Dynamical Model Analysis of Hadron Resonances (III)

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Lecture III:

1. Explain how resonances are determined from experiments and defined theoretically

2. Explain the extraction of resonances in exactly soluble models

3. Nucleon resonances extracted from 8-channels analysis of data of πN, $\gamma N \to \pi N$, ηN, $K\Lambda$, $K\Sigma$

4. Interpretations of extracted resonance parameters

What are nucleon resonances ?

Experimental fact:

Excited Nucleons (N*) are unstable and coupled with meson-baryon continuum to form nucleon resonances

→

Nucleon resonances contain information on

a. Structure of N*
b. Meson-baryon Interactions

How are Nucleon Resonances extracted from data ?

Assumptions:

- ✓ Partial-wave amplitudes are analytic functions F (E) on the complex E-plane

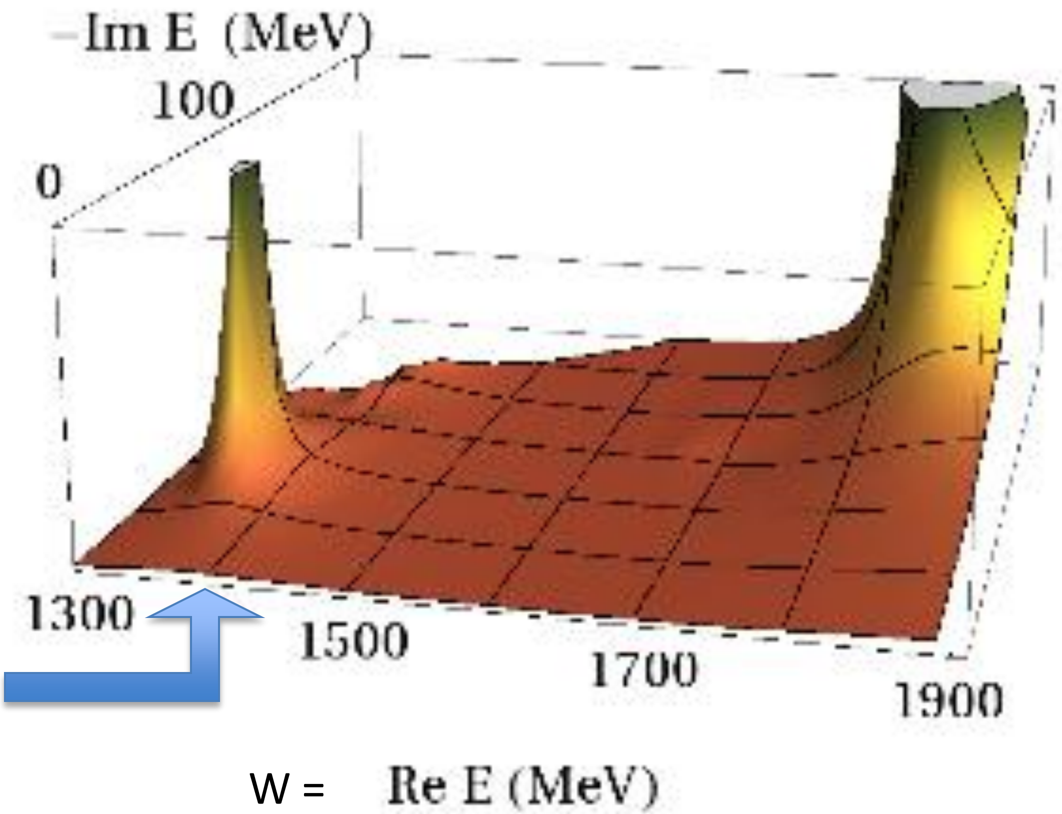
- ✓ F (E) are defined uniquely by the partial-wave amplitudes A (W) determined from accurate and complete experiments on physical W-axis

- ✓ The Poles of F(E) are the masses of Resonances of the underlying fundamental theory (QCD).

Analytic continuation

F (E) ← A (W) ← Data

F (E)



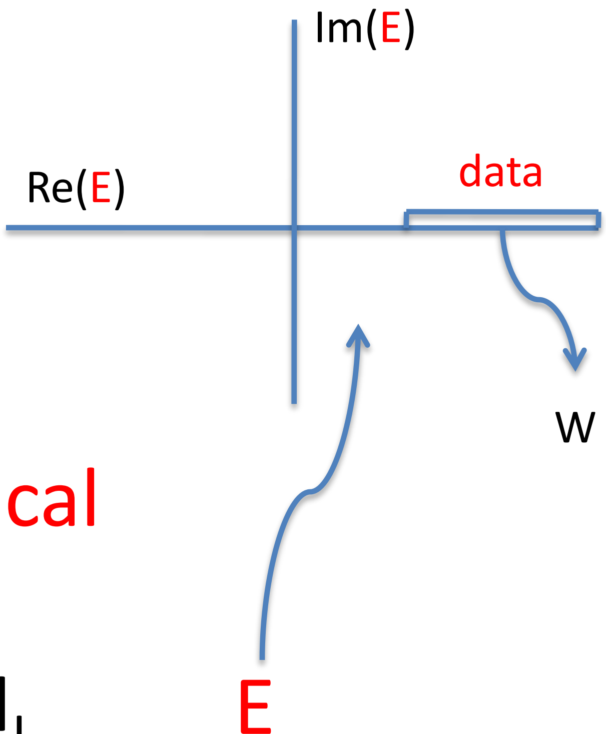
A (W)

W = Re E (MeV)

In Dynamical Model Analysis:

a. Partial scattering amplitude A(W) is defined by Hamiltonian

$$A(W) = H_I + H_I \frac{1}{W-H+i\varepsilon} H_I$$



b. Analytic continuation : dynamical

Solve $$F(E) = H_I + H_I \frac{1}{E-H+i\varepsilon} H_I$$

Theoretical framework :

(Gamow, Peierls, Dalitz, Moorhouse, Bohm….)

Resonances are the eigenstates of the Hamiltonian with outgoing-wave boundary condition

$$H|\psi> = E|\Psi>$$

$E= k^2/2m$ scattering state

$E= (k_R + ik_I)^2/2m$ resonance

$$\psi(r) \longrightarrow f(\theta)\frac{e^{i(k_R + ik_I)r}}{r}$$

Resonance

$$\psi(r) \longrightarrow e^{-ikr} + f(\theta)\frac{e^{ikr}}{r}$$

Scattering

Since $E_R = (k_R + ik_I)^2/(2m)$

$\Longrightarrow$

Eigenstate with E_R can have momentum either $(k_R + ik_I)$ or $(-k_R - ik_I)$

Resonance : $k_R > 0,\ k_I < 0$

$$E_R = (k_R + ik_I)^2/2m$$
$$= (k^2_R - k^2_I)/2m + ik_R k_I/m$$

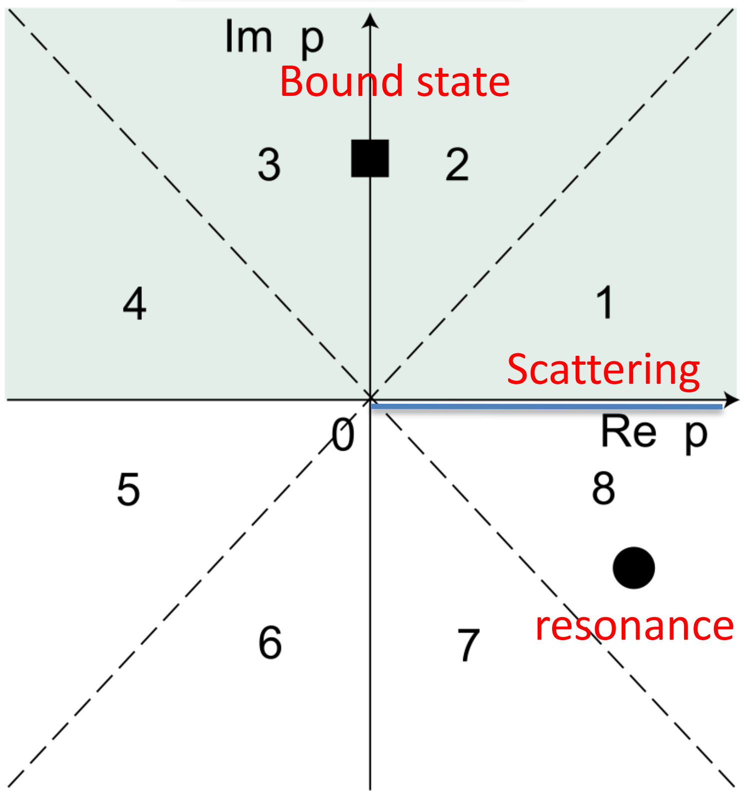
$\mathrm{Im}(E_R) < 0$

$p = p_R + i\, p_I$

$p_I > 0$: physical sheet

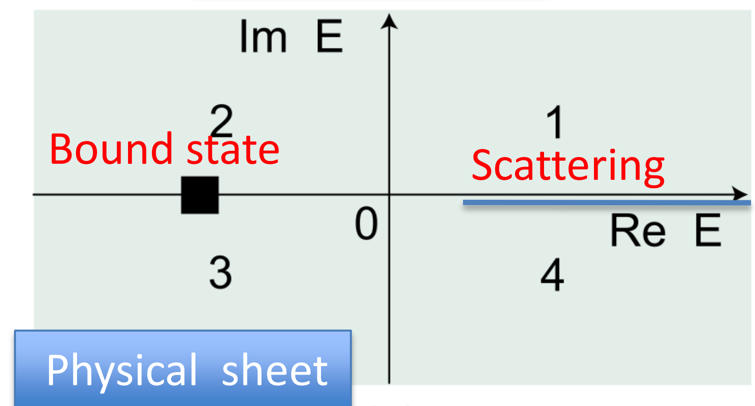
< 0 : unphysical sheet

p - plane

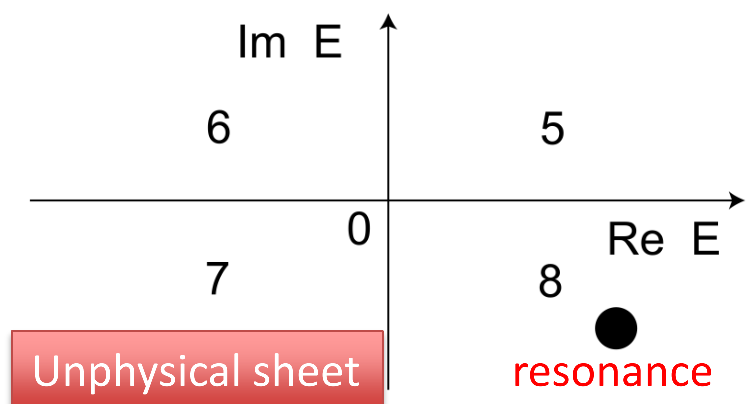


(a)

E – plane



(b)



(c)

Resonance position can be found from solutions on unphysical sheet ($k_I < 0$)

$$F(E) = H_I + H_I \frac{1}{E-H} H_I$$

Eigenstates of H

$$= H_I + \sum_i H_I \frac{|\psi_i\rangle\langle\psi_i|}{E-E_i} H_I$$

$$\xrightarrow{E \to E_P} \frac{R}{E - E_P}$$

Pole on unphysical sheet

$H|\psi_R\rangle = E_P|\Psi_R\rangle$; $E_P = (k_R + ik_I)^2/2m$ resonance

$k_R > 0,\ k_I < 0$

Interpretations :

1. Time dependence of Resonance state

$$\Psi_{E_p}(r,t) = e^{-iE_pt}\psi_{E_p}(r) = e^{-iE_rt}e^{-E_it}\psi_{E_p}(r)$$

$\rightarrow$ $E_P = E_R - iE_i, \quad E_R, E_I > 0$

It is a "virtual" decaying state :

$$P(t) \quad = \quad |<\Psi_{E_p}(t)|\Psi_{E_p}(t)>|^2 = e^{-2E_it}|<\psi_{E_p}|\psi_{E_p}>|^2$$

2. E_p is on unphysical sheet : $E_P = k^2_P/2m; \quad k_P = k_R - i\,k_I, \quad k_I > 0$

$$\begin{aligned} u_{E_p}(r \rightarrow \infty) &= Ne^{ik_pr} \\ &= Ne^{ik_rr}e^{k_ir} \rightarrow \infty \end{aligned}$$

$\rightarrow$

The wavefunction is not square integrable

$\rightarrow$

$|\psi_{Ep}>$ is not in the Hilbert space of H, and therefore is "virtual" and unstable.

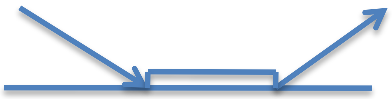
To illustrate, consider several exactly soluble model Hamiltonians

1. one-channel, one resonance
2. two-channel, one resonance
3. two-channel, two resonances

One-channel, one-resonance

$$t(p',p;E) = v(p',p;E) + \int_{C_0} dq\, q^2 \frac{v(p',q;E)t(q,p;E)}{E - E_1(q) - E_2(q)}$$

$$v(p',p;E) = \frac{g(p')g(p)}{E - M_0}$$



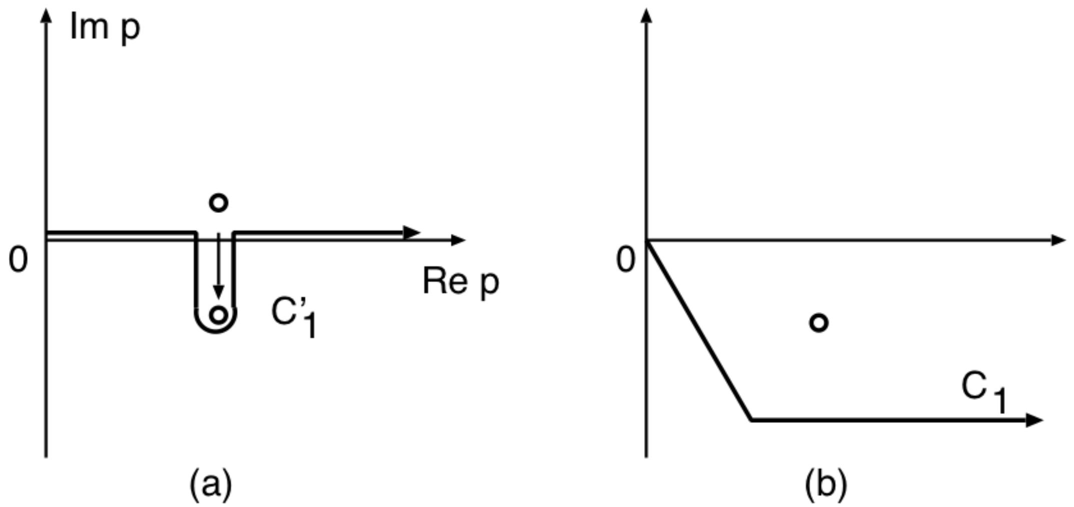
bare

$$g(p) = \frac{\lambda}{1 + p^2/\beta^2}$$

Exact solution :

$$t(p',p;E) = \frac{g(p')g(p)}{E - M_0 - \Sigma(E)}$$ = 0 at E = E_P of resonance

$$\Sigma(E) = \int_{C_0} dp\, p^2 \frac{g^2(p)}{E - E_1(p) - E_2(p)}$$



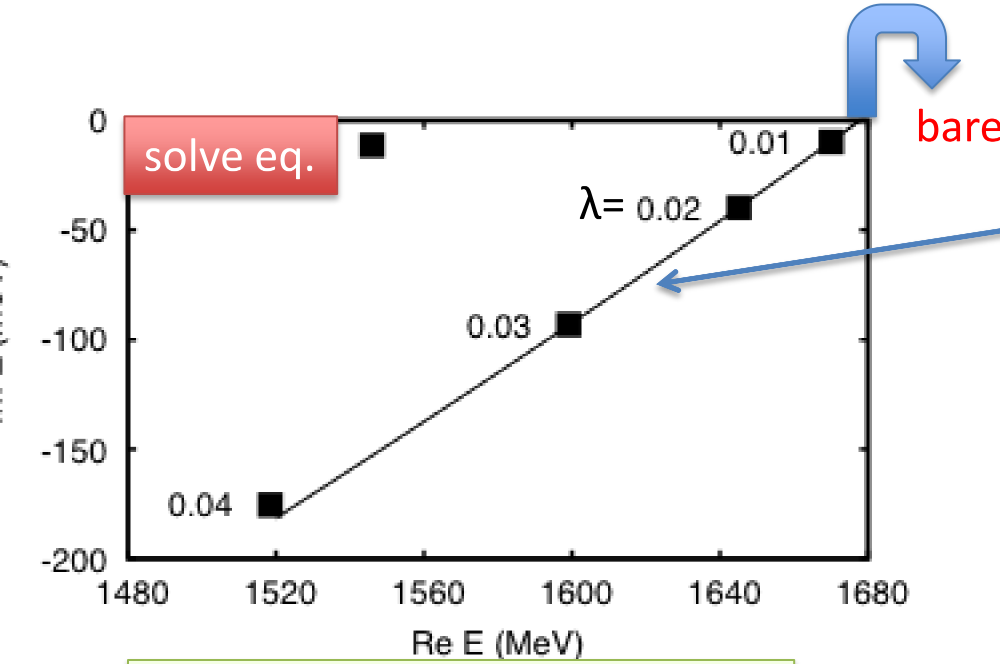
To find resonance pole, need to choose $C_0 = C'_1$ or C_1

$$t(p',p;E) = v(p',p;E) + \int_{C_0} dq\, q^2 \frac{v(p',q;E)t(q,p;E)}{E - E_1(q) - E_2(q)}$$

$$v(p',p;E) = \frac{g(p')g(p)}{E - M_0}$$

$$g(p) = \frac{\lambda}{1 + p^2/\beta^2}$$

Set E(p) = m + p_2/2m → Exactly soluble



Trajectory of pole as λ=0 -> 0.04

$$E_P - M_0 - \Sigma(E_P) = 0$$

$$\Sigma(E) = \frac{\pi\mu\beta^3\lambda^2}{2(p_0 + i\beta)^2}$$

$$t(p', p; E) = v(p', p; E) + \int_{C_0} dq\, q^2 \frac{v(p', q; E)t(q, p; E)}{E - E_1(q) - E_2(q)}$$

$$v(p', p; E) = \frac{g(p')g(p)}{E - M_0}$$

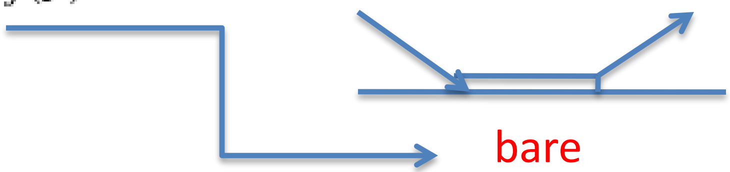
$$g(p) = \frac{\lambda}{1 + p^2/\beta^2}$$

solve integral equation with deformed path C_0= C_1

Two-channels, one-resonance

$$t_{ij}(p',p;E) = v_{ij}(p',p;E) + \sum_{k=1,2}\int_{C_0} dq\, q^2 \frac{v_{ik}(p',q)t_{kj}(q,p;E)}{E - E_{k1}(q) - E_{k2}(q)}$$

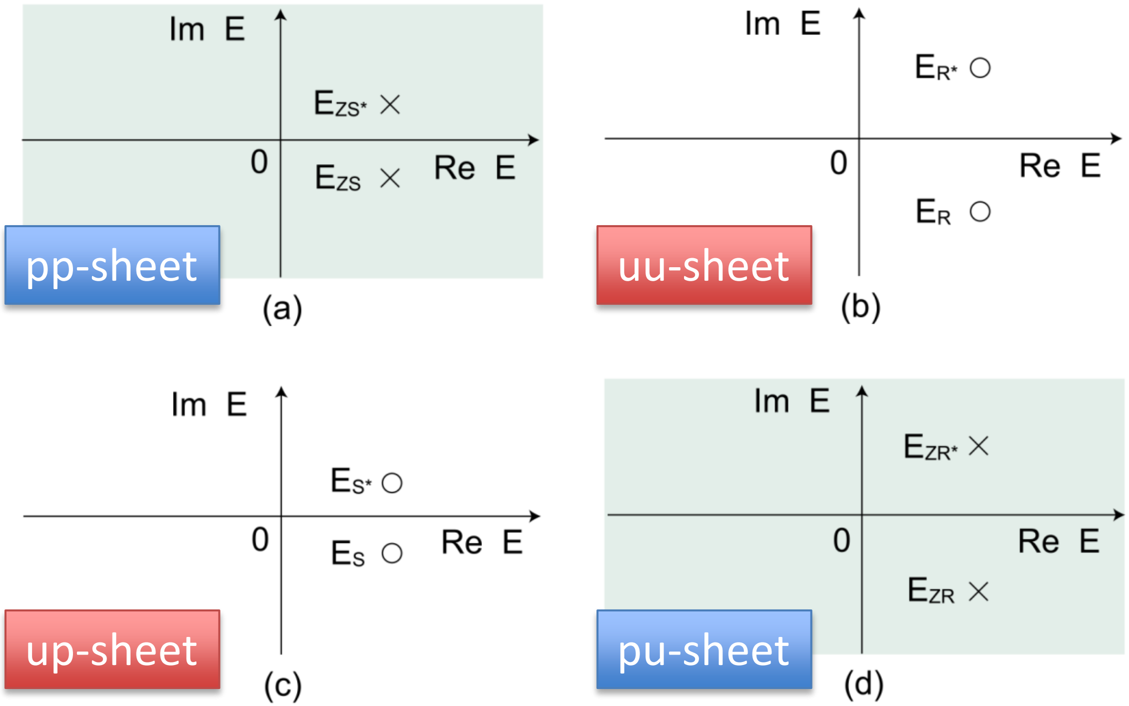
$$v_{ij}(p',p;E) = g_i(p')\frac{1}{E - M_0} g_j(p)$$



$$g_i(p) = \frac{\lambda_i}{1 + p^2/\beta_i^2}$$

$$t_{ij}(p',p;E) = \frac{g_i(p')g_j(p)}{E - M_0 - \Sigma_1(E) - \Sigma_2(E)}$$ = 0 at E = E_p of resonance

$$\Sigma_k(E) = \int_{C_0} dp\, p^2 \frac{g_k^2(p)}{E - E_{k1}(p) - E_{k2}(p)}\,.$$



Energy-sheet of two-channel

$$E_P - M_0 - \Sigma_1(E_P) - \Sigma_2(E_P) = 0$$

E_P can be on

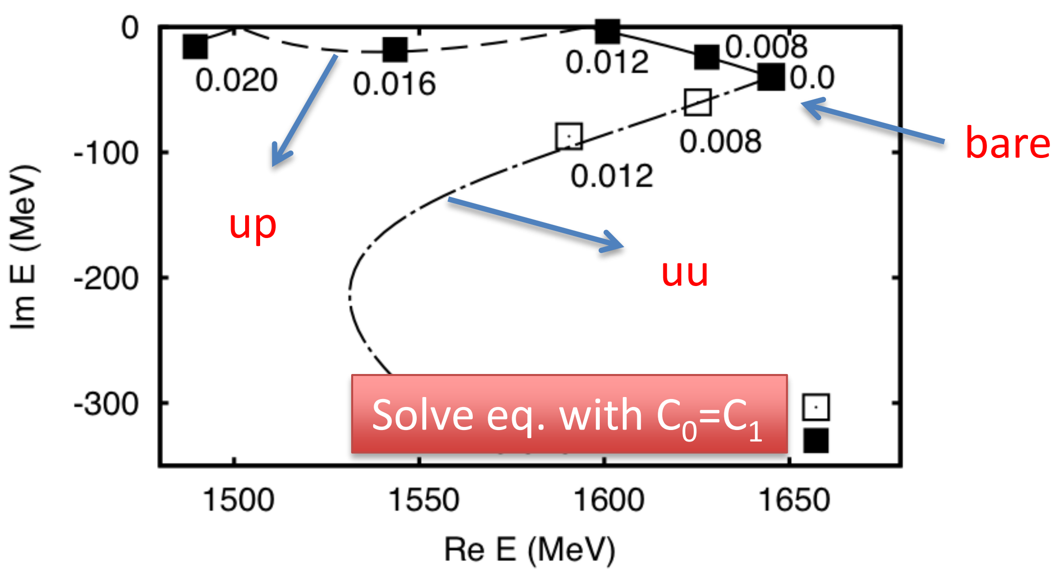
uu, up, pp, pu sheets

Exactly soluble

$$E_P - M_0 - \Sigma_1(E_P) - \Sigma_2(E_P) = 0$$

$$\Sigma_i(E) = \frac{\pi \mu_i \beta_i^3 \lambda_i^2}{2(p_i + i\beta_i)^2}$$

Set $E(p) = m + p_2/2m$



Trajectories of poles as $\lambda_1 = 0$ -> 0.02

Finding :

1. One bare can evolve into two resonances

2. one resonance on uu-sheet
   one resonance on up-sheet

Two-channels, two-esonances

$$t_{ij}(p',p;E) = v_{ij}(p',p;E) + \sum_{k=1,2} \int_{C_0} dq\, q^2 \frac{v_{ik}(p',q)t_{kj}(q,p;E)}{E - E_{k1}(q) - E_{k2}(q)}$$

$$v_{ij}(p',p;E) = g_{i1}(p')\frac{1}{E - M_1}g_{j1}(p) + g_{i2}(p')\frac{1}{E - M_2}g_{j2}(p)$$

Bare states

$$t_{ij}(p',p;E) = \sum_{\alpha,\beta}[D^{-1}(E)]_{\alpha,\beta}\, g_{j,\beta}(p)$$

$$[D(E)]_{\alpha,\beta} = [E - M_\alpha]\delta_{\alpha,\beta} - \Sigma_{\alpha,\beta}(E)$$

Det [D(E)]= 0 at E = E_P of resonance

$$\Sigma_{\alpha,\beta}(E) = \sum_i \int_{C_0} dq\, q^2 \frac{g_{i,\alpha}(q)g_{i,\beta}(q)}{E - E_{i1}(q) - E_{i2}(q) + i\varepsilon}$$

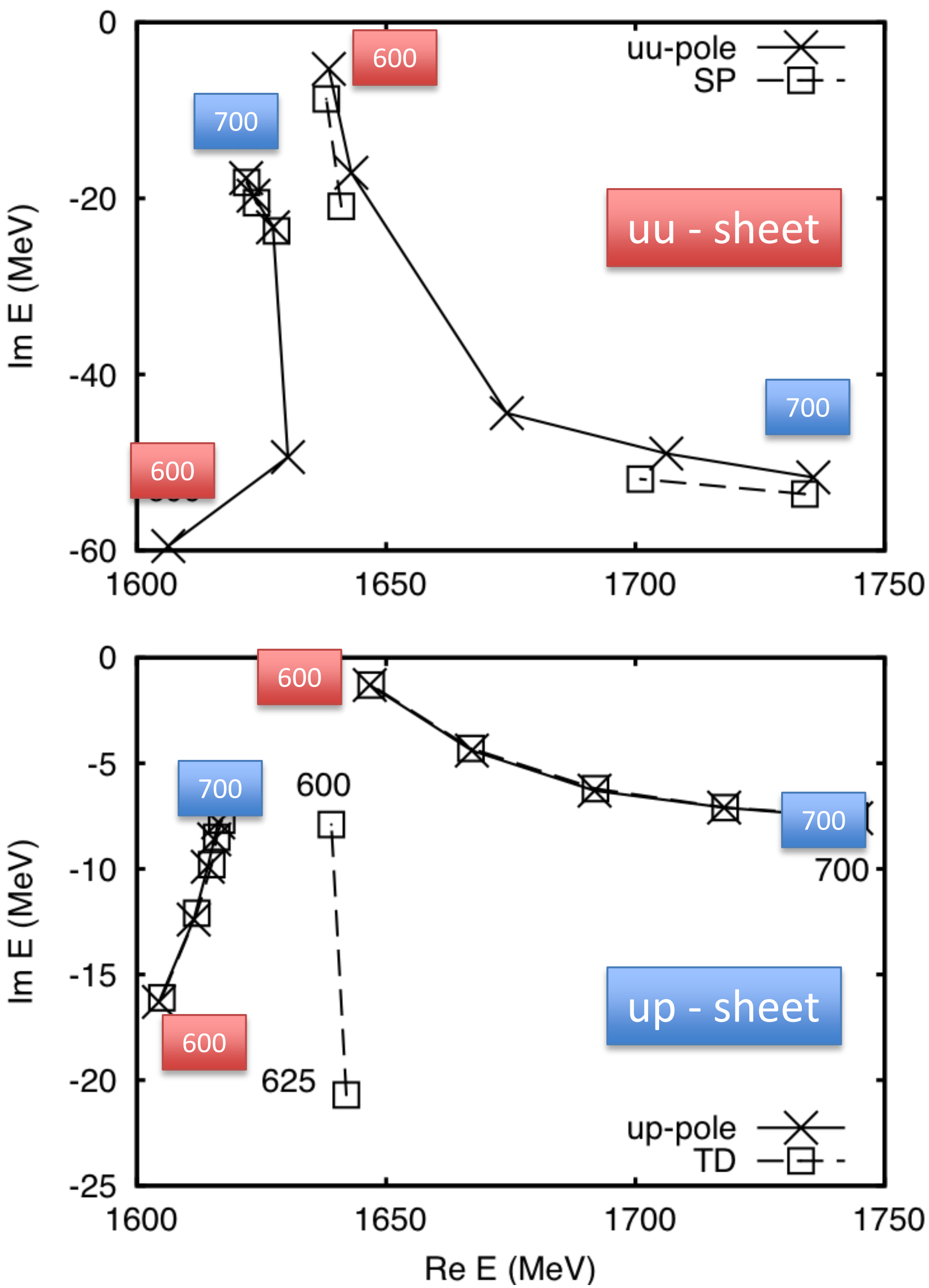
$$g_{i\alpha}(p) = \frac{\lambda_{i\alpha}}{1 + p^2/\beta_{i\alpha}^2}$$

$$\begin{aligned} \text{Det}\, D(E) &= [E - M_1 - \Sigma_{11}(E)][E - M_2 - \Sigma_{22}(E)] - \Sigma_{12}(E)\Sigma_{21}(E) \\ &= 0 \end{aligned}$$

At E = E_p , Resonance pole

$$M_2 = M_1 + \Delta M$$

Mass difference between two bare stste



Trajectories of resonances as $\Delta M = 600 \rightarrow 700$

SP : speed-plot method

TD : time-delayed method

Finding :

1. Two bare can evolve into four resonances

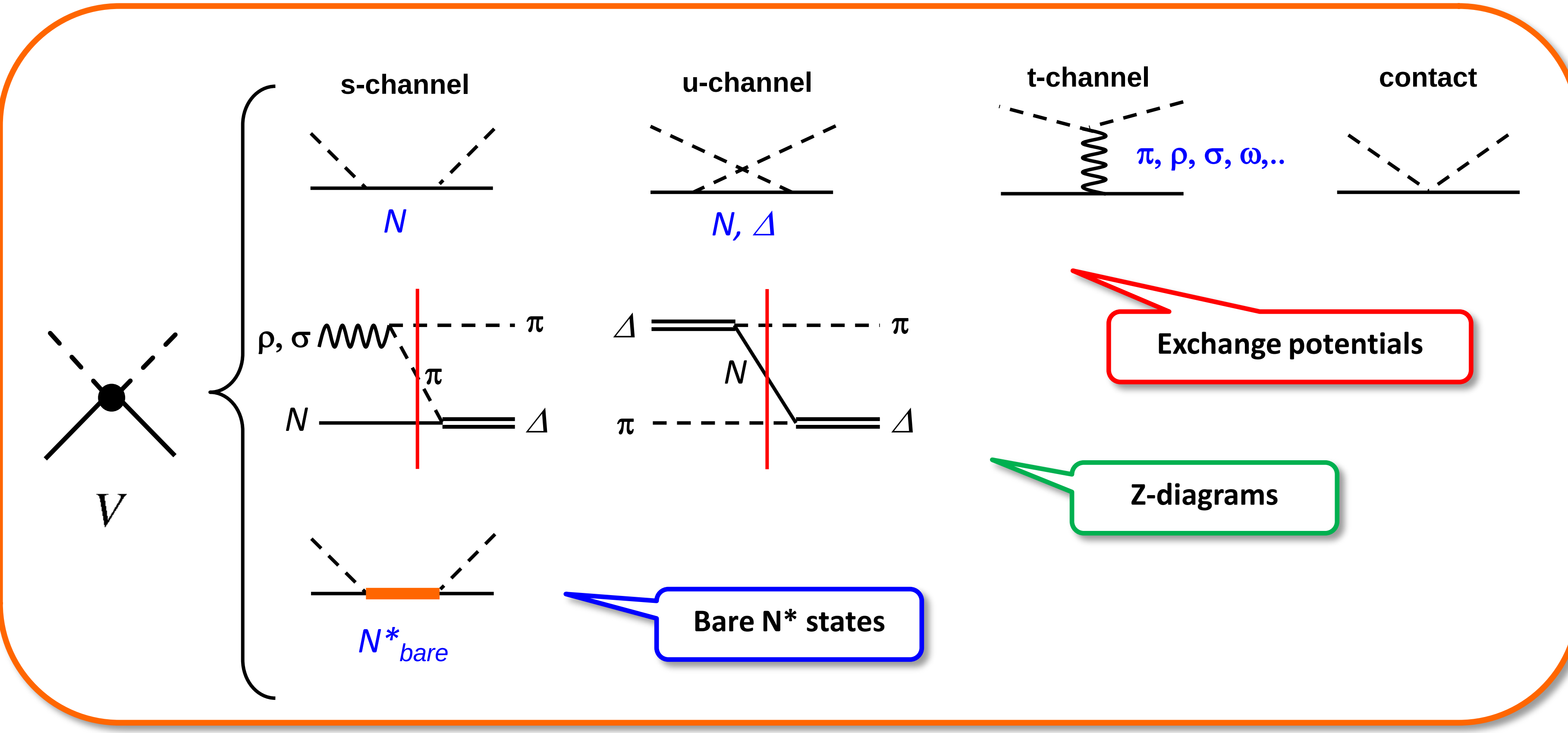
2. two resonances on uu-sheet
   two resonances on up-sheet

For realistic coupled-channel models, the extraction of resonance is further complicated by the mechanisms due to meson-exchange $v_{MB,M'B'}$, $v_{MB,\pi\pi N}$

Need to choose path C_0 carefully

Dynamical coupled-channels (DCC) model for meson production reactions

For details see Matsuyama, Sato, Lee, Phys. Rep. 439,193 (2007)



$$V_{a,b} = v_{a,b} + Z_{a,b} + \sum_{N^*} \frac{\Gamma^{\dagger}_{N^*,a}\Gamma_{N^*,b}}{E - M_{N^*}}$$

Exchange potentials | Z-diagrams | bare N* states

8-channel model parameters have been determined by the fits to the data of

πN, γN -> πN, ηN, $K\Lambda$, $K\Sigma$

→

Extract nucleon resonances

Extraction of N* information

Definitions of

✓ N* masses (spectrum) ➔ Pole positions of the amplitudes

✓ N* → MB, γN decay vertices ➔ Residues$^{1/2}$ of the pole

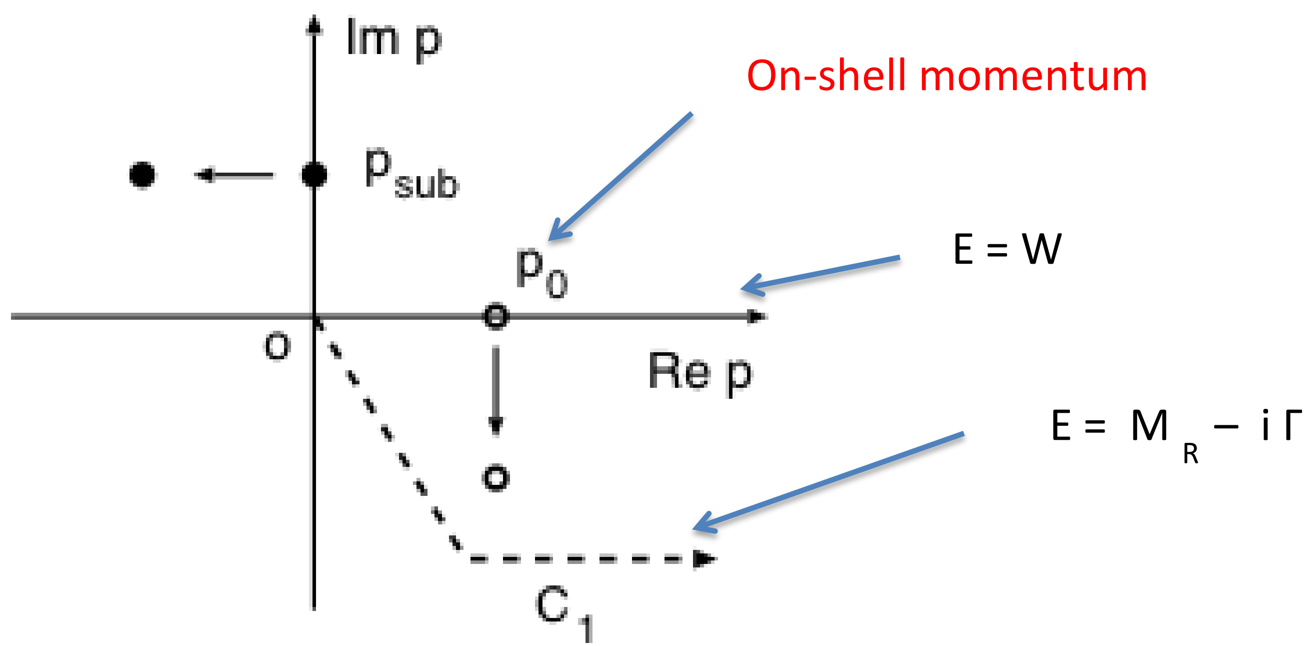
N* → b
decay vertex

$$\langle p_a|\hat{T}(E)|p_b\rangle\Big|_{E\to E_0} \to \frac{\bar{\Gamma}(E_0,p_a)\bar{\Gamma}(E_0,p_b)}{E-E_0} + (\text{regular terms})$$

N* pole position
($Im(E_0) < 0$)

Suzuki, Sato, Lee, Phys. Rev. C79, 025205 (2009)
Phys. Rev. C 82, 045206 (2010)

$$T^{(LS\,J)}_{a,b}(p_a, p_b; E) = V^{(LS\,J)}_{a,b}(p_a, p_b; E) + \sum_c \int_0^\infty q^2 dq V^{(LS\,J)}_{a,c}(p_a, q; E) G_c(q; E) T^{(LS\,J)}_{c,b}(q, p_b; E)$$



Search poles on 2^n sheets of Riemann surface

$n = 8$

Search on the sheets where

a. close channels: physical ($k_I > 0$)

b. open channels: unphysical ($k_I < 0$)

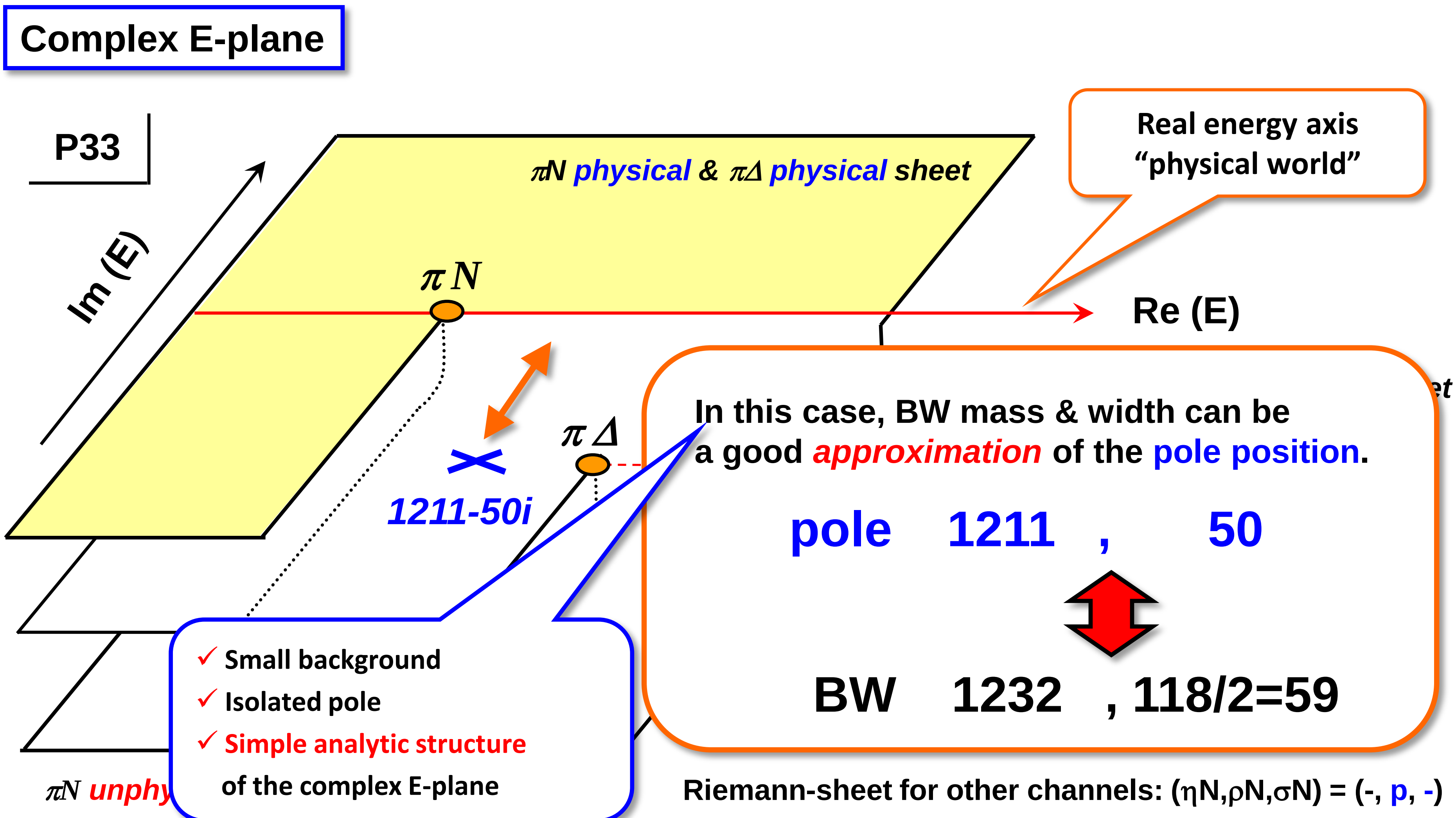
Near threshold :

search on both physical and unphysical

$k = k_R + i\,k_I$ on-shell momentum

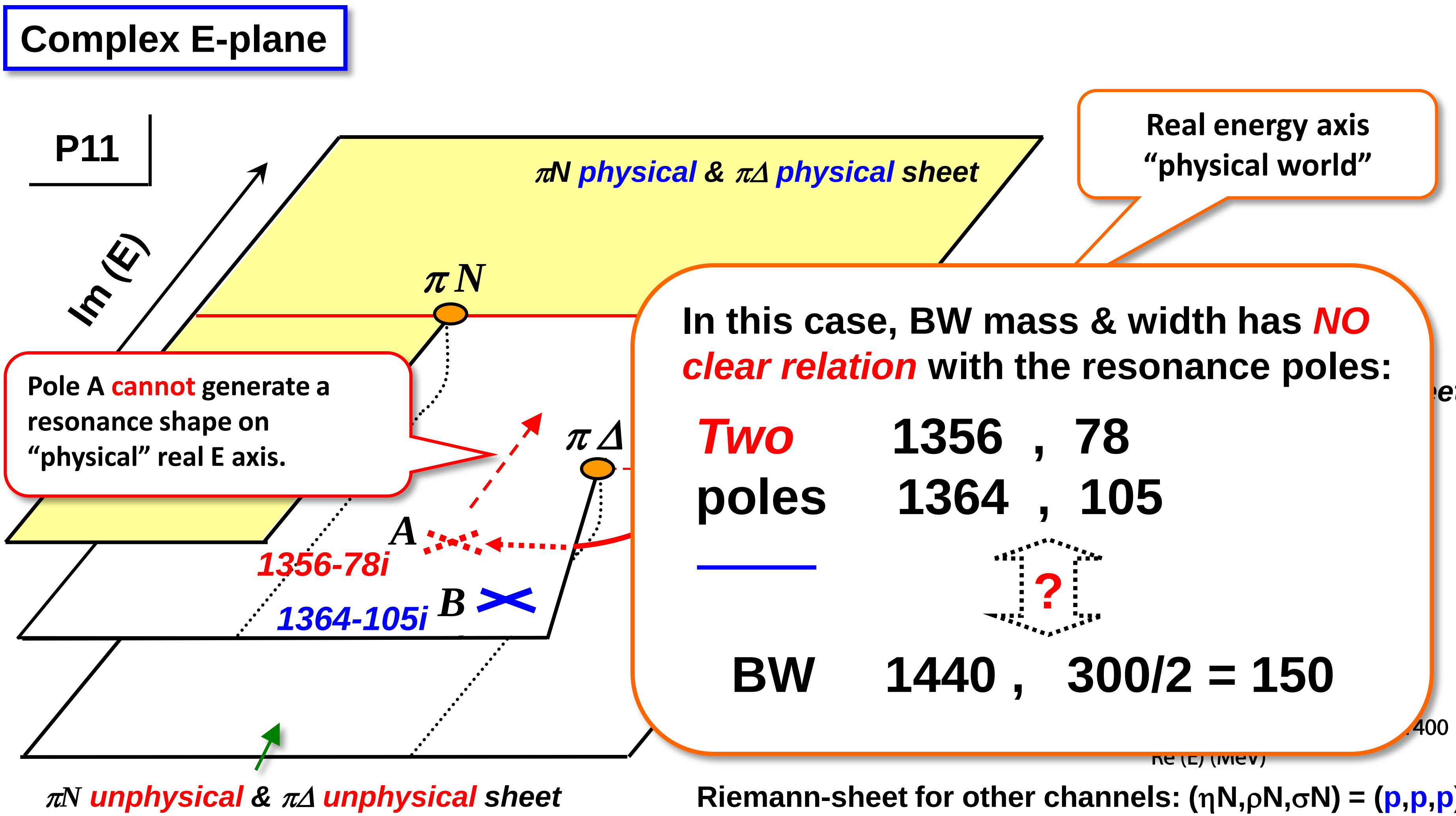
Delta(1232) : The 1st P33 resonance

Suzuki, Julia-Diaz, Kamano, Lee, Matsuyama, Sato, PRL104 042302 (2010)



Two-pole structure of the Roper P11(1440)

Suzuki, Julia-Diaz, Kamano, Lee, Matsuyama, Sato, PRL104 042302 (2010)

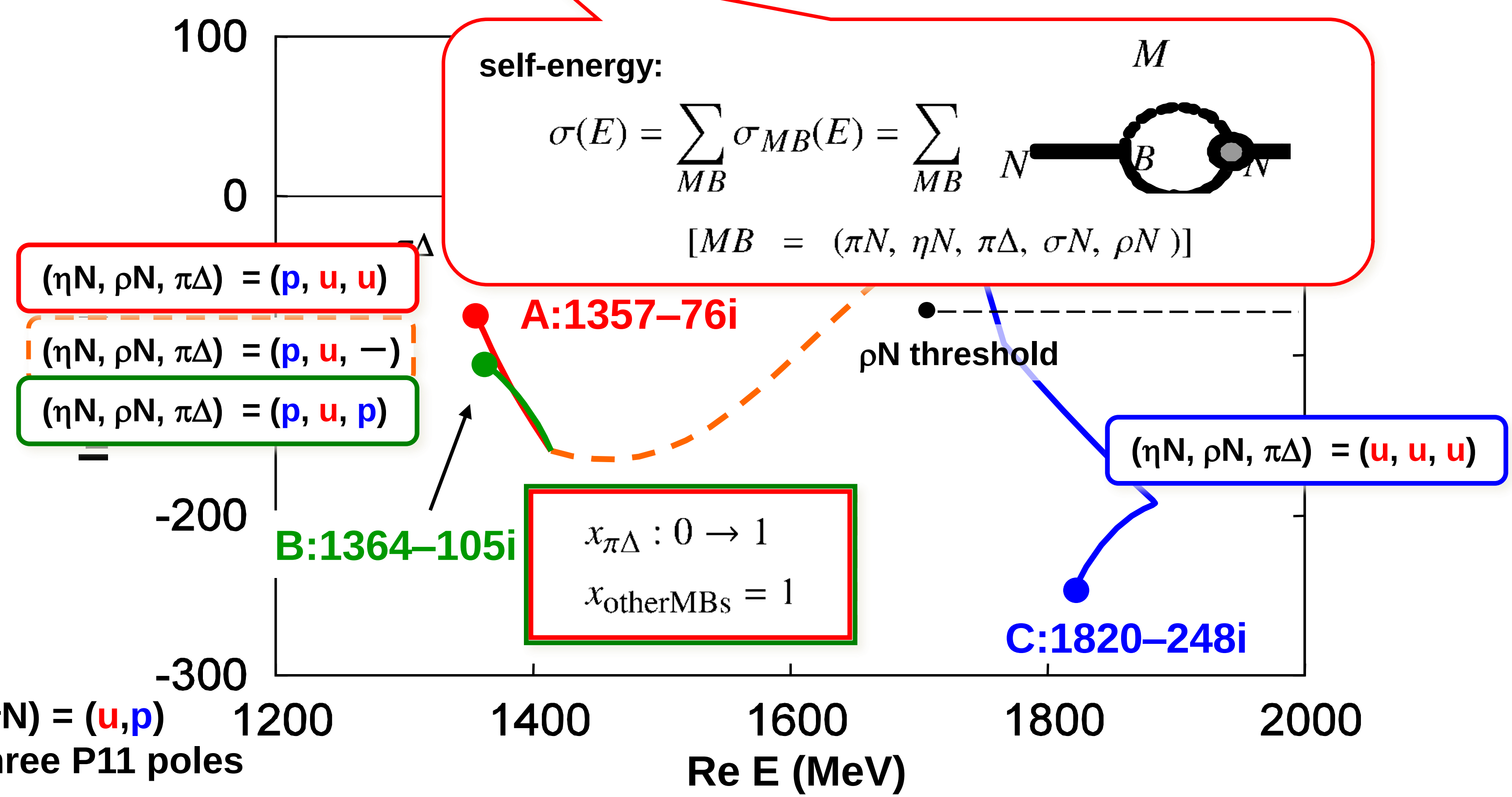


Dynamical origin of P11 resonances

Suzuki, Julia-Diaz, Kamano, Lee, Matsuyama, Sato, PRL104 042302 (2010)

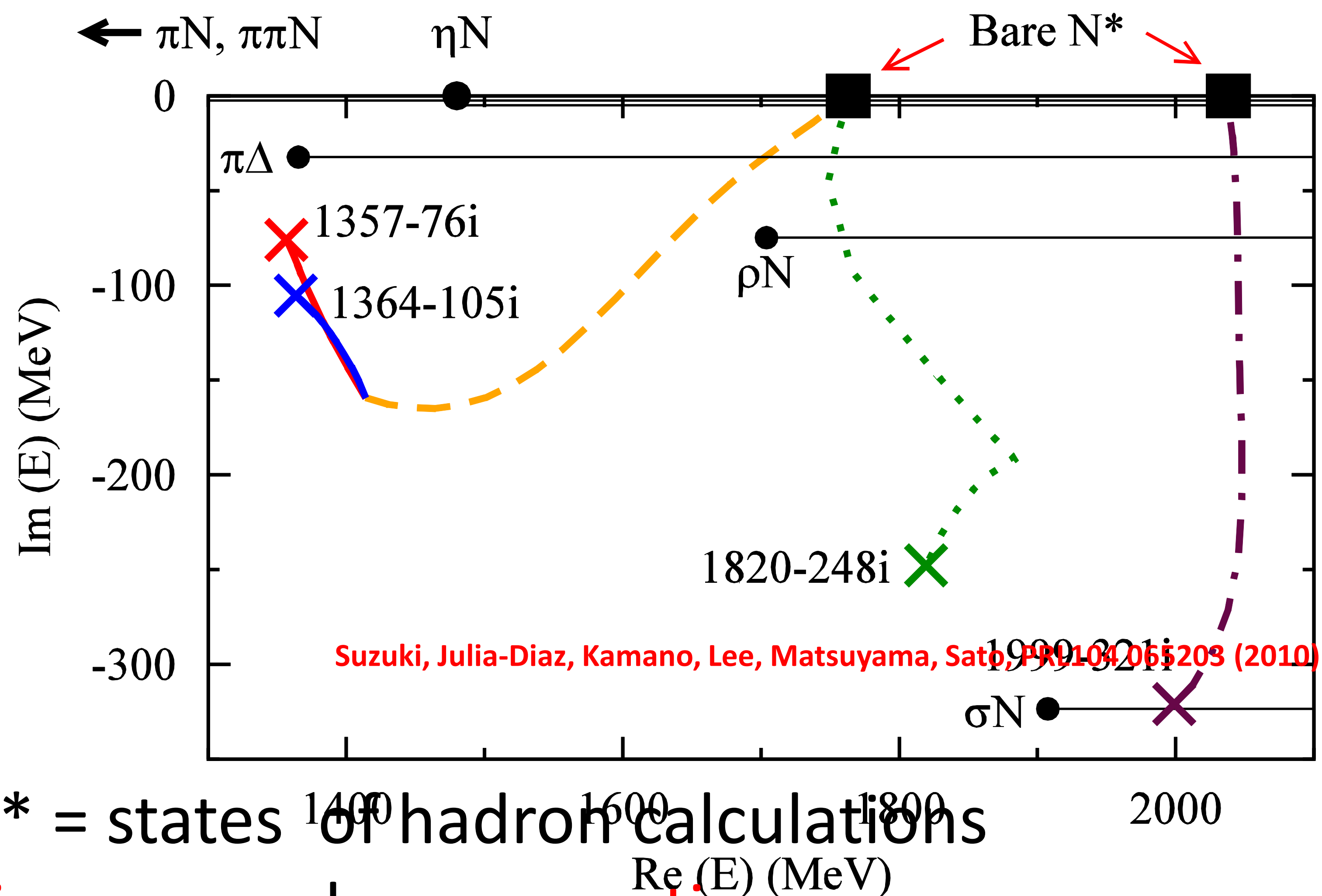
Pole trajectory of N* propagator

$$\frac{1}{E - m^0_{N^*} - \sigma(E)} \to \frac{1}{E - m^0_{N^*} - \sum_{MB} x_{MB}\sigma_{MB}(E)} \qquad x_{MB} : 0 \to 1$$



(πN,σN) = (u,p) for three P11 poles

Dynamical origin of P11 resonances



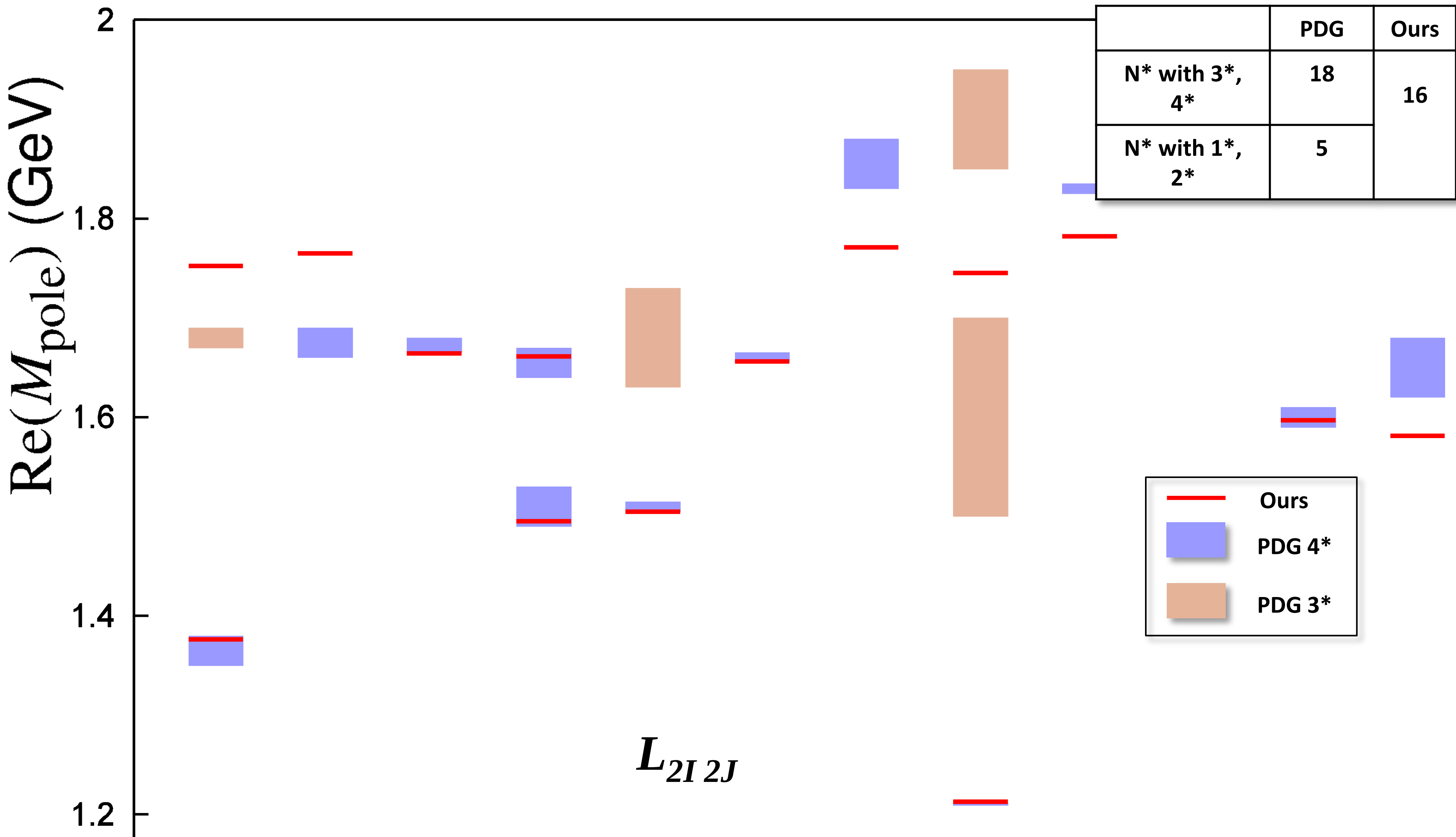
Bare N* = states of hadron calculations
excluding meson-baryon continuum
(quark models, DSE, etc..)

Spectrum of N* resonances

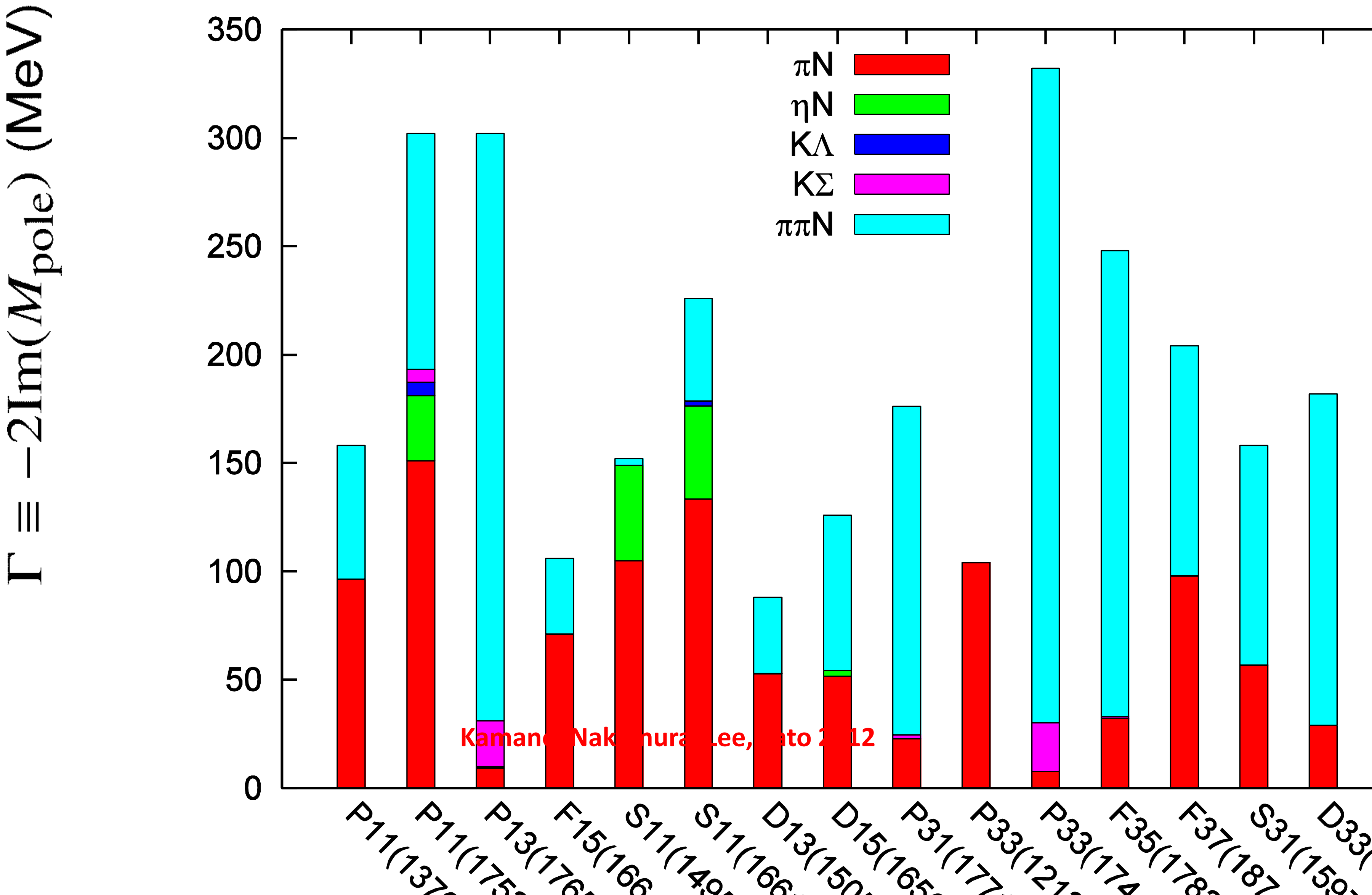
Kamano, Nakamura, Lee, Sato ,2012

Real parts of N* pole values

	PDG	Ours
N* with 3*, 4*	18	16
N* with 1*, 2*	5	



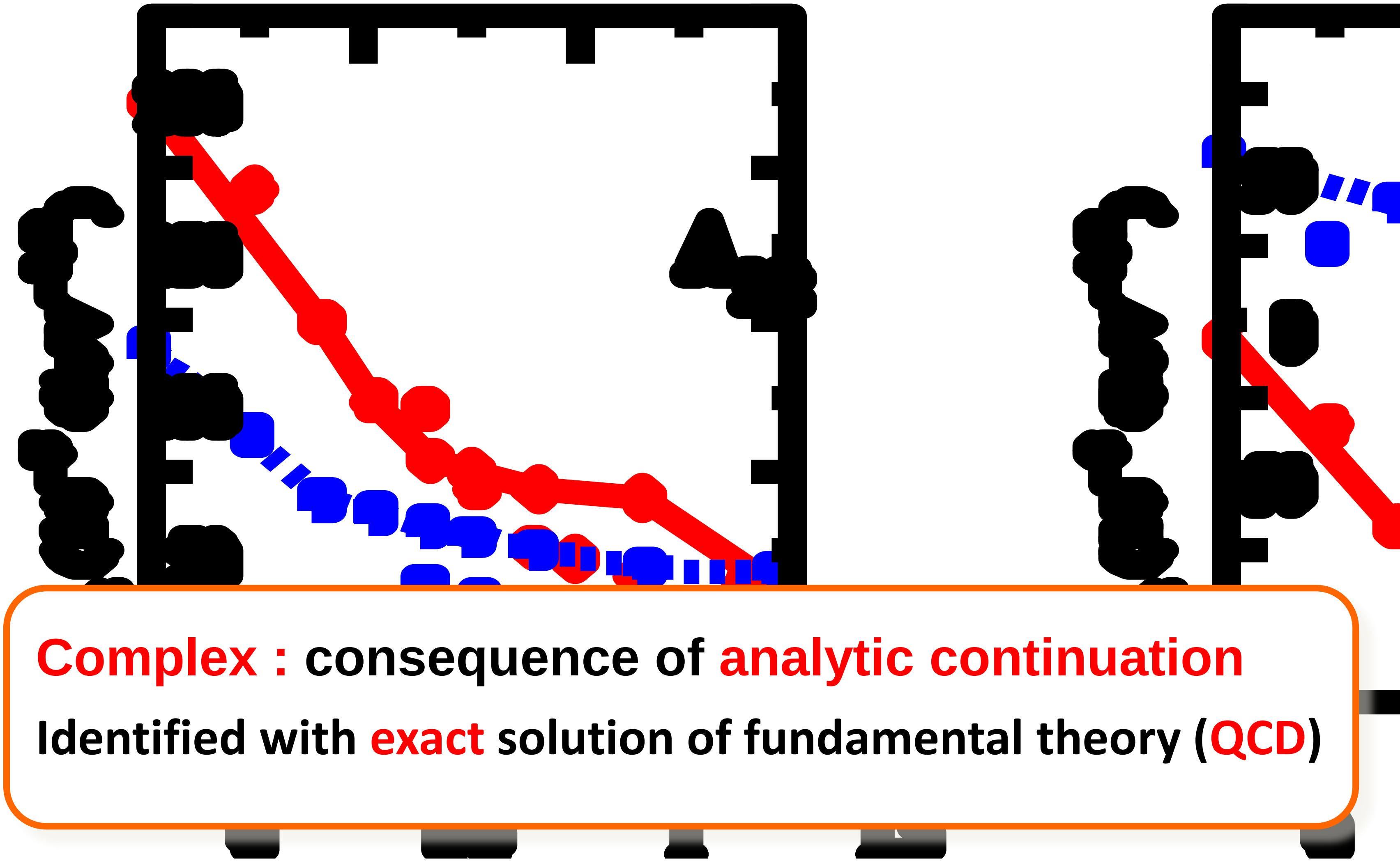
Width of N* resonances



N-N* form factors at Resonance poles

Suzuki, Julia-Diaz, Kamano, Lee, Matsuyama, Sato, PRL104 065203 (2010)
Suzuki, Sato, Lee, PRC82 045206 (2010)

Nucleon - 1st D13 e.m. transition form factors



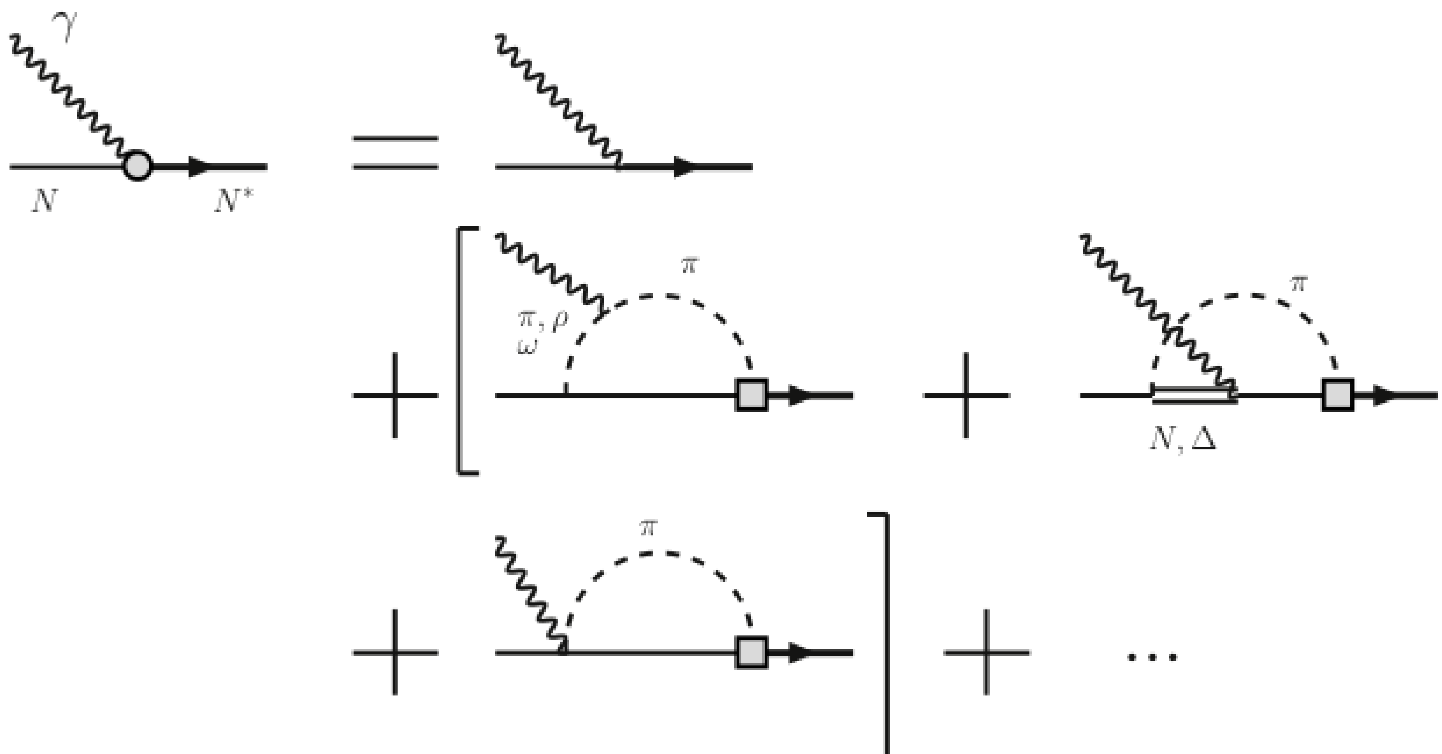
Complex : consequence of **analytic continuation**

Identified with **exact** solution of fundamental theory (**QCD**)

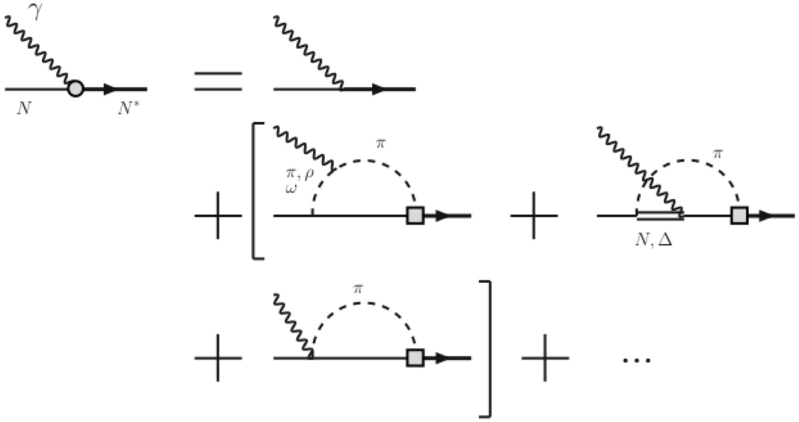
Interpretations :

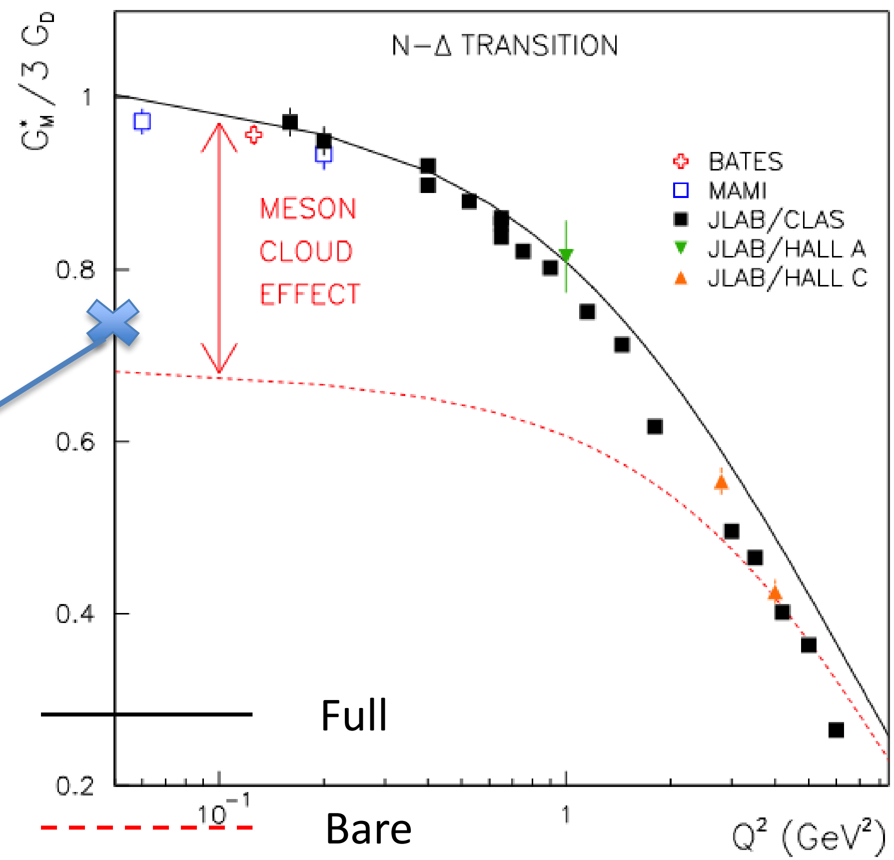
Meson cloud effects are essential for testing structure calculations of

- Delta (1232)
- Roper(1440)



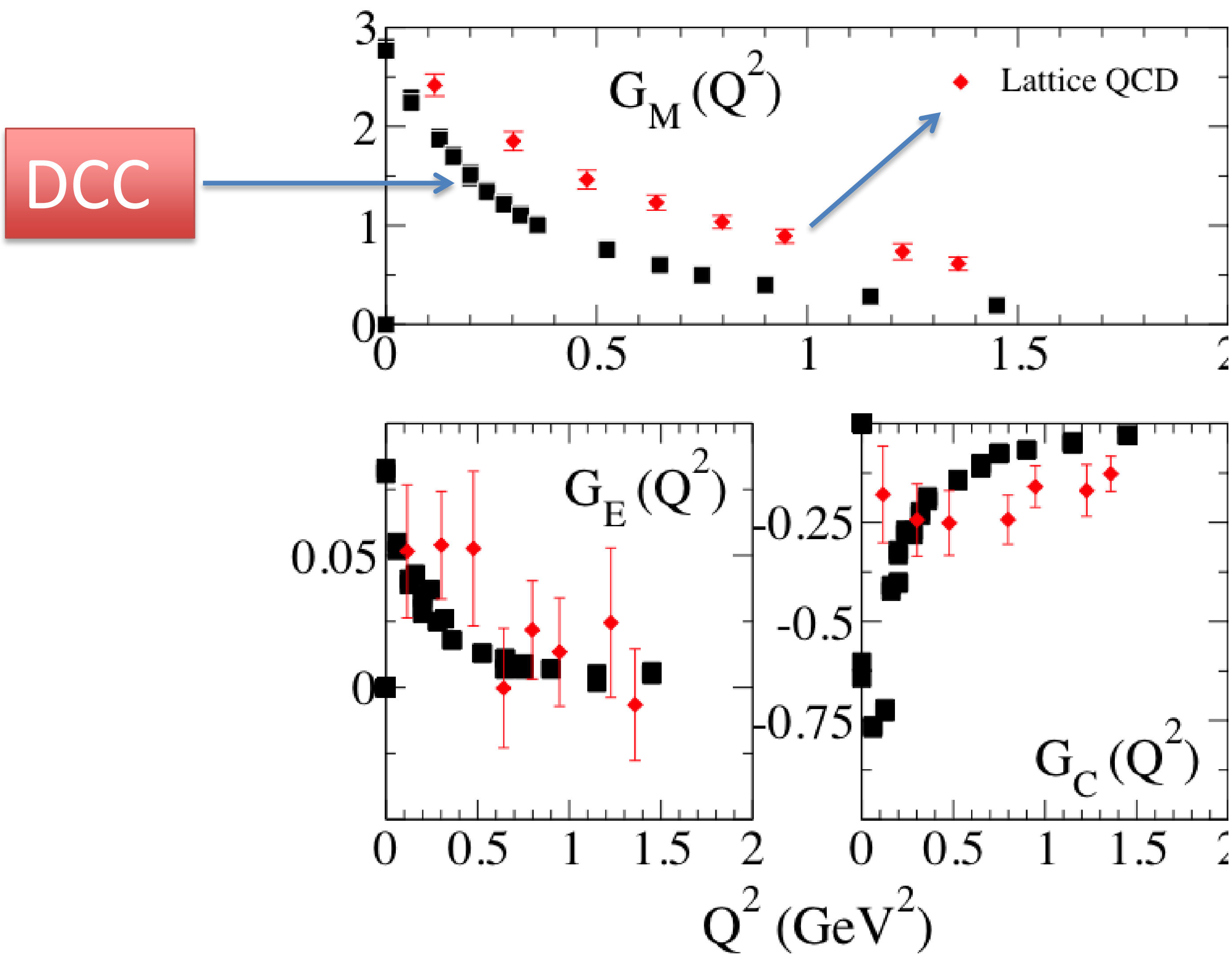
$G_M(Q^2)$ for γ N → Δ (1232) transition



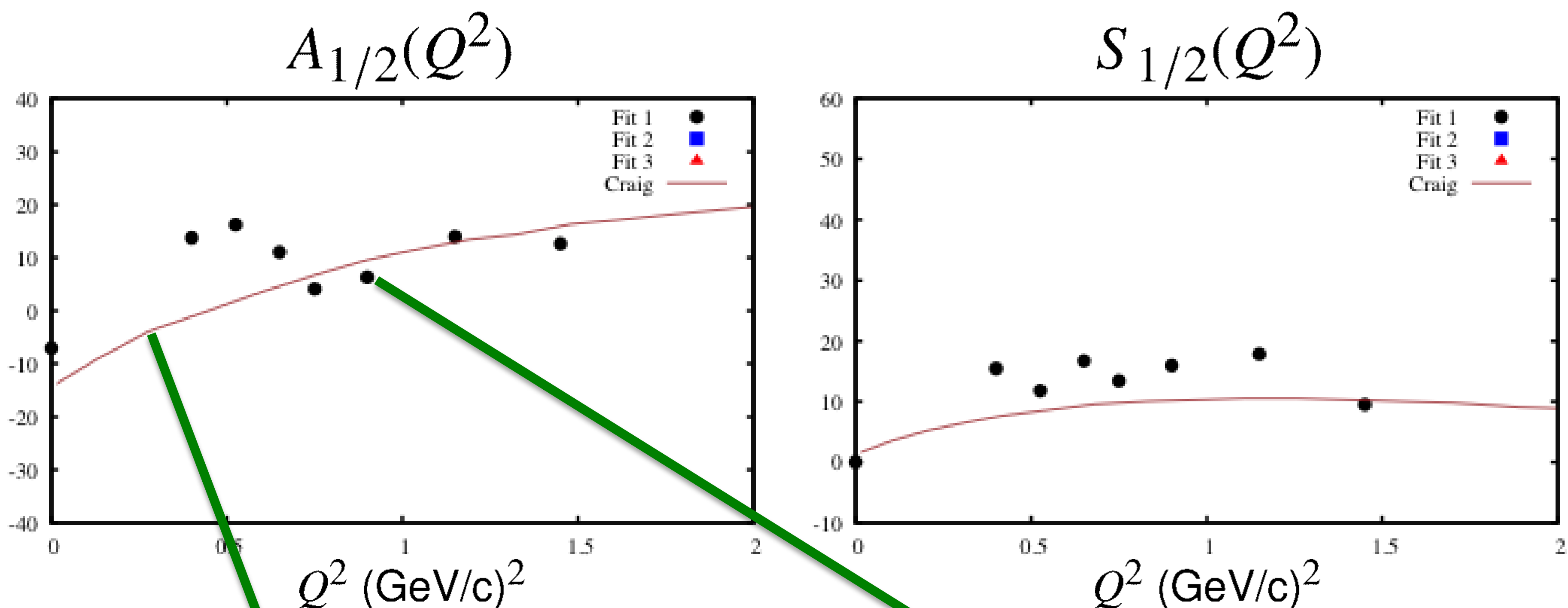


Note:
Most of the available static hadron models give $G_M(Q^2)$ close to “Bare” form factor.

γ N → Δ(1232) form factors compared with Lattice QCD data (2006)



γ p → Roper e.m. transition



"Static" form factor from DSE-model calculation. (C. Roberts et al, 2011)

"Bare" form factor determined from our DCC analysis (2010).

Back up

$$
\begin{aligned}
S(E) &= e^{2i\delta} \\
S(E) &= 1 + 2iT(E)
\end{aligned}
$$

Speed:

$$
\mathrm{sp}(E) = \left| \frac{dT}{dE} \right| . \tag{1}
$$

The SP method defines the resonance mass M_R by

$$
\frac{d}{dE}\mathrm{sp}(E) \mid_{E=M_R} = 0
$$

and

$$
\mathrm{sp}(M_R \pm \Gamma_R/2) = \mathrm{sp}(M_R)/2
$$

This can be easily by the usual Breit-Wigner parameterization:

$$
T(E) = \frac{\Gamma_R/2}{E - M_R + i\Gamma_R/2}
$$