Dynamical Model Analysis of Hadron Resonances (I)

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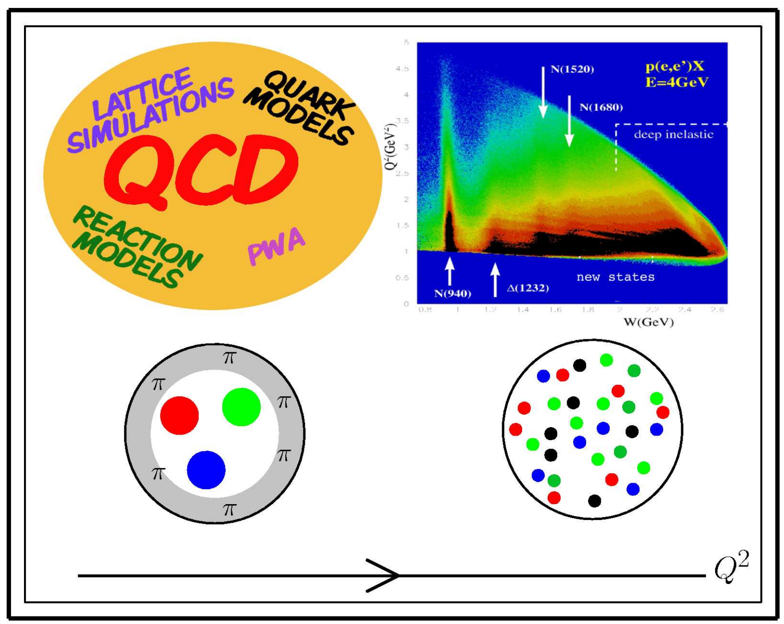


Lecture I :

- Introduction
- Theoretical Formulations of Dynamical Models

Lecture II :

- Dynamical Model for the Δ (1232) resonance
- Dynamical Coupled-channel Model for Nucleon Resonances (N^*)

Lecture III:

- Extractions and Interpretations of Hadron Resonances

Lecture IV:

- Dynamical coupled-channel Models for heavy/exotic mesons with 3-mesons decays
- Summary and future developments

Introduction

Why do we need dynamical model analysis?

Experimental fact :

Excited hadrons are unstable and coupled with continuum few-particle states to form resonances

$\longrightarrow$

Nucleon resonances contain information on

- Structure of excited nucleon states
- Meson-nucleon Interactions

$\longrightarrow$

The extracted parameters of resonances are due to

- Excitation of quark sub-structure of hadrons
- Reaction mechanisms of hadron-hadron reactions

$\longrightarrow$

Objective of dynamical model analysis :

Separate the contributions from structure and reaction mechanisms

$\longrightarrow$

Provide interpretations of the extracted resonance parameters in terms of hadron structure models and reaction mechanisms

What are the essential ingredients of dynamical model analysis?

- Unitarity Condition

  $\longrightarrow$

  crucial in extracting the spectrum and structure of N^* and (M^*)

- The formulation must be consistent with the well established non-perturbative physics of QCD :

  1. Confinement
  2. Dynamical Chiral Symmetry Breaking

  $\longrightarrow$

  crucial in interpreting the hadron resonance parameters within QCD .

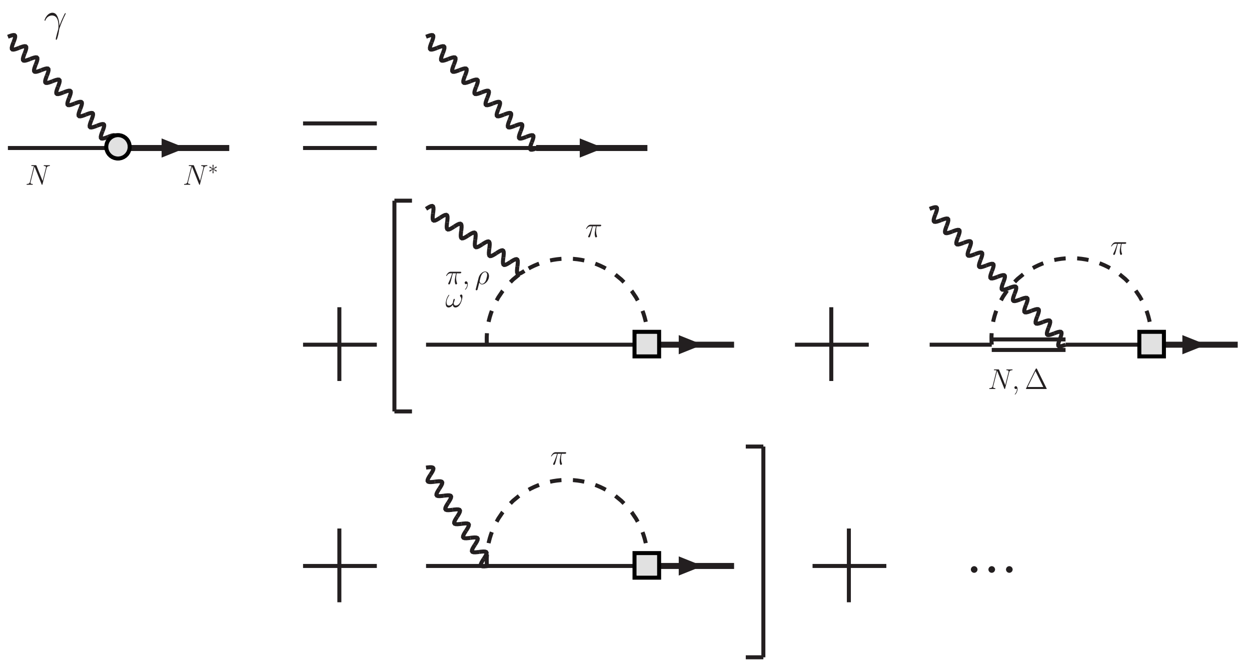
$\longrightarrow$

Most of Dynamical Reaction Models are based on the assumption:

Baryon is made of a confined quark-core and meson cloud

Example:

Argonne-Osaka approach (will be detailed later) :



Formulations of Dynamical Models

Starting point :

Relativistic local quantum field theory for mesons and baryons

$\longrightarrow$

To illustrate, consider the pseudo-scalar Lagrangian density for πN interaction:

$$L(x) = L_0(x) + L_I(x)\,,$$

$$L_I(x) = \bar{\psi}_N(x)\Gamma_0\psi_N(x)\phi_\pi(x)\,.$$

$$\Gamma_0 = g\gamma_5$$

$\longrightarrow$

Equation for πN scattering :

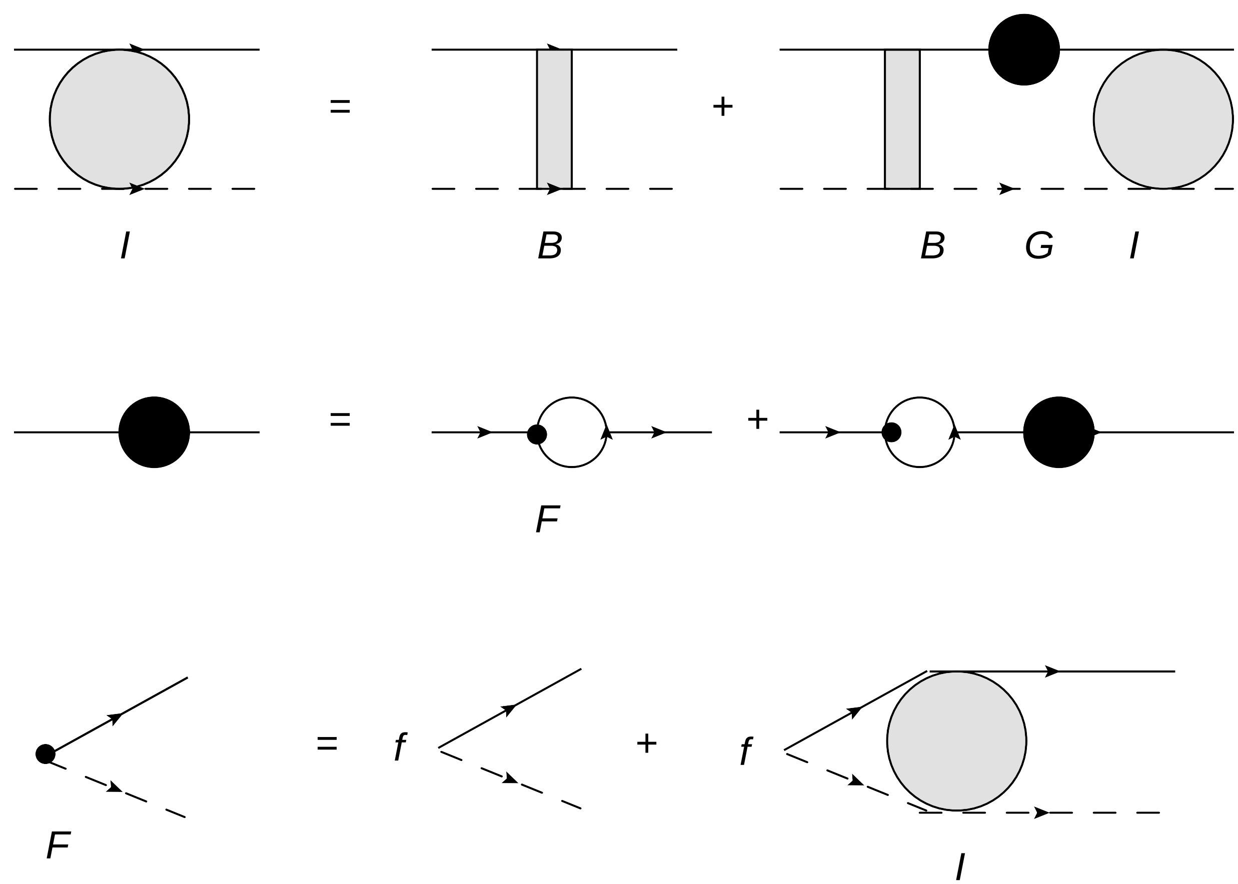
$$I(k',k;P) = B(k',k;P) + \int d^4k'' B(k',k'';P) G(k'';P) I(k'',k;P)$$
$$G(k;P) = \bar{S}_N(p) D_\pi(q)$$

where $p = \eta_N(s)P + k$, $k = \eta_\pi(s)P - k$, and $\eta_N(s) + \eta_\pi(s) = 1$.

$$\bar{S}_N(p) = \frac{1}{\not{p} - m_N^0 - \tilde{\Sigma}(p^2) + i\epsilon} \quad ; D_\pi(q) = \frac{1}{q^2 - m_\pi^2} \quad ;$$

$$\tilde{\Sigma}(p^2) = \int d^4\Gamma_0 G(k;P)\tilde{\Gamma}(k;p)$$
$$\tilde{\Gamma}(k;p) = \Gamma_0 + \int d^4k'\Gamma_0 G(k';P) I(k',k;P)$$

$B(k',k;P)$: sum of all irreducibale Feynman diagrams of $\pi N \rightarrow \pi N$



=
+
I
B
B
G
I
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Since

B = sum of infinite number of irreducible Feynman diagrams

$\longrightarrow$

The πN scattering problem can not be solved exactly !

$\longrightarrow$

Introduce approximations to obtain managable scattering equation for amplitude T which satisfy unitarity condition

$$S^\dagger S = 1$$

$$S = 1 + 2iT$$

$$Im(T) = -2|T|^2$$

This Lecture:

Explain two most commonly used approaches:

- Models derived from Bethe-Salpeter Equation
- Models based on the unitary tranformation method

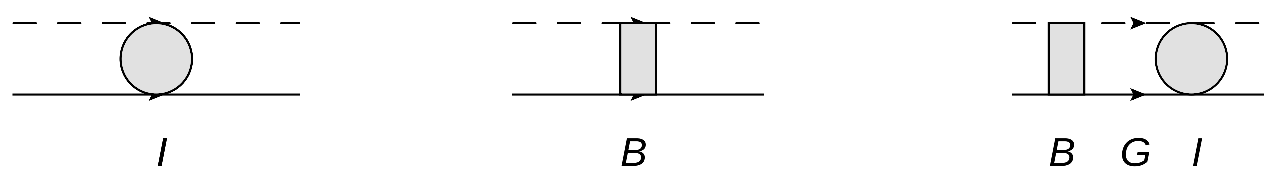
Models based on the Bethe-Salpeter Equation

Ladder Bethe-Salpeter Equation for πN scattering:

$$I(k',k;P) = B(k',k;P) + \int d^4k^{''} B(k',k^{''};P) G(k^{''};P) I(k^{''},k;P)$$

$$G(k;P) = S_N(p) D_\pi(q) \;\; ; \;\; B(k,k';P) = \text{tree-diagrams}$$

$$S_N(p) = \frac{1}{i\not{p} - m_N^0 - +i\epsilon} \;\; ; D_\pi(q) = \frac{1}{q^2 - m_\pi^2} \;\; ;$$



Bethe−Salpter Equation

Complications (due to pinched singularities) of solving Ladder Bethe-Salpeter equation can be found in A.D. Lahiff and I.R. Afnan, Phys. Rev. C60, 024608 (1999); Phys. Rev. C66, 044001 (2002)

A. Factorized on-shell approximation (k_0: on-shell momentum)

$$I(k_0,k_0;P) = B(k_0,k_0;P) + \int d^4k^{''} B(k_0,k^{''};P)G(k^{''};P)I(k^{''},k_0;P)$$
$$\rightarrow B(k_0,k_0;P) + \int d^4k^{''} B(k_0,k_0;P)G(k^{''};P)I(k_0,k_0;P)$$

$\longrightarrow$

Unitary Chiral model

$$I(k_0,k_0;P) = [1 - [\int d^4k^{''} G(k^{''};P)]_{regularized}]^{-1} B(k_0,k_0;P)$$

$regularized$: dimensional regularization or setting $\int_0^\infty dk \rightarrow \int_0^\Lambda dk$

$B(k_0,k_0;P)$ = contact terms of Chiral Lagrangians

B. Three-dimensional Reduction

Replace propagator $G(k;P)$ by

$$G_0(k;P) = \frac{1}{(2\pi)^3}\int \frac{ds'}{s-s'+i\epsilon} f(s,s')[\alpha(s,s')\not{P}+\not{k}+m_N]$$
$$\times\delta^+([\eta_N(s')P'+k]^2-m_N^2)\delta^+([\eta_\pi(s')P'-k]^2-m_\pi^2)$$

where

$$s = P^2 \quad ; \quad P' = \sqrt{(s'/s)}P$$

with the conditions

$$f(s,s) = 1 \ ; \ \alpha(s,s)=1 \quad ; \ \eta_N(s)+\eta_\pi(s)=1$$

$\longrightarrow$

Give correct πN elastic cut of unitarity condition

$$Disc[G_0(k;P)] = \frac{-i}{(2\pi)^2}[\eta_N(s)\not{P}+\not{k}+m_N]$$
$$\times\delta^+([\eta_N(s)P'+k]^2-m_N^2)\delta^+([\eta_\pi(s)P'-k]^2-m_\pi^2)$$

Note :

$G_0(k;P) \propto \delta(k^0 - f(\vec{k},P))$, k^0 : time component

$\longrightarrow$

Bethe-Salpeter Equation with the replacement $G(k;P) \longrightarrow G_0(k;P)$ becomes a three-dimensional integral equation

$\longrightarrow$

The form of the resulting three-dimensional scattering equation depends on the choice of $f(s,s')$ and $\alpha(s,s')$.

Examples:

Blankenbeckler-Sugar(Bbs) : $\alpha(s,s') = \eta_N(s')\sqrt{\frac{s'}{s}}$; $f(s,s') = 1$

Kadyshevsky(Kady) : $\alpha(s,s') = \eta_N(s')\sqrt{\frac{s'}{s}}$; $f(s,s') = \frac{\sqrt{s}+\sqrt{s'}}{2\sqrt{s'}}$

In center of mass system $P = (\sqrt{s}, \vec{0})$ and $\vec{p} = -\vec{q} = \vec{k}$, One can show that the resulting three-dimensional Bethe-Salpeter euqation can be cast into

$$
\begin{aligned}
t_R(\vec{k}, \vec{k}\,'; \sqrt{s}) &= v(\vec{k}, \vec{k}\,'; \sqrt{s}) \\
&+ \int d\vec{k}\,'' v(\vec{k}, \vec{k}\,''; \sqrt{s}) g_R(\vec{k}\,''; \sqrt{s}) t(\vec{k}\,'', \vec{k}\,'; \sqrt{s})
\end{aligned}
$$

where

$$
\begin{aligned}
t_R(\vec{k}, \vec{k}\,'; \sqrt{s}) &= \int dk_0' \int dk_0 \delta(k_0)) I(k, k'; \sqrt{s}) \delta(k_0') \\
v(\vec{k}, \vec{k}\,'; \sqrt{s}) &= \int dk_0' \int dk_0 \delta(k_0) B(k, k'; \sqrt{s}) \delta(k_0')
\end{aligned}
$$

with

$$
\begin{aligned}
g_{Bbs}(\vec{k}\,''; \sqrt{s}) &= \frac{1}{(2\pi)^3} \frac{u(\vec{k})\bar{u}(\vec{k})}{\sqrt{s} - \sqrt{s_{\vec{k}}} + i\epsilon} \frac{2\sqrt{s_{\vec{k}}}}{\sqrt{s} + \sqrt{s_{\vec{k}}}} \frac{m_N}{E_N(\vec{k})} \frac{1}{2E_\pi(\vec{k})} \\
g_{Kady}(\vec{k}\,''; \sqrt{s}) &= \frac{1}{(2\pi)^3} \frac{u(\vec{k})\bar{u}(\vec{k})}{\sqrt{s} - \sqrt{s_{\vec{k}}} + i\epsilon} \frac{m_N}{E_N(\vec{k})} \frac{1}{2E_\pi(\vec{k})} \\
\sqrt{s_{\vec{k}}} &= E_N(\vec{k}) + E_\pi(\vec{k})
\end{aligned}
$$

Define

$$T_R(k.k';\sqrt{s}) = N_R(k,s)C(\vec{k})[\bar{u}(\vec{k})t_R(k,k';\sqrt{s})u(\vec{k'})]C(\vec{k}\,')N_R(k',s)$$
$$V_R(k.k';\sqrt{s}) = N_R(k,s)C(\vec{k})[\bar{u}(\vec{k})v(k,k';\sqrt{s})u(\vec{k'})]C(\vec{k}\,')N_R(k',s)$$

where

$$C(k) = \frac{1}{(2\pi)^{3/2}}\sqrt{\frac{m_N}{E_N(k)}}\frac{1}{\sqrt{2E_\pi(k)}}$$
$$N_{Bbs}(k,s) = \frac{2\sqrt{s_{\vec{k}}}}{\sqrt{s}+\sqrt{s_{\vec{k}}}}$$
$$N_{Kady}(k,s) = 1$$

We then have the following scattering equation

$$T_R(\vec{k},\vec{k}\,';\sqrt{s}) = V_R(\vec{k},\vec{k}\,';\sqrt{s}) + \int d\vec{k}\,'' V_R(\vec{k},\vec{k}\,'';\sqrt{s}) \frac{1}{\sqrt{s}-\sqrt{s_{\vec{k}}}+i\epsilon} T_R(\vec{k}\,'',\vec{k}\,';\sqrt{s})$$

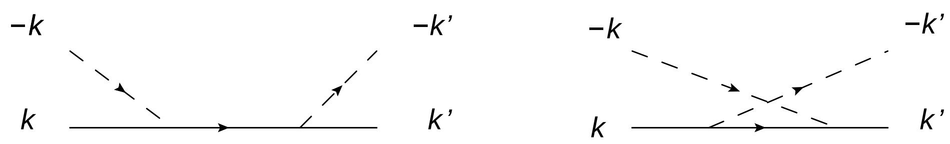
$$V_R(k.k';\sqrt{s}) = N_R(k,s)C(\vec{k})[v^s(k,k';\sqrt{s})+v^u(k,k';\sqrt{s})]C(\vec{k}\,')N_R(k',s)$$

where

$$v^s(k,k';\sqrt{s}) = \bar{u}(\vec{k})\frac{\sqrt{s}\gamma_0+m_N}{s-m_N^2}u(\vec{k}\,')$$

$$v^u(k,k';\sqrt{s}) = \bar{u}(\vec{k})\frac{(E_N(k_0)-E_\pi(k_0))\gamma_0+(\vec{k}+\vec{k}\,')\cdot\gamma+m_N}{(E_N(k_0)-E_\pi(k_0))^2-(\vec{k}+\vec{k}\,')^2-m_N^2}u(\vec{k}\,')$$

$$\sqrt{s} = E_N(k_0)+E_\pi(k_0)$$



In center of mass frame

Models based Unitary Transformation Method

To illustrate, consider pseudo-scalar Lagrangian density for πN interaction:

$$L(x) = L_0(x) + L_I(x)\,,$$

$$L_I(x) = \bar{\psi}_N(x)\Gamma_0\psi_N(x)\phi_\pi(x)\,.$$

$$\Gamma_0 = g\gamma_5$$

Canonical quantization : Interaction Hamiltonian density

$$H_I(x) = -L_I(x) = -\bar{\psi}_N(x)\Gamma_0\psi_N(x)\phi_\pi(x)$$

$$\psi_N(x) = \frac{1}{(2\pi)^{3/2}}\int d\vec{p}\sqrt{\frac{m_N}{E_N(p)}}[u(\vec{p})b_{\vec{p}}e^{-ip\cdot x} + v(\vec{p})d^\dagger_{\vec{p}}e^{ip\cdot x}]$$

$$\phi_\pi(x) = \frac{1}{(2\pi)^{3/2}}\int d\vec{k}\frac{1}{\sqrt{2E_\pi(k)}}[a_{\vec{k}}e^{-ik\cdot x} + a^\dagger_{\vec{k}}e^{ik\cdot x}]$$

with

$$\{b_{\vec{p}\,'}, b^\dagger_{\vec{p}}\} = \delta(\vec{p}\,' - \vec{p}) \quad ; \quad [a_{\vec{k}\,'}, a^\dagger_{\vec{k}}] = \delta(\vec{k}\,' - \vec{k})$$

The Hamiltonian (instant form) is then defined by

$$\begin{aligned} H &= \int H(\vec{x}, t=0) d\vec{x}. \\ &= H_0 + H_I \end{aligned}$$

with

$$\begin{aligned} H_0 &= \int d\vec{k}[E_N(k) b^\dagger_{\vec{k}} b_{\vec{k}} + E_\pi(k) a^\dagger_{\vec{k}} a_{\vec{k}}], \\ H_I &= \Gamma_{N\leftrightarrow \pi N} \\ &= \int d\vec{k}_1 d\vec{k}_2 d\vec{k} \delta(\vec{k} - \vec{k}_1 - \vec{k}_2)[\Gamma_{N,\pi N}(\vec{k}_1, \vec{k}_2) b^\dagger_{\vec{k}} b_{\vec{k}_1} a_{\vec{k}_2}) + (h.c)], \end{aligned}$$

where

$$\Gamma_{N,\pi N}(\vec{k}_1, \vec{k}_2) = \frac{1}{(2\pi)^{3/2}} \sqrt{\frac{m_N}{E_N(\vec{k}_1 + \vec{k}_2)}} \bar{u}(\vec{k}_1 + \vec{k}_2) \Gamma_0 u(\vec{k}_1) \sqrt{\frac{m_N}{E_N(k_1)}} \frac{1}{\sqrt{2E_\pi(k_2}}$$

Our task is to solve

$$H|\alpha> \quad = \quad E_\alpha|\alpha>$$

It is a many-body problem since the the $N \leftrightarrow \pi N$ interaction in H_I can generate infinite number of pions.

$\longrightarrow$

Derive equations for obtain solutions $|\bar{\alpha}>$ in a finte Fock space S_{Fock} spanned by few-particle states : $|N>$, $|\pi N>$, $|\pi\pi N>$.

Procedure:

Introduce a unitary transformation to define interactions in S_{Fock} :

$$H'|\bar{\alpha}>= E_\alpha|\bar{\alpha}>,$$

where

$$\begin{aligned} H' &= UHU^\dagger\,, \\ |\bar{\alpha}> &= U|\alpha> \end{aligned}$$

Decompose the interaction Hamiltonian into two parts

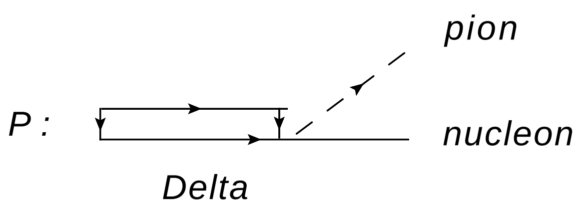
$$H_I \quad = \quad H_I^P + H_I^Q$$

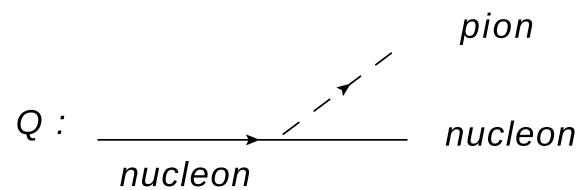
- H_I^P : processes can take place in free space

  $a \longrightarrow b + c$ with $m_a \geq m_b + m_c$

- H_I^Q : processes can not take place in free space

  $a \longrightarrow b + c$ with $m_a < m_b + m_c$





Defining $U = exp(-iS) = 1 - iS + ...$ (S is an hermitian operator)

$$\begin{aligned} H' &= UHU^\dagger \\ &= U(H_0 + H_I^P + H_I^Q)U^\dagger \\ &= H_0 + H_I^P + H_I^Q + [H_0, iS\,] + [H_I, iS\,] + \frac{1}{2!}\Big[[H_0, iS\,], iS\Big] + \cdots. \end{aligned}$$

The method of Kobayashi, Sato, and Ohtsubo (KSO) is :

- Choose S to eliminate H_I^Q

$$H_I^Q + [H_0, iS\,] = 0$$

- The transformed Hamiltonian then becomes

$$H' = H_0 + H_I'\,,$$

with

$$H_I' = H_I^P + [H_I^P, iS\,] + \frac{1}{2}[H_I^Q, iS\,] + \text{higher order terms}\,.$$

To find S , define the few-particle states of the free Hamiltonian

$$H_0|N >= m_N|N > ,$$
$$H_0|\vec{k},\vec{p} >= (E_\pi(k) + E_N(p))|\vec{k},\vec{p} > ,$$
$$H_0|\vec{k}_1,\vec{k}_2,\vec{p} >= ((E_\pi(k_1) + E_\pi(k_2) + E_N(p))|\vec{k}_1,\vec{k}_2,\vec{p} >$$
$$\cdots .$$

Evaluate the matrix elements

$$< \alpha|H_I^Q + [H_0, iS]|\beta >= 0$$

$$|\alpha >, \ |\beta >= |N >, \quad |\vec{k},\vec{p} >, \quad |\vec{k}_1,\vec{k}_2,\vec{p} >$$

$\longrightarrow$

Matirx elements of S in $S_{Fock} = N \oplus \pi N \oplus \pi\pi N$ are determined by the matrix elements of vertex function $\Gamma_{N,\pi N}(\vec{p},\vec{k})$ of the original Hamiltonian H

In the center of mass frame $\vec{p} = -\vec{k}$, we have

$$< \vec{k}, \vec{p}|(iS)|N >= -\Gamma_{N,\pi N}(k)\frac{\delta(\vec{k}+\vec{p})}{E_\pi(k)+E_N(p)-m_N} \,,$$

$$< N|(iS)|\vec{k}\,', \vec{p}\,' >= -\frac{\delta(\vec{k}\,'+\vec{p}\,')}{m_N - E_\pi(k') - E_N(p')}\Gamma^*_{N,\pi N}(\vec{k}\,') \,,$$

$$< \vec{k}_1, \vec{k}_2, \vec{p}|(iS)|\vec{k}', \vec{p}\,' >= \Gamma^*_{N,\pi N}(k_1)\frac{-\delta(\vec{k}' - \vec{k}_2)\delta(\vec{p}\,' - \vec{k}_1 - \vec{p})}{E_\pi(k_1) + E_N(p) - E_N(p')} + (1 \leftrightarrow 2) \,,$$

$$< \vec{k}, \vec{p}|(iS)|\vec{k}_1, \vec{k}_2, \vec{p} >= \Gamma_{N,\pi N}(k_2)\frac{-\delta(\vec{k} - \vec{k}_1)\delta(\vec{p}\,' - \vec{k}_2 - \vec{p})}{E_N(p) - E_\pi(k_2) - E_N(p)} + (1 \leftrightarrow 2)$$

The above allow us to evaluate the matrix elements of the transformed Hamiltonian

$$H_I' = H_I^P + [H_I^P, iS] + \frac{1}{2}[H_I^Q, iS] \tag{1}$$

Example :

Matrix element between πN states

$$
\begin{aligned}
< \vec{p}\vec{k}|\frac{1}{2}[H_I^Q, iS]||\vec{p}\,'\vec{k}\,' > \quad &= \quad \frac{1}{2}\sum_I [(< \vec{p}\vec{k}|\Gamma_{N\leftrightarrow\pi N}|I >< I|(iS)|\vec{p}\,'\vec{k}\,' > \\
&\quad - < \vec{p}\vec{k}|(iS)|I >< I|\Gamma_{N\leftrightarrow\pi N}|\vec{p}\,'\vec{k}\,' >]
\end{aligned}
$$

From the matrix elements of S, one can see that the intermediate states which can contrinute are

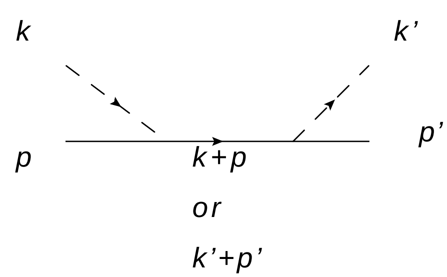
$$
|I >= |N > + |\pi(k_1)\pi(k_2)N(P_I) >
$$

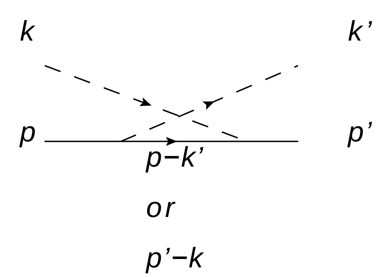
In the center of mass frame ($\vec{p} = -\vec{k}$ and $\vec{p}' = -\vec{k}'$), we then obtain

$$< -\vec{k}\vec{k}|\frac{1}{2}[H_I^Q, iS]|| - \vec{k}'\vec{k}' > \quad = \quad v^{(s)}(\vec{k}, \vec{k}') + v^{(u)}(\vec{k}, \vec{k}')\,.$$

where

$$\begin{aligned}
v^{(s)}(\vec{k}, \vec{k}') \quad &= \quad \frac{1}{2}\Gamma^*_{N,\pi N}(k)[\frac{1}{E_\pi(k) + E_N(k) - m_N} \\
&+ \frac{1}{E_\pi(k') + E_N(k') - m_N}]\Gamma^*_{N,\pi N}(k')\,, \\
v^{(u)}(\vec{k}, \vec{k}') \quad &= \quad \frac{1}{2}\Gamma^*_{N,\pi N}(k')[\frac{1}{E_N(k) - E_\pi(k') - E_N(\vec{k} + \vec{k}')} \\
&+ \frac{1}{E_N(k') - E_\pi(k) - E_N(\vec{k} + \vec{k}')}]\Gamma_{N,\pi N}(k)\,.
\end{aligned}$$





Extend the derivation to include anti-nucleon components, the transformed Hamiltonian is

$$H' = H_0 + H_I$$

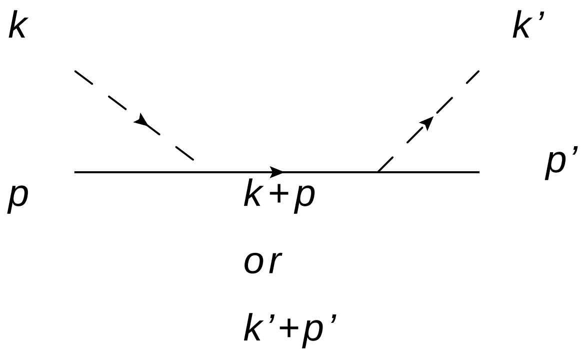
with

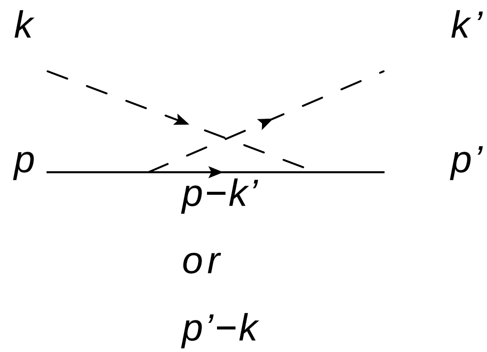
$$\begin{aligned} H_0 &= \int d\vec{k}[E_N(k)b^\dagger_{\vec{k}}b_{\vec{k}} + E_\pi(k)a^\dagger_{\vec{k}}a_{\vec{k}}], \\ H_I &= \int d\vec{k}d\vec{p}d\vec{k}\,'d\vec{p}\,'\delta(\vec{k}+\vec{p}-\vec{k}\,'-\vec{p}\,')b^\dagger_{\vec{p}}a^\dagger_{\vec{k}}b_{\vec{p}\,'}a_{\vec{k}\,'} \\ &\times[<\vec{k}\vec{p}|v^s|\vec{k}\,'\vec{p}\,'> + <\vec{k}\vec{p}|v^u|\vec{k}\,'\vec{p}\,'>] \end{aligned}$$

$$<\vec{k}\vec{p}|v^\alpha|\vec{k}\,'\vec{p}\,'> = \frac{1}{(2\pi)^3}\frac{1}{\sqrt{2E_\pi(k')}}\sqrt{\frac{m_N}{E_N(k')}}\frac{1}{\sqrt{2E_\pi(k)}}\sqrt{\frac{m_N}{E_N(k)}}\bar{u}_{\vec{p}}[I^\alpha]u_{\vec{p}},$$

$$I^s = \left(\frac{f_{\pi NN}}{m_\pi}\right)^2 \gamma^5 \frac{1}{2}[S_N(p+k) + S_N(p'+k')]\gamma^5$$

$$I^u = \left(\frac{f_{\pi NN}}{m_\pi}\right)^2 \gamma^5 \frac{1}{2}[S_N(p-k') + S_N(p'-k)]\gamma^5$$





All particles are on-mass-shell : $p_0 = E_N(\vec{p})$, $k_0 = E_\pi(\vec{k})$

Collisions can be off-energy-shell $p_0 + k_0 \neq p'_0 + k'_0$, the unitary transformation chooses the averaged ; i.e.

$$\frac{1}{2}[S_N(p+k) + S_N(p'+k')] = \frac{1}{2}\left[\frac{1}{\gamma_0(E_N(\vec{p}) + E_\pi(\vec{k})) - \vec{\gamma}\cdot(\vec{k}+\vec{p})} + \frac{1}{\gamma_0(E_N(\vec{p}\,') + E_\pi(\vec{k}\,')) - \vec{\gamma}\cdot(\vec{k}\,'+\vec{p}\,')}\right]$$

To calculate $\pi N \rightarrow \pi N$ scattering amplitude $T(E)$ from transformed Hamiltonian $H' = H_0 + H_I$, we solve

$$T(E) = H_I + H_I \frac{1}{E - H_0 + i\epsilon} T(E)$$

In the center of mass frame ($\vec{p} = -\vec{k}$, $\vec{p}\,' = -\vec{k}\,'$), the matrix element of $T(E)$ is then defined by

$$\begin{aligned} T(\vec{k}, \vec{k}\,'; E) &= v(\vec{k}, \vec{k}\,') \\ &+ \int d\vec{k}\,'' v(\vec{k}, \vec{k}\,'') \frac{1}{E - E_N(\vec{k}\,'') - E_\pi(\vec{k}\,'') + i\epsilon} T(\vec{k}\,'', \vec{k}\,'; E) \end{aligned}$$

where

$$v(\vec{k}, \vec{k}\,'; E) = < -\vec{k}\vec{k}|v^s| - \vec{k}\,'\vec{k}\,' > + < -\vec{k}\vec{k}|v^u| - \vec{k}\,'\vec{k}\,' >$$

Next Lecture:

The application of the Unitary transformation method to obtain a model for investigation nucleon resonances (N^*) in πN and γN reactions