Dynamical Model Analysis of Hadron Resonances (IV)

T.-S. Harry Lee

Argonne National Laboratory

Lecture IV:

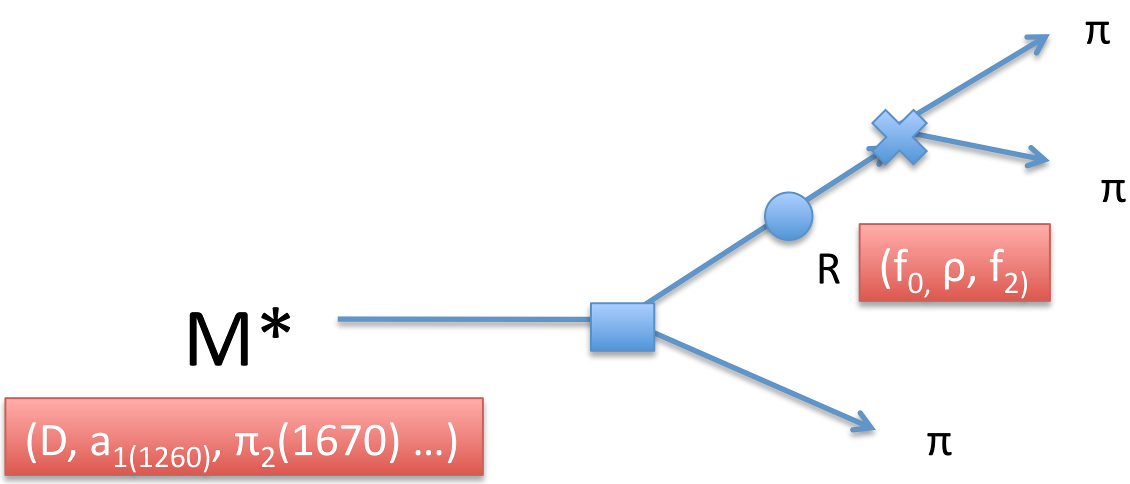
1. Dynamical model for extracting heavy/exotic mesons from 3-mesons production reactions

2. Summary

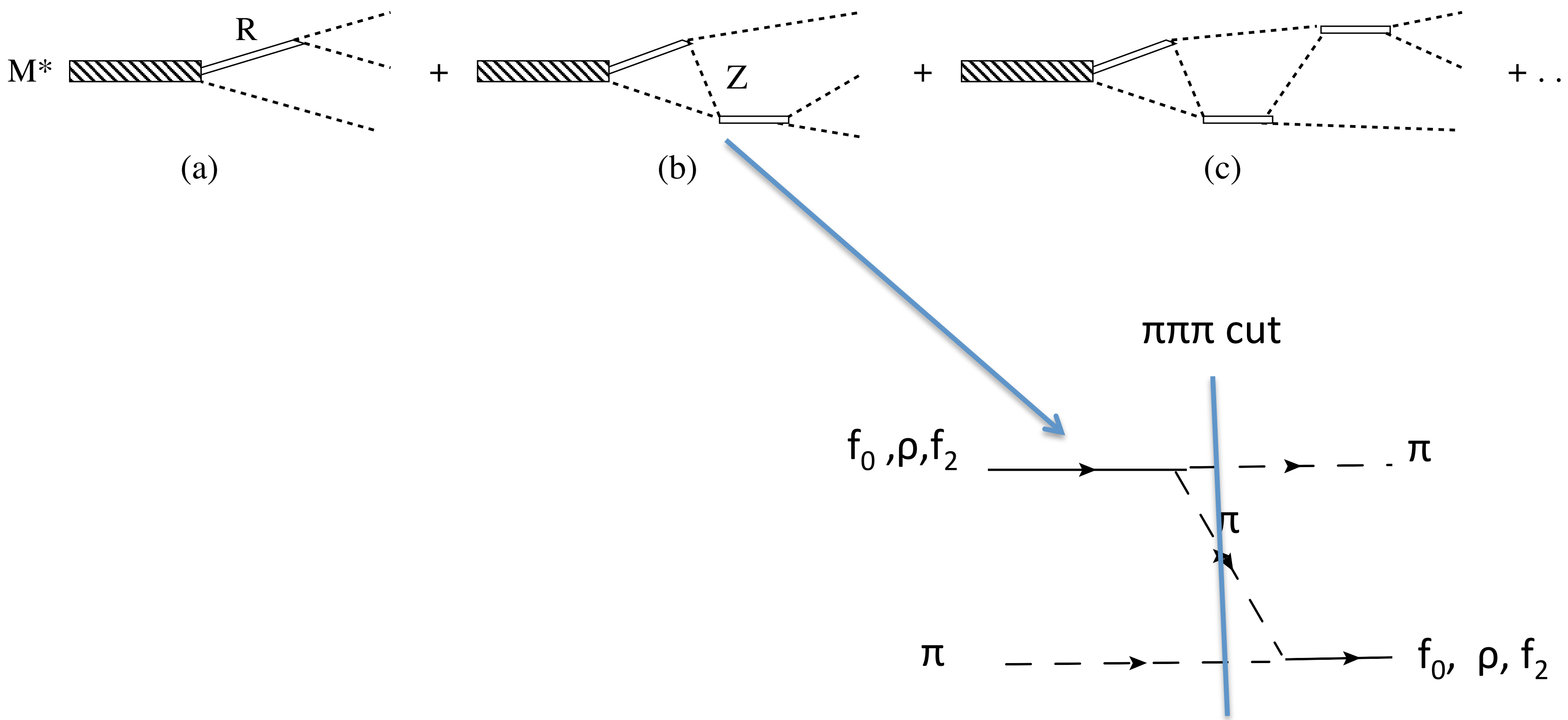
_

Most of current analysis :

Isobar model



Three-Body Unitarity Condition



What are the corrections on isobar-model analysis??

→

Need to investigate within a unitary model

Consider a model Hamiltonian

(Kamano, Nakamura, Lee, Sato, Phys. Rev. C (2011))

M^* = heavy/exotic mesons

$$H = H_0 + H',$$

$$H' = \sum_{cR} [\Gamma_{cR,M^*} + \Gamma^{\dagger}_{cR,M^*}] + H'',$$

R, R' = f_0 , ρ , f_2 ,

a,b,c =π, or K

$$H'' = \sum_{c'R',cR} v_{c'R',cR} + \sum_{R} \sum_{ab} [f_{ab,R} + f^{\dagger}_{ab,R}] \, ,$$

First step :

Fit $\pi\pi$ -> $\pi\pi$, KK scattering to determine

$H_{int} = f_{\pi\pi,\,R} + f_{kk,R}$

R = bare f_0 , ρ, f_2

Solve

$$T(E) = H_{int} + H_{int} \frac{1}{E - H + i\varepsilon} H_{int}$$

π π scattering amplitude

$$T^{LI}_{\pi\pi,\pi\pi}(q',q;E) = \sum_{R',R} \bar{f}^{LI}_{\pi\pi,R'}(q')\tau^{LI}_{R',R}(E)\bar{f}^{LI}_{R,\pi\pi}(q),$$

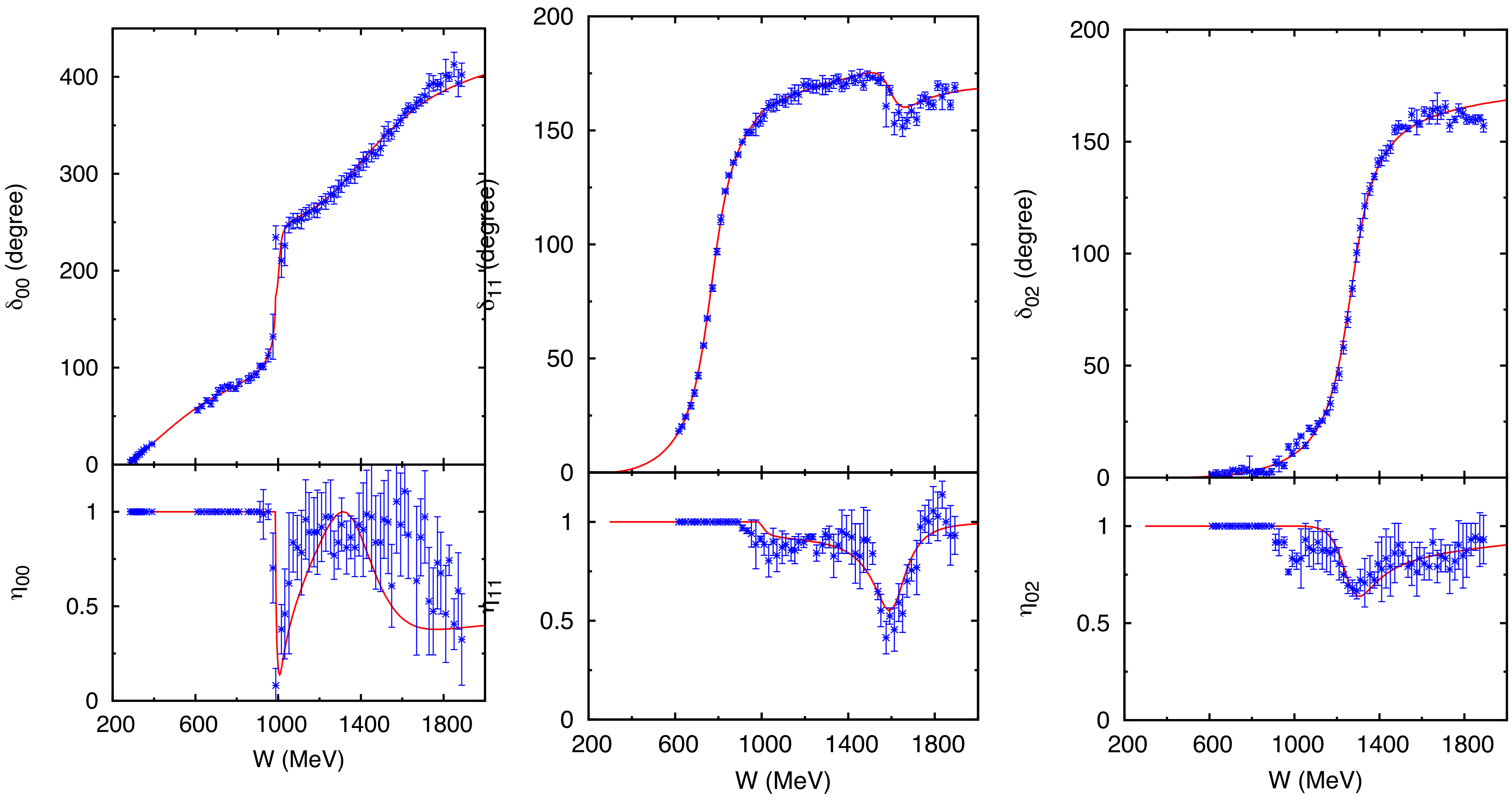
$$[(\tau^{LI})^{-1}(E)]_{R'R} = (E - m_R)\delta_{R',R} - \Sigma^{LI}_{R',R}(E),$$

$$\Sigma^{LI}_{R',R}(E) = \sum_{ab=\pi\pi,K\bar{K}} \int q^2 dq \frac{\bar{f}^{LI}_{R',ab}(q)\bar{f}^{LI}_{ab,R}(q)}{E - E_a(q) - E_b(q) + i\epsilon},$$

$\bar{f}^{L_{ab},I_{ab}}_{ab,R}(q) =\ R \rightarrow \pi\pi, K\bar{K}$ interaction

Coupled-channel model for ππ and K $\bar{K}$ scattering

Fits of ππ amplitudes



Coupled-channel model for $\pi\pi$ and $K\bar{K}$ scattering

Bare masses

$R\ (L, I)$	M_{R_1} (MeV)	M_{R_2} (MeV)
f_0 (0, 0)	1220	2400
ρ (1, 1)	891	1840
f_2 (2, 0)	1607	—

II : uu
III :up

L	I	Pole position (GeV) [Riemann sheet]		
0	0	$0.43 - 0.27i$ [II]	$1.00 - 0.009i$ [II]	$1.35 - 0.17i$ [III]
1	1	$0.77 - 0.081i$ [II]	$1.61 - 0.12i$ [III]	—
2	0	$1.25 - 0.10i$ [III]	—	—

Investigate 3-π decays of heavy mesons

$e^+ e^- \to \bar{D} D \to \bar{D} + \pi\pi\pi$

$\pi N, \gamma N \to M^* N$

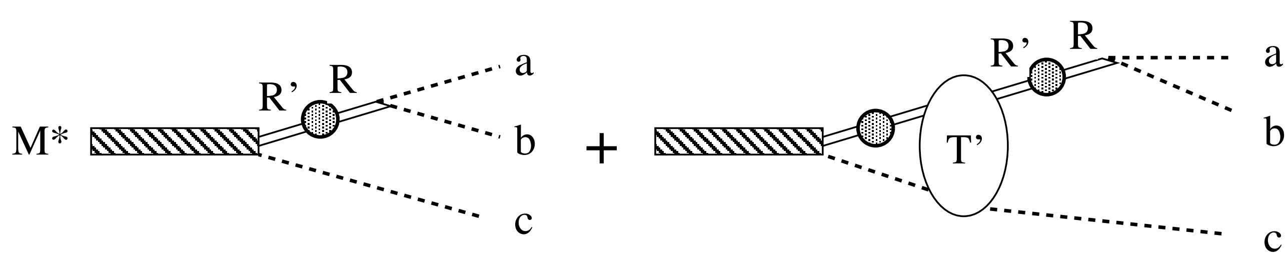
$f_0,\ a_1,$ .. Exotic $\to \pi\pi\pi$

M^* -> 3-mesons amplitude

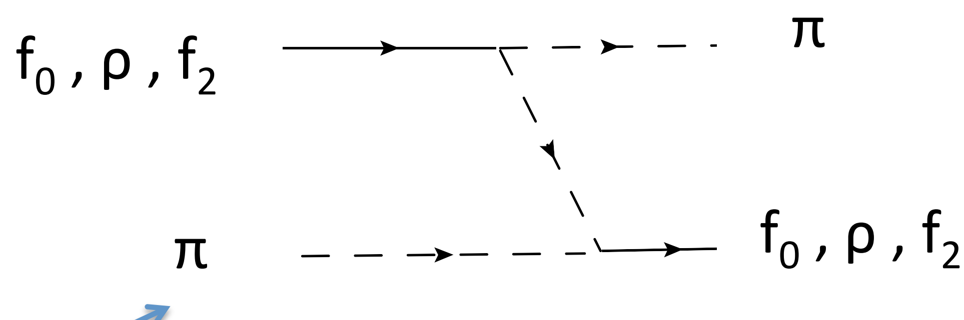
$$T_{(ab)c,M^*}(E) = T^{\text{Isobar}}_{(ab)c,M^*}(E) + T^{\text{FSI}}_{(ab)c,M^*}(E),$$

$$T^{\text{Isobar}}_{(ab)c,M^*}(E) = \sum_{R} \sum_{c'R'} \langle ab| f_{ab,R} G_{cR,c'R'}(E) \Gamma_{c'R',M^*} |M^*\rangle,$$

$$T^{\text{FSI}}_{(ab)c,M^*}(E) = \sum_{R} \sum_{c'R'} \sum_{c'''R''',c''R''} \langle ab| f_{ab,R} G_{cR,c'R'}(E) T'_{c'R',c'''R'''}(E) G_{c'''R''',c''R''}(E) \Gamma_{c''R'',M^*} |M^*\rangle.$$



Use Spline method



$$T'_{c'R',cR}(E) = Z_{c'R',cR}(E) + \sum_{c'''R''',c''R''} Z_{c'R',c'''R'''}(E) G_{c'''R''',c''R''}(E) T'_{c''R'',cR}(E).$$

$$Z_{c'R',cR}(E) = \sum_{c''} f_{R',cc''} \frac{1}{E - E_c - E_{c'} - E_{c''} + i\epsilon} f_{c'c'',R}.$$

$$[G^{-1}(E)]_{c'R',cR} = \delta_{c',c}\left[(E - E_c - E_R)\delta_{R',R} - \Sigma_{R',R}(E - E_c)\right].$$

$$\Sigma_{R',R}(w) = \sum_{ab} \langle R'| f_{R',ab} \frac{\mathcal{B}_{ab}}{w - E_a - E_b + i\epsilon} f_{ab,R}|R\rangle,$$

Effect of three-body unitarity on pole positions

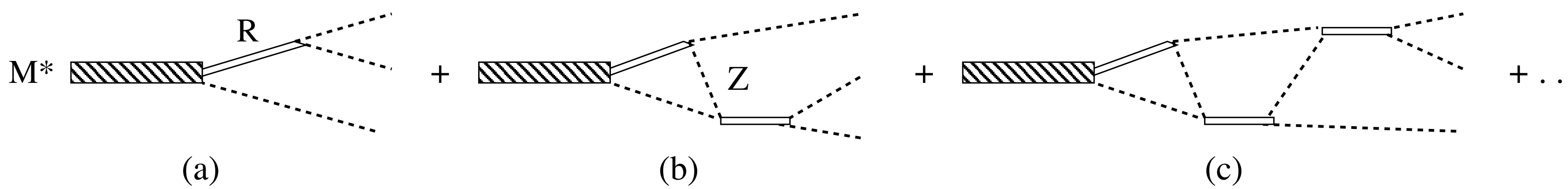
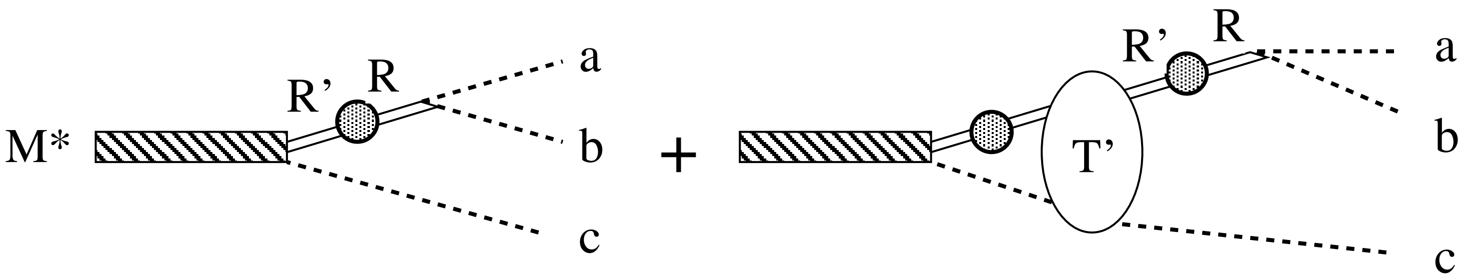
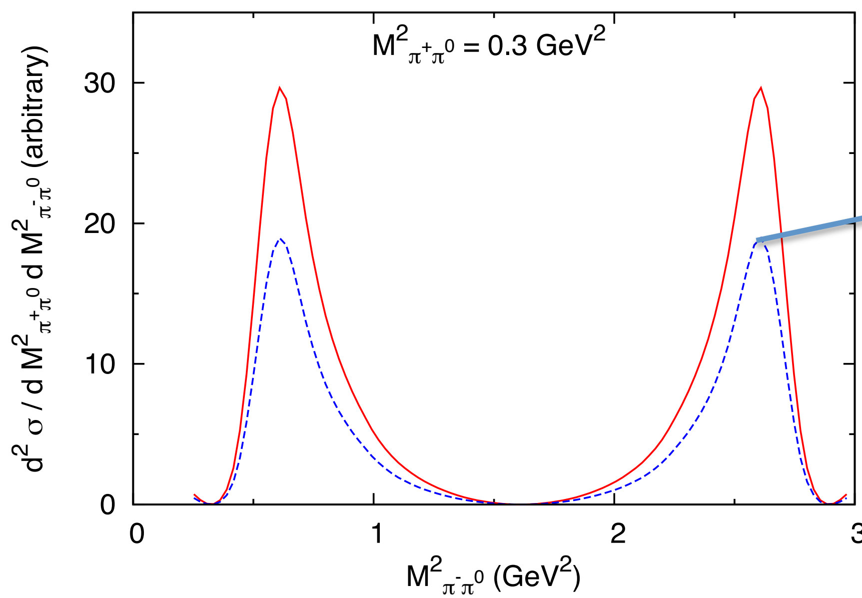


TABLE I. Pole masses of $a_1(1260)$, $\pi_2(1670)$ and $\pi_2(2100)$. Here "with Z" denotes the results of the full unitary model, while "w/o Z" denotes the results in which the Z-diagrams are turned off from the full unitary model.

	Pole masses (MeV)		
	$a_1(1260)$	$\pi_2(1670)$	$\pi_2(2100)$
with Z	$1230 - 213\ i$	$1672 - 130\ i$	$2090 - 313\ i$
No Z	$1122 - 148\ i$	$1661 - 127\ i$	$2044 - 398\ i$

$\Delta\ \mathrm{Im}(M_R) = +65$ $+3$ -85


M*
R'
R
a
b
c
+
R'
R
T'
a
b
c

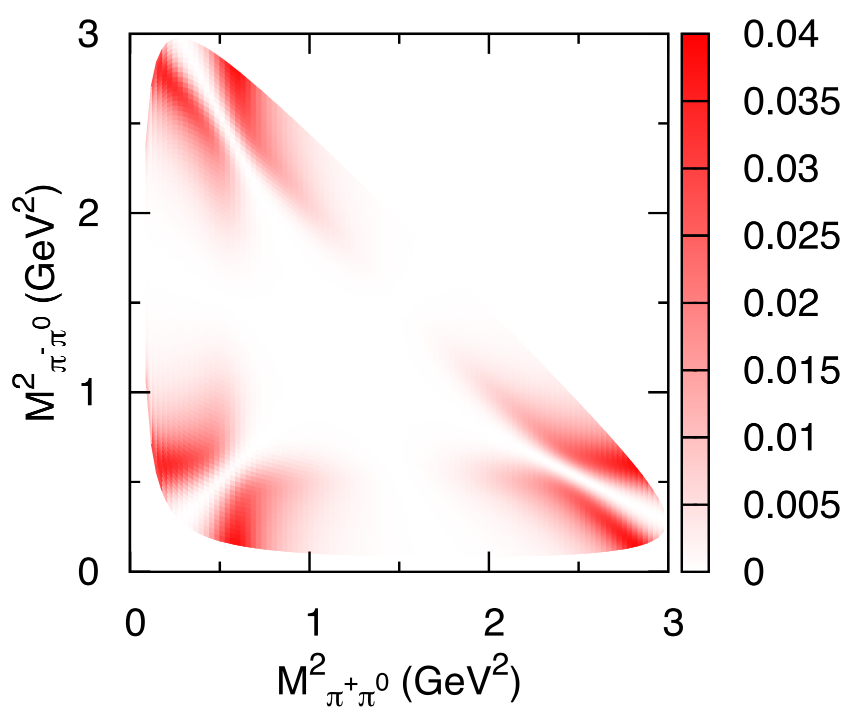

$M^2_{\pi^+\pi^0}$ = 0.3 GeV^2
$d^2\sigma / d M^2_{\pi^+\pi^0} d M^2_{\pi^-\pi^0}$ (arbitrary)
$M^2_{\pi^-\pi^0}$ (GeV^2)
0
10
20
30
0
1
2
3

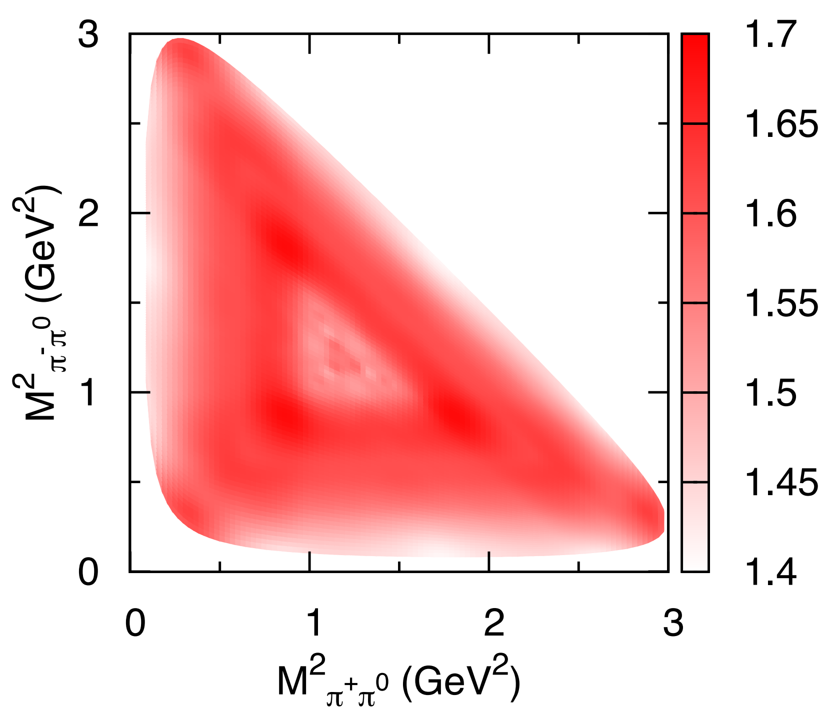
T'=0

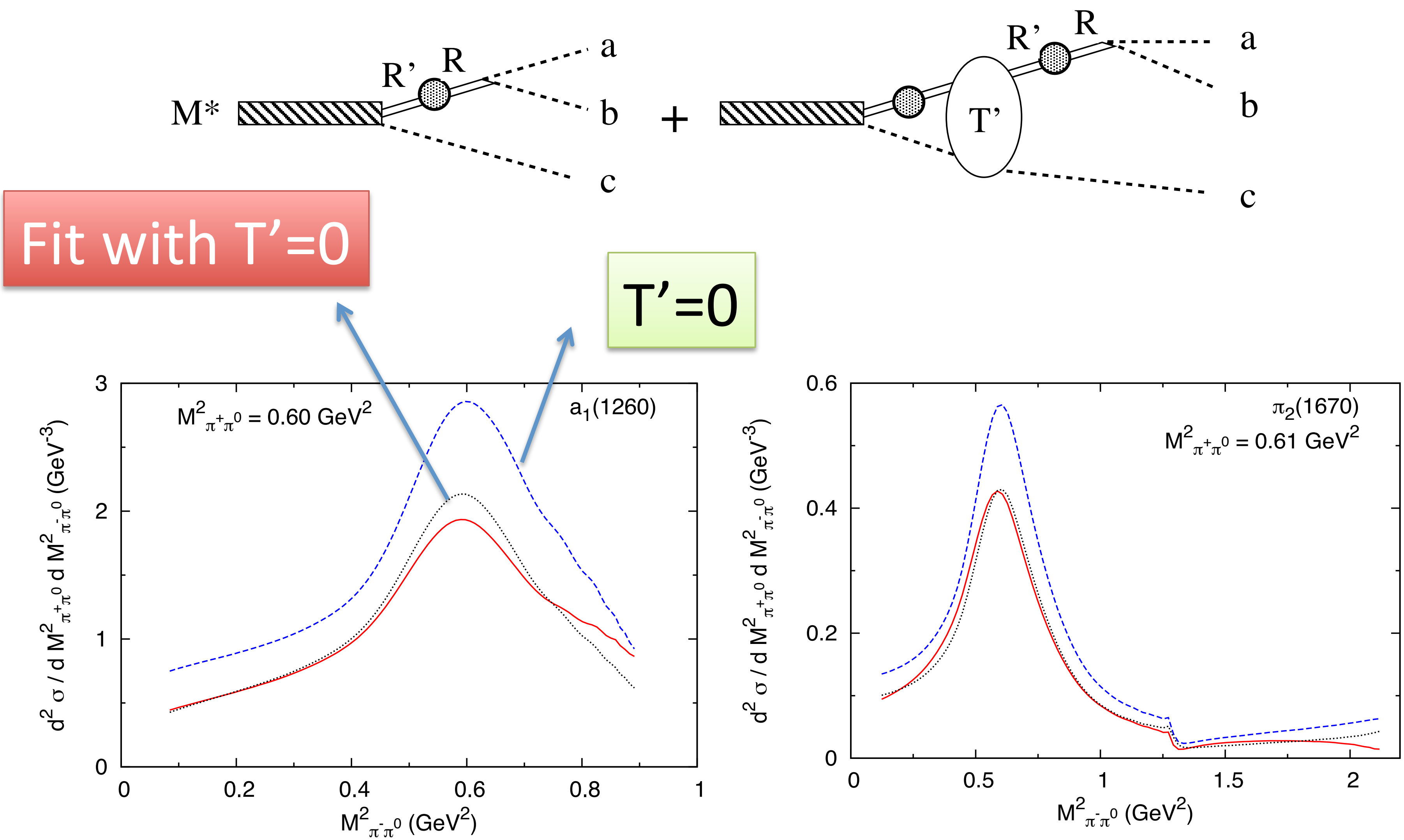
D ->3π

Dalitz plott of D->3π

σ(full)/σ(no T’)






M*
R’
R
a
b
c
+
T’
R’
R
a
b
c
Fit with T’=0
T’=0
M²π+π0 = 0.60 GeV²
a1(1260)
d² σ / d M²π+π0 d M²π-π0 (GeV⁻³)
M²π-π0 (GeV²)
π2(1670)
M²π+π0 = 0.61 GeV²
d² σ / d M²π+π0 d M²π-π0 (GeV⁻³)
M²π-π0 (GeV²)

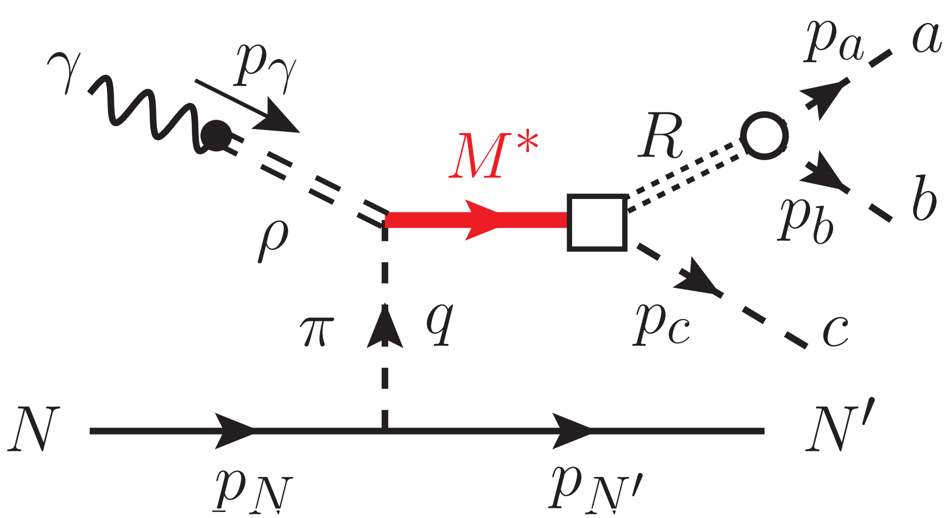
M^*	$a_1(1260)$ Pole mass (MeV)	$\pi_2(1670)$ Pole mass (MeV)	$\pi_2(2100)$ Pole mass (MeV)
Full	$1230 - 213i$	$1672 - 130i$	$2090 - 313i$
Isobar fit	$1230 - 100i$	$1672 - 97i$	$2090 - 261i$

Δ Im(M_R)= +113 +33 +52

Note: Re($M_{R)}$ is kept the same in the fits

fit with T'=0

Experiment at Jlab



M^* = $a_1(1230)$, $a_2(1320)$

$\pi_1(1600)$, $\pi_2(1670)$

$a_1(1700)$, $a_2(1700)$, $\pi_2(1800)$

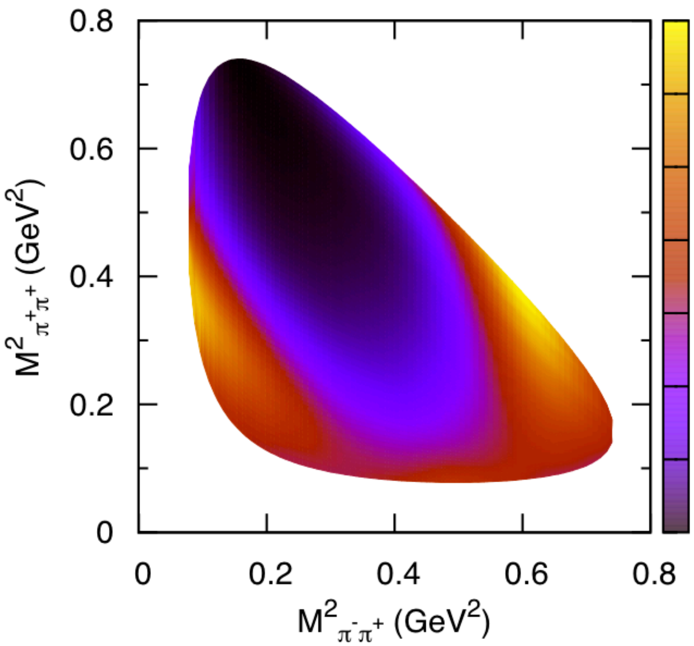
Bare paremeters of M* decays are taken from

$^3P_0$ model of Barnes, Close, Page, Swanson

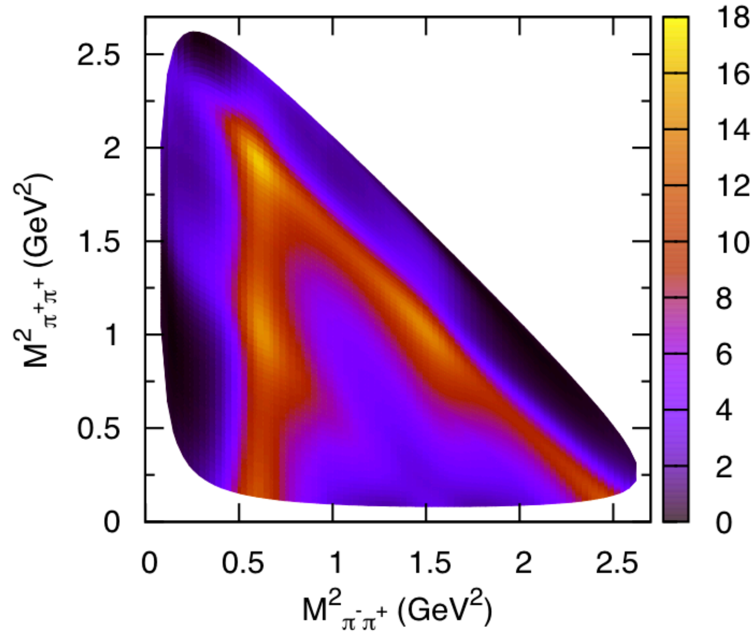
(Phys. Rev. D 55, 4157 (1997))

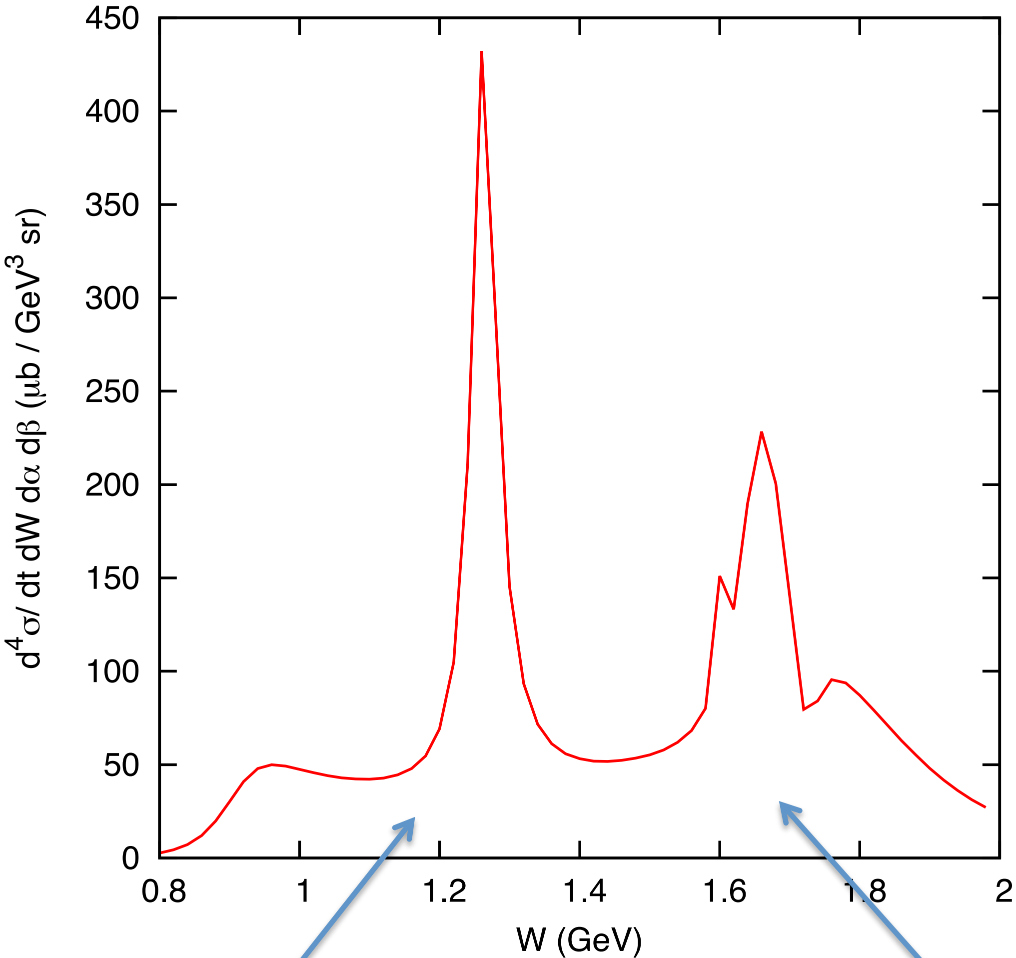
Dalitz plot of γp -> M* n ->$\pi^+ \pi^+ \pi^-$ n

W= 1.0 GeV



W=1.7 GeV





Overlapping resonances

$a_1(1230)$, $a_2(1320)$

$\pi_1(1600)$, $\pi_2(1670)$

$a_1(1700)$, $a_2(1700)$, $\pi_2(1800)$

Isobar-fit to pseudo-data from unitary model:

1. Set T' = 0

2. Complex M* -> $\pi f_{0,}$ $\pi\rho$, πf_2 coupling constant

3. Add compex W-dependent background constants

Similar to the usual isobar-model fit

Accurate fit to Dalitz plot data can be obtained.
But the interpretations of the data are
very different:

M* can decay into more partial waves than
those included in generating the Dalitz plot data

	$a_1(1260)$	
	Unitary model	Isobar-fit model
$M^0_{M^*}$	1230.	1230.
$C([\pi f_0(1)]_{L=1})$	-	$-2.00+8.17i$
$C([\pi f_0(1)]_{L=2})$	-	$7.51-2.31i$
$C([\pi \rho(1)]_{L=0})$	24.55	$31.87-3.74i$
$C([\pi \rho(1)]_{L=2})$	-	-
$C([\pi \rho(2)]_{L=0})$	-	$11.01+5.18i$
$C([\pi \rho(2)]_{L=2})$	-	$-0.32-1.85i$
$C([\pi f_2(1)]_{L=1})$	-	$-2.72+4.36i$
$C([\pi f_2(1)]_{L=3})$	-	$0.28+0.01i$

Coupling with M*

Leaking decay channels

Final state interactions due to three-body unitarity must be accounted for in extracting structure information from future experiments for testing hadron models of heavy/exotic mesons.

Current works and plans:

1. Include meson-meson interactions calculated from tree diagrams of Chiral Lagrangian

2. Analyze 3-meson decay data from KEK to determine CP violation phases

3. Plan to analyze CLAS data of $\gamma N \rightarrow \pi\pi\pi N$ (presented at St. Petersburg, Florida) when become accessible

Summary

1. Information on excited hadrons are from hadron resoances extracted from experiments

2. Hadron resonances contain information
   a. Structure of excited hadrons
   b. Hadron-hadron Interactions

3. Resonances correspond to solutions of full theory (QCD)

Lessons from recent analysis of N* and M* :

Current hadron calculations from

Constituent quark models,

Dyson-Schwinger-Equation models

…….

can be properly tested only when the reaction mechanisms have been accounted for within a dynamical reaction model