Dynamical Model Analysis of Hadron Resonances (II-A)

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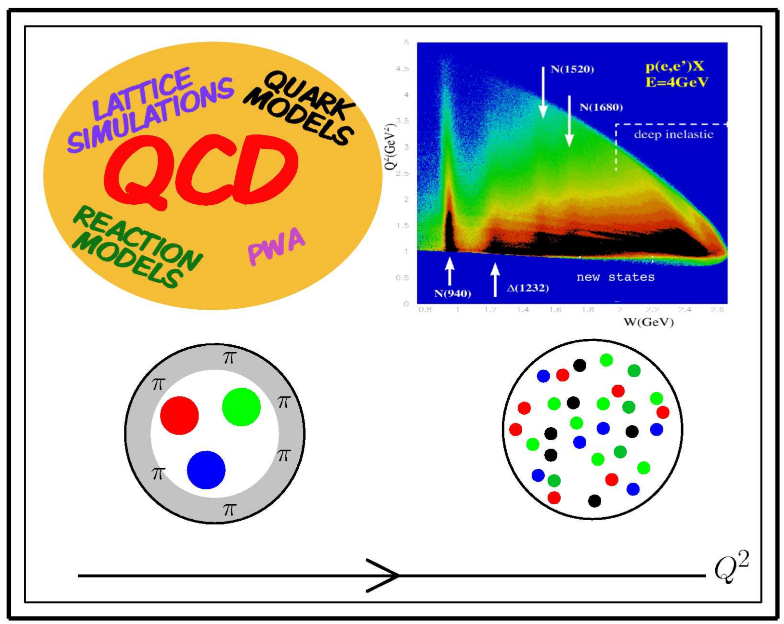


Lecture II-A :

- Dynamical Model for the Δ (1232) resonance
- Coupled-channel Model for N^* with $M_R \leq 2$ GeV

Dynamical model analysis of Δ (1232)

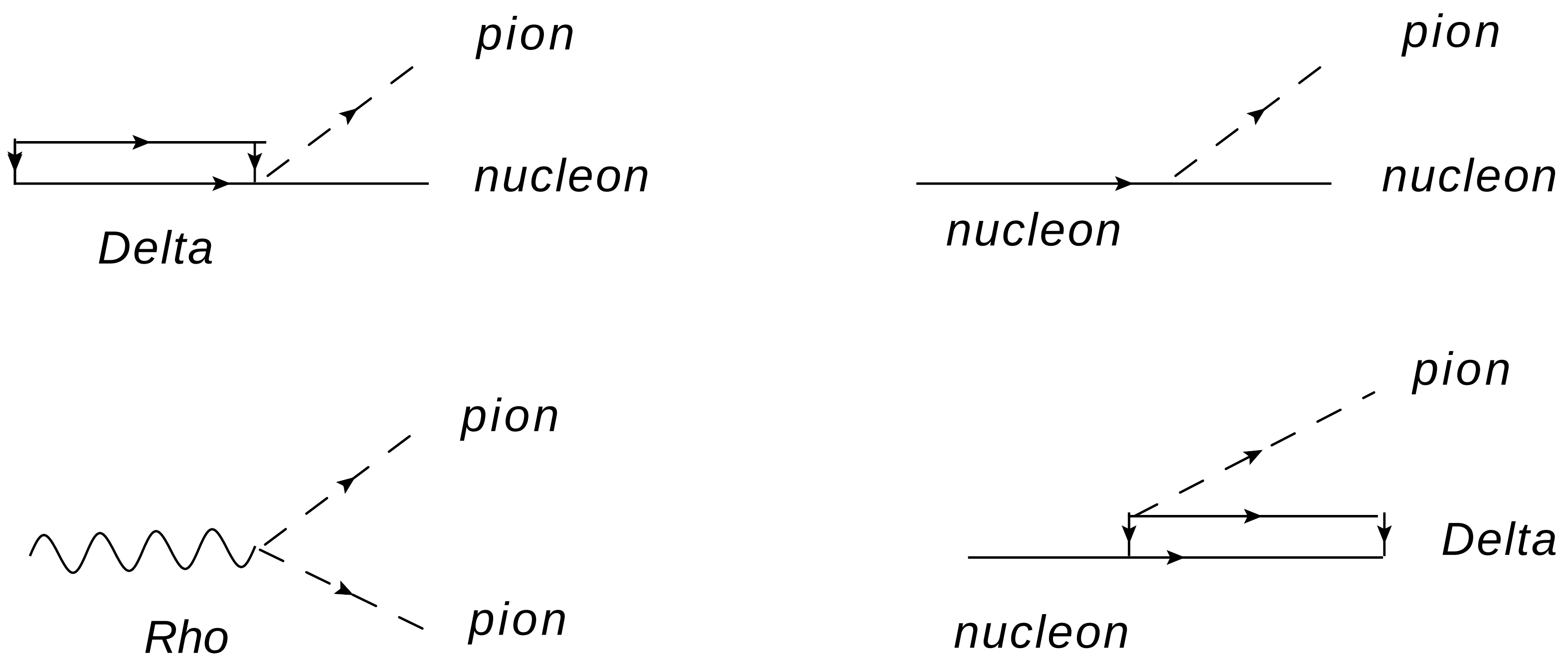
T. Sato, and T.-S. H. Lee, **Phys. Rev. C54,2660 (1996), C63, 055201(2001)**

Start with the interaction Lagrangian with N, π, Δ, ρ

$$L_I(x) = L_{\pi NN}(x) + L_{\pi N\Delta}(x) + L_{\rho NN}(x) + L_{\rho\pi\pi}(x)$$

$$\begin{aligned}
L_{\pi NN}(x) &= -\frac{f_{\pi NN}}{m_\pi}\bar{\psi}_N(x)\gamma_5\gamma_\mu\vec{\tau}\psi_N(x)\partial^\mu\cdot\vec{\phi}_\pi(x) \\
L_{\pi N\Delta} &= \frac{f_{\pi N\Delta}}{m_\pi}\bar{\psi}^\mu_\Delta(x)\vec{T}\psi_N(x)\cdot\partial_\mu\vec{\phi}_\pi(x) + [\text{h.c.}] \\
L_{\rho NN}(x) &= g_{\rho NN}\bar{\psi}_N(x)\frac{\vec{\tau}}{2}\cdot[\gamma_\mu\vec{\phi}^\mu_\rho(x) - \frac{\kappa_\rho}{2m_N}\sigma_{\mu\nu}\partial^\nu\vec{\phi}^\mu_\rho(x)]\psi_N(x) \\
L_{\rho\pi\pi}(x) &= g_{\rho\pi\pi}(\vec{\phi}_\pi\times\partial_\mu\vec{\phi}_\pi)\cdot\vec{\phi}^\mu_\rho
\end{aligned}$$

Interaction mechanisms in H^P (physical) and H^Q (unphysical)



P−mechanisms

Q−mechanisms

By appling the unitary transformation method up to the order of $g^2_{\pi NN}$, the model Hamiltonian for πN scattering is

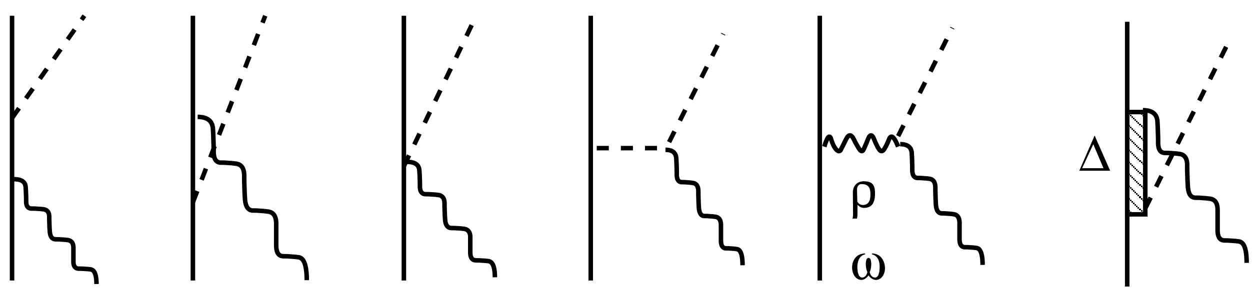
$$H = H_0 + \Gamma + \Gamma^\dagger + v$$

$$\Gamma = \Gamma_{\Delta,\pi N} + \Gamma_{\Delta,\gamma N}$$

$$v = v_{\pi N} + v_{\pi\gamma}$$

Γ : Excitation of the bare Δ (quark core)

v : Meson-exchange mechanisms :



In the center of mass frame $\vec{q} = -\vec{p} = \vec{k}$, the matrix element of $v_{\pi N}$ is

$$
\begin{aligned}
&< \vec{k}' i' | v_{\pi N} | \vec{k} i > \\
&= \sum_{\alpha} \frac{1}{(2\pi)^3} \frac{1}{\sqrt{2E_\pi(k')}} \sqrt{\frac{m_N}{E_N(k')}} [\bar{u}_{-\vec{k}'} [I_\alpha(\vec{k}' i', \vec{k} i) u_{-\vec{k}}] \frac{1}{\sqrt{2E_\pi(k)}} \sqrt{\frac{m_N}{E_N(k)}}
\end{aligned}
$$

i, i' : pion siospin component

Rules for calculating $I_\alpha(\vec{k}' i', \vec{k} i)$ are fixed by the unitary transformation :

1. All particles in the external legs of the Feynman amplitude $I_\alpha(\vec{k}' i', \vec{k} i)$ are on-mass-shell : $p_0 = E_N(\vec{p})$, $k_0 = E_\pi(\vec{p})$
2. Use averaged propagators to evaluate $I_\alpha(\vec{k}' i', \vec{k} i)$

$$I_{N_D}(\vec{k}'i',\vec{k}i) = (\frac{f_{\pi NN}}{m_\pi})^2 \tau_{i'}\gamma^5 \not{k}' \frac{1}{2}[S_N(p+k)+S_N(p'+k')]\tau_i\gamma^5 \not{k}$$

$$I_{N_E}(\vec{k}'i',\vec{k}i) = (\frac{f_{\pi NN}}{m_\pi})^2 \tau_{i}\gamma^5 \not{k} \frac{1}{2}[S_N(p-k')+S_N(p'-k)]\tau_{i'}\gamma^5 \not{k}',$$

$$I_{\rho}(\vec{k}'i',\vec{k}i) = \frac{ig_{\rho NN}g_{\rho\pi\pi}}{4}\Big\{[\gamma_\mu - \frac{\kappa_\rho}{2m_N}i\sigma_{\mu\nu}(p-p')^\nu]D_\rho^{\mu\lambda}(p-p')(k+k')_\lambda] + [(p-p')\leftrightarrow(k'-k)]\Big\}\epsilon_{ii'k}\tau_k$$

$$I_{\Delta_D}(\vec{k}'i',\vec{k}i) = (\frac{f_{\pi N\Delta}}{m_\pi})^2 T^\dagger_{i'}k'_\mu \frac{1}{2}[S_\Delta^{\mu\nu}(p+k)+S_\Delta^{\mu\nu}(p'+k') - S_\Delta^{(+)\mu\nu}(p+k) - S_\Delta^{(+)\mu\nu}(p'+k')]T_i k_\nu$$

$$I_{\Delta E}(\vec{k}'i',\vec{k}i) = (\frac{f_{\pi N\Delta}}{m_\pi})^2 T^\dagger_{i}k_\mu \frac{1}{2}[S_\Delta^{\mu\nu}(p-k')+S_\Delta^{\mu\nu}(p'-k)]T_{i'} k'_\nu$$

The propagators are

$$
\begin{aligned}
S_N(p) &= \frac{1}{\not p - m_N} \\
S_\Delta^{\mu\nu}(p) &= \frac{1}{3(\not p - m_\Delta)}\left[2(-g^{\mu\nu} + \frac{p^\mu p^\nu}{m_\Delta^2}) + \frac{\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu}{2} - \frac{p^\mu\gamma^\nu - p^\nu\gamma^\mu}{m_\Delta}\right] \\
D_\rho^{\mu\nu}(p) &= -\frac{g^{\mu\nu} - p^\mu p^\nu/m_\rho^2}{p^2 - m_\rho^2} \\
S_\Delta^{(+)\mu\nu}(p) &= \frac{m_\Delta}{E_\Delta(p)}\ \frac{\omega_p^\mu \bar{\omega}_p^\nu}{p_0 - E_\Delta(p)}
\end{aligned}
$$

The matrix element of the vertex interaction $\Gamma_{\Delta\leftrightarrow\pi N}$ is

$$< \Delta|\Gamma_{\Delta\leftrightarrow\pi N}|\vec{k}i >= -\frac{f_{\pi N\Delta}}{m_\pi}\frac{i}{\sqrt{(2\pi)^3}}\frac{1}{\sqrt{2E_\pi(k)}}\sqrt{\frac{E_N(k)+m_N}{2E_N(k)}}\,\vec{S}\cdot\vec{k}T_i$$

$\vec{S},\vec{T}$ arwe the $N\rightarrow\Delta$ transition spin and isospin operators.

Similar procedure is used to obtain the matrix elements of electromagnetic interactions $v_{\pi\gamma}$ and $\Gamma_{\Delta,\gamma N}$.

To obtain a formula for identifing the Δ resonance apply the projection operator technique, as described in Feshbach's textbook

Projection operator method:

- Divide the Hilbert space : $1 = P + Q$

  $P = |\pi N >< \pi N| + |\gamma N >< \gamma N|$

  $Q = |\Delta >< \Delta|$

  $\rightarrow$

  Hamiltonian in the P-space

$$
\begin{aligned}
H &= H_0 + v + w \\
v &= v_{\pi N} + v_{\gamma \pi} \\
w &= \Gamma^\dagger \frac{Q}{E - H_0} \Gamma
\end{aligned}
$$

Derivation of reaction amplitude

Scattering amplitude $T(E)$ in the P-space is defined by

$$
\begin{aligned}
T(E) &= (v+w)+(v+w)\frac{1}{E-H_0}T(E) \\
&= (v+w)+(v+w)\frac{1}{E-H_0-v-w}(v+w)
\end{aligned}
$$

Objective : isolate the amplitude due to the non-Δ interaction v.

Use the relations for any operators A, B, and C (E is a scalar):

$$
\begin{aligned}
\frac{1}{E-C-B} &= \frac{1}{E-C}+\frac{1}{E-C}B\frac{1}{E-C-B} \\
\text{define } A &= B+B\frac{1}{E-C}A = B+A\frac{1}{E-C}B \\
\rightarrow A &= [1-B\frac{1}{E-C}]^{-1}B = B[1-\frac{1}{E-C}B]^{-1}
\end{aligned}
$$

$$
\frac{1}{E-C-B} = \frac{1}{E-C}+\frac{1}{E-C}A\frac{1}{E-C}
$$

Detailed derivation of scattering equations using projection operator technique is explained in Appendix I of this lecture

Dynamical scattering equations for $\Delta(1232)$ resonance

$\pi N \rightarrow \pi N$ amplitude:

$$T_{\pi N}(E) = t_{\pi N}(E) + \frac{\bar{\Gamma}_{\Delta\rightarrow\pi N}(E)\bar{\Gamma}_{\pi N\rightarrow\Delta}(E)}{E - m_\Delta - \Sigma_\Delta(E)}$$

meson-exchange amplitude : $t_{\pi N}(E) = v_{\pi N} + v_{\pi N} G_{\pi N}(E) t_{\pi N}(E)$

Dressed $\Delta \leftrightarrow \pi N$ vertex:

$$\bar{\Gamma}_{\pi N\rightarrow\Delta}(E) = \Gamma_{\pi N\rightarrow\Delta}(1 + G_{\pi N}(E) t_{\pi N}(E))$$

$$\bar{\Gamma}_{\Delta\rightarrow\pi N}(E) = (1 + G_{\pi N}(E) t_{\pi N}(E))\Gamma_{\Delta\rightarrow\pi N}$$

Δ self-energy : $\Sigma_\Delta(E) = \Gamma_{\pi N\rightarrow\Delta} G_{\pi N}(E) \bar{\Gamma}_{\Delta\rightarrow\pi N}(E)$

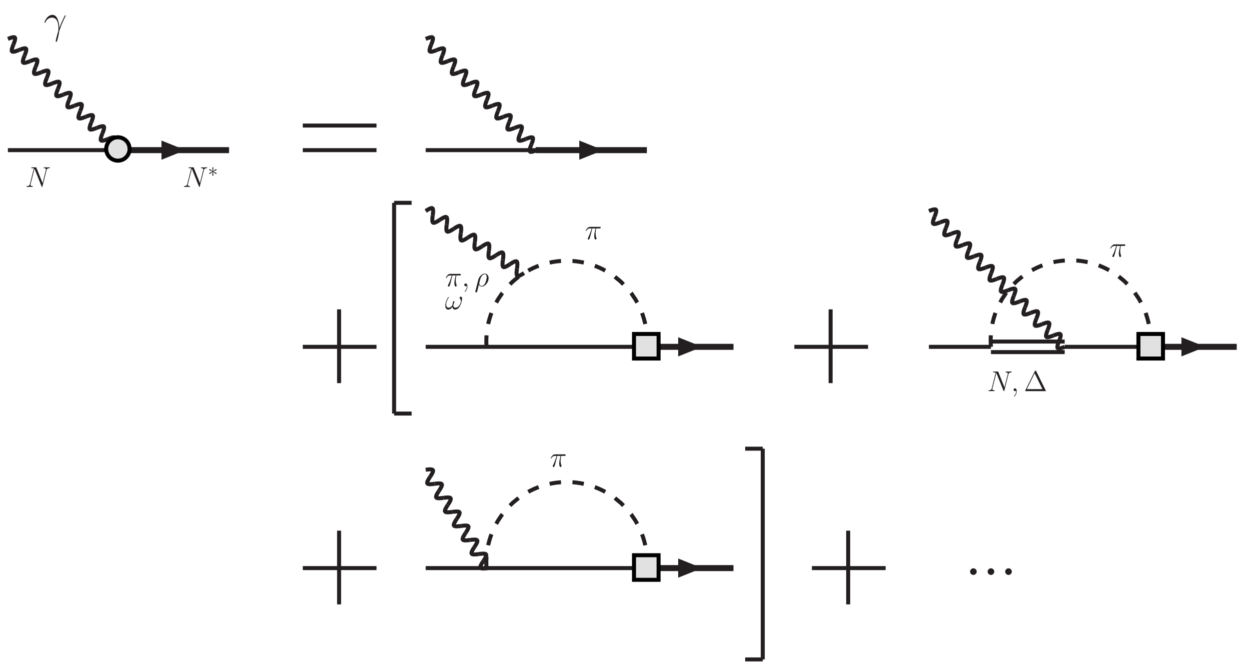
πN propagator : $G_{\pi N}(E) = \dfrac{1}{E - E_N(k) - E_\pi(k) + i\epsilon}$

$\gamma N \to \pi N$ amplitude:

$$T_{\gamma\pi}(E) = t_{\gamma\pi}(E) + \frac{\bar{\Gamma}_{\Delta\to\pi N}(E)\bar{\Gamma}_{\gamma N\to\Delta}(E)}{E - m_\Delta - \Sigma_\Delta(E)}$$

Meson-exchange amplitude : $t_{\gamma\pi}(E) = v_{\gamma\pi} + t_{\pi N}(E) G_{\pi N}(E) v_{\gamma\pi}$

Dressed $\Delta \leftrightarrow \gamma N$: $\bar{\Gamma}_{\gamma N\to\Delta}(E) = \Gamma_{\gamma N\to\Delta} + \bar{\Gamma}_{\pi N\to\Delta}(E) G_{\pi N}(E) v_{\gamma\pi}$



Numerical task : solve the πN scattering equation.

In each partial wave, it is of the following form

$$
\begin{aligned}
T(k,k',E) &= V(k,k') + \int_0^\infty q^2 dq V(k,q) G(q,E) T(q,k',E) \\
G(q,E) &= \frac{1}{E - E_1(q) - E_2(q) + i\epsilon} \\
&= \frac{P}{E - E_1(q) - E_2(q)} - i\pi\delta(E - E_1(q) - E_2(q))
\end{aligned}
$$

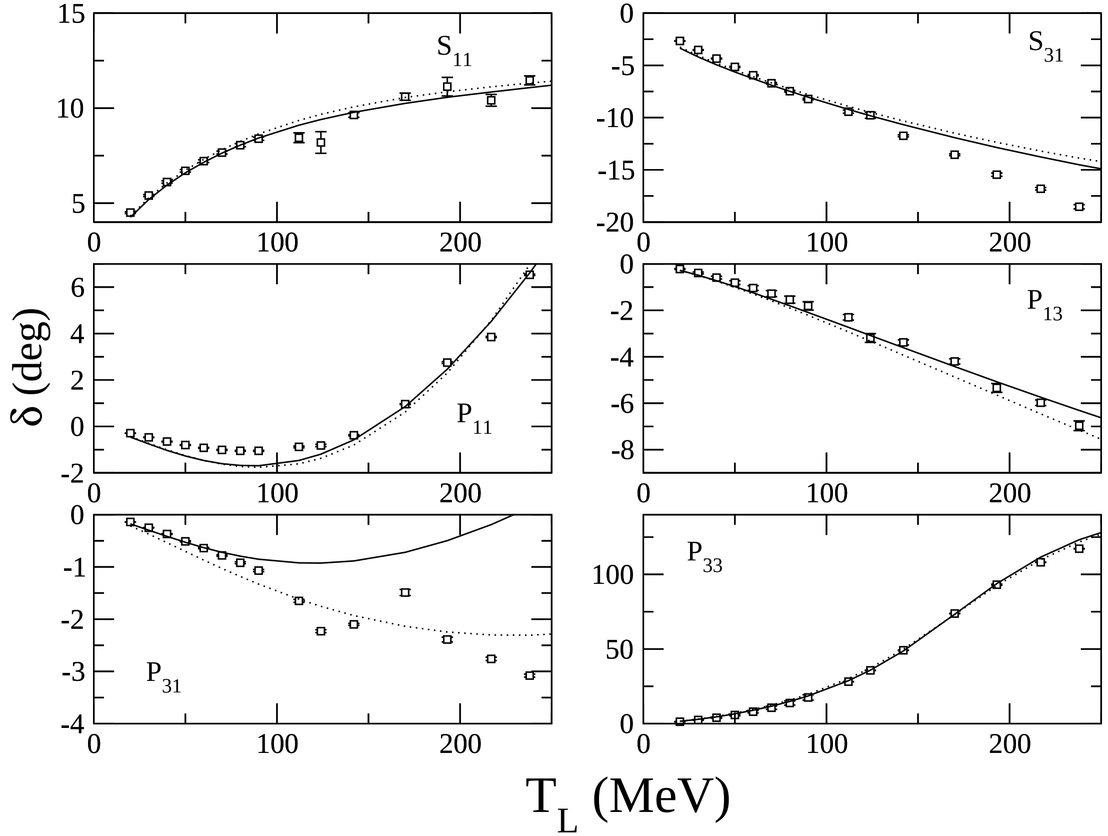
P : taking the principal-value of the integration

Can be solved by using the standard matrix method

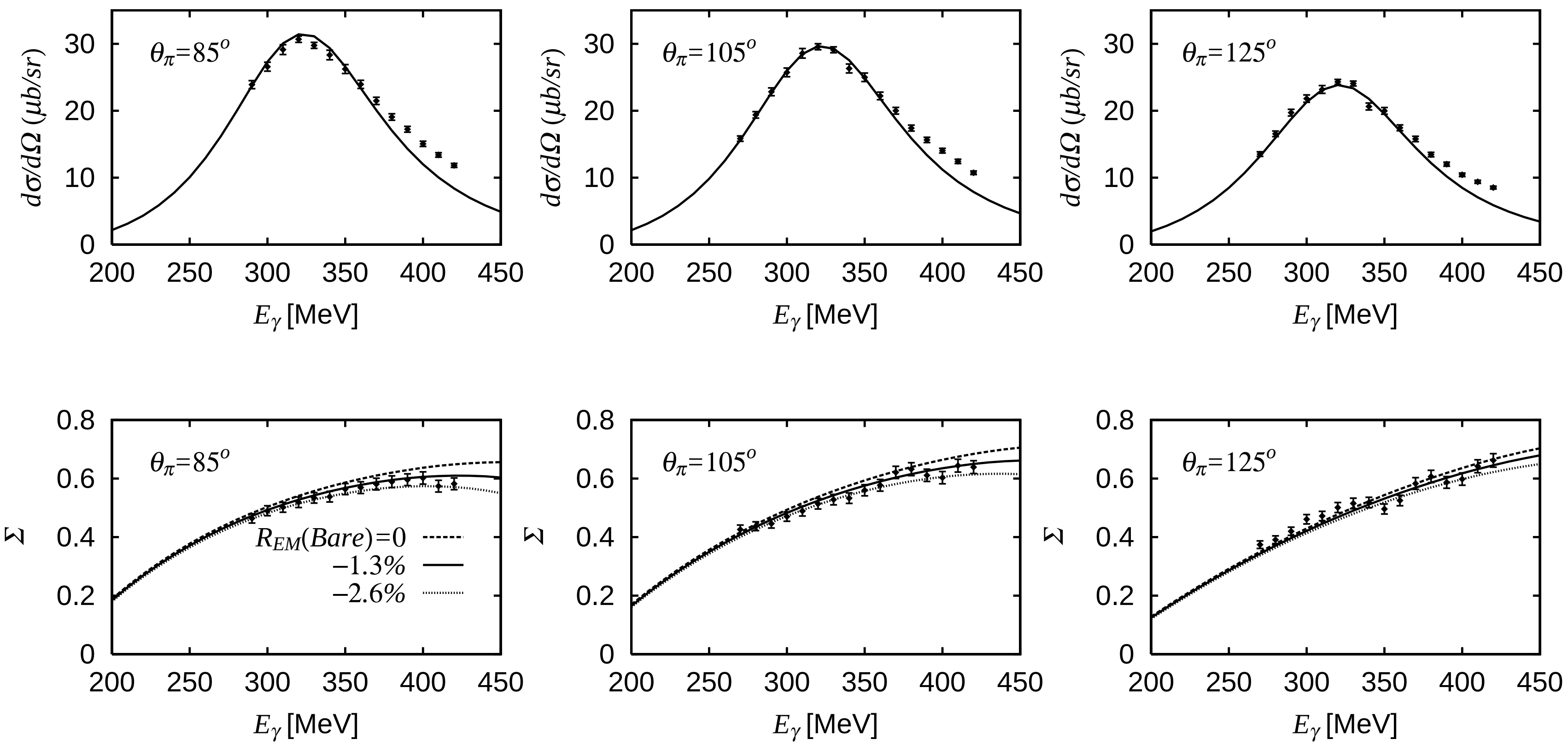
(explained in the Appendix II of this lecture)

Sample results (T. Sato and T.-S. H. Lee, 1996, 2001)

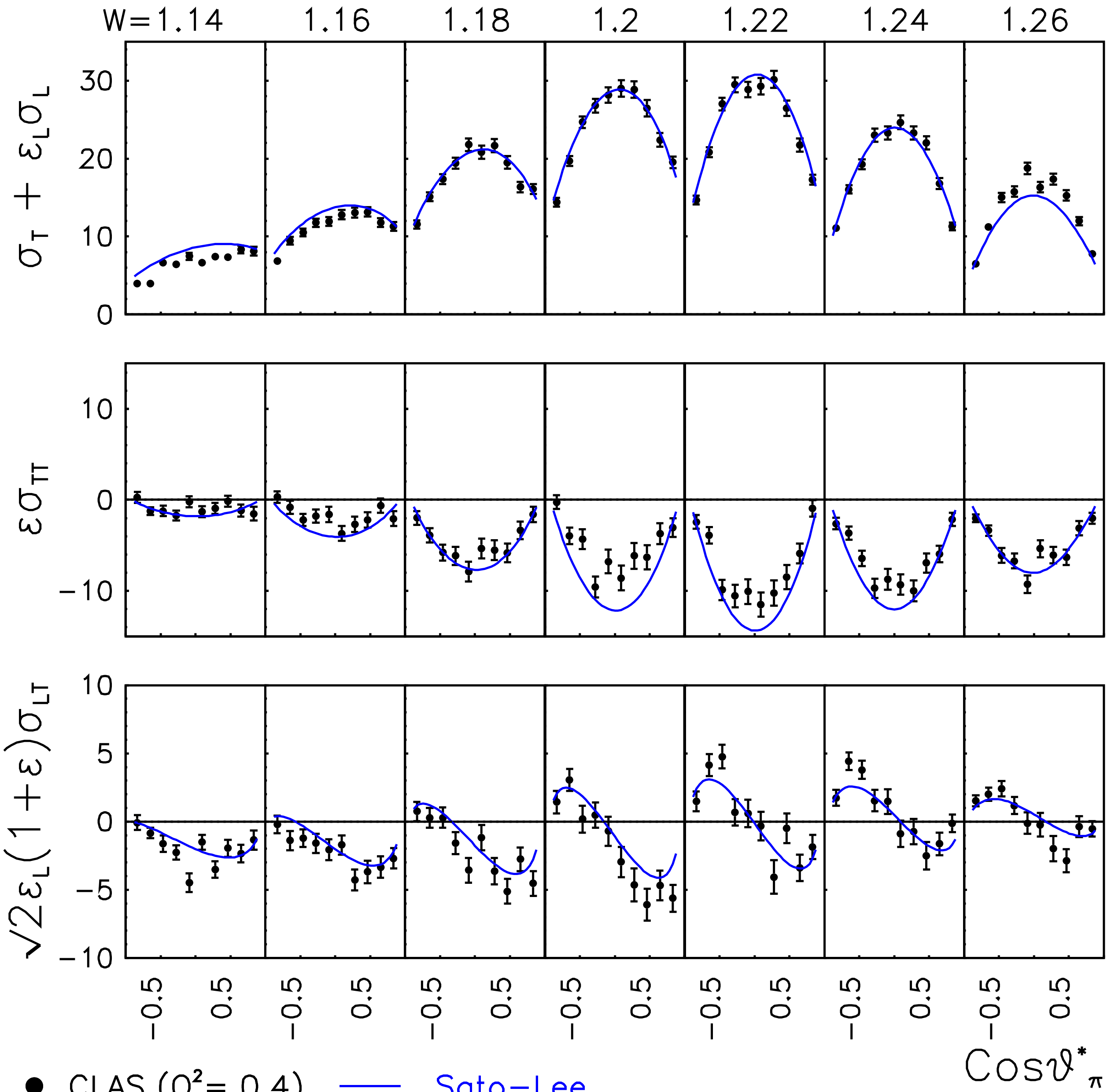
πN scattering phase shifts



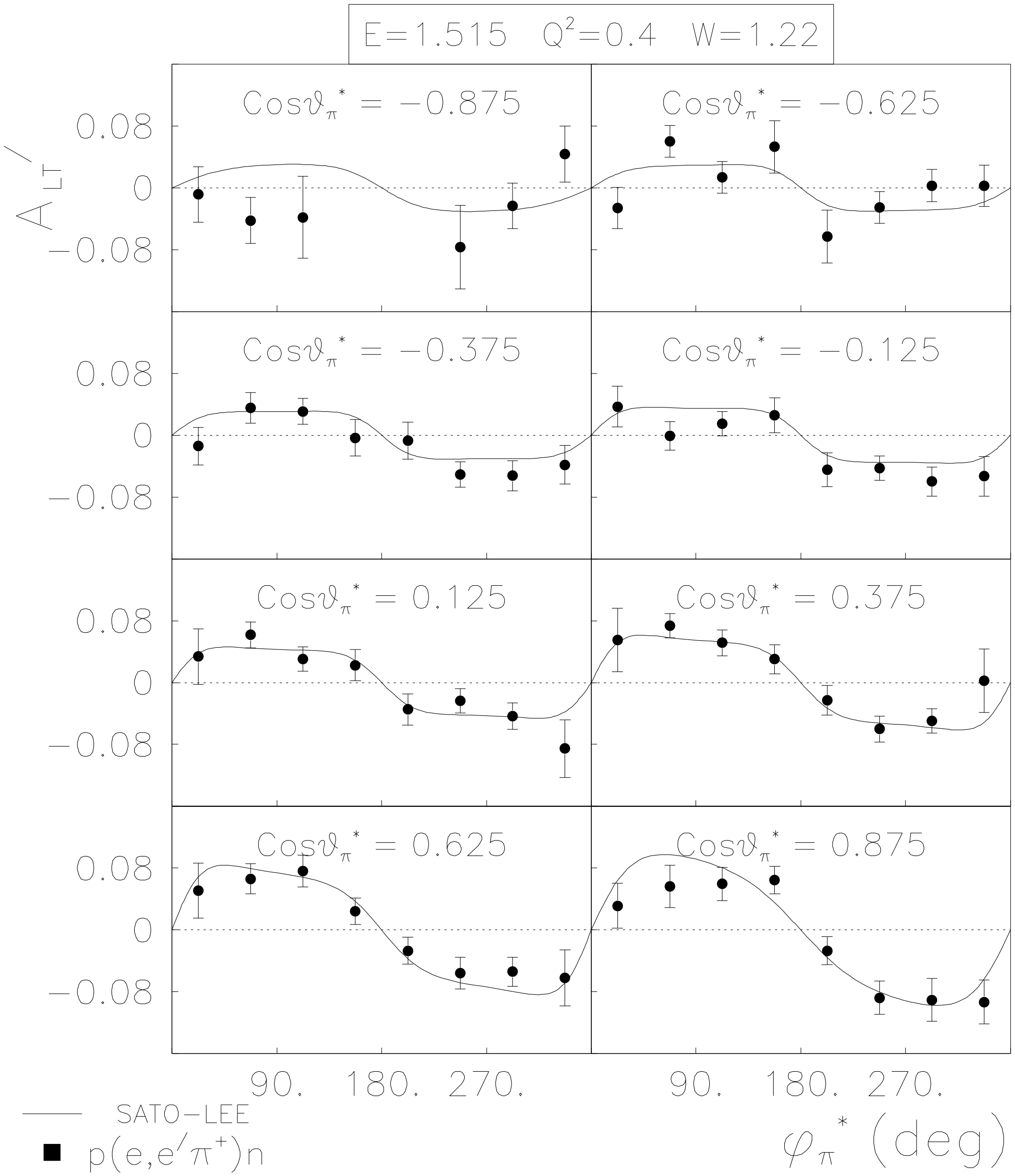
Differential cross section $d\sigma/d\Omega$ and photon asymmetry Σ of $p(\gamma, \pi^0)p$



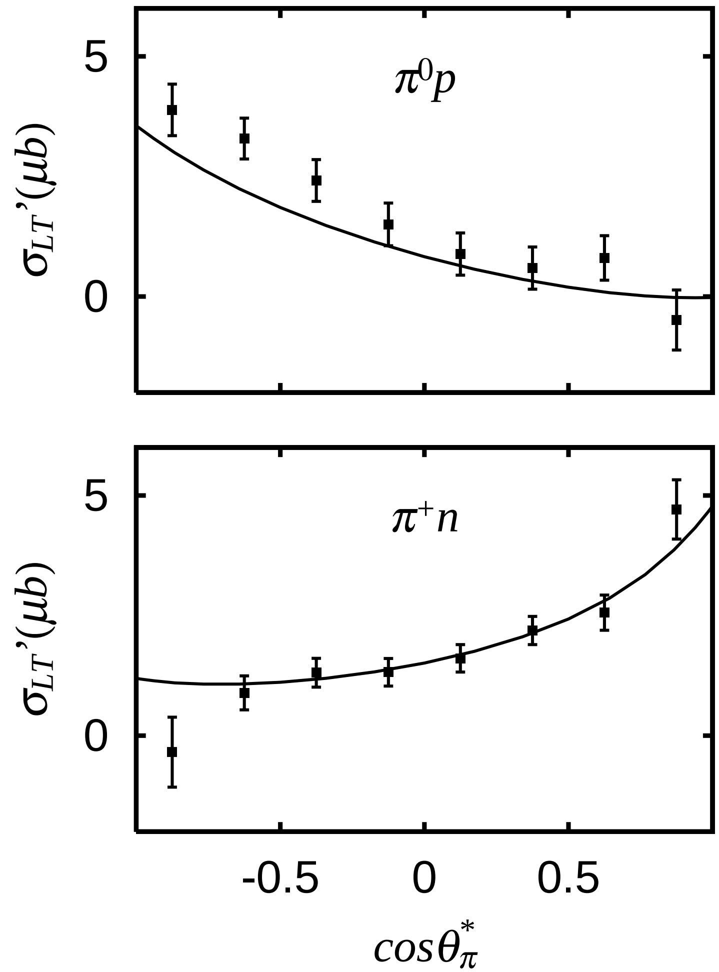
unpolarized $p(e, e'\pi^0)p$ data from JLAB (2001)



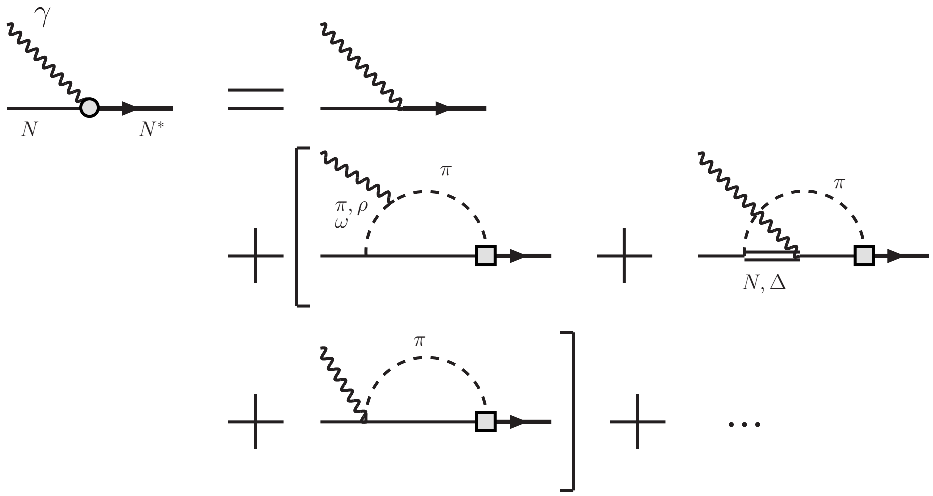
$A_{LT'}$ from $p(\vec{e}, e'\pi)$ data of JLab (2004)

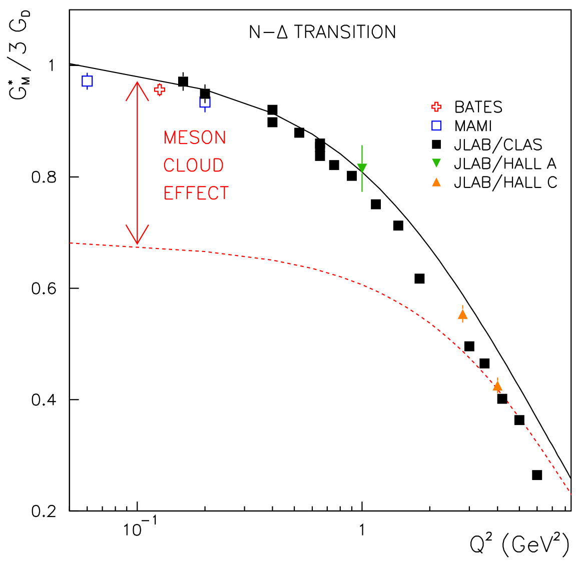


σ'_{LT} from $p(\vec{e}, e'\pi)$ data of JLab (2004)



Determinations of $\gamma^* N \to N^*$ form factors





2001-2005 :

Extend the dynamical model to investigate weak pion production reactions

(T. Sato, D. Uno, T.-S. H. Lee, Phys. Rev C67, 065201 (2003))

(K. Matsui, T. Sato, T.-S. H. Lee, Phys. Rev. C72, 025204 (2005))

$\longrightarrow$

1. Determine the axial form facor $G^A_{N,\Delta}(Q^2)$
2. Predict parity-violation in $e + p \rightarrow e' + X$: examine the neutral currents

Task: Construct axial currents by using unitary transformation

Procedures :

$$j_\mu^{em} = V_\mu^3 + V_\mu^{IS}$$

- Vctor currents V_μ : determined in $(e, e'\pi)$ studies

$\longrightarrow$

$$\text{CC} \quad j_\mu^{CC} = V_\mu^{1+i2} - A_\mu^{1+i2}$$

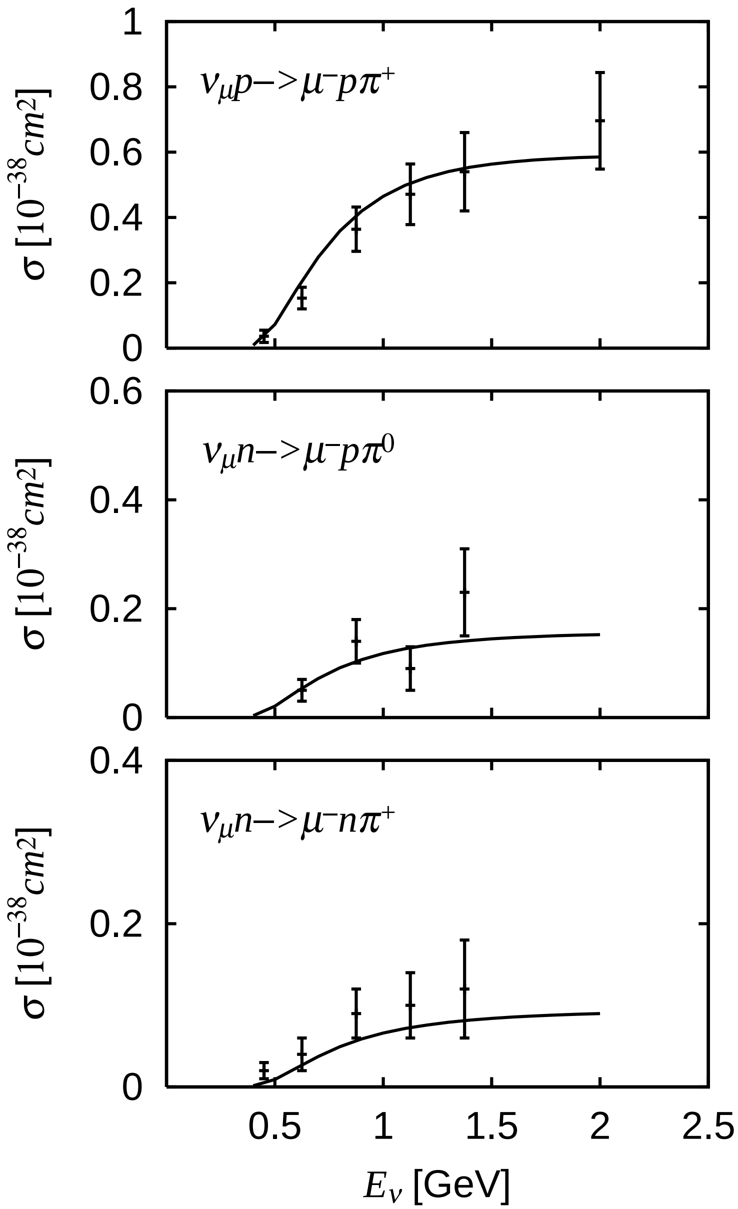
$$\text{NC} \quad j_\mu^{NC} = (1 - 2\sin^2\theta_W)j_\mu^{em} - V_\mu^{IS} - A_\mu^3$$

- Non-resonant axial current A_μ : derived from effective Lagrangians

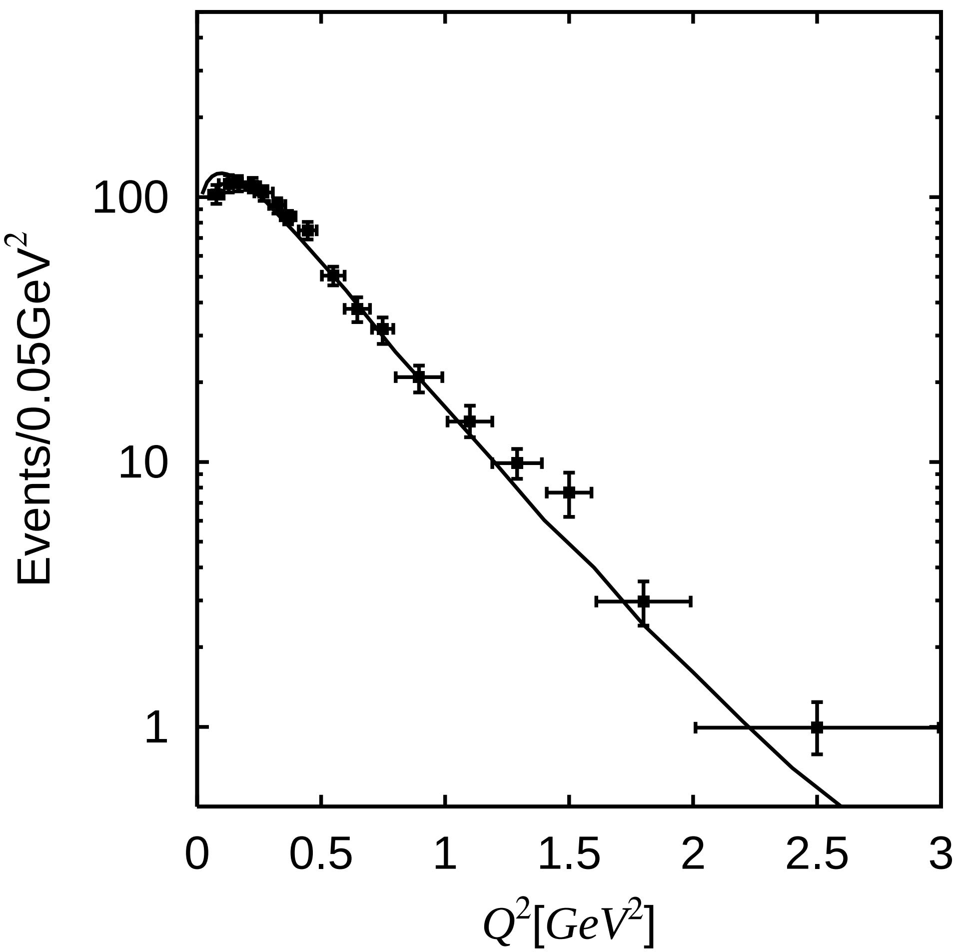
$\longrightarrow$

Extract $G^A_{N,\Delta}(Q^2)$ from $N(\nu, \mu\pi)$ data at few GeV

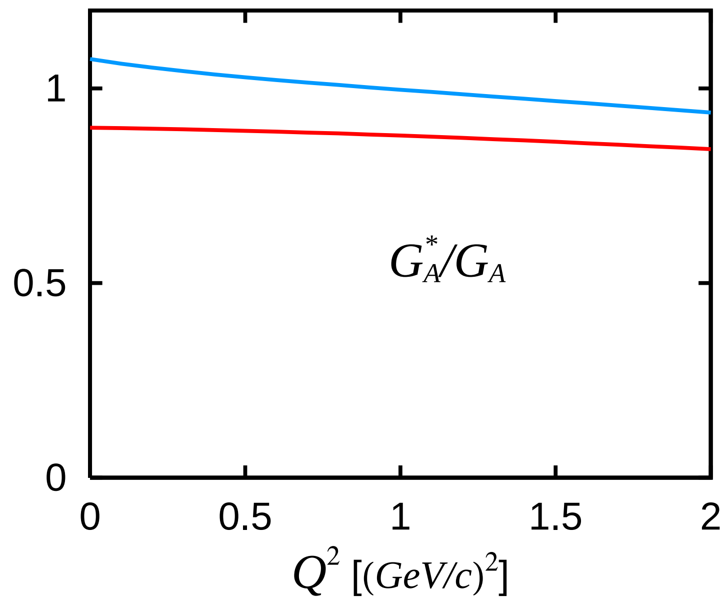
Total cross sections of $p(\nu_\mu, \mu^- \pi)$ results from SL model



$d\sigma/dQ^2$ of $p(\nu_\mu, \mu^-\pi^+)$



Determined axial N-Δ form factor G^*_A



Red curves : no pion cloud effect

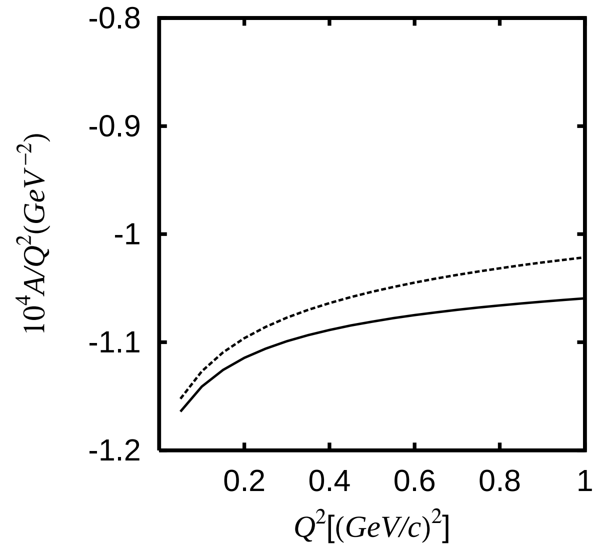
Role of neutral currents

- Consider Parity-violating asymmetry (A)of $e + p \to e' + X$

$$
\begin{aligned}
A &= \frac{d\sigma(h_e = +1) - d\sigma(h_e = -1)}{d\sigma(h_e = +1) + d\sigma(h_e = -1)} \\
&= -\frac{Q^2 G_F}{\sqrt{2}(4\pi\alpha)}[2 - 4\sin^2\theta_W + \Delta_V + \Delta_A]
\end{aligned}
$$

$$
\begin{aligned}
\Delta_V &: \quad determined(SL - model) \\
\Delta_A &\propto \sin^2\frac{\theta}{2}(1 - 4\sin^2\theta_W)W_3(em - nc)
\end{aligned}
$$

$W_3(em - nc) \leftarrow$ isoscalar axial form factor $A_{isoscalar}$



$E_e = 1$ GeV, $\theta = 110^o$, $W = 1.232$ GeV

Recent JLab E08-11 experiment results

(From a JLab seminar by Xiaochao Zheng, April 13,2012) :

Data on deuteron : $10^6 A \quad = \quad -66.258 \pm 7.768$ (preliminary)

SL model prediction (2003) (n + p) : $10^6 A \quad = \quad -88.5$

Start 2004 by Argonne-Osaka Collaboration :

Extend the dynamical model for Δ (1232) to N^* region

Guided by experimental data:

$N^*$with mass $M_R \leq 2$ GeV are in the data of

$\pi N, \gamma N \rightarrow \pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma, \omega N$

$\longrightarrow$

Need to develop coupled-channel formulation

Dynamical coupled-channel model (MSL Model)

A. Matsuyama, T. Sato, and T.-S. H. Lee, **Phys. Rept. 439, 193-263 (2007)**

- P space : Meson-Baryon (MB) states

$$P = \sum_{MB} |MB >< MB| + |\pi\pi N >< \pi\pi N|$$

$$MB = \pi N, \gamma N, \eta N, \pi\Delta, \rho N, \sigma N, K\Lambda, K\Sigma$$

- Q-spapce : Bare N^* states

$$Q = \sum_{i} |N_i^* >< N_i^*|$$

- Meson-exchange interactions

$$v = v_{22} + v_{23} + v_{32} + v_{33}$$

$$v_{22} = \sum_{MB,M'B'} v_{MB,M'B'}$$

$$v_{23} + v_{32} = \sum_{MB} [v_{MB,\pi\pi N} + v_{\pi\pi N,MB}]$$

$$v_{33} = v_{\pi\pi N,\pi\pi N}$$

- Coupling of bare N^* with the meson-baryon channels

$$\Gamma_V = \sum_{N^*} [(\sum_{MB} \Gamma_{N^*,MB} + \Gamma_{N^*,\pi\pi N}]$$

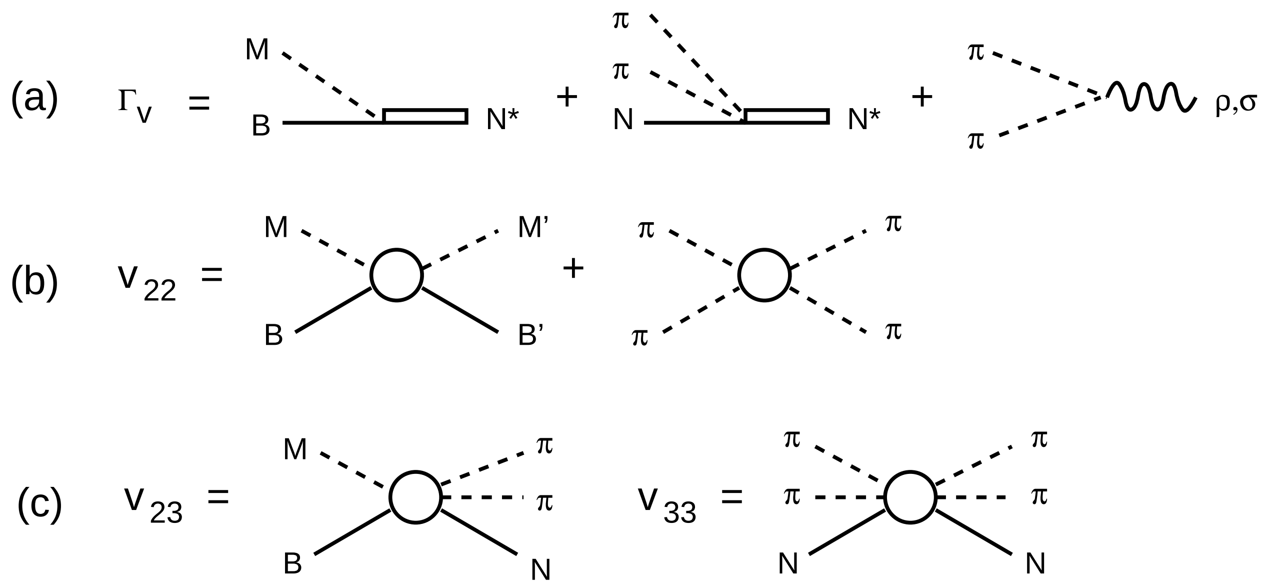
Derive H from Lagrangians by appling the unitary transformation

$\longrightarrow$

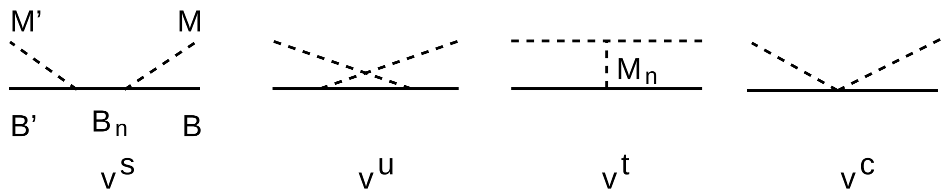
$$H = H_0 + H_I$$

$$H_0 = \sum_i \sqrt{\vec{p}_i^{\,2} + m_i^2}$$

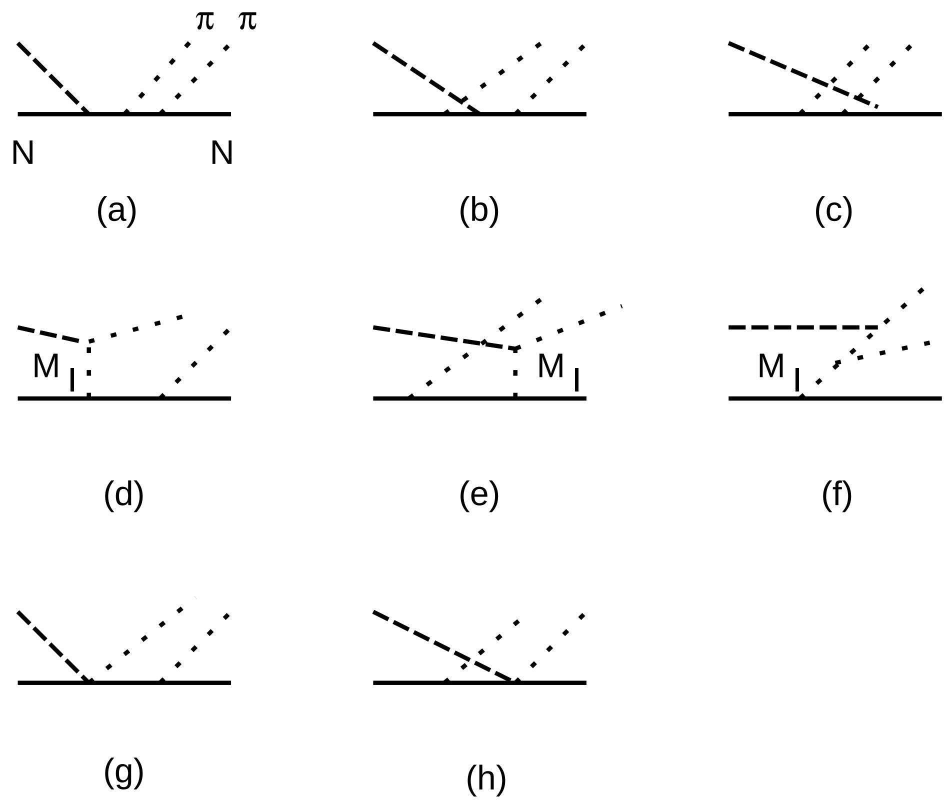
$$H_I = \Gamma_V + v_{22} + v_{23} + v_{33}$$



- $2 \rightarrow 2$ interaction v_{22} :



- $2 \rightarrow 3$ interaction v_{23} :



Examples of about 150 $MB \rightarrow M'B'$ interactions

$$\bar{V}_{\pi\Delta,\pi N} = \bar{V}_a^{11} + \bar{V}_b^{11} + \bar{V}_c^{11} + \bar{V}_d^{11} + \bar{V}_e^{11}$$

with

$$\bar{V}_a^{11} = \frac{f_{\pi NN} f_{\pi N\Delta}}{m_\pi^2} T^j \epsilon_\Delta^* \cdot k' S_N(p+k) \not{k} \gamma_5 \tau^i$$

$$\bar{V}_b^{11} = \frac{f_{\pi NN} f_{\pi N\Delta}}{m_\pi^2} T^i \epsilon_\Delta^* \cdot k S_N(p-k') \not{k}' \gamma_5 \tau^j$$

$$\bar{V}_c^{11} = i \frac{f_{\rho N\Delta} f_{\rho\pi\pi}}{m_\rho} \frac{\epsilon_{jil} T^l}{q^2 - m_\rho^2} [\epsilon_\Delta^* \cdot q(\not{k} + \not{k}')\gamma_5 - \epsilon_\Delta^* \cdot (k+k') \not{q}\gamma_5]$$

$$\bar{V}_d^{11} = -\frac{f_{\pi\Delta\Delta} f_{\pi N\Delta}}{m_\pi^2} [\epsilon_\Delta^*]_\mu \not{k}' \gamma_5 T_\Delta^j S_\Delta^{\mu\nu}(p'+k') T^i k_\nu$$

$$\bar{V}_e^{11} = -\frac{f_{\pi\Delta\Delta} f_{\pi N\Delta}}{m_\pi^2} [\epsilon_\Delta^*]_\mu \not{k} \gamma_5 T_\Delta^i S_\Delta^{\mu\nu}(p-k') T^j k'_\nu$$

Examples of $v_{\pi\pi N,\gamma N} = j^\mu \epsilon_\mu$

$$j^\mu = j^\mu(1) + j^\mu(2) + j^\mu(3) + j^\mu(4) + j^\mu(5) + j^\mu(6)$$

$$\begin{aligned}
j^\mu(1) &= i[\frac{f_{\pi NN}}{m_\pi}]^2[\not{k}^i\gamma_5\tau^i S_N(p' + k^i)\gamma^\mu\gamma_5\epsilon_{kj3}\tau^k \\
&+\gamma^\mu\gamma_5\epsilon_{kj3}\tau^k S_N(p - k^i)\ \not{k}^i\gamma_5\tau^i]\,, \\
j^\mu(2) &= -[\frac{f_{\pi NN}}{m_\pi}]^2[\not{k}^i\gamma_5\tau^i S_N(p' + k^i)\ \not{k}^j\gamma_5\tau^j S_N(p' + k^i + k^j)J_N^\mu \\
&+\ \not{k}^i\gamma_5\tau^i S_N(p' + k^i)J_N^\mu S_N(p - k^j)\ \not{k}^j\gamma_5\tau^j \\
&+J_N^\mu S_N(p - k^i - k^j)\ \not{k}^i\gamma_5\tau^i S_N(p - k^j)\ \not{k}^j\gamma_5\tau^j]\,, \\
j^\mu(3) &= -i[\frac{f_{\pi NN}}{m_\pi}]^2[\not{k}^i\gamma_5\tau^i S_N(p' + k^i)(\not{p} - \not{p}' - \not{k}^i)\gamma_5\epsilon_{kj3}\tau^k \frac{(p - p' - k^i + k^j)^\mu}{(p - p' - k^i)^2 - m} \\
&+(\not{p} - \not{p}' - \not{k}^i)\gamma_5\epsilon_{kj3}\tau^k S_N(p - k^i)\ \not{k}^i\gamma_5\tau^i \frac{(p - p' - k^i + k^j)^\mu}{(p - p' - k^i)^2 - m_\pi^2}]\,,
\end{aligned}$$

Reaction Ampliudes within MSL model:

$$T_{a,b}(E) = t_{a,b}(E) + t^R_{a,b}(E)$$

$a, b = \gamma N, \pi N, \eta N, \pi\Delta, \rho N, \sigma N, K\Lambda, K\Sigma$

Meson-exchange term :

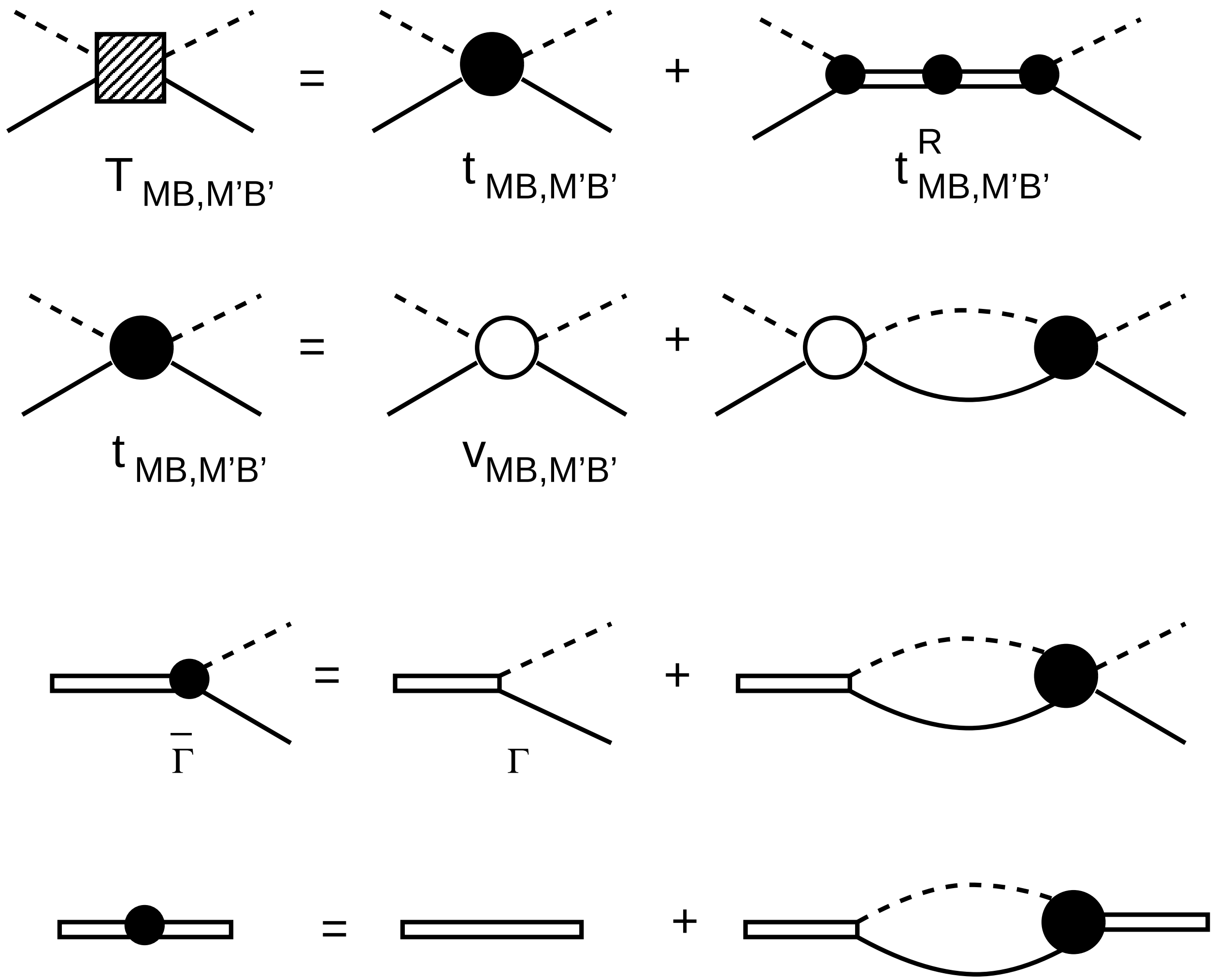
$$t_{a,b}(E) = v_{a,b} + \sum_c v_{a,c} G_c(E) t_{c,b}(E)\,,$$

N^*-excitation term:

$$t^R_{a,b}(E) = \sum_{N^*_i, N^*_j} \bar{\Gamma}^\dagger_{N^*_i,a}(E)[G^*(E)]_{i,j} \bar{\Gamma}_{N^*_j,b}(E)\,.$$

Dressed vertex:

$$\bar{\Gamma}_{N^*,a}(E) = \Gamma_{N^*,a} + \sum_b \Gamma_{N^*,b} G_b(E) t_{b,a}(E)\,,$$



=
+
T MB,M'B'
t MB,M'B'
t R MB,M'B'
=
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t MB,M'B'
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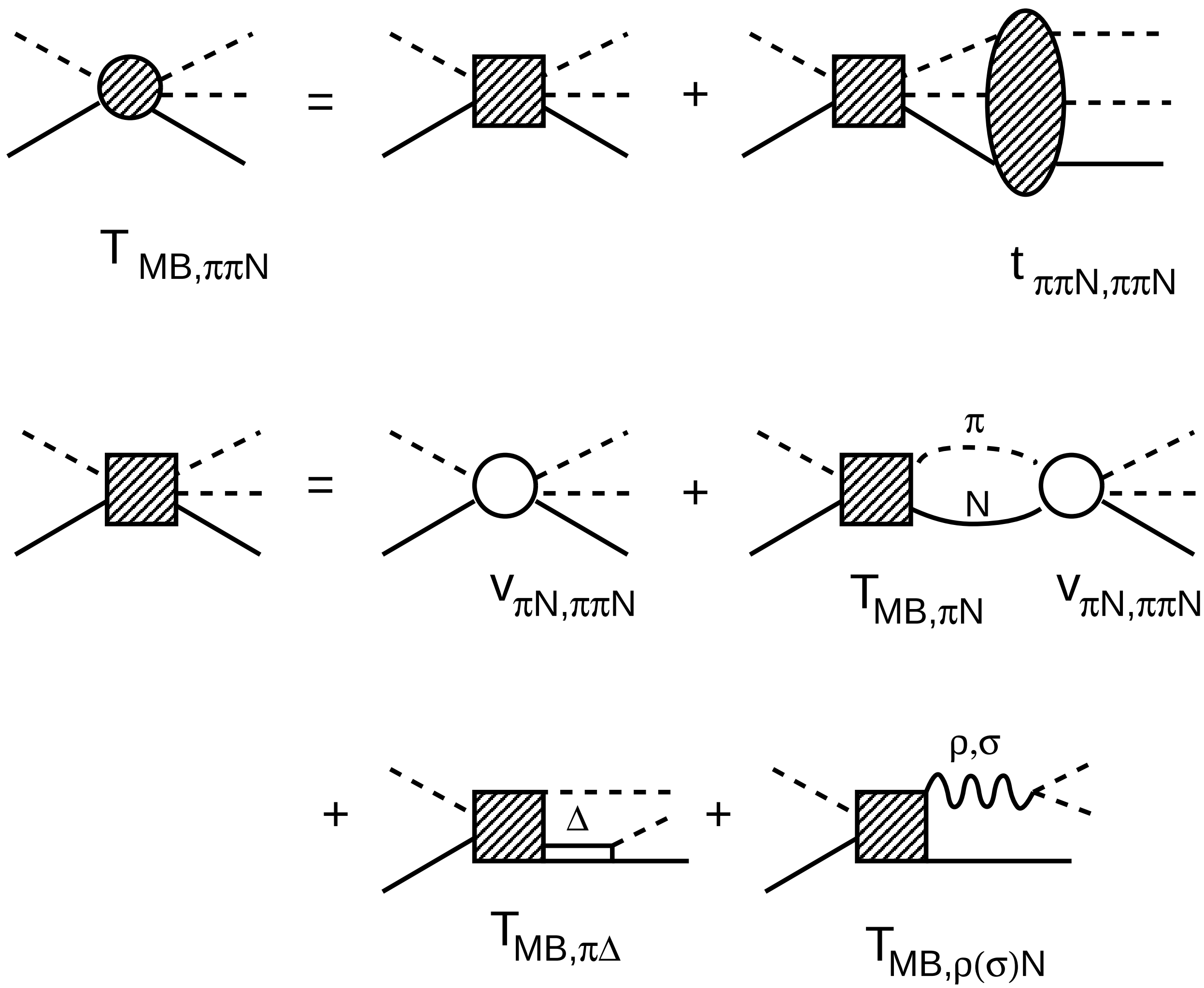

T MB,ππN
=
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t ππN,ππN
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vπN,ππN
TMB,πN
vπN,ππN
+
Δ
+
ρ,σ
TMB,πΔ
TMB,ρ(σ)N

Main numerical task:

Develop method for solving coupled-channel equation with $\pi\pi N$ cut :

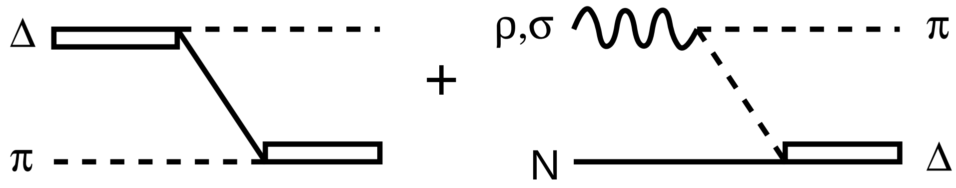
$$T_{a,b}(p,p';E) = V_{a,b}(p,p';E) + \sum_{c=1,n_c} \int_0^\infty q^2 dq V_{a,c}(p,q) G_c(q,E) T_{c,b}(q,p';E)$$

$$V_{a,b}(p,p';E) = v_{a,b}(p,p') + Z^{(E)}_{a,b}(p,p';E)$$

$v_{a,b}(p,p') =$ energy independent meson-exchange interactions

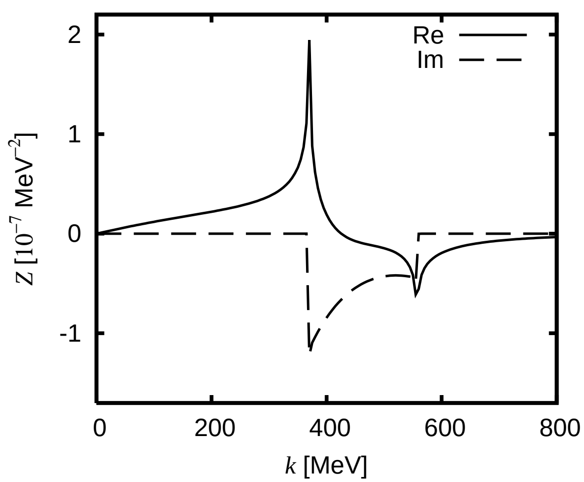
$Z^{(E)}_{a,b}(p,p';E)$: consequency of $\pi\pi N$ unitarity cut

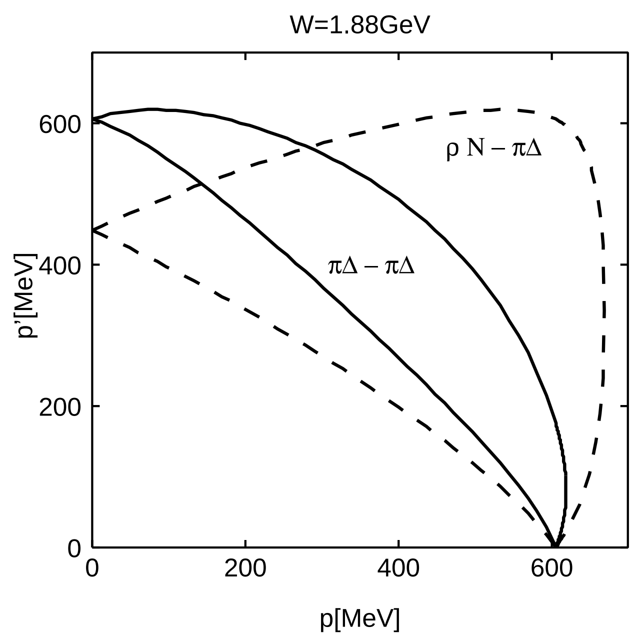
$Z^{(E)}_{MB,M'B'} =$



$$Z^{(E)}_{\pi\Delta,\pi\Delta}(p,p';E) \quad \sim \quad \int_{-1}^{+1} dx \frac{P_L(x)F(\vec{p},\vec{p}\,')}{E - E_\pi(\vec{p}) - E_\pi(\vec{p}\,') - E_N(\vec{p}+\vec{p}\,') + i\epsilon}$$

$\longrightarrow$ diverge logarithmically in the moon-shape region





Apply Spline function method (A. Matsuyama):

- solve coupled-channel equations with $\pi\pi N$ cut

- include $\pi\pi N$ cut effects exactly to calculate

  $\pi N \rightarrow \pi\pi N$

  $\gamma N \rightarrow \pi\pi N$

  (Note : very difficult by contour rotation/deformation)

- Explained in MSL paper

Remarks :

- K-matrix models can be derived from taking on-shell approximation :

$$T_{\alpha,\beta}(p_0,p_0,E) \quad \rightarrow \quad \sum_{\gamma} V_{\alpha,\gamma}(p_0,p_0)[\delta_{\alpha,\gamma} + iT_{\gamma,\beta}(p_0,p_0,E)]$$

$$\Uparrow$$

$$on - shell$$

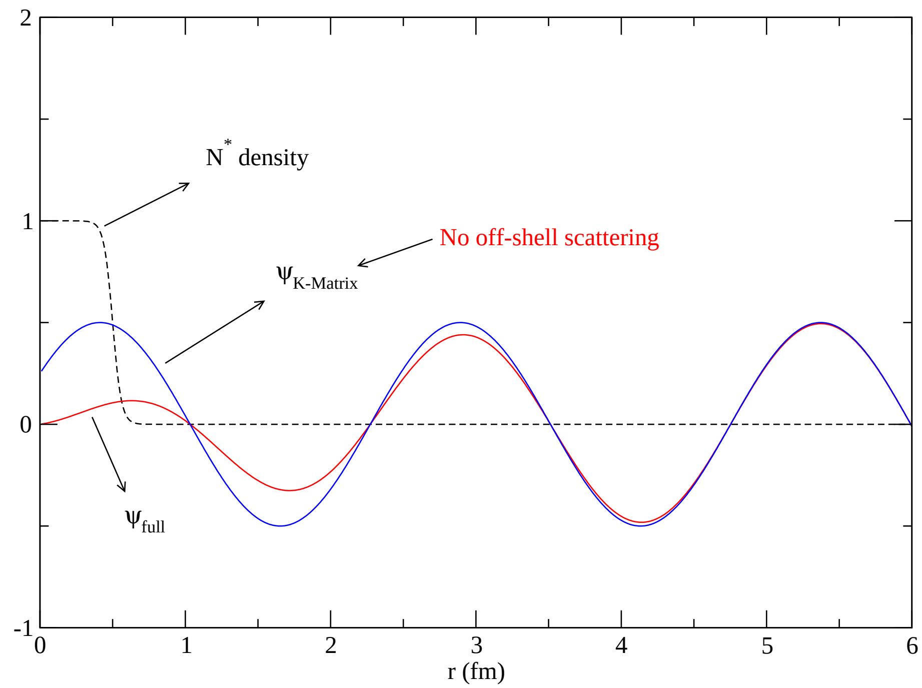
$\longrightarrow$

Main feature of a dynamical approach :

Account for reaction mechanisms in the short-range region where we want to map out N^* structure

$\longrightarrow$

– important for developing interpretations of the extracted N^* parameters



$N^* \longrightarrow MB$ amplitude :

$A = \int dr F_{N-N^*}(r)\Psi_{full}(r)$

$\Psi_{full}(r \rightarrow \infty) = \frac{sin(kr+\delta)}{kr}$

$\Psi_{K-Matrix}(r) = \frac{sin(kr+\delta)}{kr}$ ← short range mechanisms are neglected

Dynamical coupled-channel Model analysis of nucleon resonances (N^*)

Argonne-Osaka collaboration (since 1996)

Collaboration@EBAC (2006-2012)

1. 2004-2010:
   - Extend the model for Δ (1232) to include ηN, $\pi\pi N$ ($\pi\Delta, \sigma N, \rho N$)
   - Perform 6-channel analysis of the data $\pi N, \gamma N \rightarrow \pi N$ and $N(e, e'\pi)$.
   - Examine the coupled-channel effects on $\pi N, \gamma N \rightarrow \pi\pi N$ reactions
   - Develop analytic continuation method for extracting nucleon resonances within the dynamical coupled-channel model
2. 2010 - 2012:
   - Also include $K\Lambda, K\Sigma$
   - Perform 8-channel simultaneous fits to world data of $\pi N, \gamma N \rightarrow \pi N, \eta N, K\Lambda, K\Sigma$
   - Extract positions and residues of the nucleon resonances

Go to Lecture II-B

Appendix I

Derivation of scattering equation using projection operator technique

Scattering amplitude $T(E)$ in the P-space is defined by

$$T(E) = (v+w) + (v+w)\frac{1}{E-H_0}T(E) \tag{1}$$

$$= (v+w) + (v+w)\frac{1}{E-H_0-v-w}(v+w) \tag{2}$$

From Eq.(2), we have

$$T(E) = (v+w)[1 + \frac{1}{E-H_0-v-w}(v+w)]$$

$$= (v+w)\frac{1}{E-H_0-v-w}(E-H_0) \tag{3}$$

Eqs.(1)-(3) gives

$$[1 + \frac{1}{E-H_0}T(E)] = \frac{1}{E-H_0-v-w}(E-H_0)) \tag{4}$$

Eq.(1) leads to

$$[1 - v\frac{1}{E-H_0}]T(E) = v + w[1 + \frac{1}{E-H_0}T(E)] \tag{5}$$

Using Eq.(4)

$\rightarrow$

$$
\begin{aligned}
T(E) &= [1 - v\frac{1}{E-H_0}]^{-1}v + [1 - v\frac{1}{E-H_0}]^{-1}w[1 + \frac{1}{E-H_0}T(E)] \\
&= [1 - v\frac{1}{E-H_0}]^{-1}v + [1 - v\frac{1}{E-H_0}]^{-1}w\frac{1}{E-H_0-v-w}(E-H_0)
\end{aligned}
\tag{6}
$$

define

$$
\begin{aligned}
t_w(E) &= w[1 + \frac{1}{E-H_0-v}t_w(E)] \\
&\text{or} \\
t_w(E) &= w[1 + \frac{1}{E-H_0-v-w}w]
\end{aligned}
\tag{7}
$$

$\rightarrow$

$$
\frac{1}{E-H_0-v-w} = \frac{1}{E-H_0-v} + \frac{1}{E-H_0-v-w}w\frac{1}{E-H_0-v}
$$

$$= \quad [1 + \frac{1}{E - H_0 - v} t_w] \frac{1}{E - H_0 - v} \tag{8}$$

Using Eq.(8), Eq.(6) becomes

$$\begin{aligned} T(E) &= [1 - v\frac{1}{E - H_0}]^{-1} v \\ &\quad + [1 - v\frac{1}{E - H_0}]^{-1} w [1 + \frac{1}{E - H_0 - v} t_w] \frac{1}{E - H_0 - v} [E - H_0] \\ &= [1 - v\frac{1}{E - H_0}]^{-1} v + [1 - v\frac{1}{E - H_0}]^{-1} t_w \frac{1}{E - H_0 - v} [E - H_0] \end{aligned} \tag{9}$$

Similarly define

$$t(E) = v[1 + \frac{1}{E - H_0} t(E)]$$

or

$$\begin{aligned} t(E) &= v[1 + \frac{1}{E - H_0 - v} v] \\ &= v \frac{1}{E - H_0 - v} (E - H_0) \end{aligned} \tag{10}$$

From above, we have

$$t(E) = [1 - v \frac{1}{E - H_0}]^{-1} v \tag{11}$$

$$\frac{1}{E - H_0 - v} (E - H_0) = [1 + \frac{1}{E - H_0} t(E)] \tag{12}$$

$$[1 - v \frac{1}{E - H_0}]^{-1} = [1 + t(E) \frac{1}{E - H_0}] \tag{13}$$

Using Eqs.(11)-(13), Eq.(9) becomes

$$T(E) = t(E) + [1 + t(E)\frac{1}{E - H_0}]t_w[1 + \frac{1}{E - H_0}t(E)] \quad (14)$$

Recall w to write

$$t_w(E) = \Gamma^\dagger \frac{1}{E - H_0}\Gamma + \Gamma^\dagger \frac{1}{E - H_0}\Gamma \frac{1}{E - H_0 - v}]t_w(E) \quad (15)$$

Write

$$t_w(E) = \Gamma^\dagger(E)\gamma$$

$$\frac{1}{E - H_0 - v} = \frac{1}{E - H_0} + \frac{1}{E - H_0}t(E)\frac{1}{E - H_0} \quad (16)$$

We then have

$$\begin{aligned} D(E) &= [1 - \frac{1}{E - H_0}\Sigma]^{-1}\frac{1}{E - H_0} \\ &= \frac{1}{E - H_0 - \Sigma(E)} \end{aligned} \quad (17)$$

where

$$\Sigma(E) = \Gamma \frac{1}{E - H_0}\bar{\Gamma}^\dagger(E)$$

$$\bar{\Gamma}^{\dagger}(E) \quad = \quad [1 + t(E)\frac{1}{E - H_0}]\Gamma^{\dagger}$$

Define

$$\bar{\Gamma}(E) = \Gamma(1 + \frac{1}{E - H_0}t(E) \tag{18}$$

Eq.(14) becomes

$$T(E) = t(E) + \bar{\Gamma}^{\dagger}(E)\frac{1}{E - H_0 - \Sigma(E)}\bar{\Gamma}(E) \tag{19}$$

Appendix II

Numerical methods for sloving scattering equations

1 single-channel case

For each partial wave, the scattering equation is of the following form

$$T(k,k',E) = V(k,k') + \int_0^\infty q^2 dq V(k,q) G(q,E) T(q,k',E) \quad (20)$$

where the propagator is

$$\begin{aligned} G(q,E) &= \frac{1}{E - E_1(q) - E_2(q) + i\epsilon} \\ &= \frac{P}{E - E_1(q) - E_2(q)} - i\pi\delta(E - E_1(q) - E_2(q)) \end{aligned} \quad (21)$$

P : taking the principal-value of the integration

$E_i(q) = \sqrt{m_i^2 + q^2}$

For any function $f(q)$, write

$$\int_0^\infty q^2 dq f(q) G(q,E) = P\int_0^\infty q^2 dq \frac{f(q)}{E - E_1(q) - E_2(q)} - i\pi\rho(E) f(q_0) \tag{22}$$

where the on-shell momentum q_0 is

$$q_0 = \frac{[(E^2 - m_1^2 - m_2^2)^2 - 4m_1^2 m_2^2]^{1/2}}{2E} \tag{23}$$

$$\rho(E) = \frac{q_0 E_1(q_0) E_2(q_0)}{E} \tag{24}$$

Use the property

$$P\int_0^\infty dq \frac{1}{q_0^2-q^2} = 0 \tag{25}$$

to write

$$\begin{aligned}
& P\int_0^\infty q^2 dq f(q) \frac{1}{E-E_1(q)-E_2(q)} \\
&= P\int_0^\infty dq \frac{1}{q_0^2-q^2}[\frac{q_0^2-q^2}{E-E_1(q)-E_2(q)}]q^2 f(q) \\
&= P\int_0^\infty q^2 dq \frac{1}{q_0^2-q^2}[\frac{q_0^2-q^2}{E-E_1(q)-E_2(q)}q^2 f(q)] \\
& \quad -[\frac{q_0^2-q^2}{E-E_1(q)-E_2(q)}q^2 f(q)]_{q\to q_0} P\int_0^\infty dq \frac{1}{q_0^2-q^2}
\end{aligned} \tag{26}$$

Note that the second term in the right-hand-side of the above equation, which is zero because of Eq.(25), is to make the integrand at any q in the integration finite numerically. This is the standard substraction method for performing principal-value integration of a singular integrand.

By taking $q \to q_0$ limit, we find that

$$[\frac{q_0^2 - q^2}{E - E_1(q) - E_2(q)} q^2 f(q)]_{q \to q_0} = f(q_0) q_0^2 [\frac{2q}{q/E_1(q) + q/E_2(q)}]_{q \to q_0}$$
$$= f(q_0) q_0^2 \frac{2\rho(E)}{q_0} \quad (27)$$

If we choose N mesh points q_i with weights w_i ($q_i \neq q_0$ for any i) to perform an integration

$$\int_0^\infty g(q) dq = \sum_{i=1,N} g(q_i) w_i \,, \quad (28)$$

Eq.(26) is converted into a sum

$$P \int_0^\infty q^2 dq f(q) \frac{1}{E - E_1(q) - E_2(q)} = \sum_{i=1,N} \frac{f(q_i)}{E - E_1(q_i) - E_2(q_i)} q_i^2 w_i$$
$$- f(q_0) q_0^2 \frac{2\rho(E)}{q_0} [\sum_{i=1,N} \frac{1}{q_0^2 - q_i^2} w_i] \quad (29)$$

If we define the on-shell momentum q_0 as the $(N+1)$-th mesh point $q_{N+1} = q_0$, the above equation becomes

$$P \int_0^\infty q^2 dq f(q) \frac{1}{E - E_1(q) - E_2(q)} \quad = \quad \sum_{i=1,N+1} f(q_i) \hat{W} i \tag{30}$$

where

$$\begin{aligned} \hat{W}_i &= \frac{q_i^2 w_i}{E - E_1(q_i) - E_2(q_i)}, \qquad i = 1, N \\ \hat{W}_{N+1} &= -q_0^2 \frac{2\rho(E)}{q_0} [\sum_{i=1,N} \frac{1}{q_0^2 - q_i^2} w_i] \end{aligned} \tag{31}$$

By using Eq.(30), Eq.(22) can then be calculated by the following sum

$$\begin{aligned} \int_0^\infty q^2 dq f(q) G(q, E) &= \int_0^\infty q^2 dq f(q) \frac{1}{E - E_1(q) - E_2(q) + i\epsilon} \\ &= \sum_{i=1,N+1} f(q_i) W_i \end{aligned} \tag{32}$$

with

$$\begin{aligned} W_i &= \hat{W}_i\,, \qquad i=1,N \\ W_{N+1} &= \hat{W}_{N+1} - i\pi\rho(E) \end{aligned} \tag{33}$$

We are interested in getting the solutions of Eq.(20) for $k = q_i$ and $k' = q_j$ with $i, j = 1, 2 \cdots N+1$. With the property Eq.(32), Eq.(20) clearly can be written as a sum

$$T_{i,j} = V_{i,j} + \sum_{k=1,N+1} V_{i,k} W_k T_{k,j}\,, \qquad i,j = 1, N+1 \tag{34}$$

where $T_{i,j} = T(q_i, q_j, E)$ and $V_{i,j} = V(q_i, q_j)$. We can rewrite the above equation as

$$\sum_{k=1,N+1} [\delta_{i,k} - V_{i,k} W_k] T_{k,j} = V_{i,j} \tag{35}$$

This is just a matrix equation and the solution can be found from

$$T_{i,j} = \sum_{k=1,N+1} [F^{-1}]_{i,k} V_{k,j} \tag{36}$$

where

$$F_{i,k} = \delta_{i,k} - V_{i,k} W_k \tag{37}$$

So the task is to invert a $(N+1) \times (N+1)$ matrix F.

The on-shell element $T(q_0, q_0, E) = T(q_{N+1}, q_{N+1}, E)$ can be used to calculate the cross section. The half-off-shell matrix elements $T(k, q_0, E)$ define the scattering wavefunction in momentum space

$$< k|\chi_E^{(+)} >= \chi_E^{(+)}(k) = \frac{1}{q_0^2}\delta(k - q_0) + \frac{1}{E - E_1(k) - E_2(k) + i\epsilon} T(k, q_0, E) \tag{38}$$

$$< \chi_E^{(-)}|k >= \chi_E^{(-)*}(k) = \frac{1}{q_0^2}\delta(k - q_0) + T(q_0, k, E)\frac{1}{E - E_1(k) - E_2(k) + i\epsilon} \tag{39}$$

By using Eq.(32), we can convert any integration over scattering wavefunctions as sums

$$< k|O|\chi_E^{(+)} > = \int q^2 dq < k|O|q > \chi_E^{(+)}(q)$$

$$= \; < k|O|q_{N+1} > + \sum_{i=1,N+1} < k|O|q_i > W_i T_{i,N+1} \quad (40)$$

and

$$< \chi_E^{(-)}|O|k > \; = \; \int q^2 dq \chi_E^{(-)*}(q) < q|O|k >$$

$$= \; < q_{N+1}|O|k > + \sum_{i=1,N+1} T_{N+1,i} W_i < q_i|O|k > \quad (41)$$

Here we note that the above method is appicable for ”any” E. For E below the theshold $E_{th} = m_1 + m_2$, the $\delta-$function term of Eq.(21) is absence and there is no singularity in the propagator. Thus we can get the solution by simply setting $W_{N+1} = 0$.

2 Coupled-channels case

We now consider a set of coupled-channel scattering equations with n_c chnnels

$$T_{a,b}(k,k',E) \quad = \quad V_{a,b}(k,k') + \sum_{c=1,n_c} \int_0^{\infty} q^2 dq V_{a,c}(k,q) G_c(q,E) T_{c,b}(q,k',E) \tag{42}$$

with $a, b = 1, n_c$. The numerical method described in section I can be directly extended to solve the above equation. If we choose N mesh points for channel c, we then simply add a sub-indix c for all equations in section I. Explicitly, for channel c, we have an on-shell momentum $q_{c,N+1} = q_{c,0}$, defined by $E = E_{c,1}(q_{c,0}) + E_{c,2}(q_{c,0})$, and have the mesh points and weights, $(q_{c,i}, W_{c,i})$, $i = 1, (N+1)$ for an integration, of the form of Eq.(32), over the propagator of channel c.

We next define $N_C = n_c \times (N+1)$ mesh points by setting

$$(q_1, \cdot\cdot q_{N_C}) \quad : \quad ([q_{1,1}, \cdot\cdot, q_{1,(N+1)}], [q_{2,1}, \cdot\cdot q_{2,(N+1)}], \cdot\cdot, [q_{n_c,1}, \cdot\cdot q_{n_c,(N+1)}]) \tag{43}$$

$$(W_1, \cdot\cdot W_{N_c}) \quad : \quad ([W_{1,1}, \cdot\cdot W_{1,(N+1)}], [W_{2,1}, \cdot\cdot W_{2,(N+1)}], \cdot\cdot, [W_{n_c,1}, \cdot W_{n_c,(N+1)}]) \tag{44}$$

We then can cast Eq.(42) into

$$T_{m,n} = \sum_{l=1,N_c} [F^{-1}]_{m,l} V_{l,n} \tag{45}$$

with $m, n = 1, N_c$, and

$$F_{m,l} = \delta_{m,l} - V_{m,l} W_l \tag{46}$$

where

$$T_{m,n} = T(q_{a,i}, q_{b,j}, E) \tag{47}$$

$$V_{m,n} = V(q_{a,i}, q_{b,j}, E) \tag{48}$$

The indices are related by

$$m = (n_c - a) + i \tag{49}$$

$$n = (n_c - b) + j \tag{50}$$

where $a, b = 1, n_c$ and $i, j = 1, (N + 1)$.

Dynamical Model Analysis of Hadron Resonances (II-B)

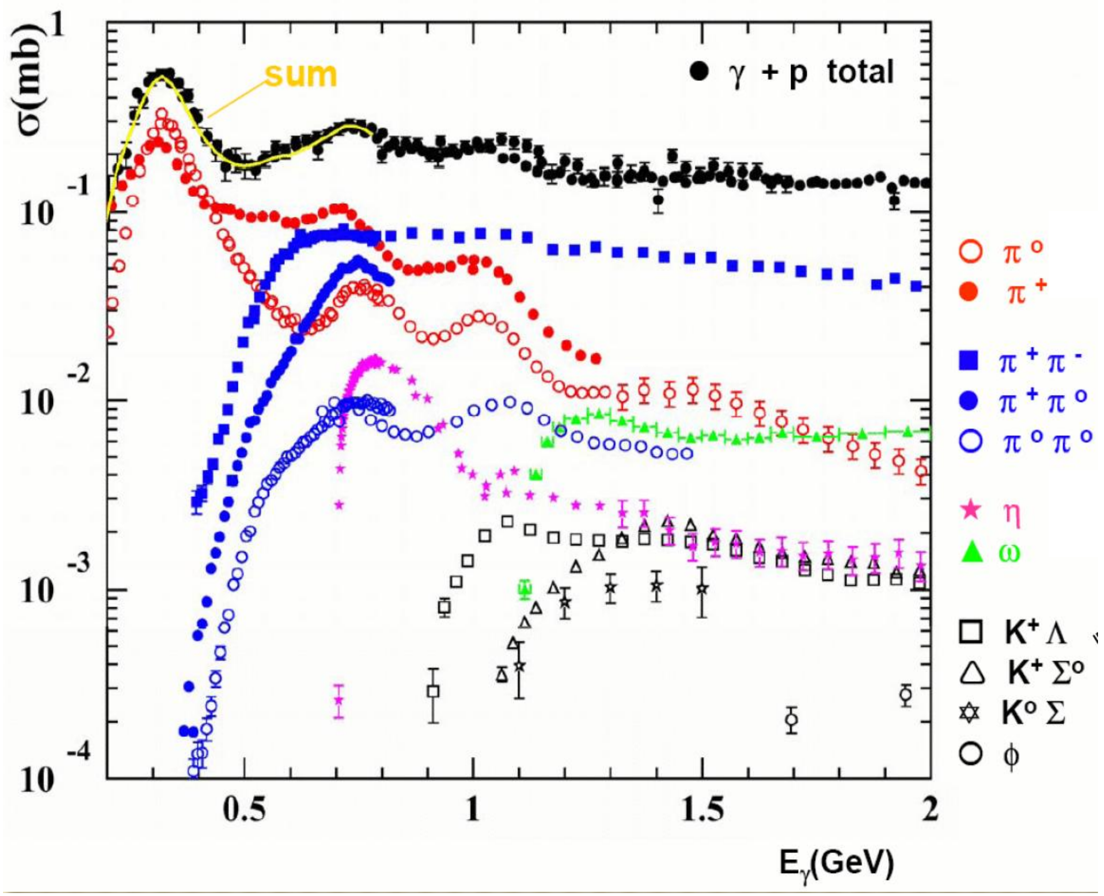
T.-S. Harry Lee
Argonne National Laboratory

Total cross sections of meson photoproduction

Unitarity Condition

→

Coupled-channel approach is needed



→

MB : γN, πN, 2π-N, ηN, KΛ, KΣ, ωN

Dynamical Coupled-Channels analysis

Fully combined **analysis of γN , πN → πN , ηN , KΛ, KΣ reactions !!**

	2006-2009	2010-2012
✓ # of coupled channels	6 channels (γN,πN,ηN,πΔ,ρN,σN)	8 channels (γN,πN,ηN,πΔ,ρN,σN,KΛ,KΣ)
✓ πp → πN	< 2 GeV	< 2.1 GeV
✓ γp → πN	< 1.6 GeV	< 2 GeV
✓ π-p → ηn	< 2 GeV	< 2 GeV
✓ γp → ηp	—	< 2 GeV
✓ πp → KΛ, KΣ	—	< 2.2 GeV
✓ γp → KΛ, KΣ	—	< 2.2 GeV

Kamano, Nakamura, Lee, Sato, 2012

Analysis Database

Pion-induced reactions (purely strong reactions)

	Waves	# of data	Waves	# of data
$\pi N \to \pi N$ PWA	S_{11}	56×2	D_{13}	52×2
	S_{31}	56×2	D_{15}	52×2
	P_{11}	56×2	D_{33}	50×2
	P_{13}	52×2	D_{35}	31×2
	P_{31}	52×2	F_{15}	39×2
	P_{33}	56×2	F_{17}	23×2
			F_{35}	34×2
			F_{37}	35×2
			Sum	1288

SAID

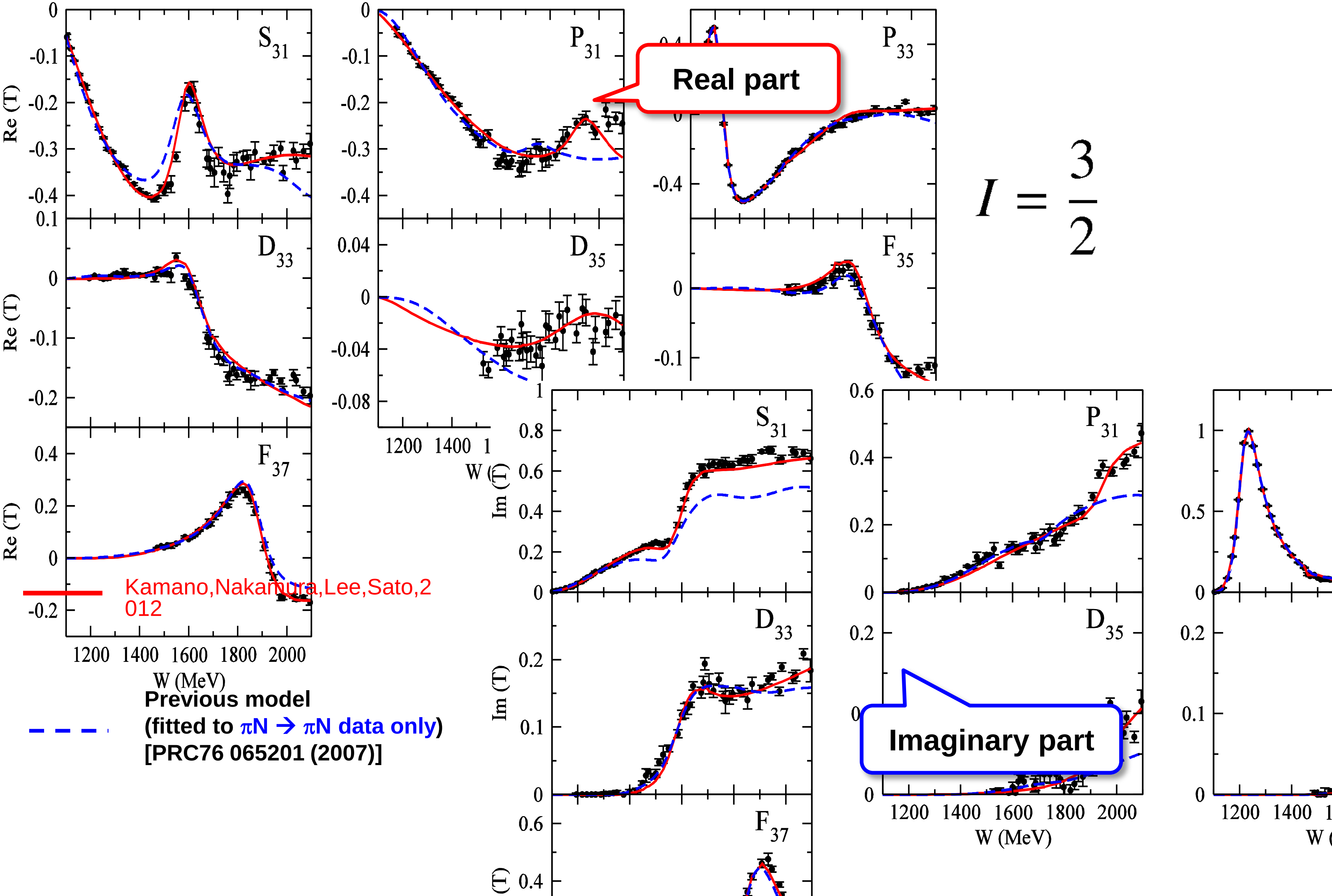
	$d\sigma/d\Omega$	P	R	a	Sum
$\pi^- p \to \eta p$	294	-	-	-	294
$\pi^- p \to K^0 \Lambda$	544	262	-	-	806
$\pi^- p \to K^0 \Sigma^0$	215	70	-	-	285
$\pi^+ p \to K^+ \Sigma^+$	552	312	-	-	864
Sum	1605	644	-	-	2249

~ 28,000 data points to fit

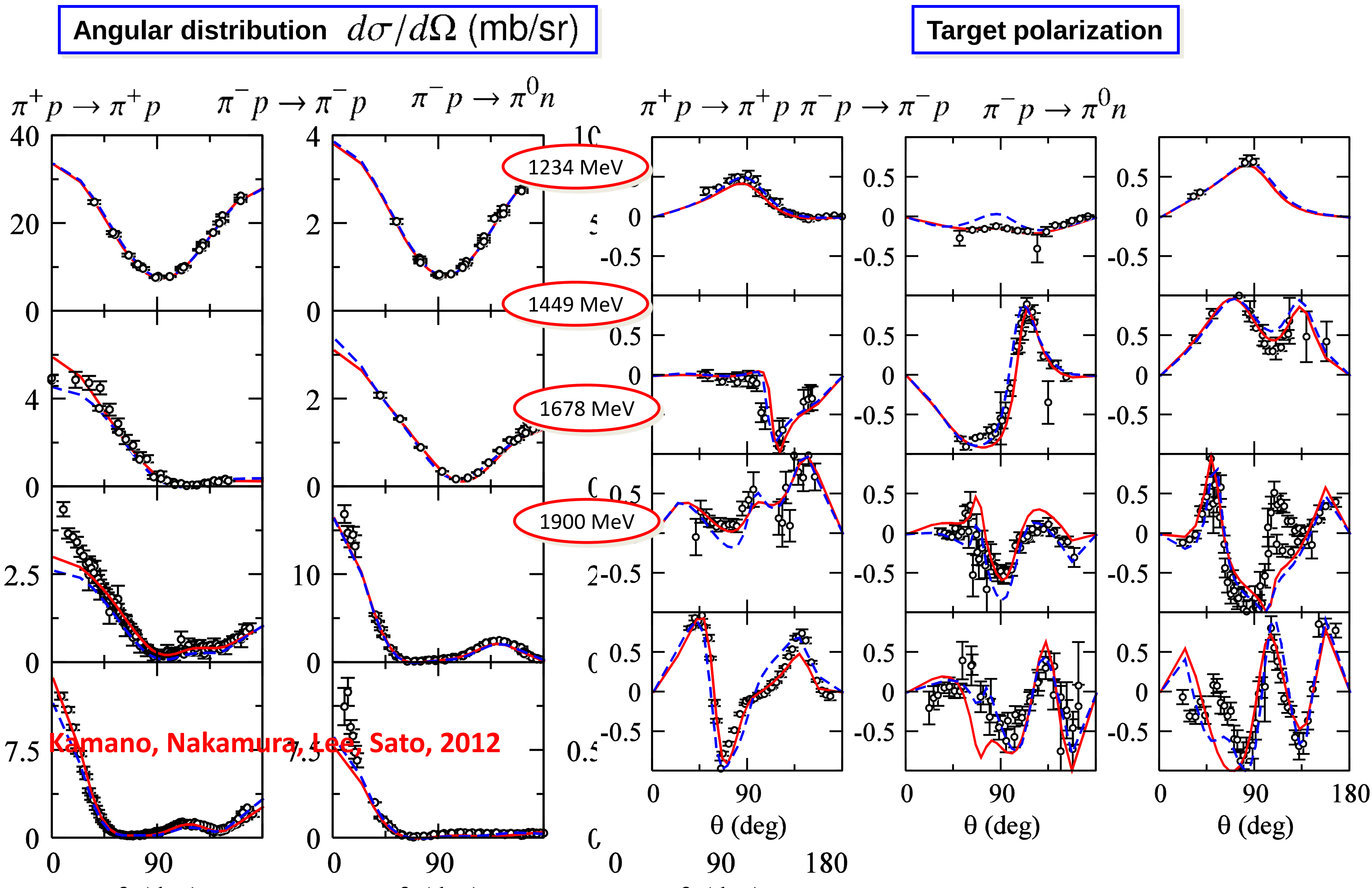
Photo-production reactions

	$d\sigma/d\Omega$	Σ	T	P	G	H	E	F	$O_{x'}$	$O_{z'}$	$C_{x'}$	$C_{z'}$	$T_{x'}$	$T_{z'}$	$L_{x'}$	$L_{z'}$	sum
$\gamma p \to \pi^0 p$	8290	1680	353	557	28	24	-	-	-	-	-	-	-	-	-	-	10860
$\gamma p \to \pi^+ n$	5384	1014	661	221	75	123	-	-	-	-	-	-	-	-	-	-	7478
$\gamma p \to \eta p$	1076	197	50	-	-	-	-	-	-	-	-	-	-	-	-	-	1323
$\gamma p \to K^+ \Lambda$	611	118	69	410	-	-	-	-	66	66	89	89	-	-	-	-	1518
$\gamma p \to K^+ \Sigma^0$	2949	116	-	320	-	-	-	-	-	-	52	52	-	-	-	-	3489
Sum	18310	3043	1133	1508	103	147	-	-	66	66	141	141	-	-	-	-	24668

Partial wave amplitudes of pi N scattering



Pion-nucleon elastic scattering

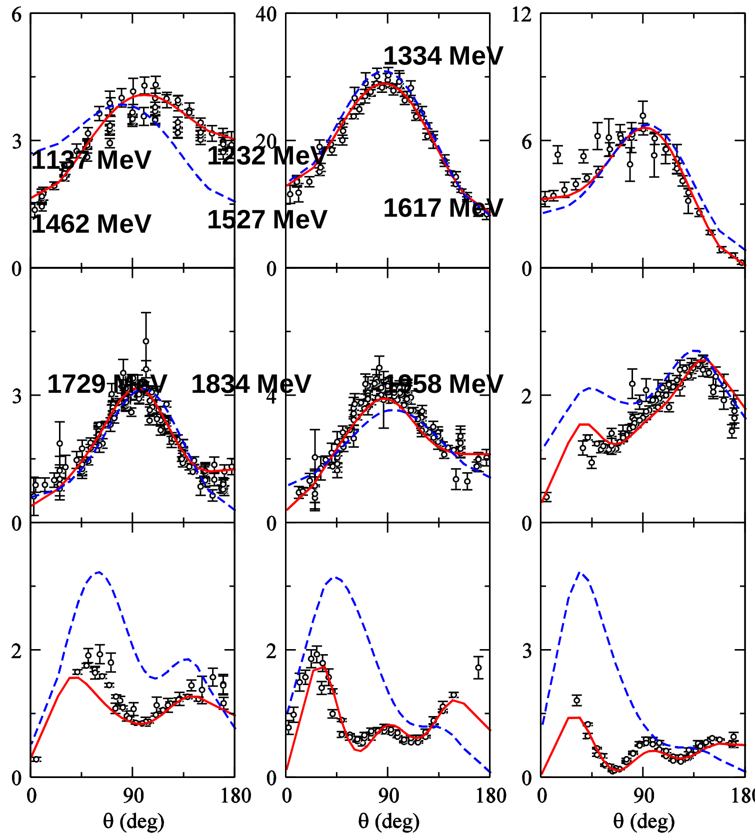


Single pion photoproduction

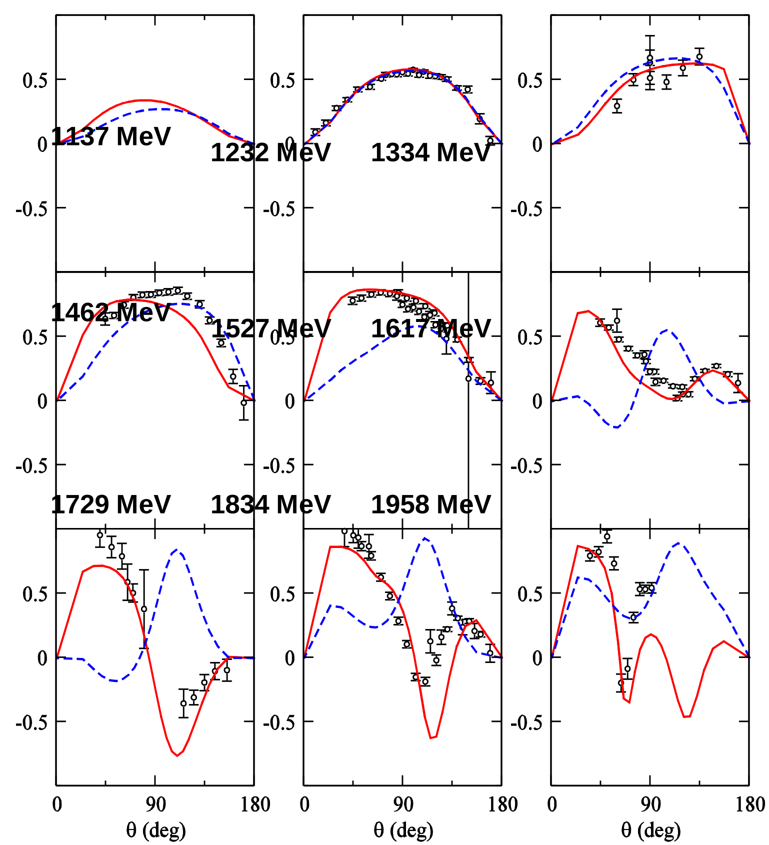
Kamano, Nakamura, Lee, Sato, 2012

$$\gamma p \rightarrow \pi^0 p$$

Angular distribution $d\sigma/d\Omega$ (μb/sr)



Photon asymmetry Σ



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Previous model (fitted to γN → πN data *up to 1.6 GeV*) [PRC77 045205 (2008)]

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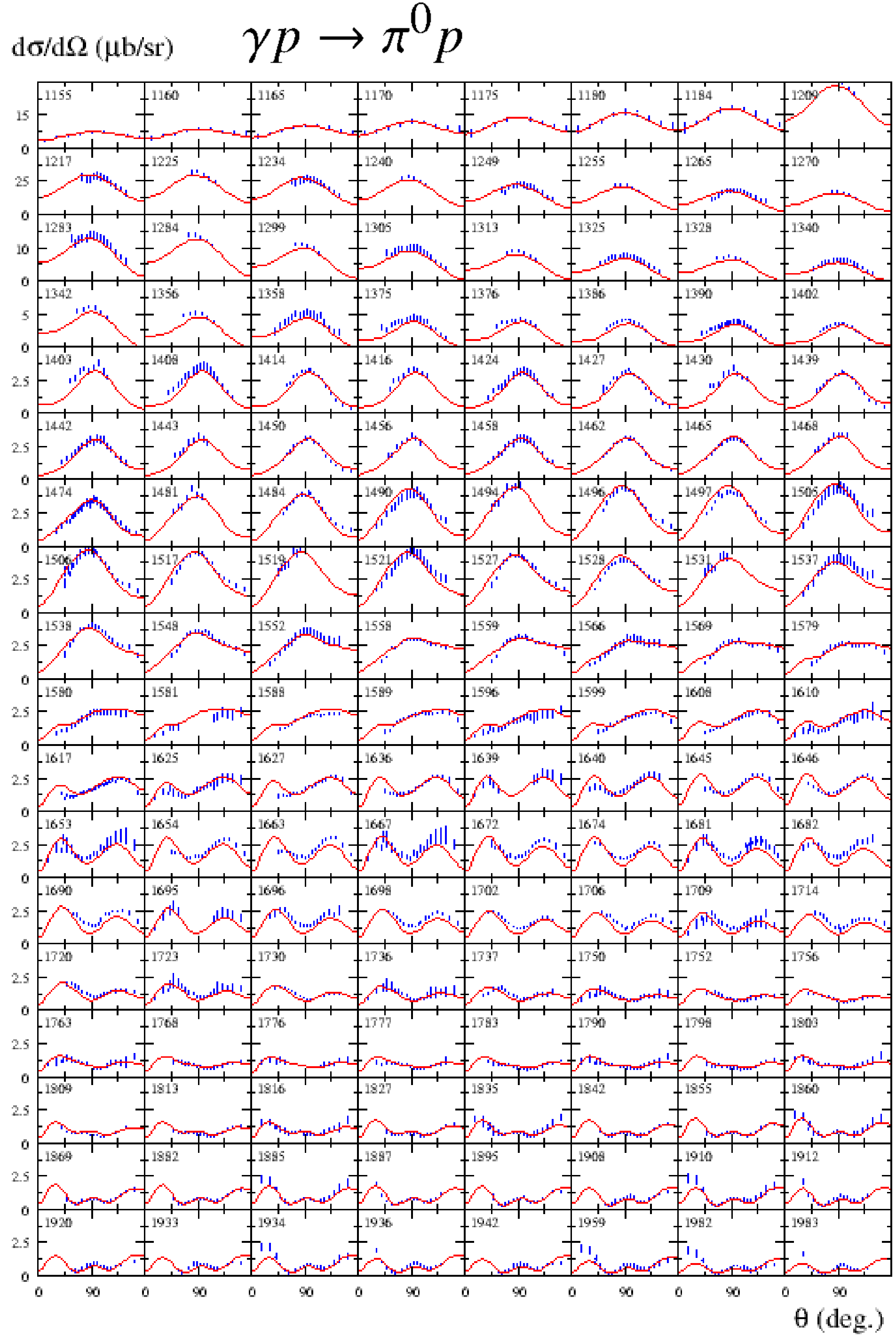
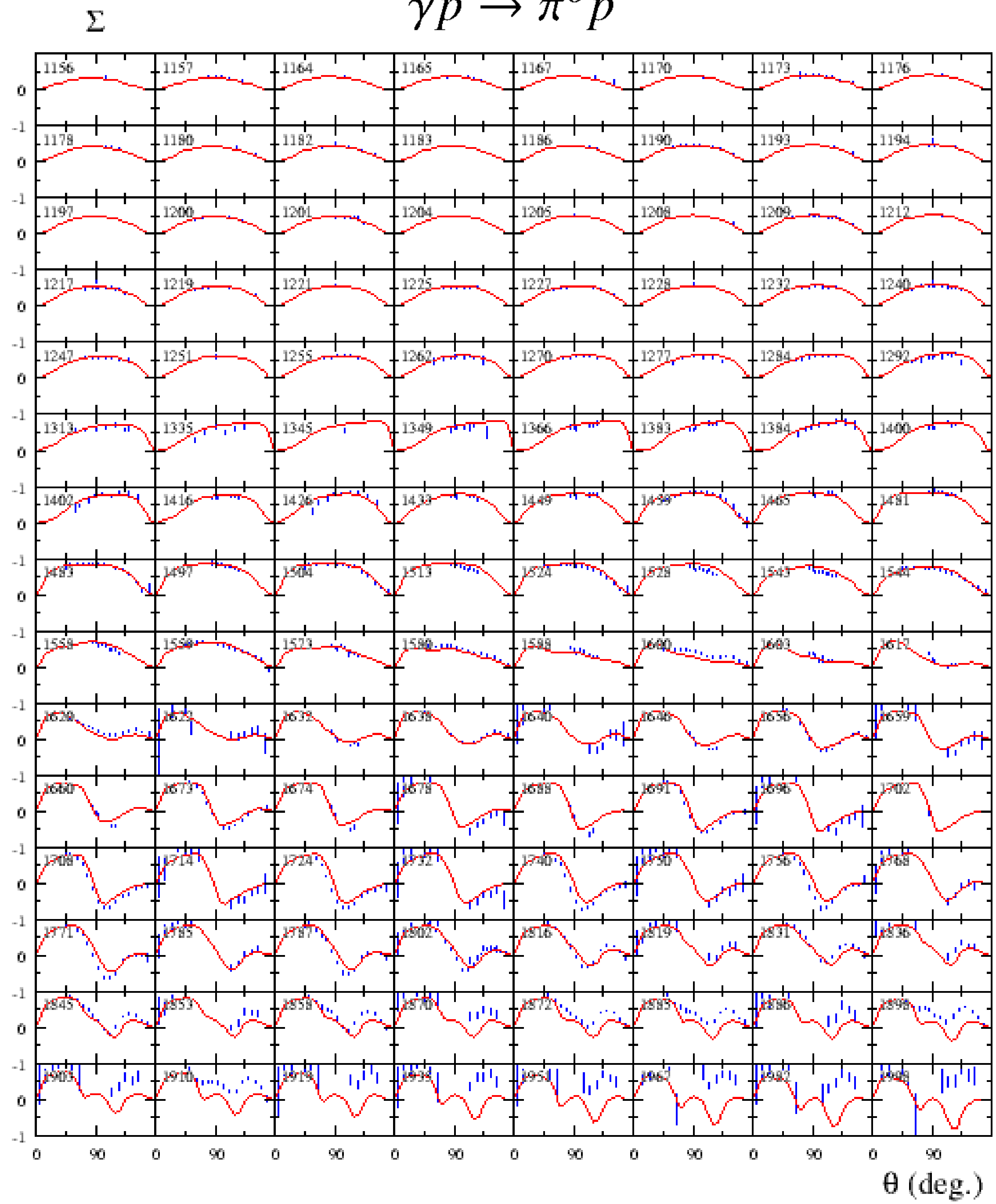


$\gamma p \rightarrow \pi^0 p$

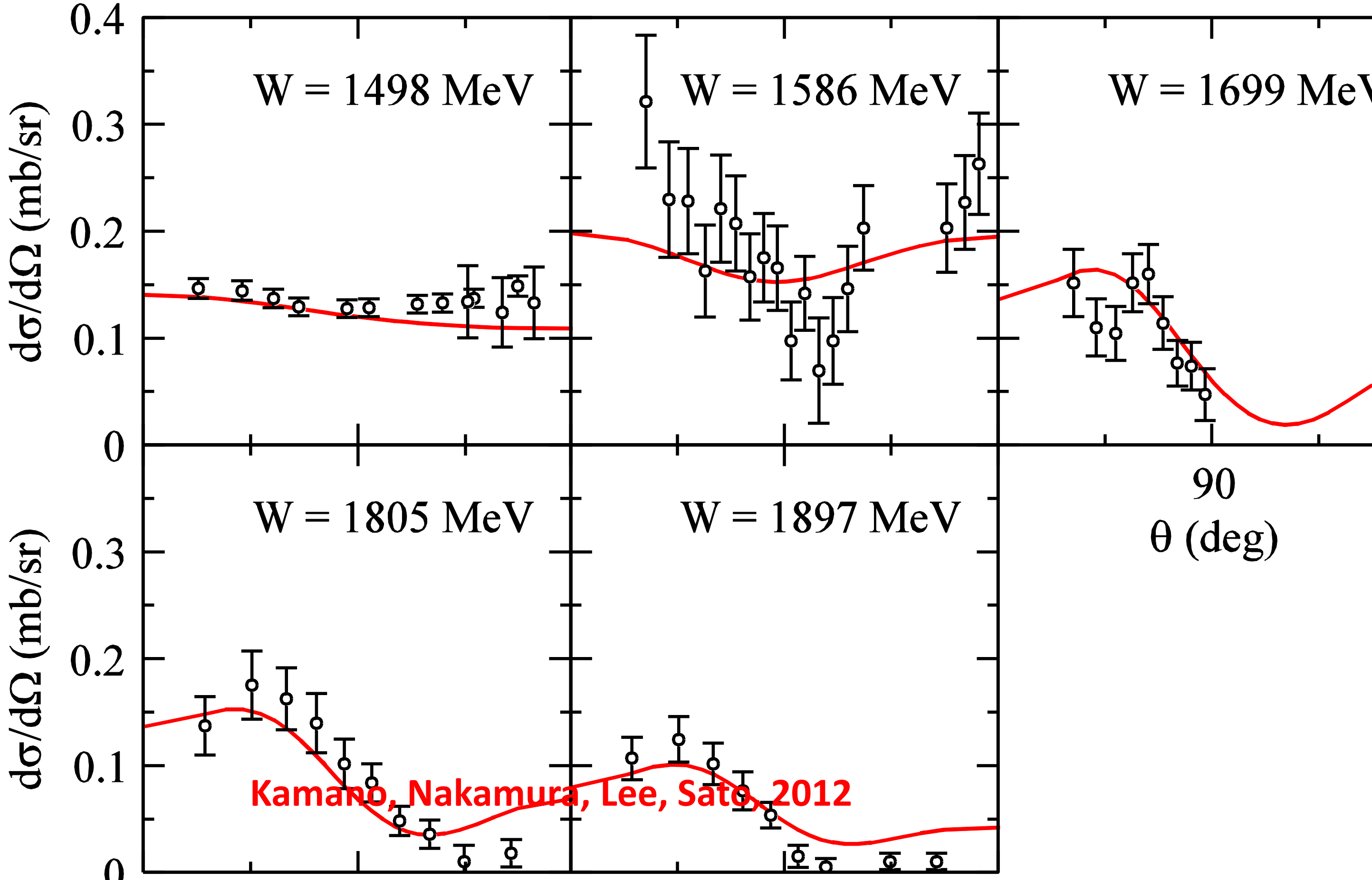
Kamano, Nakamura, Lee, Sato, 2012



Eta production reactions

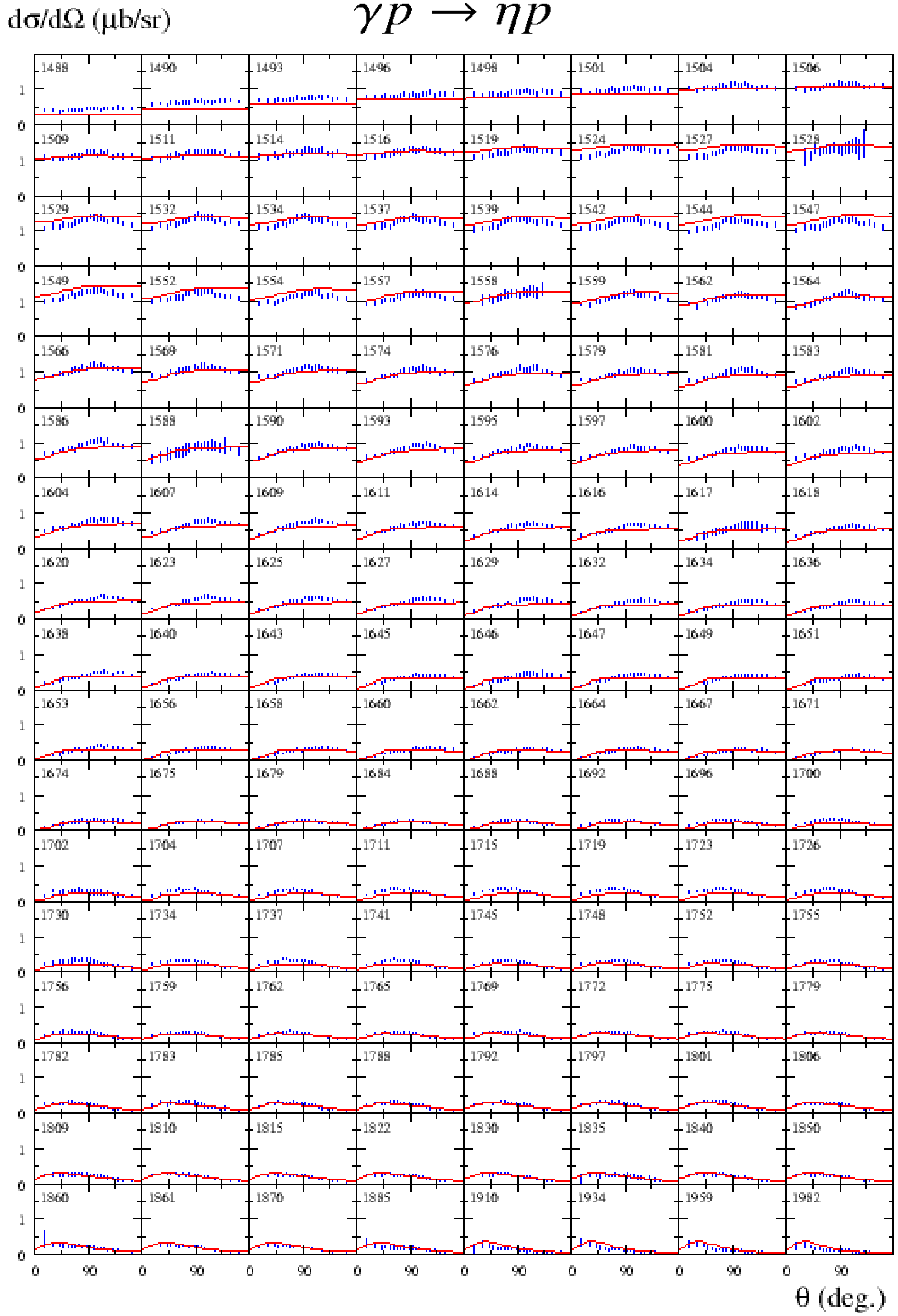
$\pi^- p \to \eta n$

$d\sigma/d\Omega$ (mb/sr)



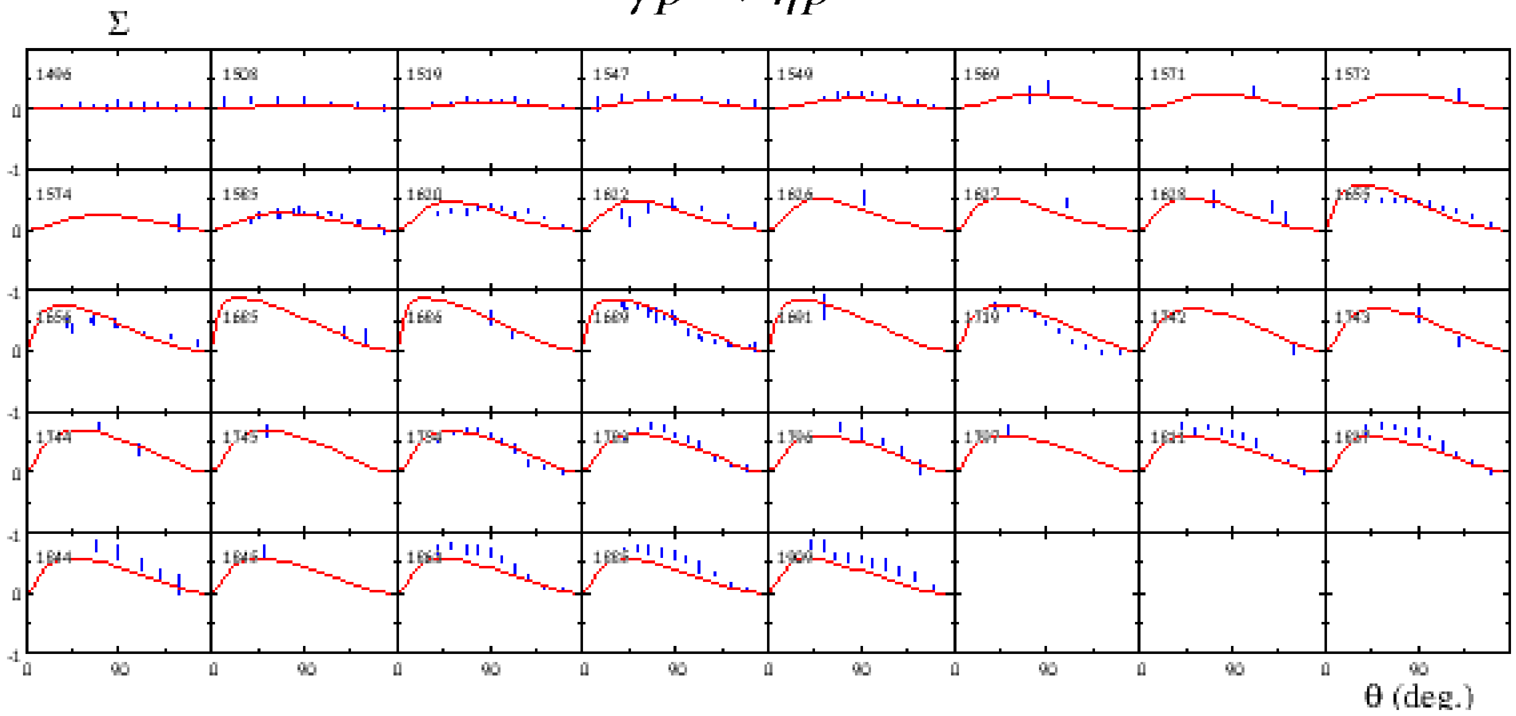
$\gamma p \rightarrow \eta p$

Kamano, Nakamura, Lee, Sato, 2012



Kamano, Nakamura, Lee, Sato, 2012

$\gamma p \rightarrow \eta p$

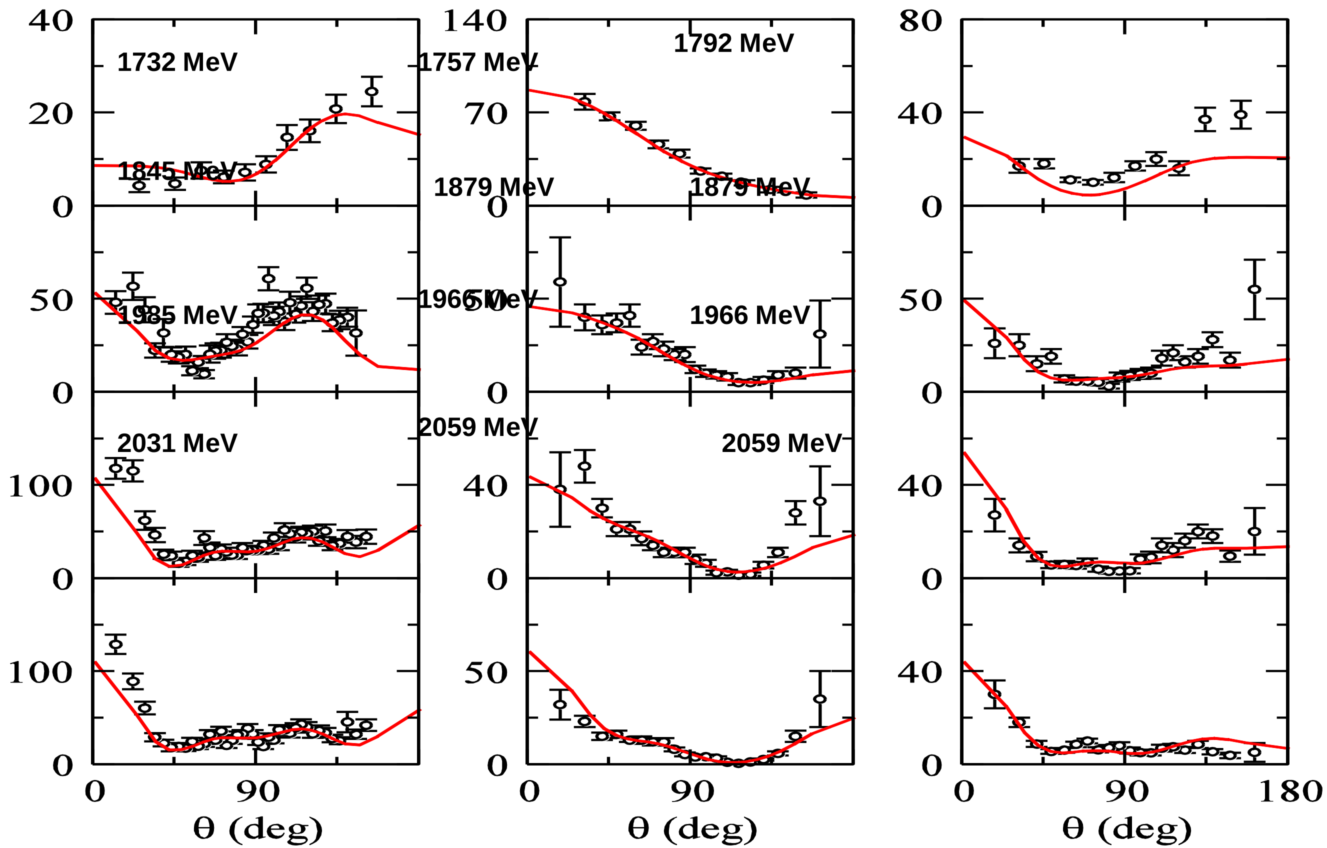


KY production reactions

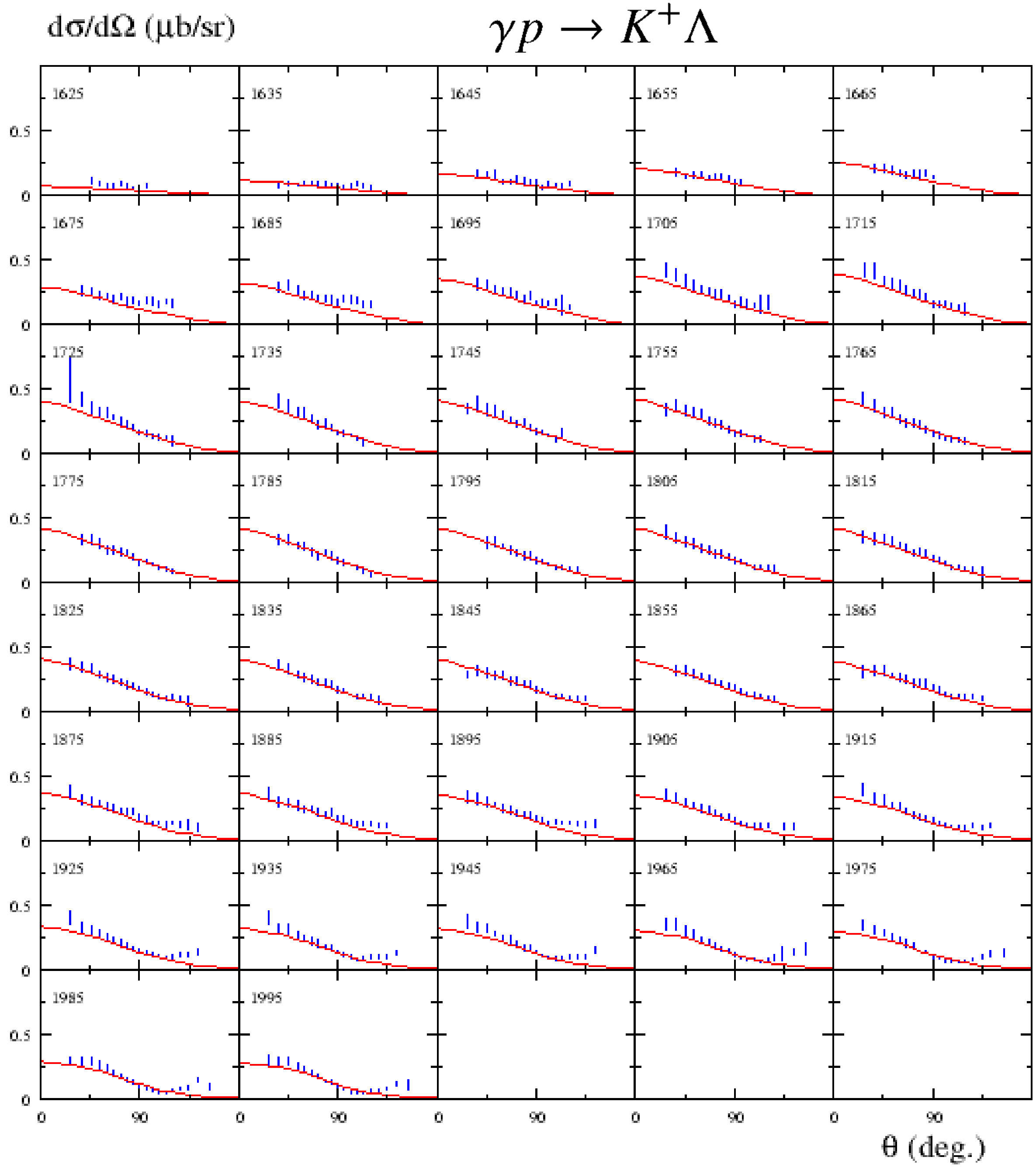
Kamano, Nakamura, Lee, Sato, 2012

Pion-Nucleon -> K+ Lambda

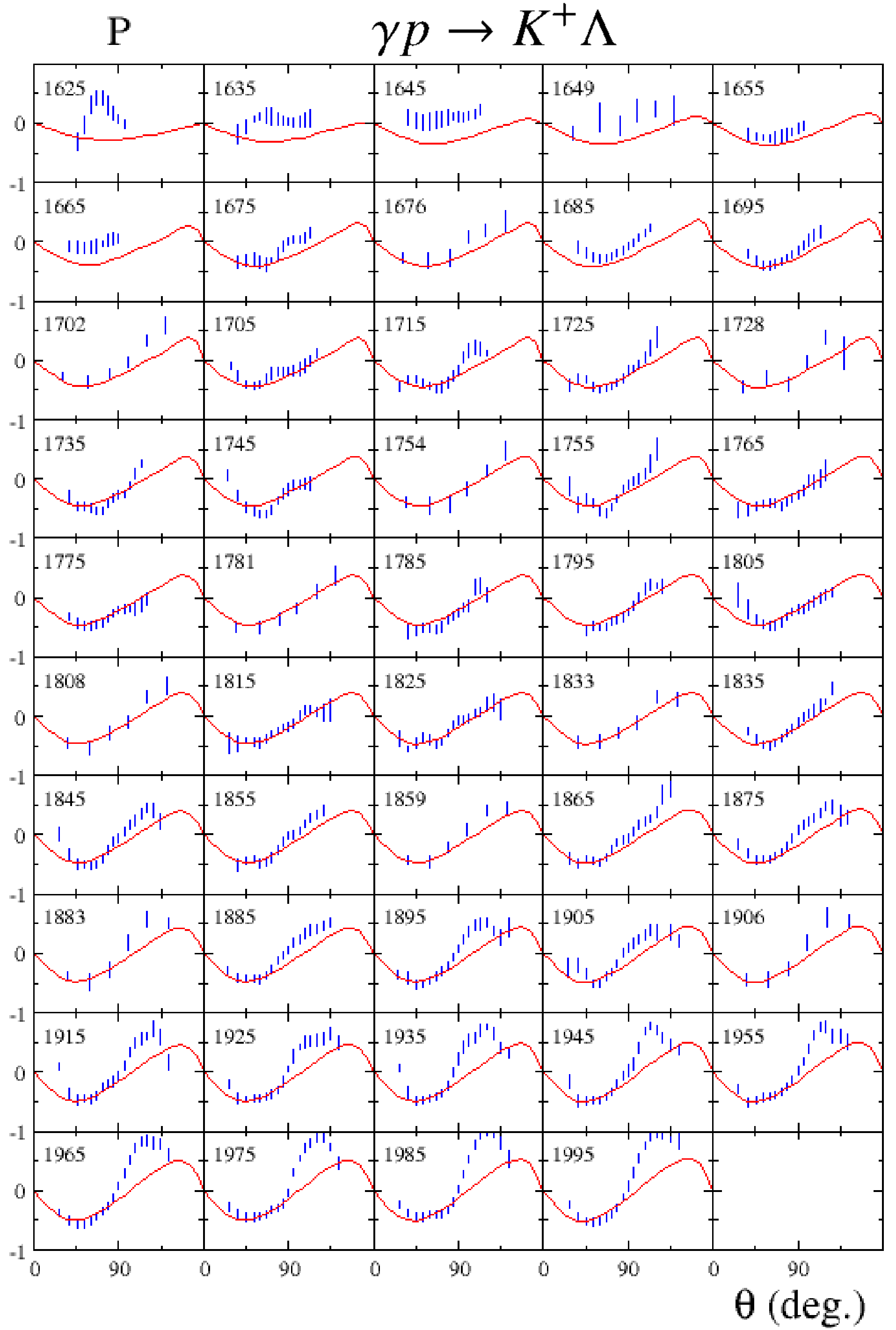
$d\sigma/d\Omega$ (μb/sr)



Kamano, Nakamura, Lee, Sato, 2012



Kamano, Nakamura, Lee, Sato, 2012



Single pion electroproduction ($Q^2 > 0$)

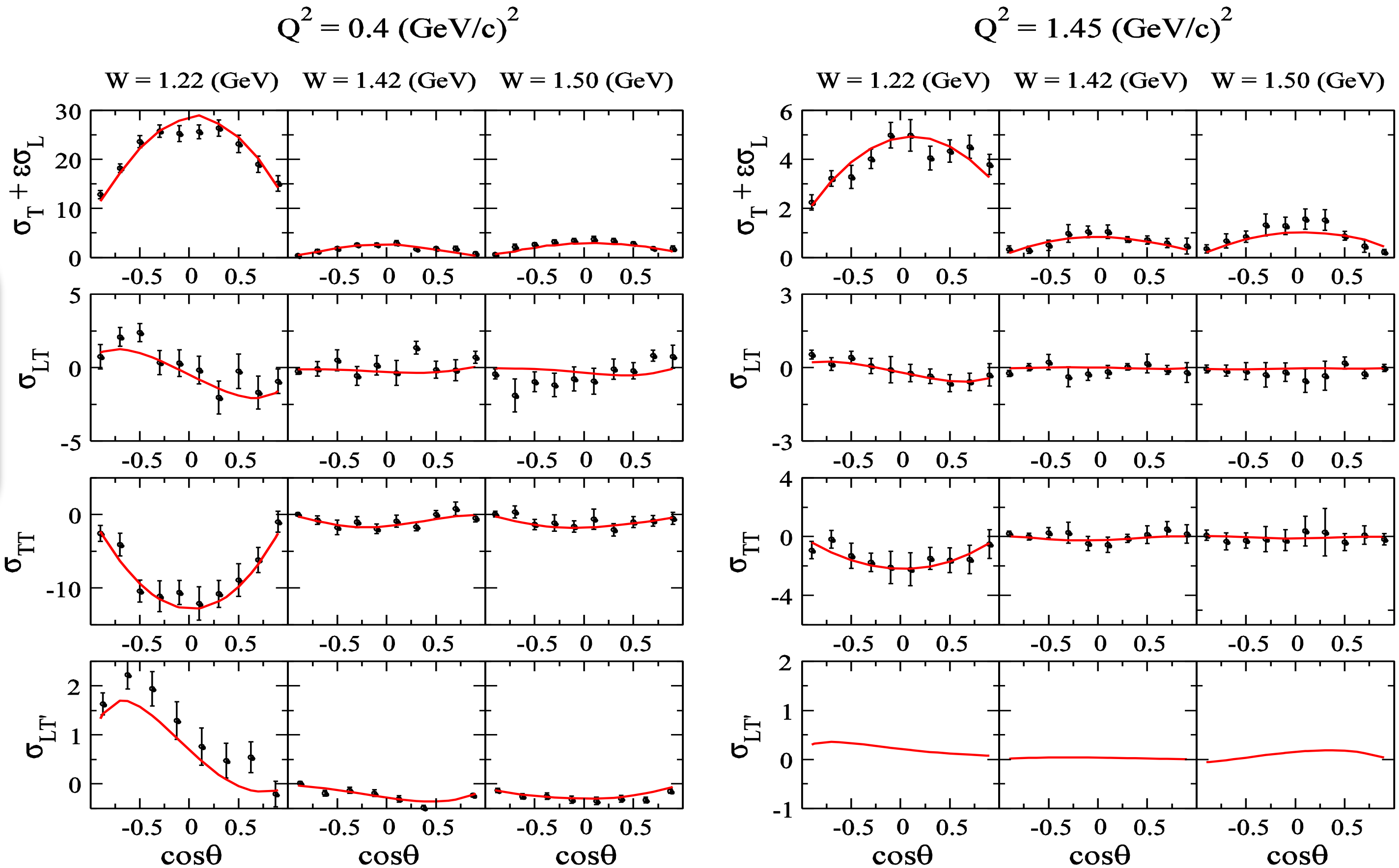
Julia-Diaz, Kamano, Lee, Matsuyama, Sato, Suzuki, PRC80 025207 (2009)

Fit to the structure function data from CLAS

$$\sigma_\alpha = \sigma_\alpha(W, Q^2, \cos\theta^*_\pi)$$

$$\frac{d\sigma^5}{dE_{e'}d\Omega_{e'}d\Omega^*_\pi} = \Gamma_\gamma\left[\sigma_T + \epsilon\sigma_L + \sqrt{2\epsilon(1+\epsilon)}\sigma_{LT}\cos\phi^*_\pi + \epsilon\sigma_{TT}\cos 2\phi^*_\pi + h_e\sqrt{2\epsilon(1-\epsilon)}\sigma_{LT'}\sin\phi^*_\pi\right].$$

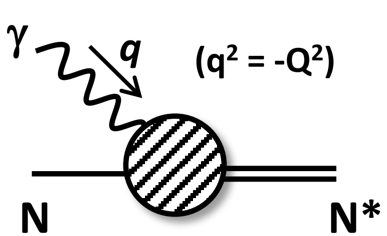
p (e,e' π^0) p



W < 1.6 GeV

Q^2 < 1.5 $(\text{GeV/c})^2$

$\Gamma^{\text{bare}}_{\gamma N \to N^*}$ is determined at each Q^2.



N-N* e.m. transition form factor

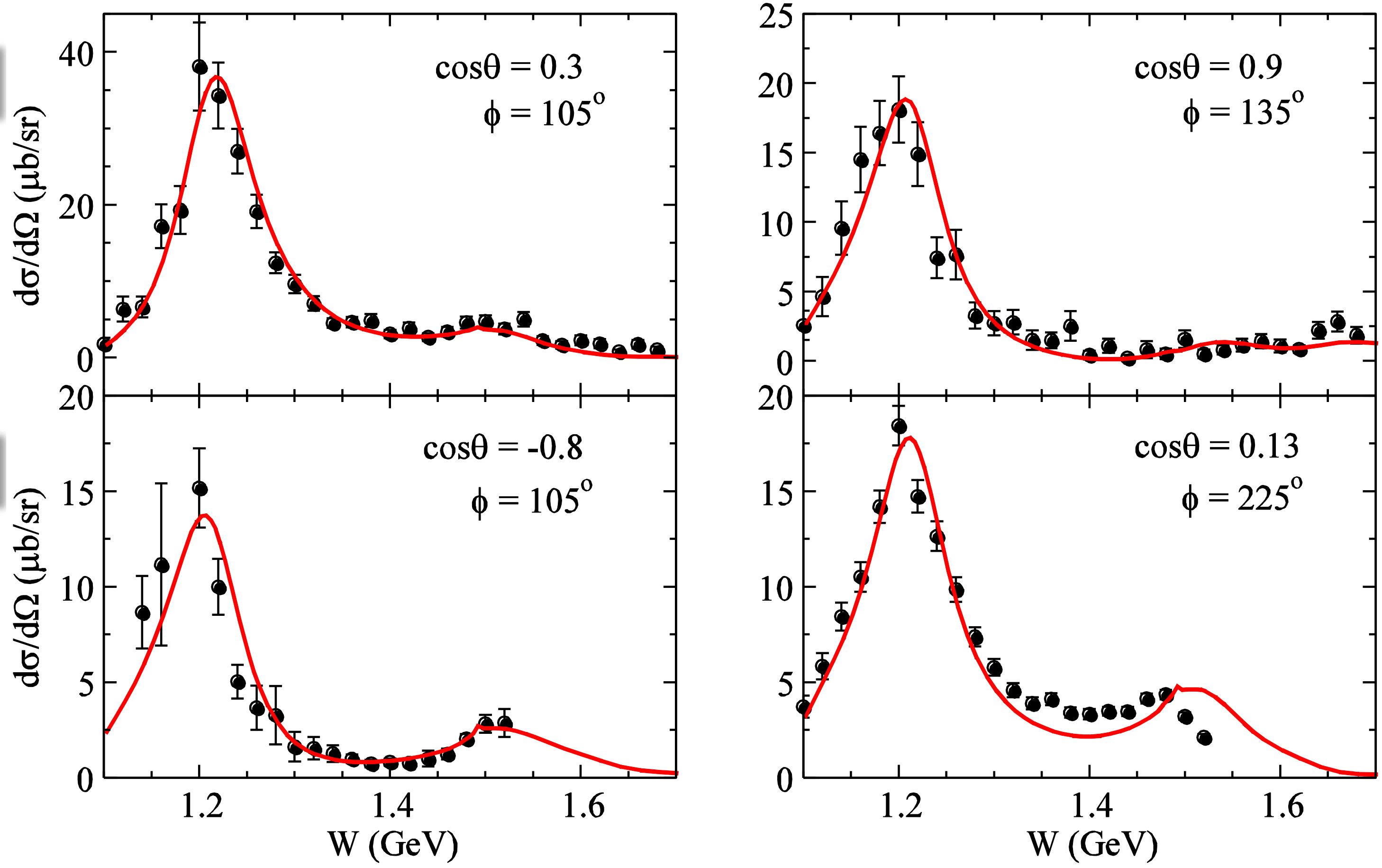
Single pion electroproduction ($Q^2 > 0$)

Julia-Diaz, Kamano, Lee, Matsuyama, Sato, Suzuki, PRC80 025207 (2009)

Five-fold differential cross sections at $Q^2 = 0.4$ (GeV/c)^2

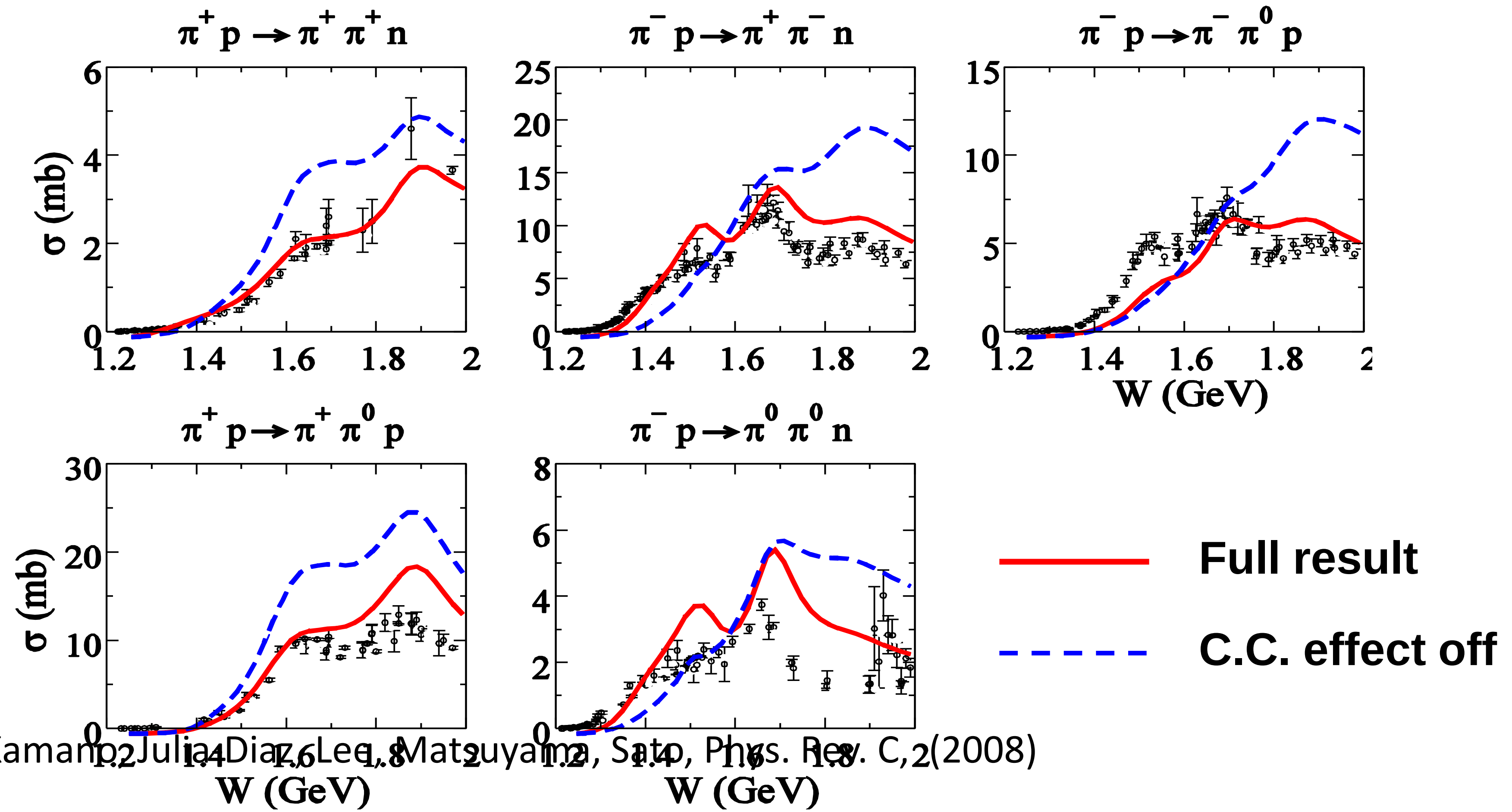
p (e,e' π^0) p

p (e,e' π^+) n



pi N → pi pi N reaction

Parameters used in the calculation are from πN → πN analysis.

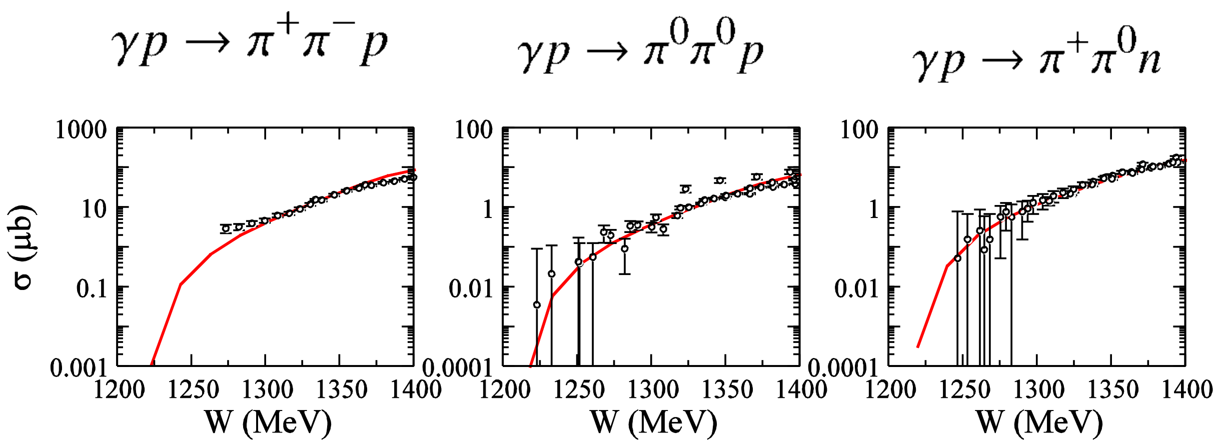


Kamano, Julia-Diaz, Lee, Matsuyama, Sato, Phys. Rev. C, (2008)

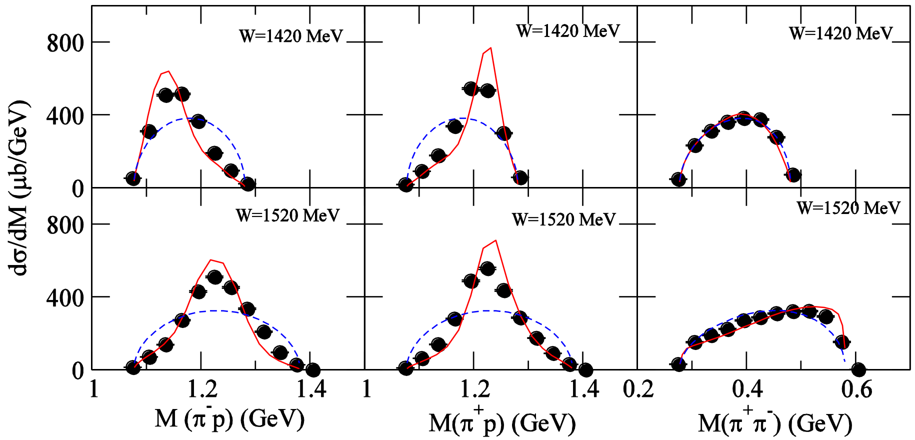
Double pion photoproduction

Kamano, Julia-Diaz, Lee, Matsuyama, Sato, PRC80 065203 (2009)

Parameters used in the calculation are from $\pi N \rightarrow \pi N$ & $\gamma N \rightarrow \pi N$ analyses.



- ✓ **Good description near threshold**
- ✓ **Reasonable shape of invariant mass distributions**
- ✓ **Above 1.5 GeV, the total cross sections of $p\pi^0\pi^0$ and $p\pi^+\pi^-$ overestimate the data.**



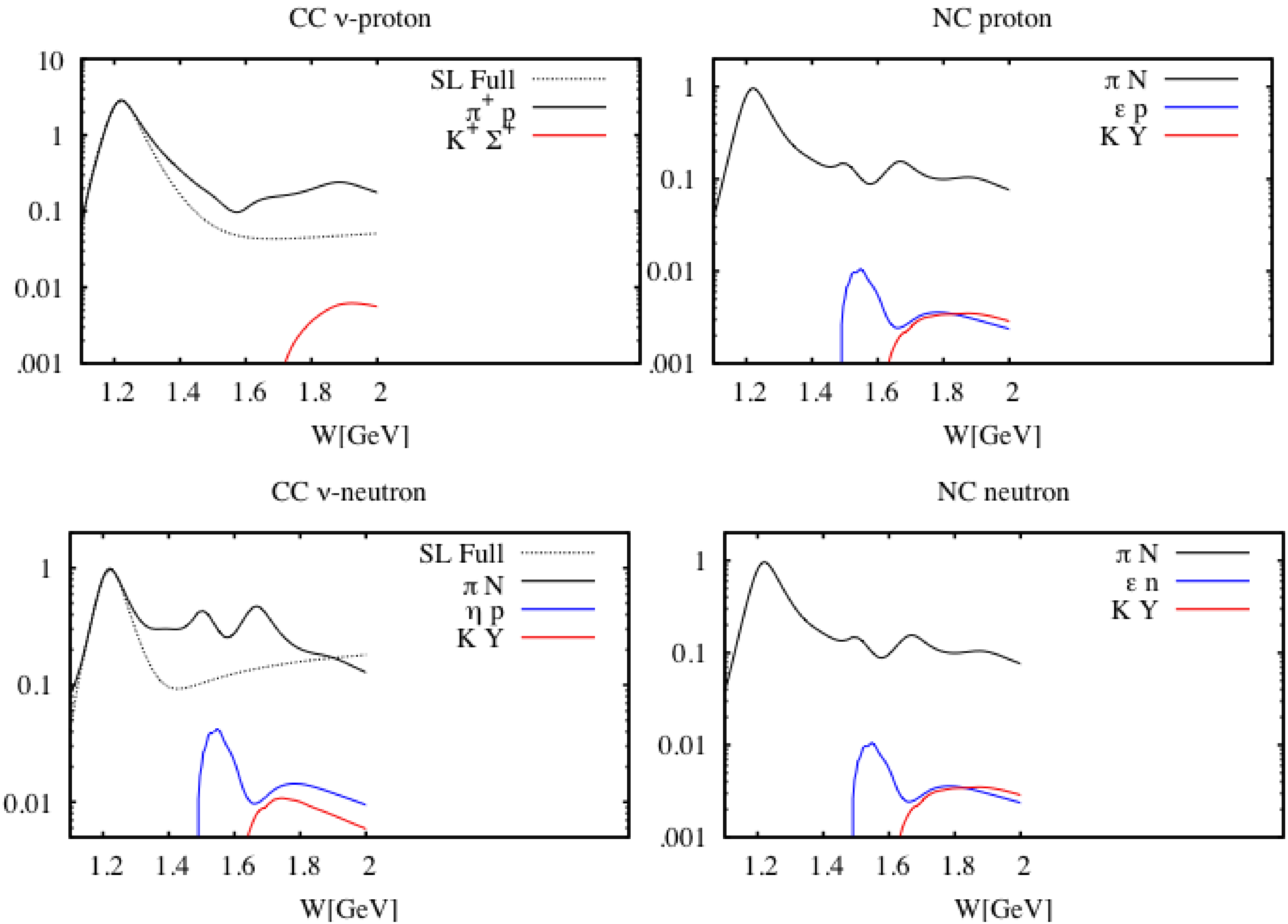
2012 : extended to include axial current A^{μ}

→

Predict neutrino-nucleon total cross sections

→

Provide input to analyze neutrino oscillation experiments



CC ν-proton
SL Full
$\pi^+ p$
$K^+ \Sigma^+$
10
1
0.1
0.01
.001
1.2
1.4
1.6
1.8
2
W[GeV]
NC proton
π N
ε p
K Y
1
0.1
0.01
.001
1.2
1.4
1.6
1.8
2
W[GeV]
CC ν-neutron
SL Full
π N
η p
K Y
1
0.1
0.01
1.2
1.4
1.6
1.8
2
W[GeV]
NC neutron
π N
ε n
K Y
1
0.1
0.01
.001
1.2
1.4
1.6
1.8
2
W[GeV]

Remarks on numerical tasks :

1. DCC is not an algebraic approach like analysis based on polynomials or K-matrix

   Solve coupled integral equations with 8 channels by inverting 400 × 400 complex matrix formed by about 150 Feynman diagrams for each partial waves (about 20 partial waves up to L=5)

2. Fits to about 28,000 data points

3. New mechanisms are usually needed to develop theoretically to improve the fits, not just blindly vary the parameters

4. Analytic continuation for extracting resonances requires careful analysis of the analytic structure of driving terms (150 Feynman amplitudes) of the coupled integral equations with 8 channels

5. Typically, we need

240 processors
using supercomputer Fusion at ANL
NERSC at LBL

We have used 200,000 hours in January-February, 2012 for 8-channel analysis