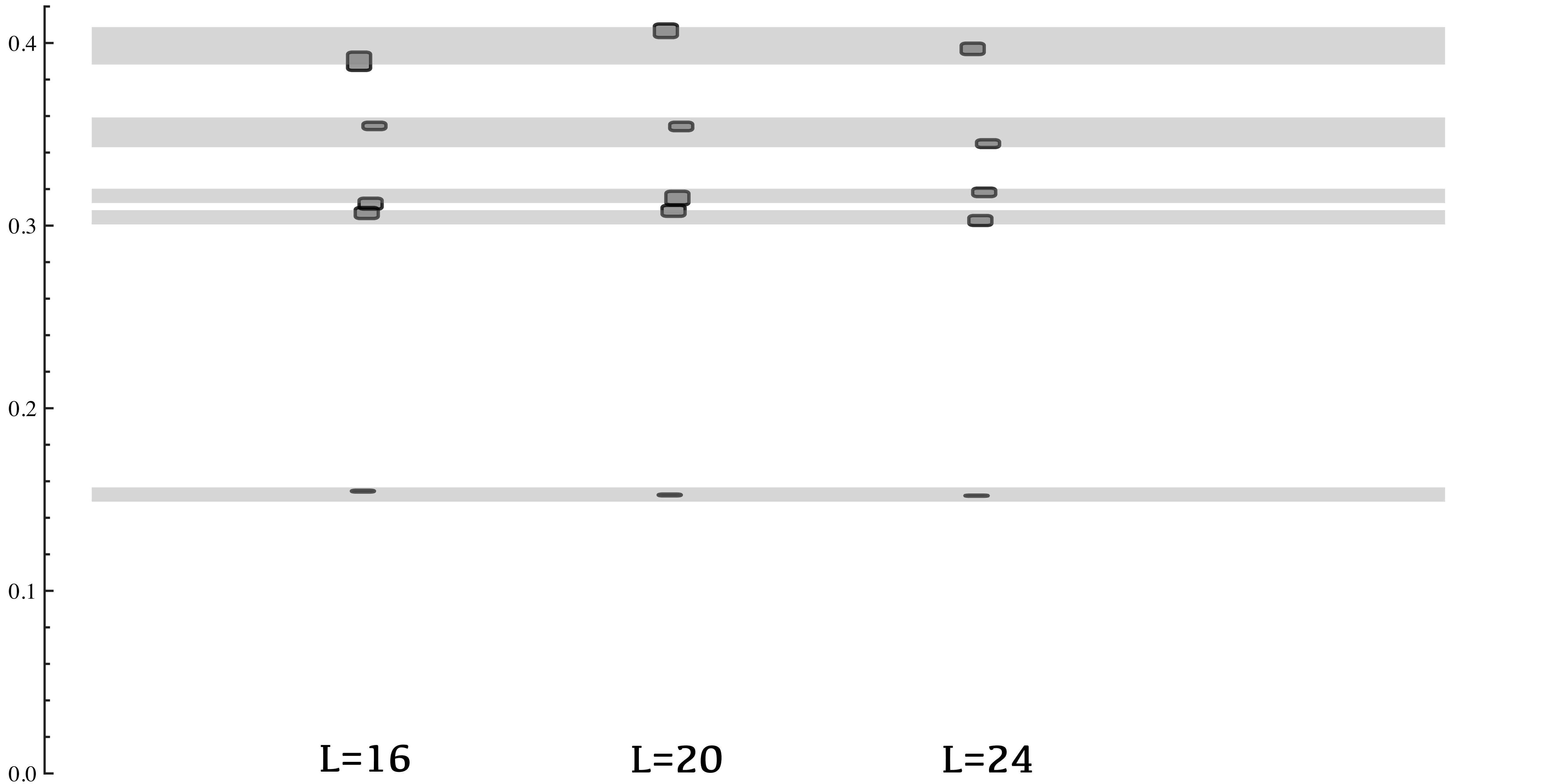


strongly volume dependent spectrum

the spectrum using 'local' operators - T_1^{--}

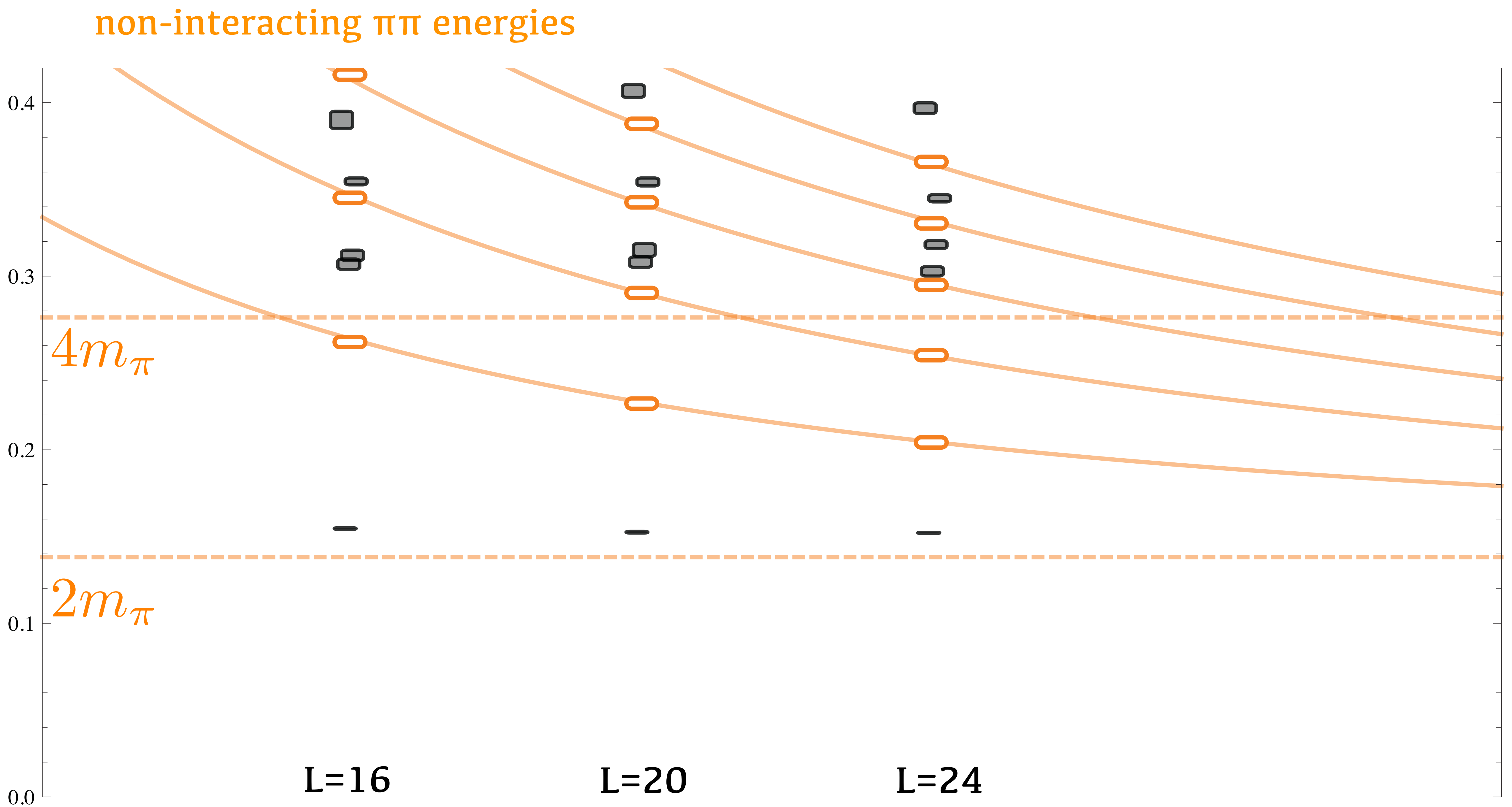
m_π~400 MeV

no systematic volume dependence observed in the spectrum



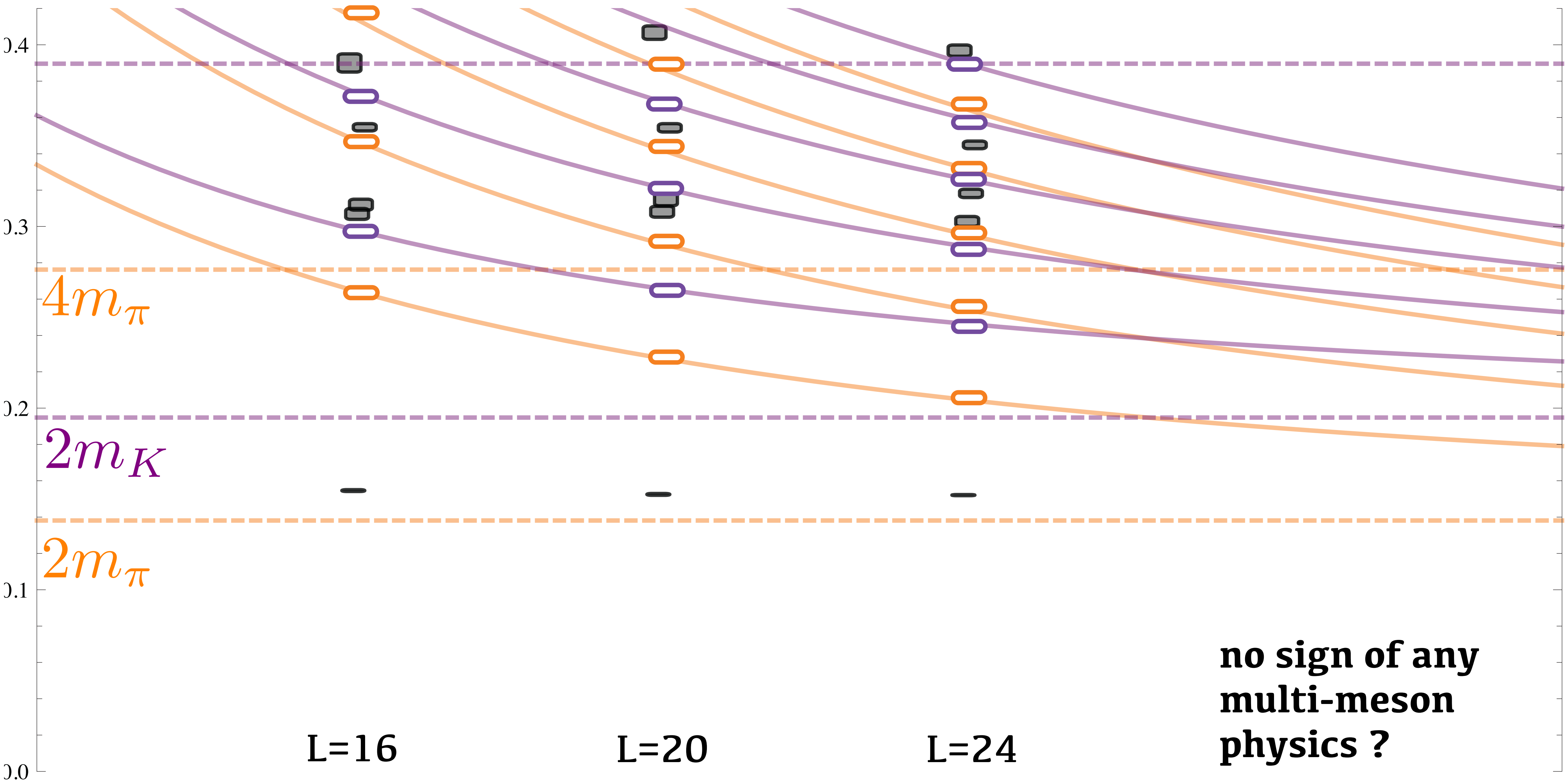
the spectrum using ‘local’ operators - T_1^{--}

m_π~400 MeV



the spectrum using ‘local’ operators - T_1^{--}

m_π~400 MeV



no sign of any multi-meson physics ?

the spectrum using 'local' operators

we suspect that this effect can be understood

➟ hypothesise that energy eigenstates are superpositions of

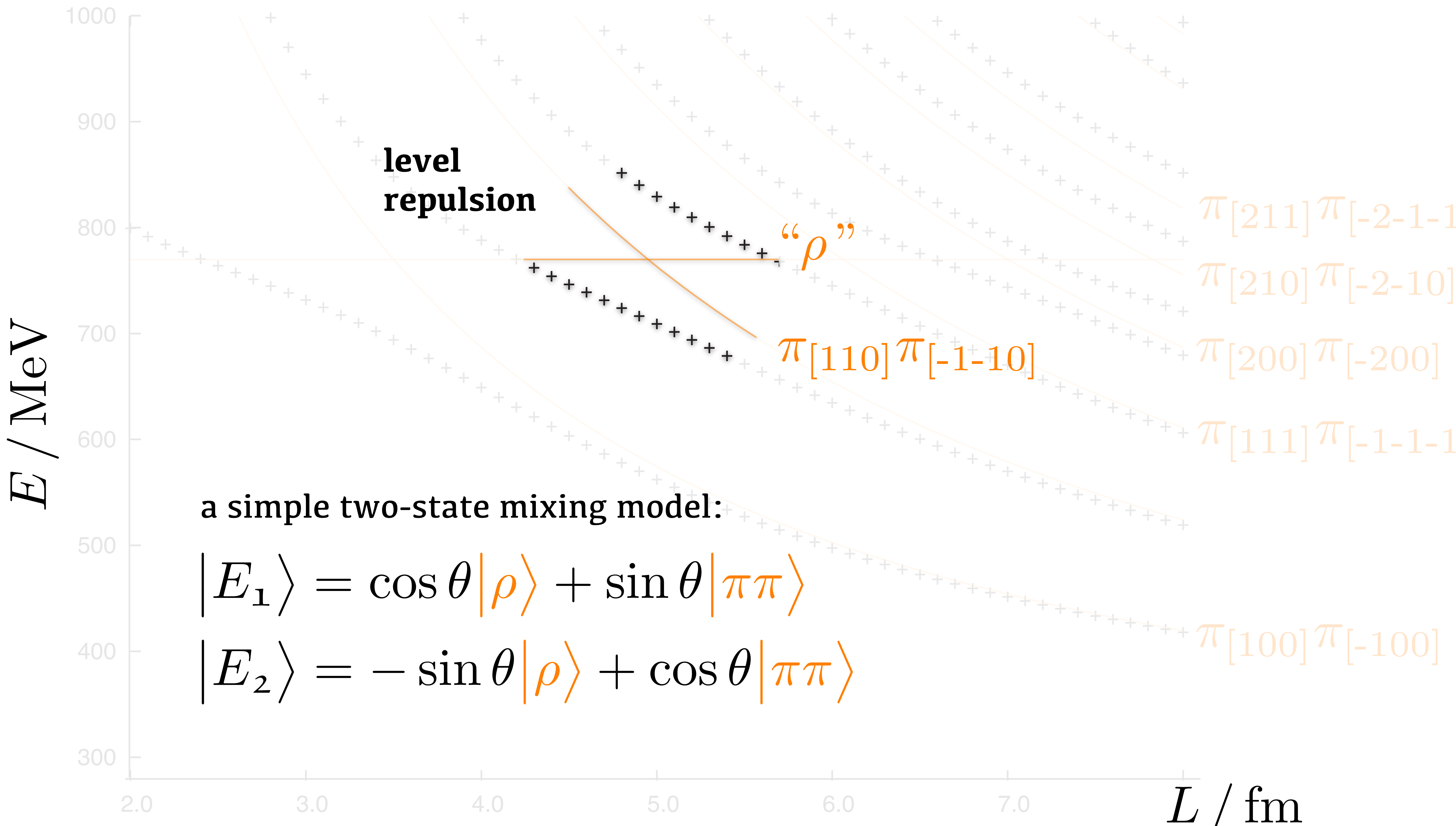
- ⋆ a $q\bar{q}$ state, call it $\left|\rho\right\rangle$
- ⋆ "non-interacting" $\left|\pi\pi\right\rangle$ basis states

➟ hypothesise that local $q\bar{q}$ operators have a suppressed overlap onto $\left|\pi\pi\right\rangle$

by at least a factor of $1/L^3$

ππ isospin-1 scattering

expected finite-volume spectrum given a ρ resonance



a simple two-state mixing model:

$$\big|E_1\big\rangle = \cos\theta\big|\rho\big\rangle + \sin\theta\big|\pi\pi\big\rangle$$

$$\big|E_2\big\rangle = -\sin\theta\big|\rho\big\rangle + \cos\theta\big|\pi\pi\big\rangle$$

the spectrum using 'local' operators

a simple two-state mixing model:

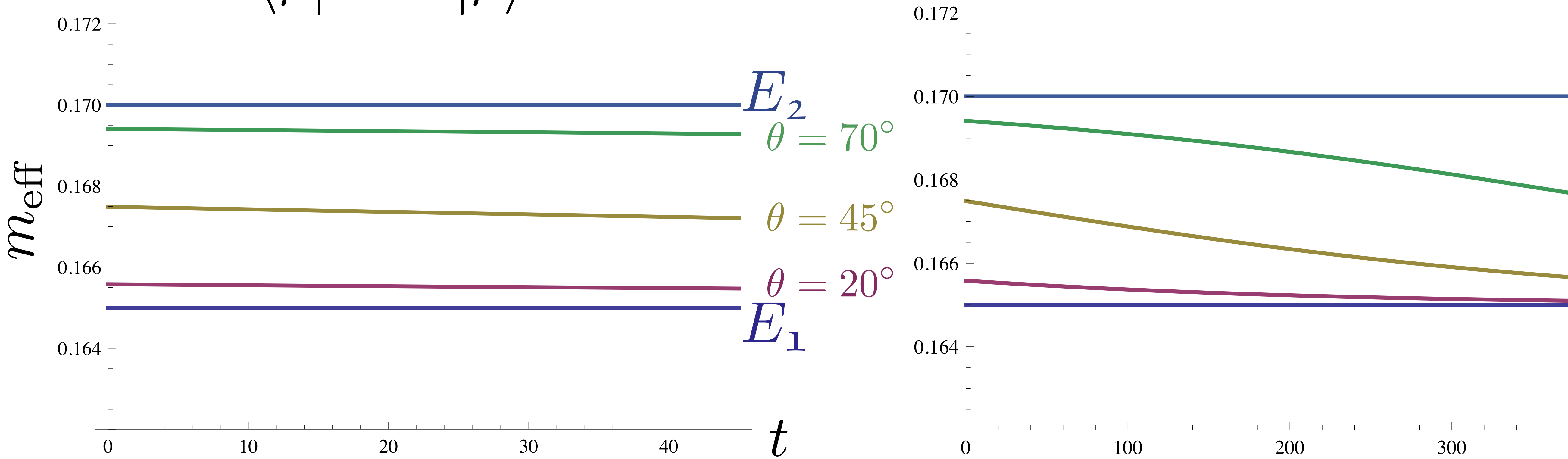
$$|E_1\rangle = \cos\theta\,|\rho\rangle + \sin\theta\,|\pi\pi\rangle$$
$$|E_2\rangle = -\sin\theta\,|\rho\rangle + \cos\theta\,|\pi\pi\rangle$$

if we only use operators which overlap well with $|\rho\rangle$ and not with $|\pi\pi\rangle$

then a variational solution won't be able to find the orthogonal combinations

the principal correlator will behave like

$$\langle\rho|e^{-Ht}|\rho\rangle = \cos^2\theta\, e^{-E_1 t} + \sin^2\theta\, e^{-E_2 t}$$



ππ isospin-1 scattering - lattice calculation

do it properly !

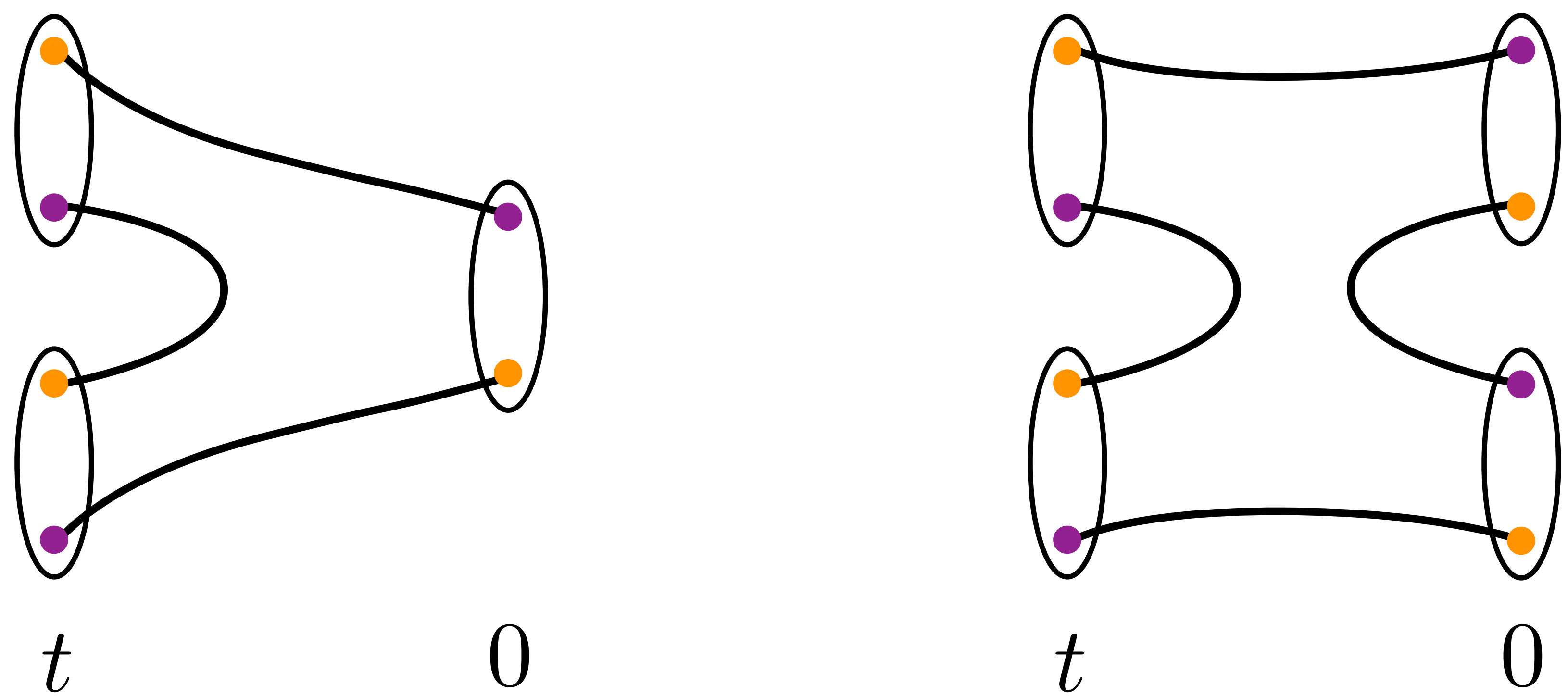
operator basis:

➟ usual big set of derivative-based fermion bilinears

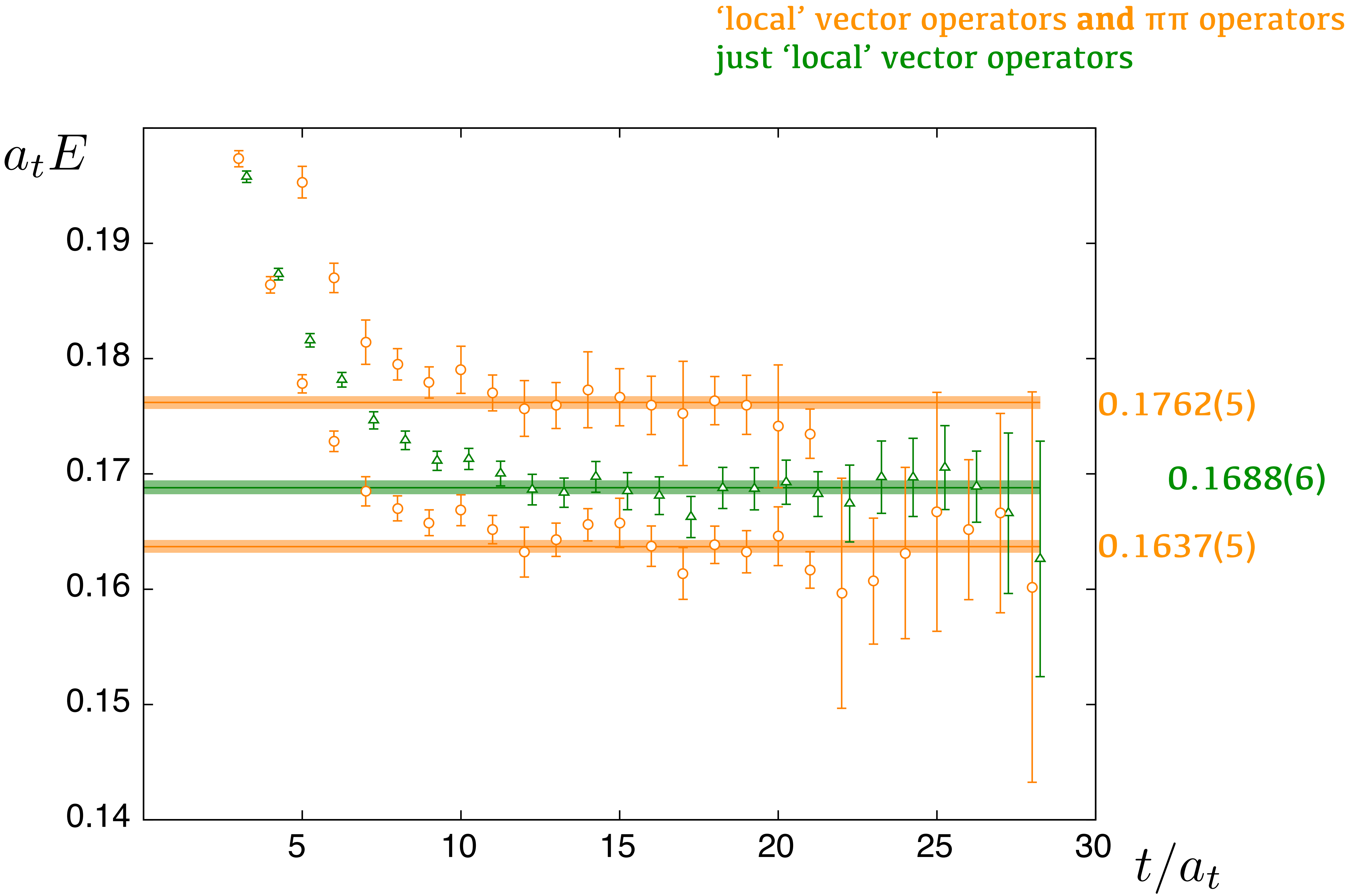
➟ ππ-like operators of definite relative momentum

need quark annihilation diagrams

e.g.

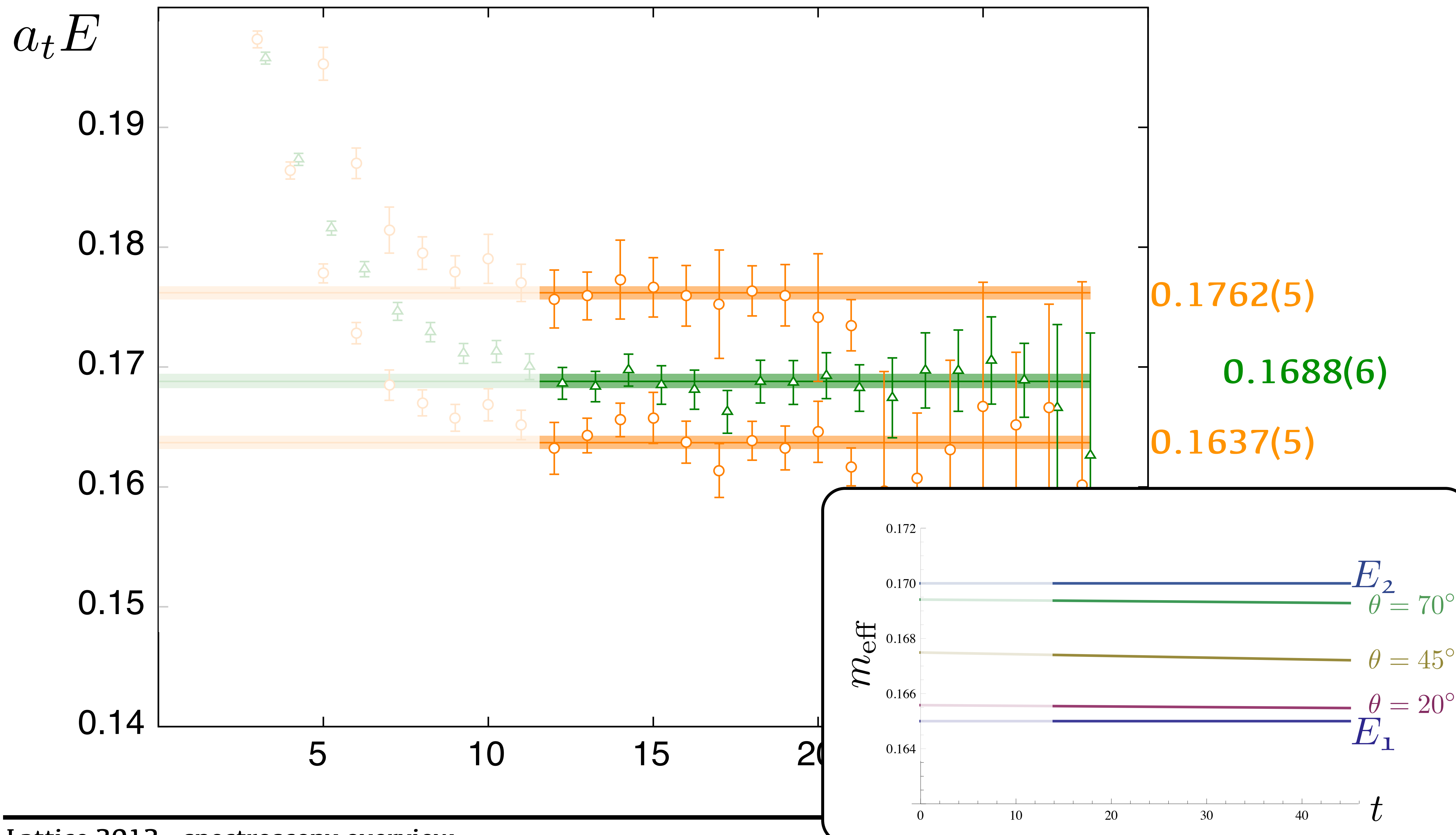


[100] A1 - with & without ππ operators

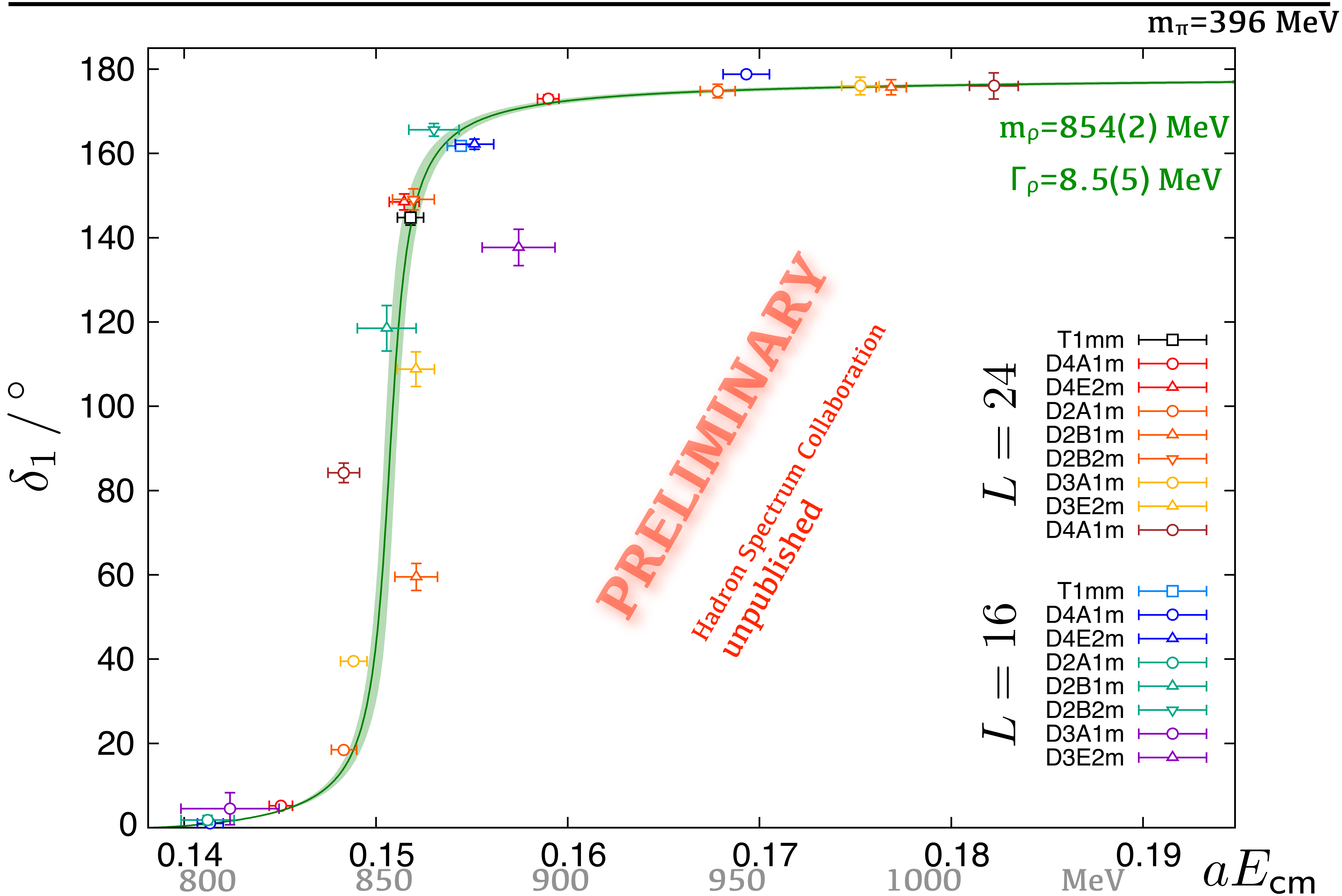


[100] A1 - with & without ππ operators

‘local’ vector operators **and** ππ operators
just ‘local’ vector operators

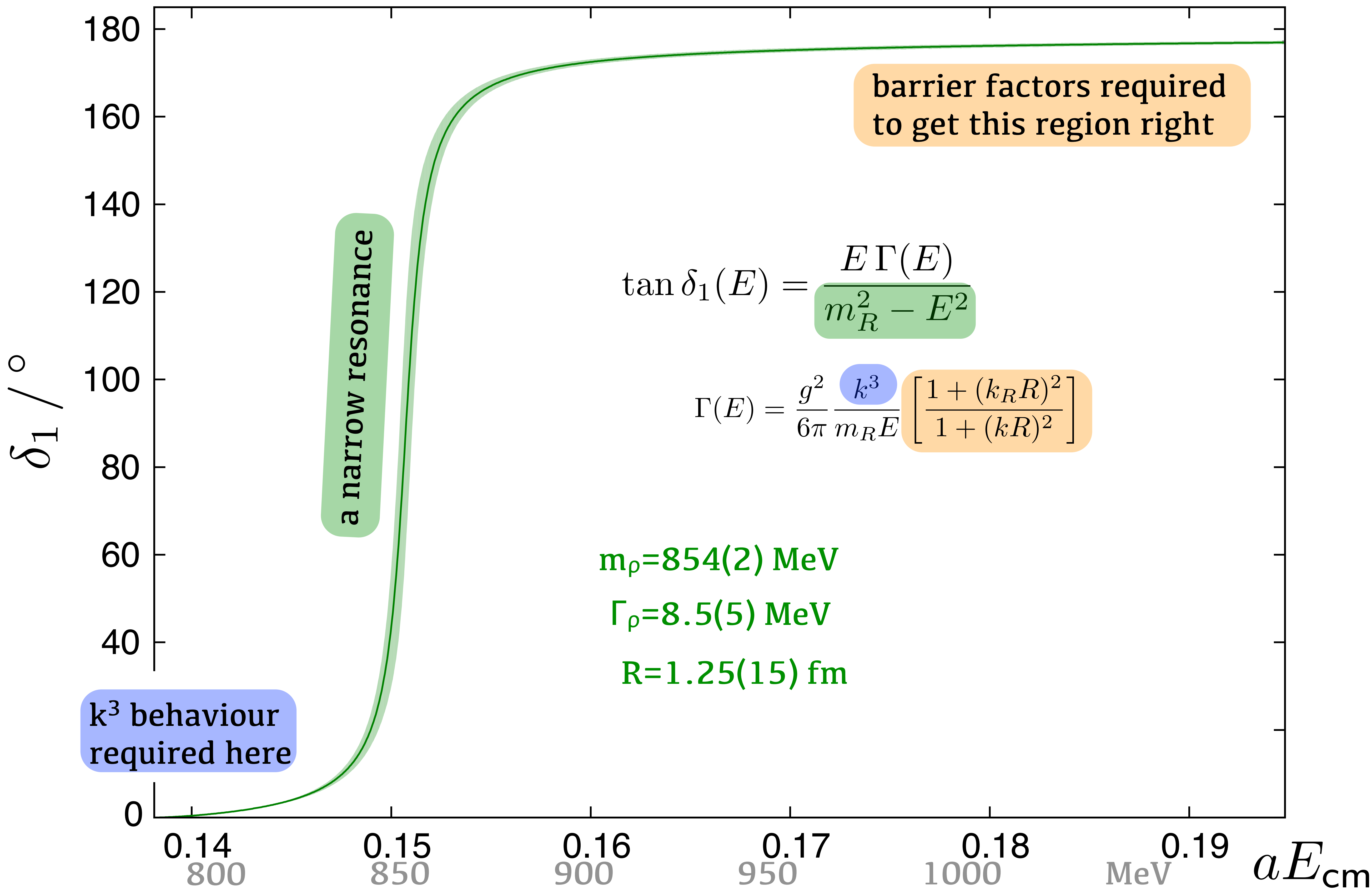


ππ isospin-1 scattering - a lattice calculation



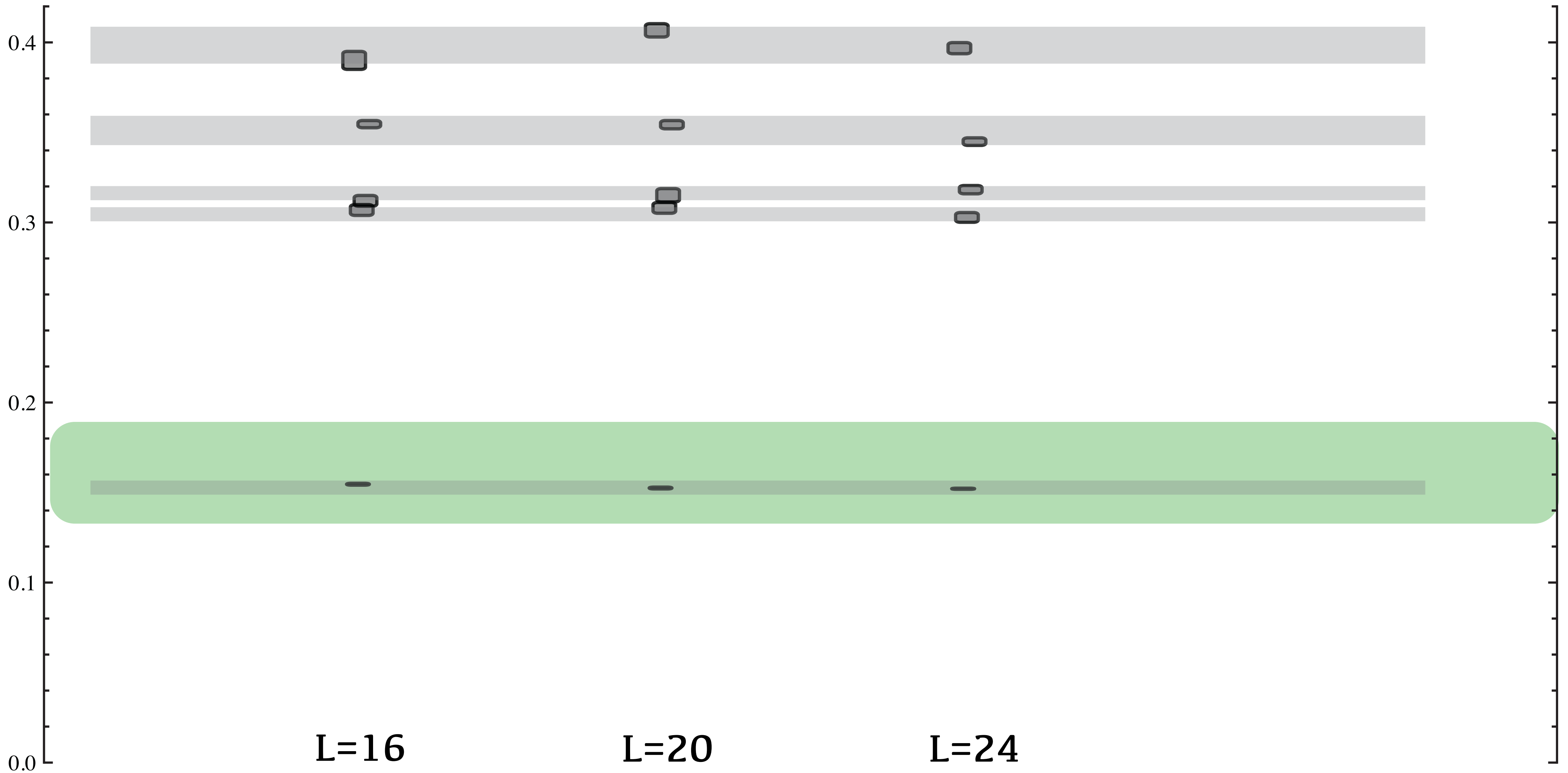
ππ isospin-1 scattering - a lattice calculation

m_π=396 MeV



the spectrum using 'local' operators - T_1^{--}

m_π~400 MeV



ππ isospin-1 scattering - pion mass dependence

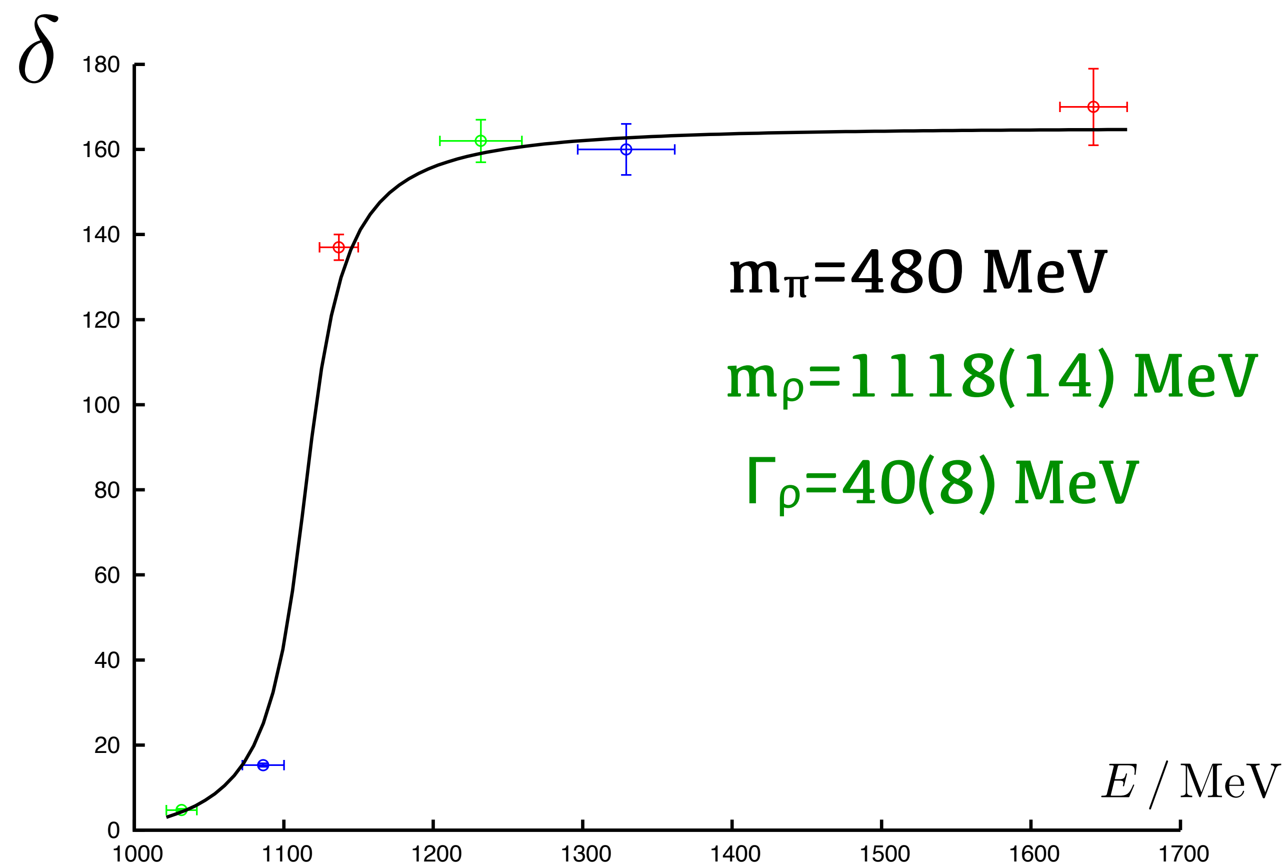
European Twisted Mass Collaboration
PRD83 (2011) 094505

➟ two-flavour calculation (no strange quarks)

➟ four different quark masses

➟ setting the lattice scale ?

➟ computed in relatively few frames

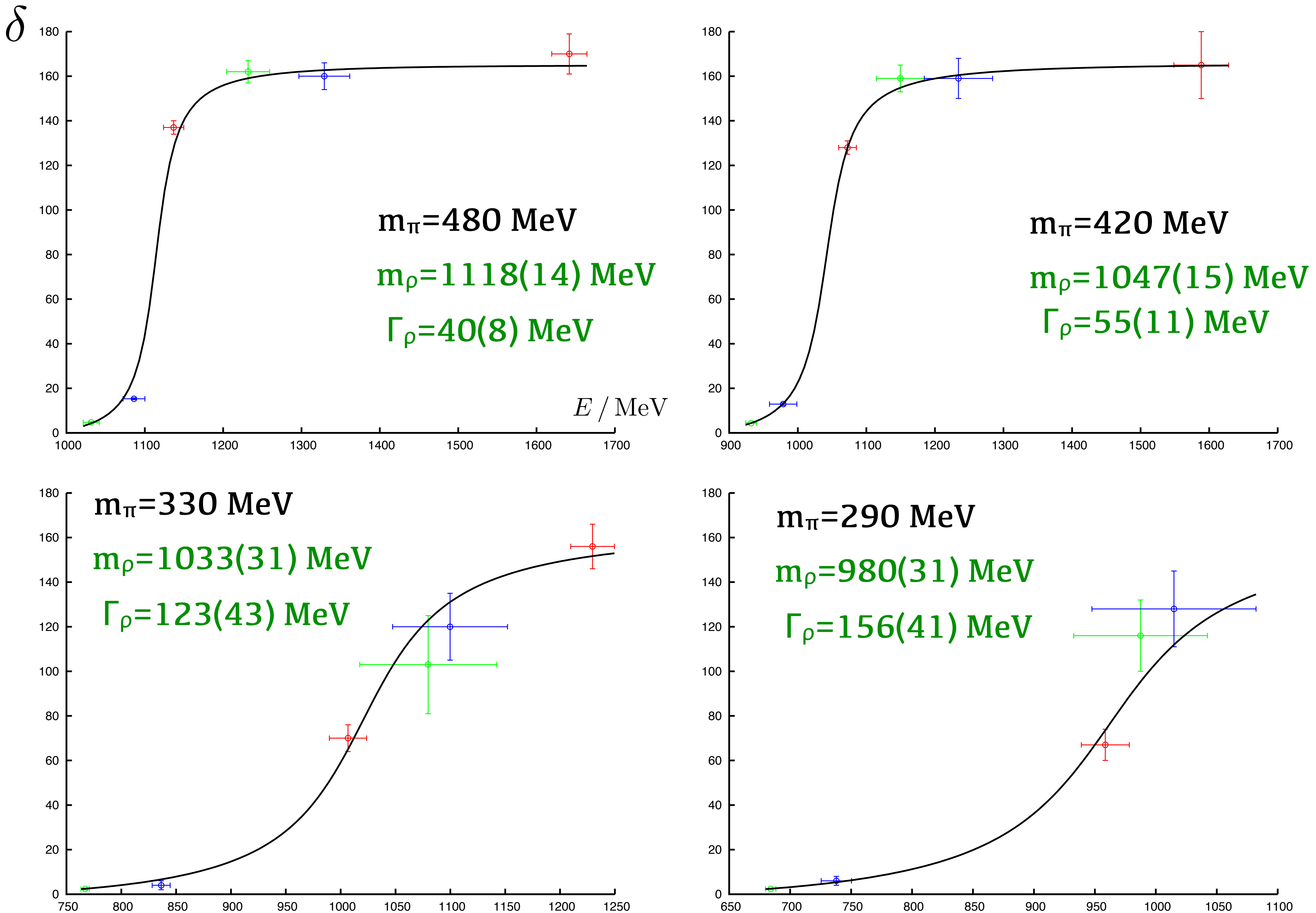


m_π=480 MeV

m_ρ=1118(14) MeV

Γ_ρ=40(8) MeV

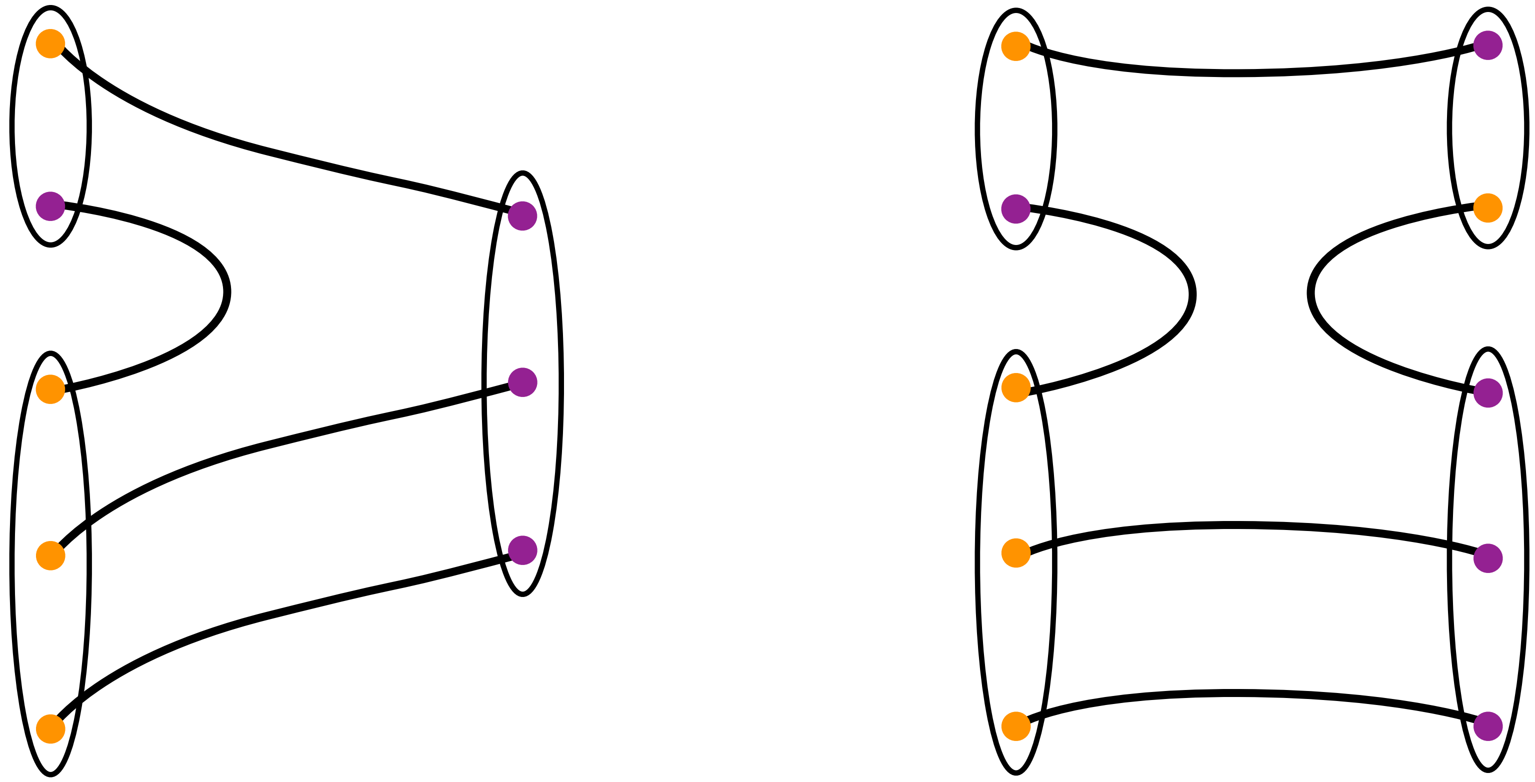
ππ isospin-1 scattering - pion mass dependence



forthcoming resonance calculations

computationally challenging:

➟ pion-nucleon elastic scattering (Δ resonance)



➟ lots of quark lines
➟ lots of matrix multiplication

... computer time

forthcoming resonance calculations

computationally challenging:

➟ pion-nucleon elastic scattering (Δ resonance)

requires untested formalism:

➟ meson-meson **inelastic** scattering (e.g a_0 in πη-K$\bar{K}$)

e.g. 2-channel inelastic scattering

$$\begin{bmatrix} \frac{4\pi}{k_1}\frac{\eta e^{2i\delta_1}-1}{2i} & \frac{4\pi}{\sqrt{k_1 k_2}}\frac{\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)}}{2} \\ \frac{4\pi}{\sqrt{k_1 k_2}}\frac{\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)}}{2} & \frac{4\pi}{k_2}\frac{\eta e^{2i\delta_2}-1}{2i} \end{bmatrix}$$

three real numbers at each scattering energy

finite-volume formalism

$$E_{\mathfrak{n}}(L) = f\big(\delta_1(E), \delta_2(E), \eta(E); L\big)$$

'measured'

three unknowns

forthcoming resonance calculations

computationally challenging:

- ➟ pion-nucleon elastic scattering (Δ resonance)

requires untested formalism:

- ➟ meson-meson **inelastic** scattering (e.g a_0 in $\pi\eta$-$K\bar{K}$)
- ➟ three-meson decays, e.g. $\omega, a_1, a_2 \ldots \rightarrow \pi\pi\pi$

all the complications of building unitary, analytic scattering amplitudes present for experiment !