CP Violation and all that.

Brian Meadows
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Lecture I Outline

- CP Violation what it is, how it happens, what is it not.
- The CKM model for CPV
  - Unitarity triangles
- A role for hadrons and amplitude analysis
- Mixing and its role in CPV
- $B$ factory measurements
- Pause for reflection
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**CP Violation (CPV) - very brief History**

- **CPV** is observable as a particle/antiparticle asymmetry in the rates of transitions:
  \[ \Gamma(i \rightarrow f) \neq \Gamma(i \rightarrow f) \quad (\text{Bars indicate antiparticle conjugate states}) \]
  
  for processes with initial state \( i \) and final state \( f \)

- **CPV** was discovered (Fitch and Cronin 1963) in decays to \( \pi^+\pi^- \) states (with \( CP=+1 \)) by long-lived \( K_L \) mesons that were formerly found to decay only to \( \pi^-\ell^+\nu_\ell \) (with \( CP=-1 \)).

- It was next seen as an asymmetry in \( K_L \rightarrow \pi^\pm\ell^\pm\nu_\ell \) decays
CPV – very brief History (2)

- Theoretically, there was much speculation on the source of CPV – whether from mixing or decay (“direct”).
- In 1973, Kobayashi and Moskawa suggested the current 3-family nature of the SM and pointed out that it led to the possibility for CPV.
- Experimentally, CPV was not observed anywhere other than in neutral K decays for ~40 years when, much as predicted by KM, the BaBar and Belle experiments observed its interference with mixing in decays of $B^0$ mesons to $J/\psi K_s$ final states ($CP=-1$).
- First evidence for direct CPV in B decays was later observed in 2004 by the BaBar and Belle experiments.
Early evolution of the universe
CPV and Baryogenesis

- CPV is needed to account for excess ($\Delta N = N_B - N_{\bar{B}}$) of baryons over anti-baryons in our universe.
- This excess is only a small fraction of the observed number of photons $\Delta N/N_\gamma \sim N_B/N_\gamma \sim 10^{-10}$.
- Sakharov (1967) held that this requires:
  - Baryon number violating processes - $H_{\Delta N \neq 0}^{\text{eff}} \neq 0$
  - CPV so that, for any baryon-violating process, $\Gamma(i \rightarrow f) \neq \Gamma(i \rightarrow \bar{f})$
  - A period in the history of the universe when it was not in thermal equilibrium.

  In thermal equilibrium ($T$ invariance), CPT invariance is equivalent to CP invariance!

[CPT violation alone, could also generate $\Delta N \neq 0$
  - This would also violate locality, causality and Lorentz invariance. ]
Consider a particle $X$ which has just two decay modes to final states $f_{1,2}$ with baryon numbers $B_{1,2}$ (not necessarily the same) and branching fractions $r$ and $(1 - r)$.

*CPT* invariance requires anti-particle $\bar{X}$ to have the same total decay rates but *CP* violation can allow $r$ to differ from $\bar{r}$.

Decay of each $X \bar{X}$ pair (initially $B = 0$):

- $X \rightarrow B = rB_1 + (1 - r)B_2$
- $\bar{X} \rightarrow B = \bar{r}(-B_1) + (1 - \bar{r})(-B_2)$

results in a change in number of baryons $\Delta B = (r - \bar{r})(B_1 - B_2)$

*Baryon-dominant universe* requires this be positive, so:

- $B_1 \neq B_2$ - baryon number is violated for at least one decay mode
- $r \neq \bar{r}$ - there is *CPV*

Differences must have same sign too.
The SM Lagrangian is Hermitian and includes terms like
\[ \mathcal{L}(x) = \sum_i \left[ a_i \mathcal{F}_i(x) + a_i^* \mathcal{F}_i^+(x) \right] \]
where the \( \mathcal{F}_i(x) \) are scalar operators defined from quark and lepton fields and the \( a_i \) are couplings.

*CP*-invariance requires that all couplings can be made real with a suitable choice of phases for the fields.

In the *SM*, charged *EW* terms are of the form
\[ \frac{g}{\sqrt{2}} \left( \overline{u}_L \overline{c}_L \overline{t}_L \right) V_{CKM} \gamma^\mu \left( \begin{array}{c} \overline{d}_L \\ \overline{s}_L \\ \overline{b}_L \end{array} \right) W_\mu^+ \]
These all involve the CKM matrix \( V_{CKM} \) that has a phase that depends on specific flavor couplings.

The effects of such a phase cannot be removed, for all flavors, simply by re-phasing the quark fields \( \rightarrow CPV \)
**CPV is not “T-violation”**

- **CP** violation is not the same thing as “T violation”. It would be only if **CPT** were an absolute symmetry.
- “T-violation” (time-reversal symmetry breaking), has yet to be found experimentally (?)
- For instance, we know, experimentally, that $B^0 \rightarrow K\pi$ is a **CP** violating process. However, we have not observed a weak scattering $K^+\pi \rightarrow B^0$ occurring at a different rate (and we probably never will, even though **CPT** would so decree!)
- An amplitude $<f|H|i>$ describes a transition from $i \rightarrow f$. Under **CP** this becomes $<f|H|i>$, not $<i|H|f>$. 

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The CKM Matrix

- This is a 3x3 unitary transformation

\[ V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \]

- Defined by rotations about 3 axes by angles \( \theta_{12}, \theta_{13}, \theta_{23} \)

\[ U_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} ; \quad U_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} ; \quad U_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} , \]

where \( s_{ij} = \sin \theta_{ij} \) and \( c_{ij} = \sin \theta_{ij} \), and (important) a phase

\[ V_{\delta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix} \]

- Then

\[ V_{\text{CKM}} = U_{23} U_{\delta}^\dagger U_{13} U_{\delta} U_{12} \]
Wolfenstein Expansion

- So

\[
V_{\text{CKM}} = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
    -s_{13}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
    s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}
\]

- The terms have a numerical hierarchy that suggests an expansion in powers of the Cabibbo angle \( \lambda = V_{us} \):

\[
V_{\text{CKM}} \approx \begin{pmatrix}
    1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\
    -\lambda & 1 - \lambda^2/2 & A\lambda^3 \\
    A\lambda^3(1 - \rho - i\eta) & A\lambda^3 & 1
\end{pmatrix} + O(\lambda^4)
\]

where \( A, \rho \) and \( \eta \) are parameters of order one.
Get to know the CKM

- **Magnitudes:**
  Memorize the power of $\lambda$ for each term.

- **Phases:**
  Remember the phases for the two terms circled.

\[ \begin{align*}
\text{arg}\{V_{ub}\} &= -\gamma \\
\text{arg}\{V_{td}\} &= -\beta
\end{align*} \]

"The" unitarity triangle

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Other Expansions

- This expansion preserves unitarity below order $\lambda^4$.
- Preserving unitarity to all orders is possible (Buras, Lautenbacher and Ostermaier, 1994) with parameters:
  
  \[
  \begin{align*}
  s_{12} &= \lambda \\
  s_{23} &= A\lambda^2 \\
  s_{13}e^{-i\delta} &= A\lambda^3(\rho - i\eta)
  \end{align*}
  \]

- At order $\lambda^5$, this leads to

\[
\begin{pmatrix}
1-\lambda^2/2-\lambda^4/8 & \lambda & A\lambda^3(\bar{\rho}-i\bar{\eta})(1+\lambda^2/2) \\
-\lambda + A^2\lambda^5(1-2(\bar{\rho}+i\bar{\eta}))/2 & 1-\lambda^2/2-\lambda^4(1+4A^2)/8 & A\lambda^2 \\
A\lambda^3[1-\bar{\rho}-i\bar{\eta}] & -A\lambda^2 + A\lambda^4[1-2(\bar{\rho}+i\bar{\eta})]/2 & 1-A^2\lambda^4/2
\end{pmatrix} + \mathcal{O}(\lambda^6).
\]

- $V_{cd}$ acquires a phase at order $\lambda^5$.
- $V_{ts}$ acquires a phase at order $\lambda^4$.
Unitarity Triangles

\[ d \cdot s^* = 0 \text{ (K system)} \]
\[ s \cdot b^* = 0 \text{ (B}_s \text{ system)} \]
\[ d \cdot b^* = 0 \text{ (B}_d \text{ system)} \]

apply unitarity constraint to pairs of columns

from P. Burchat
(Three more) Unitarity Triangles

Apply unitarity constraint to pairs of rows

\[ u \cdot c^* = 0 \text{ (D system)} \]
\[ c \cdot t^* = 0 \text{ ("T" system)} \]
\[ u \cdot t^* = 0 \text{ ("T"" system)} \]

All six triangles have the same area. A nonzero area is a measure of CP violation and is an invariant of the CKM matrix.

from P. Burchat

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The triangles

See Bigi and Sanda, hep-phy/9909479 (1999)

Smallest Angle in triangle:

$B_d$ decays

$B_s$ decays

$D$ decays

$\sim 1$

$\sim \lambda^2$

$\sim \lambda^4$
“The” (Usual) Unitarity Triangle

Apply unitarity constraint to these two columns

from P. Burchat

Orientation of triangle has no physical significance. Only relative angle between sides is significant.

[This is the “bd” triangle a.k.a. the “B_d” triangle].

V_{ub}^*V_{ud} \quad V_{tb}^*V_{td}

V_{cb}^*V_{cd}
The Usual Unitarity Triangle

Apply unitarity constraint to these two columns

from P. Burchat
The $bd$ Unitarity Triangle before $B$ Factories

- CKM parameters $\rho$ and $\eta$ predict the observables $\varepsilon_K, x_d = \Delta M_d/\Gamma, |V_{ub}|, |V_{cb}|$

CKM parameters can be used to predict more observables:

$\varepsilon_K, \Delta M_d, \Delta M_s, BF(B \rightarrow \tau \nu), V_{ub}, \sin 2\beta, \gamma, V_{cb}$, lattice, …
CKM Parameters

- CKM parameters from these fits:

<table>
<thead>
<tr>
<th></th>
<th>UTFit</th>
<th>CKM Fitter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>$0.22545 \pm 0.00065$</td>
<td>$0.22543 \pm 0.00077$</td>
</tr>
<tr>
<td>$A$</td>
<td>$0.8095 \pm 0.0095$</td>
<td>$0.812^{+0.013}_{-0.027}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$0.135 \pm 0.021$</td>
<td>$0.132 \pm 0.020$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$0.367 \pm 0.013$</td>
<td>$0.358 \pm 0.012$</td>
</tr>
<tr>
<td>$\bar{\rho}$</td>
<td>$0.144 \pm 0.025$</td>
<td>$0.342 \pm 0.016$</td>
</tr>
</tbody>
</table>

Significant discrepancies exist:
- $\sin 2\beta$ \(\sim 3\sigma\) low
- $BF(B \to \tau\nu)$ \(2.7\sigma\) high

Is CKM model in question?

- Some “tension” exists and it will be important to continue to check the **CKM paradigm** with more precise measurements.
## Measurements of CKM elements

| $|V_{ud}|$ | 0.974250 ±0.0022 | Super-allowed $\beta$ decay. Best measurement |
| $|V_{us}|$ | 0.2253 ± 0.0008 | $K_{l2}$ & $K_{l3}$ (need lattice) and $\tau \rightarrow K_{\nu \tau}$. |
| $|V_{ub}|$ | 0.00392 ± 0.00046 | $B \rightarrow X\nu$, $B \rightarrow u$ decays. Some discrepancies |
| $|V_{cd}|$ | 0.230 ± 0.011 | Charm prod by $\nu$'s. $D \rightarrow K\nu$ needs theory |
| $|V_{cs}|$ | 1.04 ± 0.06 | $D \rightarrow K\nu$, $D_s \rightarrow \nu$ (theory limited) |
| $|V_{cb}|$ | 0.0409 ± 0.0007 | $B \rightarrow D\nu$, $B \rightarrow D^*\nu$, lattice |
| $|V_{td}|$ | 0.0081 ± 0.0005 | $B_d$ Mixing, lattice prediction for $|V_{ts}|/|V_{td}|$ |
| $|V_{ts}|$ | 0.0387 ± 0.0023 | $B_s$ Mixing |
| $|V_{ts}|$ | 0.88 ± 0.07 | Single top production (CDF, D0, Atlas, CMS) |
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Anatomy of Weak Decays

**Weak (short range) $|W|e^{i\phi}$**

Neither $I$, $U$, $V$-spin, $P$ nor $CP$ are (necessarily) conserved.

**Strong (long range) $|S|e^{i\delta}$**

Everything conserved. $I$-spin, $P$, $CP$, ..

[S]mall $E/M$ component may not conserve $I$
Space-time regimes (2)

- The two space-time ranges differ greatly so that we can write overall decay amplitude $A$ as a product

  $$A = A \ e^{i(\delta + \phi)}$$

  $$[A \equiv W \times S]$$

  **Strong phase**     **Weak phase**

- Under $CP$ weak phase $\phi$ flips sign but $\delta$ does not so

  $$\bar{A} = A \ e^{i(\delta - \phi)}$$

- This can give rise to $CPV$

  *Similar considerations govern the behavior of all processes involving weak interactions.*
Hadron – Friend or Foe!

- Hadrons were once thought to be a nuisance, obscuring the “more fundamental” aspects of the short-range weak interactions which lay at the heart of CPV!
- Interference between hadronic amplitudes, though, allows relative phases to be measured.
- The phases observed include both strong and weak components, so actually provide valuable information on the short range weak phases in the amplitudes governing the decay.
Weak phases through the hadron eye

- Dalitz plot for $D^0 \to K_S \pi^+ \pi$
- The flavour of the $K_S$ (ie $K^0 \to \pi^+ \pi$) is undefined, so both $K^0$ and $K^*$ can be produced at the quark level:
  - $\overline{K}^*$ ($c\to s\bar{u}d \propto \cos \theta_c^2$)
  - $K^*$ ($c\to d\bar{s}u \propto -\sin \theta_c^2$)
- NOTE change of sign of weak (ie ”production” amplitude)
- Observe the $K^*$ (horizontal band)!

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Some things multi-hadron systems have done for “real physics”:

- Provided evidence for CPV in B decays
- Allowed measurement of weak phases
- Exposed small parameters such as x and y, relating to $D^0$-$D^0$ mixing appearing linearly in the interference between the weak interactions involved.
- Resolved ambiguities in CPV phases, $B_s$ mixing parameters, …
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Mixing - another “useful nuisance”

- Mixing is a ubiquitous phenomenon readily observed in the $K^0, D^0, B^0$ and $B_s$ neutral meson systems.
  - The $T^0$ could mix, but will decay before mixing can occur.
  - $\pi^0, \eta^0, \eta'$ mesons do not mix since they are their own anti-particles.
  - The $D^0$ system is the only up-type meson that mixes. The SM greatly suppresses this, however.

- Mixing can lead to CPV and, in bringing meson and anti-meson decays into interference, it can allow the measurement of the weak decay phases involved.

- A brief description follows
Neutral Meson ($M^0$) Systems

- Flavour eigenstates are not mass eigenstates so they mix:

\[
\begin{align*}
|M_1\rangle &= p|M^0\rangle + q|M^0\rangle = e^{i(m_1 - i\Gamma_1 t/2)} \\
|M_2\rangle &= p|M^0\rangle - q|M^0\rangle = e^{i(m_2 - i\Gamma_2 t/2)}
\end{align*}
\]

\[p^2 + q^2 = 1\]

- We define four mixing parameters as

\[
\begin{align*}
x &= (m_1 - m_2)/\Gamma ; \\
y &= (\Gamma_1 - \Gamma_2)/\Gamma ; \\
r_m &= |q/p| \\
\phi_M &= \text{arg}\{q/p\}
\end{align*}
\]

\[\Gamma = \frac{\Gamma_1 + \Gamma_2}{2}\]

- $CP$ is conserved only if $p=q$. In this case, then $M_1$ is $CP$-even and $M_2$ is $CP$-odd
Neutral Meson ($M^0$) Systems

- The flavour states oscillate in time differently

$$|M^0(t)\rangle = e^{-(\Gamma/2+im)t} \left\{ \cosh[\Gamma t/2(y+ix)]|M^0\rangle + \left(\frac{q}{p}\right) \sinh[\Gamma t/2(y+ix)]|\bar{M}^0\rangle \right\}$$

$$|\bar{M}^0(t)\rangle = e^{-(\Gamma/2+im)t} \left\{ \left(\frac{p}{q}\right) \sinh[\Gamma t/2(y+ix)]|M^0\rangle + \cosh[\Gamma t/2(y+ix)]|\bar{M}^0\rangle \right\}$$

- Unless $p = q$, these are in not phase and “mixing-induced”, time-dependent CPV asymmetry occurs.
Where does mixing come from?

- Transitions that change flavour ($\Delta S$, $\Delta C$ or $\Delta B=2$) are possible via box diagrams, illustrated for $B_d^0$ below:

$$B^0 \rightarrow W \rightarrow u, c, t$$

- The weak phase $\phi_M = \text{Arg}\{q/p\}$ can be computed in the SM.

For $B^0$ system

$$\phi_M^B = \arg \left\{ \frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}} \right\} \approx 2\beta$$

For $K^0$ system

$$\phi_M^K = \arg \left\{ \frac{V_{td}V_{ts}^*}{V_{td}^*V_{ts}} \right\} \approx 2\beta$$

For $B_s$ system

$$\phi_M^{B_s} = \arg \left\{ \frac{V_{ts}V_{tb}^*}{V_{ts}^*V_{tb}} \right\} \approx 0$$

We return to $D^0$ system later.

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Observation of Mixing

- For mixing to be observed, \( M^0 \) (or \( \bar{M}^0 \)) must be flavour-tagged at time \( t=0 \) and then again when it decays.

- Decays to final states accessible to both \( M^0 \) and \( \bar{M}^0 \) are possible, especially when the final state is comprised of hadrons.

In such cases, interference between \( M^0 \) and \( \bar{M}^0 \) decays occurs and can be used to measure mixing and CPV parameters.
Decays to states accessible to both $M^0$ and $\bar{M}^0$

- We define amplitudes

$$A_f = \langle f | H | M^0 \rangle \quad ; \quad \bar{A}_f = \langle f | H | \bar{M}^0 \rangle$$

$$A_f = \langle \bar{f} | H | M^0 \rangle \quad ; \quad \bar{A}_f = \langle \bar{f} | H | \bar{M}^0 \rangle$$

- The important parameter is

$$\lambda_f = q \frac{\bar{A}_f}{p A_f} \quad \text{The phase of } \lambda_f \text{ includes } \phi_M \text{ and the weak and strong phases in the decay.}$$

and its measurement is crucial to CPV studies.

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The effects of mixing can be included in a fit to a multi-body hadronic final state \( f \) arising from decays of neutral mesons, such as \( M^0 \).

The \( t \)-dependent decay amplitude for a state \( M^0 \) at \( t = 0 \) is:

\[
\langle f | H | M^0(t) \rangle \equiv A_f(t) = e^{-(\Gamma/2+im)t} \left\{ \cosh[\Gamma t/2(y+ix)] \langle f | H | M^0 \rangle + \left( \frac{q}{p} \right) \sinh[\Gamma t/2(y+ix)] \langle f | H | \bar{M}^0 \rangle \right\}
\]

where \( \langle f | H | M^0 \rangle \equiv A_f \) and \( \langle f | H | \bar{M}^0 \rangle \equiv \bar{A}_f \) are linear combinations of amplitudes (eg “isobar”, etc.) normally present in a fit at \( t = 0 \). They depend upon the position \( \vec{s} \) in the phase space.

This can be written in the form:

\[
A_f(\vec{s}, t) = A_f(\vec{s}, 0) e^{-(\Gamma/2+im)t} \left\{ \cosh[\frac{\Gamma}{2}(y+ix)t] + \lambda_f(\vec{s}) \sinh[\frac{\Gamma}{2}(y+ix)t] \right\}
\]

The dependences on point in phase space \( \vec{s} \) and time \( t \) factorize.
Time-dependent “Dalitz plot fits”

- The Dalitz plot density is then \( \propto |A_f(\vec{s}, t)|^2 \)

- The Dalitz plot density can be written, similarly, for \( \bar{M}^0 \).
  - It is not, in general, the same as that for the \( M^0 \).

- Note that
  - mixing brings both \( M^0 \) and \( \bar{M}^0 \) into interference in the decay of \( \bar{M}^0 \);
  - The phase space \( \vec{s} \) time \( t \) dependences factorize.
  - *The parameter* \( \lambda_f \) *becomes* \( \lambda_f(\vec{s}) e^{i\phi_W} \)

The weak phase, \( \phi_W \), of \( \lambda_f \) is assumed not to depend on \( \vec{s} \)
Three types of CPV

1. Origin is in the mixing ("indirect CPV")
   
   \[ \frac{q}{p} = r_M e^{i\phi_M} \neq 1 \quad (r_M \equiv \left| \frac{q}{p} \right|). \]

2. Origin in the decay ("direct CPV")

   \[ |\bar{A}_f| \neq |A_f| \]

3. Coming from interference between mixing and decay ("indirect CPV" – a.k.a. "mixing-induced CPV")

   \[ \lambda_f = \frac{q \bar{A}_f}{p A_f} \neq 1 \]
Direct CPV

- When two (or more) amplitudes, $T$ and $P$, for example, mediate a decay process, then decay amplitudes are

\[
\mathcal{A} = T + Pe^{i(\Delta \delta + \Delta \phi_P)} \\
\bar{\mathcal{A}} = T + Pe^{i(\Delta \delta - \Delta \phi_P)}
\]

\[
\begin{align*}
\Delta \phi & \equiv (\phi_T - \phi_P) \\
\Delta \delta & \equiv (\delta_T - \delta_P)
\end{align*}
\]

\[
|\bar{\mathcal{A}}| \neq |\mathcal{A}|
\]

This leads to a CP asymmetry in decay rates

\[
A_{CP} = \frac{|\mathcal{A}_f|^2 - |\bar{\mathcal{A}}_f|^2}{|\mathcal{A}_f|^2 + |\bar{\mathcal{A}}_f|^2} = \frac{2r \sin(\Delta \phi) \sin(\Delta \delta)}{1 + r^2 + 2r \cos \Delta \delta \cos \Delta \phi}
\]

(r \equiv P/T)

NOTE $A_{CP} = 0$ unless $\Delta \phi \neq 0$ AND $\Delta \delta \neq 0$

$A_{CP}$ is largest when $P=T$. 

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Mixing-induced CPV in $M^0$ Decays

- Since $M^0$ and $M^0$ states oscillate differently in time, so do the rates for their decays $\Gamma(\overline{\Gamma})$ for $M^0(\overline{M}^0)$ to $f$:

$$\Gamma \propto e^{-\Gamma \Delta t} \left[ \cosh(y \Gamma t) + \frac{2Re(\lambda_f)}{1 + |\lambda_f|^2} \sinh(y \Gamma t) + \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos(x \Gamma t) - \frac{2Im(\lambda_f)}{1 + |\lambda_f|^2} \sin(x \Gamma t) \right]$$

$$\overline{\Gamma} \propto e^{-\overline{\Gamma} \Delta t} \left[ \cosh(y \overline{\Gamma} t) + \frac{2Re(\lambda_f)}{1 + |\lambda_f|^2} \sinh(y \overline{\Gamma} t) - \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos(x \overline{\Gamma} t) + \frac{2Im(\lambda_f)}{1 + |\lambda_f|^2} \sin(x \overline{\Gamma} t) \right]$$

The result is a time-dependent CP asymmetry

$$A_{CP}(t) = \frac{\overline{\Gamma} - \Gamma}{\overline{\Gamma} + \Gamma} = \frac{(1 - |\lambda_f|^2) \cos(x \Gamma t) - 2Im(\lambda_f) \sin(x \Gamma t)}{(1 + |\lambda_f|^2) \cosh(y \Gamma t) + 2Re(\lambda_f) \sinh(y \Gamma t)}$$
The parameter $\lambda_f$ encodes the weak and strong phases into this asymmetry:

$$\lambda_f = \frac{q A_f}{p A_f} \propto e^{i(\phi_M - 2\phi_f + \delta_f)} ; \quad \bar{\lambda}_f = \lambda_f^{-1}$$

Mixing (weak)  Decay (weak)  Decay (strong)

So measuring this $M^0-\bar{M}^0$ asymmetry as a function of time allows measurement of the weak phase $\phi_W = \phi_M - 2\phi_f$

BUT only if we know the strong phase $\delta_f$. 

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Three possibilities for measuring $\delta_f$ in a TDCPV analysis:

1. If $f$ is a CP eigenstate $f_{CP}$ (e.g. $\pi^+\pi^-$, $\phi K_s$, $J/\psi K_L$, etc.).
   strong phase of $A_f$ same as that of $\bar{A}_f$ so $\delta_f = 0$

2. Similarly, if $f$ is CP self conjugate (sum of CP-even and CP-odd states) e.g. $K_S \pi^+\pi^-$, $\pi^+\pi^0$, etc.
   strong phases of $A$ are linked to those of $\bar{A}$ so $\delta_f = 0, \pi$

3. If $f$ is a multi-hadron system -
   Amplitude analysis of hadrons allows measurement of $\delta_f$ but there may also be an unknown phase offset too.

From Dalitz Plot, etc.
Lecture I Outline

- CP Violation what it is, how it happens, what it is not.
- The CKM model for CPV
  - Unitarity triangles
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Application to $B^0$ Decays at BaBar/Belle

- For $B^0$ mesons, $y \approx 0$ so $B^0$ decay rates are:
  
  $$
  \frac{d\Gamma}{d\Delta t} \propto e^{-\Gamma \Delta t} [1 + C \cos(x\Gamma \Delta t) - S \sin(x\Gamma \Delta t)]
  $$
  
  $$
  \frac{d\bar{\Gamma}}{d\Delta t} \propto e^{-\Gamma \Delta t} [1 - C \cos(x\Gamma \Delta t) + S \sin(x\Gamma \Delta t)]
  $$

  with time-dependent (TD) CP asymmetry
  
  $$
  A_{CP}(t) = S \sin(x\Gamma \Delta t) - C \cos(x\Gamma \Delta t)
  $$

  where
  
  $$
  S = \frac{2\text{Im}(\lambda_f)}{1 + |\lambda_f|^2} \quad ; \quad C = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}
  $$

  $S$ measurable
  
  Only in TD CPV

  $C = 0$ in absence of direct CPV

  (Belle chose to use $A'' = -C$)

- CPV analyses at the $B$ factories focus on measuring $C$
  
  (direct CPV) and $S$ (to extract the weak phase of $\lambda_f$).
Original $B$ Factory goals

- Use $Y(4S)$ decays to $B^0$-$\bar{B}^0$ pairs – one is “flavour tagged” as either $B^0$ or $\bar{B}^0$ and other $B^0$ decays in a CP eigenstate.

- Measure $\Delta t$ between the two decays and the CP asymmetry at each $\Delta t$.

- Figure shows ideal case for $J/\psi K_s$
  - no experimental effects.
  - Asymmetry is large.

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**Sin 2β:** \( B^0 \rightarrow J/ψK_s \ (b \rightarrow ccs) \)

\[
\begin{align*}
\text{b} & \rightarrow \text{c} \\
\text{W} & \rightarrow \text{c} \\
\bar{d} & \rightarrow \bar{d} \quad \bar{d} \rightarrow \bar{d} \\
\bar{d} & \rightarrow \bar{d} \\
\end{align*}
\]

**B mix phase:** \( \phi^B_M = \arg \left\{ \frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}} \right\} \)

**Tree phase:** \( 2\phi_T = \arg \left\{ \frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}} \right\} \)

**Penguin phase** \( \phi_P \) is the SAME!

**K_s mix phase:** \( \phi^K_M = \arg \left\{ \frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}} \right\} \)

Comes with \( K^0 \)

\( V_{tb} \approx 1 \); \( \text{Arg}\{V_{td}\} = \pi - \beta \); \( \text{Arg}\{V_{cb}V_{cd}^*\} \approx 0 \)

\( \rightarrow \text{Weak phase:} \quad \phi_W = -2\beta \)
Sin 2\(\alpha\): \(B^0 \rightarrow \pi^+ \pi^-\) \((b \rightarrow u\bar{u}d)\)

\[ B \text{ mix phase: } \phi_M^B = \arg \left\{ \frac{V_{td} V_{tb}^*}{V_{td}^* V_{tb}} \right\} \]

\[ \text{Tree phase: } 2\phi_T = \arg \left\{ \frac{V_{ub} V_{ud}^*}{V_{ub}^* V_{ud}} \right\} \]

\(V_{tb} \approx V_{ud} \approx 1\); \(\text{Arg}\{V_{td}\} = \pi - \beta\); \(\text{Arg}\{V_{ub}\} = \gamma\)

\[ \rightarrow \text{Weak phase: } \phi_W = 2\alpha \]
$$\sin 2\gamma: \quad B_s \rightarrow \rho K_s \quad (b \rightarrow u\bar{u}d)$$

- **$B_s$ mix phase:** 
  $$\phi_M^B = \arg \left\{ \frac{V_{ts}V_{tb}^*}{V_{ts}^*V_{tb}} \right\}$$

- **Tree phase:** 
  $$2\phi_T = \arg \left\{ \frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}} \right\}$$

- **$K_s$ mix phase:** 
  $$\phi_K^K = \arg \left\{ \frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}} \right\}$$

$$V_{tb} \approx V_{ud} \approx 1; \quad \text{Arg}\{V_{ts}V_{cs}^*\} \approx 0; \quad \text{Arg}\{V_{ub}\} \approx \gamma; \quad \text{Arg}\{V_{cd}\} \approx 0$$

- **Weak phase:** 
  $$\phi_W = 2\gamma$$

---

**Experimental:** Impossible at B factories, though!

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Many other modes were actually used, including:

- those where penguin contributions were important or even dominant
- some requiring the invocation of $SU(2)$ symmetry ($I$, $U$ and $V$-spin)
- non CP-eigenstates
- multi-hadron hadron systems requiring $TD$ Dalitz plot amplitude analyses.
- various vector-vector modes (separating out the longitudinal helicity components ($CP=+1$)

A credible set of methods to determine $\gamma$ that did not involve $TD$ studies of $B_s$ mesons were also developed.
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We might expect …

- Even more modes *(LHCb and SuperB / Belle2)*
- Multi-hadron modes with 4-body *(or higher dimension)* amplitude analyses separating out the CP-odd or CP-even helicity components *(LHCb and SuperB/Belle2)*
- Full use of more precise strong phase measurements from charm threshold data *(BES3 and SuperB)*
- *TD* studies of $B_s$ mesons *(LHCb)*.
- Much information on direct or time-integrated CPV *(mostly BES3/Panda but also LHCb/SuperB/Belle2)*.
- CPV studies of the charm triangle and further understanding of the origin for LHCb evidence of direct CPV in $D^0 \rightarrow \pi^+ \pi^-$ and $D^0 \rightarrow K^+ K^-$ decays *(LHCb and SuperB/Belle2)*.
Next time, we will cover some of the B factory methods and results in these CPV measurements.