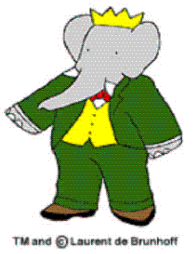
CP Violation and all that.



Brian Meadows
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Lecture I Outline

- *CP* Violation what it is, how it happens, what is it not.
- The *CKM* model for *CPV*
  - Unitarity triangles
- A role for hadrons and amplitude analysis
- Mixing and its role in *CPV*
- *B* factory measurements
- Pause for reflection

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CP Violation (*CPV*) - very brief History

- *CPV* is observable as a particle/antiparticle asymmetry in the rates of transitions:

$$\Gamma(i \to f) \neq \Gamma(\bar{i} \to \bar{f})$$

(Bars indicate antiparticle conjugate states)

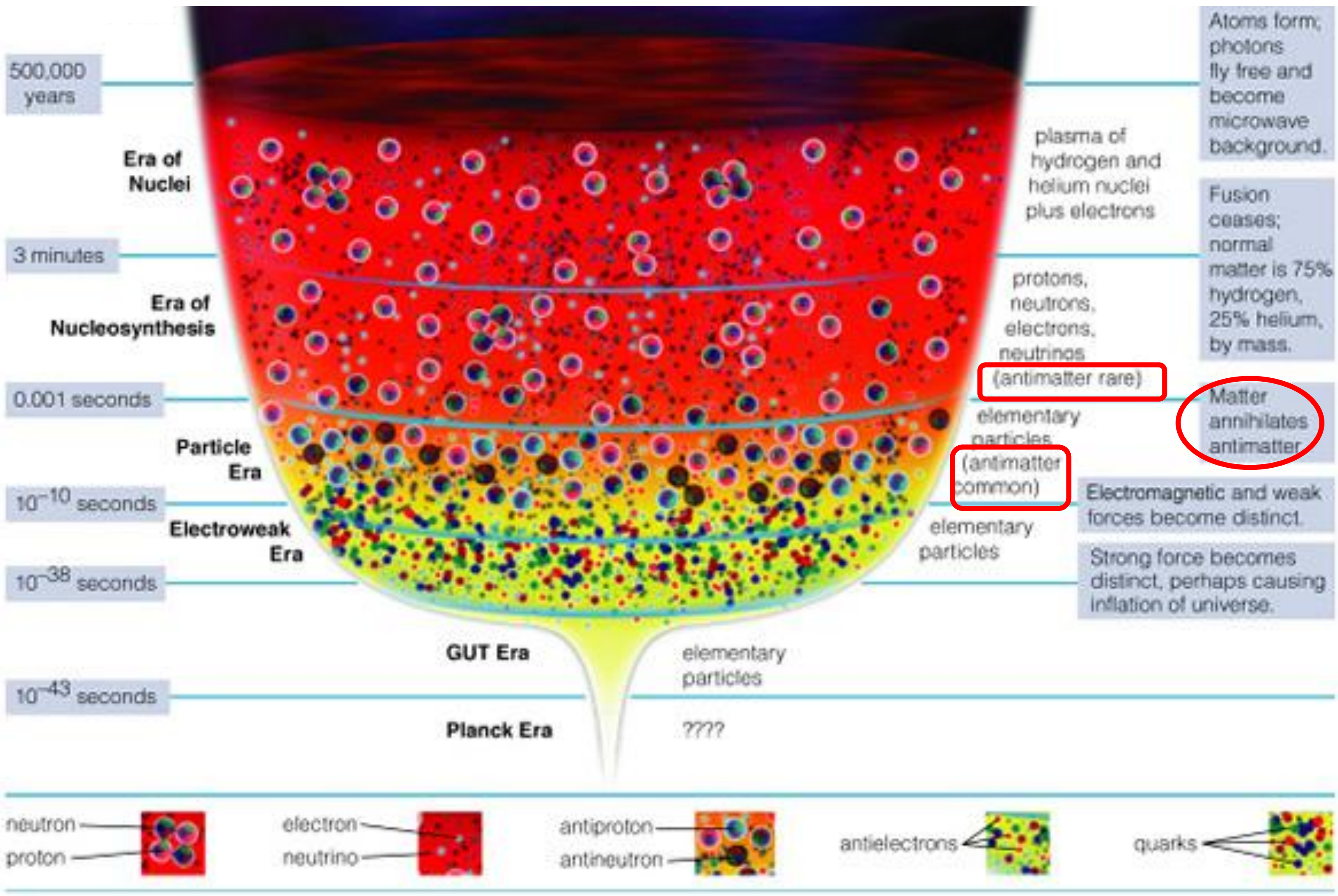
for processes with initial state i and final state f

- *CPV* was discovered (Fitch and Cronin 1963) in decays to $\pi^+\pi^-$ states (with *CP=+1*) by long-lived K_L mesons that were formerly found to decay only to $\pi^-\ell^+\nu_\ell$ (with *CP=-1*).
- It was next seen as an asymmetry in $K_L \to \pi^{\mp}\ell^{\pm}\nu_\ell$ decays

CPV – very brief History (2)

- Theoretically, there was much speculation on the source of *CPV* – whether from mixing or decay (“direct”).
- In *1973*, Kobayashi and Moskawa suggested the current 3-family nature of the SM and pointed out that it led to the possibility for *CPV*.
- Experimentally, *CPV* was not observed anywhere other than in neutral *K* decays for *~40* years when, much as predicted by *KM*, the BaBar and Belle experiments observed its interference with mixing in decays of B^0 mesons to $J/\psi\ K_s$ final states (CP=-1).
- First evidence for direct CPV in B decays was later observed in *2004* by the BaBar and Belle experiments.

Early evolution of the universe



CPV and Baryogenesis

- *CPV* is needed to account for excess $(\Delta N = N_B - N_{\bar{B}})$ of baryons over anti-baryons in our universe.
- This excess is only a small fraction of the observed number of photons $\Delta N / N_\gamma \simeq N_B / N_\gamma \sim 10^{-10}$.
- Sakharov (1967) held that this requires:
  - Baryon number violating processes - $H^{\text{eff}}_{\Delta N \neq 0} \neq 0$
  - *CPV* so that, for any baryon-violating process, $\Gamma(i \to f) \neq \Gamma(\bar{i} \to \bar{f})$
  - A period in the history of the universe when it was not in thermal equilibrium.

    In thermal equilibrium (*T* invariance), *CPT* invariance is equivalent to *CP* invariance!

[*CPT* violation alone, could also generate $\Delta N \neq 0$
  - This would also violate locality, causality and Lorentz invariance.]

Early Universe – an Illustration

- Consider a particle X which has just two decay modes to final states $f_{1,2}$ with baryon numbers $B_{1,2}$ (*not necessarily the same*) and branching fractions r and $(1 - r)$.
- *CPT* invariance requires anti-particle $\bar{X}$ to have the same total decay rates but *CP* violation can allow $\bar{r}$ to differ from r.
- Decay of each $X\bar{X}$ pair (initially $B = 0$):

$$X \longrightarrow B = r B_1 + (1 - r) B_2$$

$$\bar{X} \longrightarrow B = \bar{r}(-B_1) + (1 - \bar{r})(-B_2)$$

results in a change in number of baryons $\Delta B = (r - \bar{r})(B_1 - B_2)$

- *Baryon-dominant universe* requires this be positive, so:
  - $B_1 \neq B_2$ - baryon number is violated for at least one decay mode

AND

  - $r \neq \bar{r}$ - there is *CPV*

Differences must have same sign too

CPV and Field Theory

- The SM Lagrangian is Hermitian and includes terms like

$$\mathcal{L}(x) = \sum_i \left[a_i \mathcal{F}_i(x) + a_i^* \mathcal{F}_i^\dagger(x) \right]$$

  where the $\mathcal{F}_i(x)$ are scalar operators defined from quark and lepton fields and the a_i are couplings.

- *CP* -invariance requires that all couplings can be made real with a suitable choice of phases for the fields.
- In the *SM*, charged *EW* terms are of the form

$$\frac{g}{\sqrt{2}} \left(\bar{u}_L \; \bar{c}_L \; \bar{t}_L \right) V_{CKM} \gamma^\mu \begin{pmatrix} \bar{d}_L \\ \bar{s}_L \\ b_L \end{pmatrix} W_\mu^+$$

  These all involve the CKM matrix V_{CKM} that has a phase that depends on specific flavor couplings

  The effects of such a phase cannot be removed, for all flavors, simply by re-phasing the quark fields → *CPV*

CPV is not "*T*-violation"

- *CP* violation is not the same thing as "*T* violation". It would be only if *CPT* were an absolute symmetry.
- "*T*-violation" (time-reversal symmetry breaking), has yet to be found experimentally (?)
- For instance, we know, experimentally, that $B^0 \to K\pi$ is a *CP* violating process. However, we have not observed a weak scattering $K + \pi \to B^0$ occuring at a different rate (and we probably never will, even though *CPT* would so decree!)
- An amplitude $<f|H|i>$ describes a transition from $i \to f$. Under *CP* this becomes $<\bar{f}|H|\bar{i}>$, not $<i|H|f>$.

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The CKM Matrix

- This is a 3x3 unitary transformation

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Defined by rotations about 3 axes by angles θ_{12}, θ_{13}, θ_{23}

$$U_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix};\ U_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix};\ U_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix},$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \sin\theta_{ij}$, and (important) a phase

$$V_\delta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix}$$

- Then

$$V_{CKM} = U_{23} U_\delta^\dagger U_{13} U_\delta U_{12}$$

Wolfenstein Expansion

- So

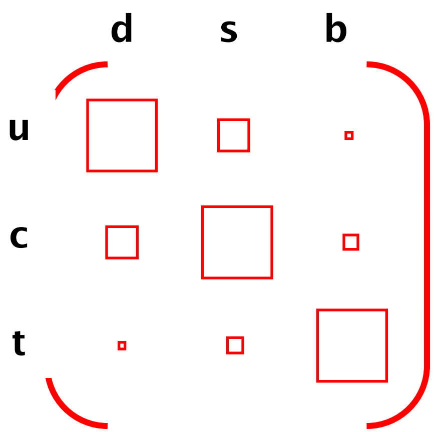
$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{13}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- The terms have a numerical hierarchy that suggests an expansion in powers of the Cabibbo angle $\lambda = V_{us}$:

$$V_{CKM} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^3 \\ A\lambda^3(1-\rho - i\eta) & A\lambda^3 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

where A, ρ and η are parameters of order one.

Get to know the CKM



$$\approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^3 \\ A\lambda^3(1-\rho-i\eta) & A\lambda^3 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$\mathrm{Arg}\{V_{ub}\} = -\gamma$

$\mathrm{Arg}\{V_{td}\} = -\beta$

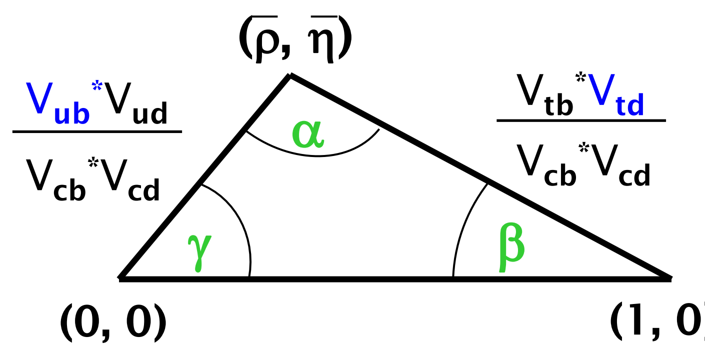
- Magnitudes:

  Memorize the power of λ for each term.

  & labels for rows/columns

- Phases:

  Remember the phases for the two terms circled



"The" unitarity triangle

Other Expansions

- This expansion preserves unitarity below order λ^4.
- Preserving unitarity to all orders is possible (Buras, Lautenbacher and Ostermaier, 1994) with parameters:

$$\begin{aligned} s_{12} &= \lambda \\ s_{23} &= A\lambda^2 \\ s_{13}e^{-i\delta} &= A\lambda^3(\rho - i\eta) \end{aligned}$$

- At order λ^5, this leads to

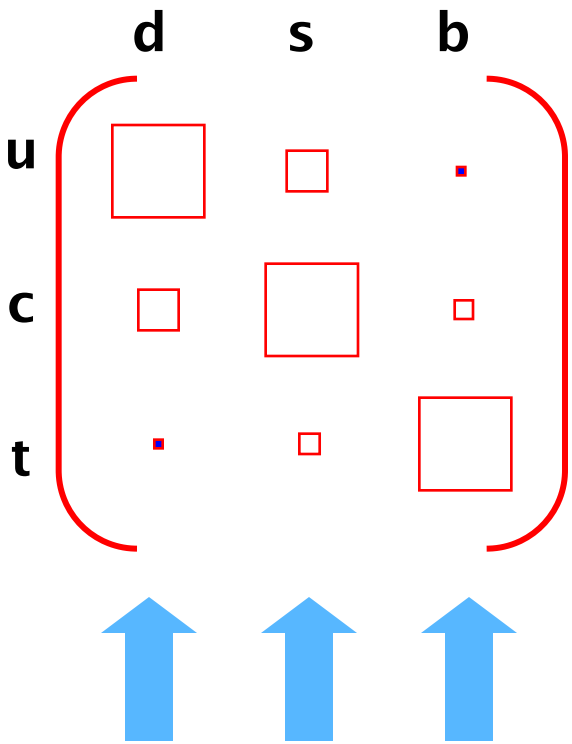
$$\begin{pmatrix} 1-\lambda^2/2-\lambda^4/8 & \lambda & A\lambda^3(\bar{\rho}-i\bar{\eta})(1+\lambda^2/2) \\ -\lambda+A^2\lambda^5[1-2(\bar{\rho}+i\bar{\eta})]/2 & 1-\lambda^2/2-\lambda^4(1+4A^2)/8 & A\lambda^2 \\ A\lambda^3[1-\bar{\rho}-i\bar{\eta}] & -A\lambda^2+A\lambda^4[1-2(\bar{\rho}+i\bar{\eta})]/2 & 1-A^2\lambda^4/2 \end{pmatrix} + \mathcal{O}(\lambda^6).$$

Phase(γ) of V_{ub} is unchanged

V_{cd} acquires a phase at order λ^5

V_{ts} acquires a phase at order λ^4

Unitarity Triangles



apply unitarity constraint
to pairs of columns

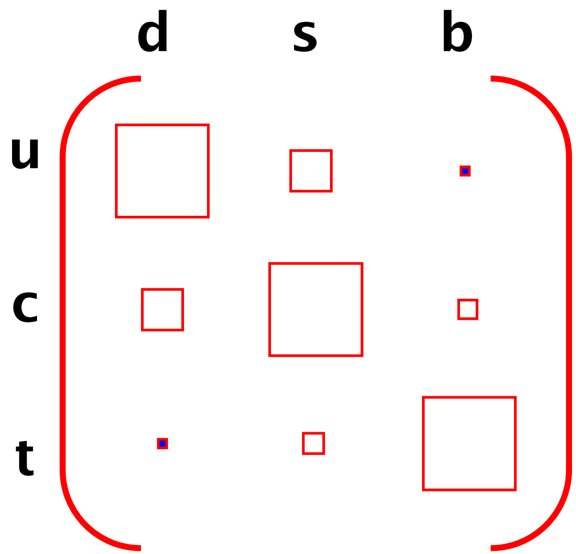
$\mathbf{d} \cdot \mathbf{s}^* = 0$ (K system)

$\mathbf{s} \cdot \mathbf{b}^* = 0$ (B_s system)

$\mathbf{d} \cdot \mathbf{b}^* = 0$ (B_d system)

from P. Burchat

(Three more) Unitarity Triangles



$\mathbf{u} \cdot \mathbf{c}^* = 0$ (D system)

$\mathbf{c} \cdot \mathbf{t}^* = 0$ ("T" system)

$\mathbf{u} \cdot \mathbf{t}^* = 0$ ("T'" system)

All six triangles have the same area. A nonzero area is a measure of CP violation and is an invariant of the CKM matrix.

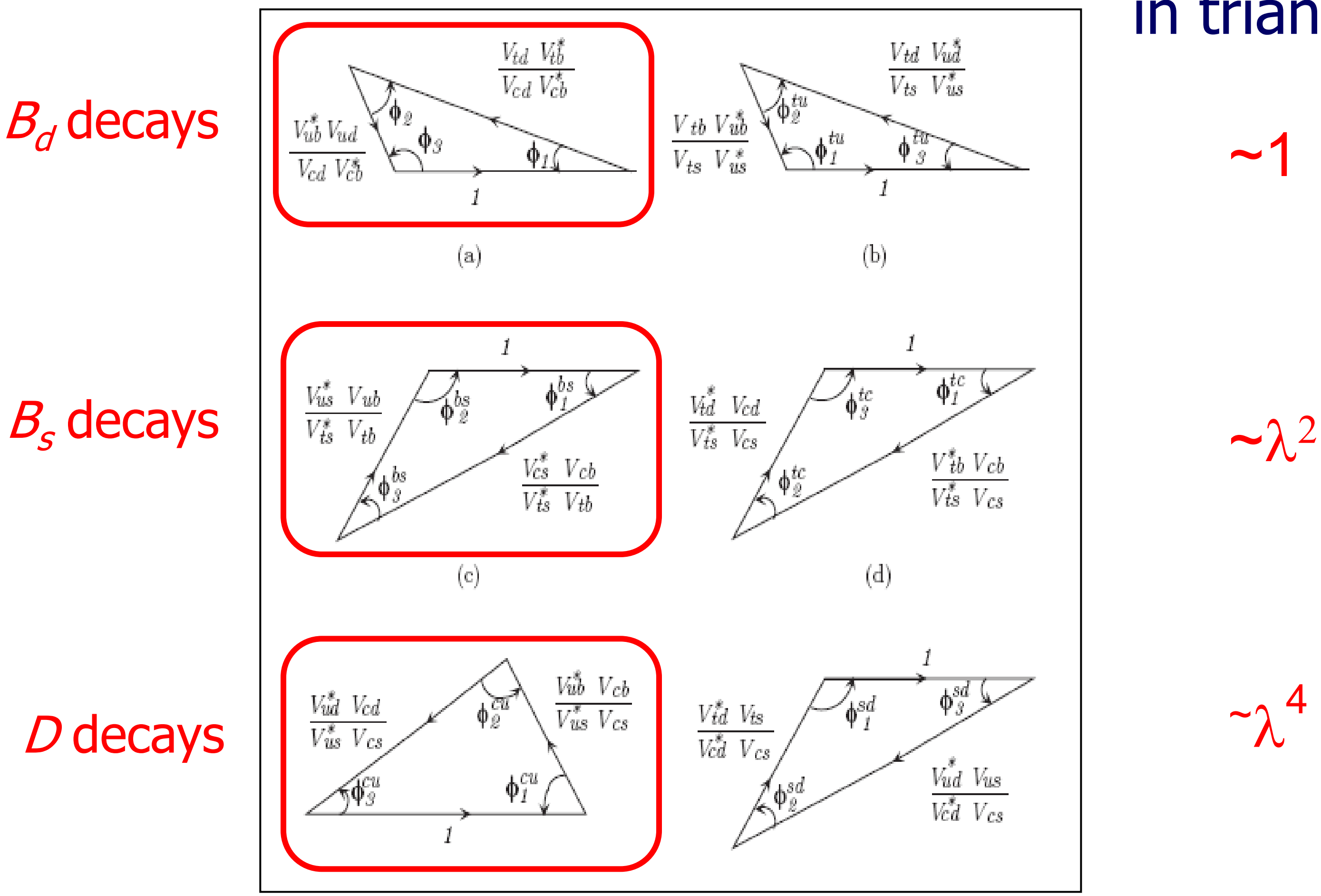
Apply unitarity constraint to pairs of rows

from P. Burchat

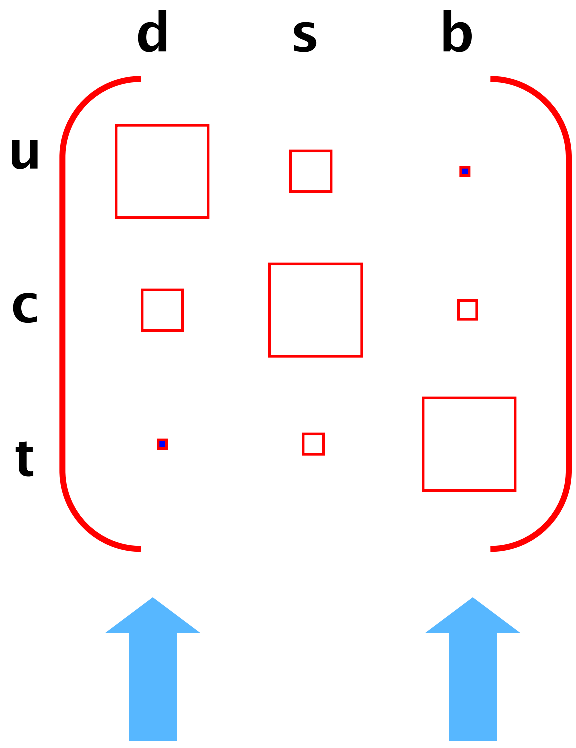
The triangles

See Bigi and Sanda, hep-phy/9909479 (1999)

Smallest Angle in triangle:

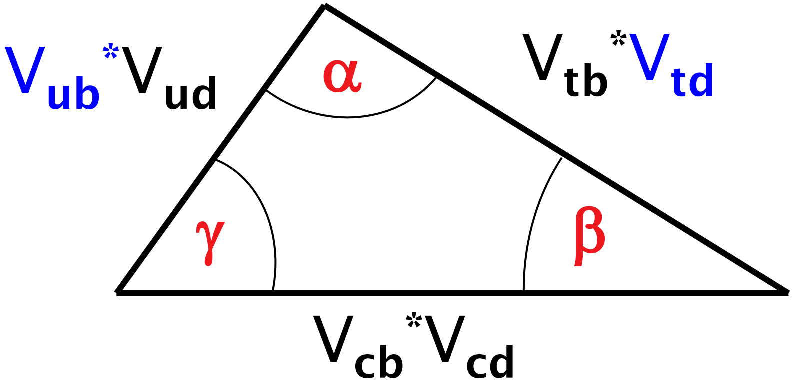


"The" (Usual) Unitarity Triangle



Apply unitarity constraint to these two columns

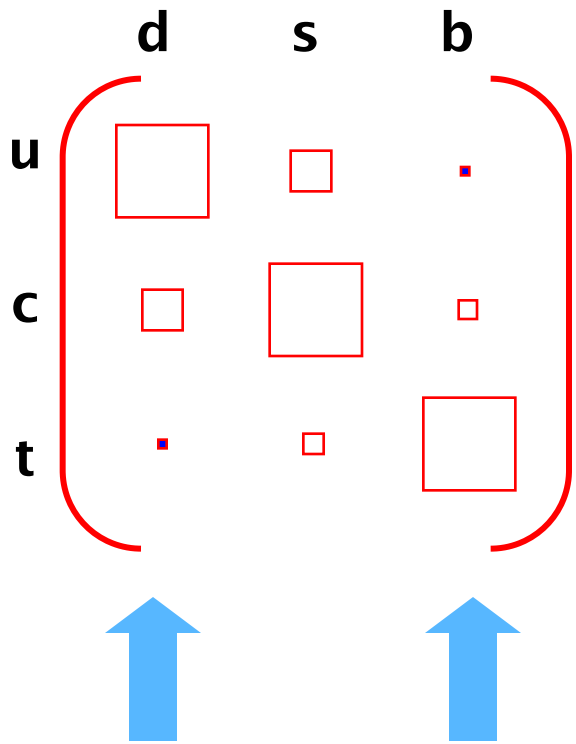
[This is the "*bd*" triangle a.k.a. the "B_d" triangle].



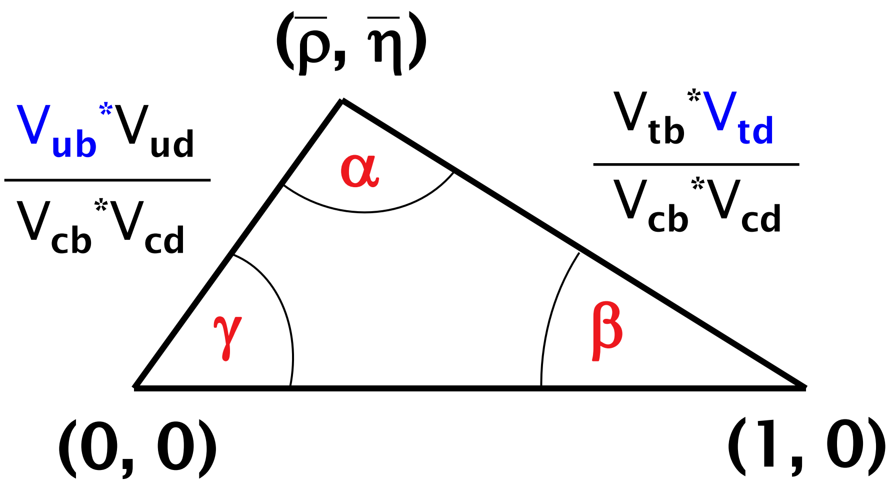
Orientation of triangle has no physical significance. Only relative angle between sides is significant.

from P. Burchat

The Usual Unitarity Triangle



Apply unitarity constraint
to these two columns



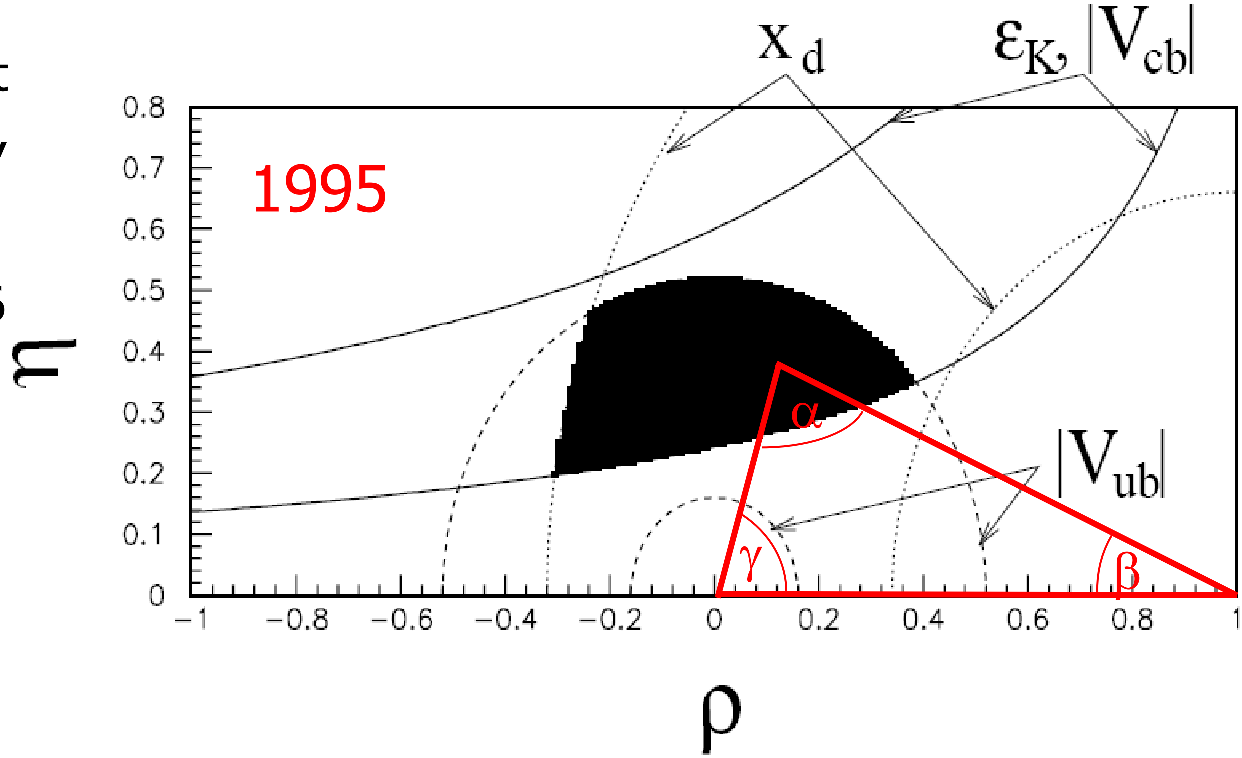
from P. Burchat

The *bd* Unitarity Triangle before *B* Factories

- CKM parameters ρ and η predict the observables

  ε_K, $x_d = \Delta M_d / \Gamma$, $|V_{ub}|$,$|V_{cb}|$

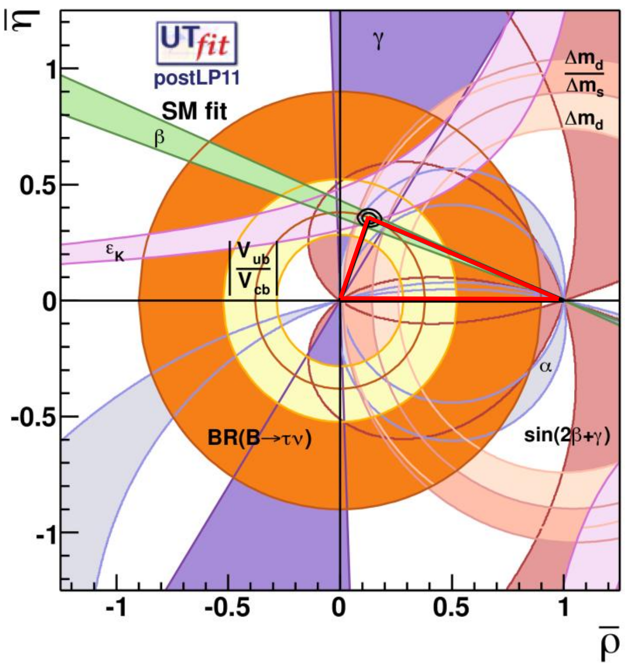
From P. Burchat and J. Richman, Rev. Mod. Phys., 67 (1995) 893-976



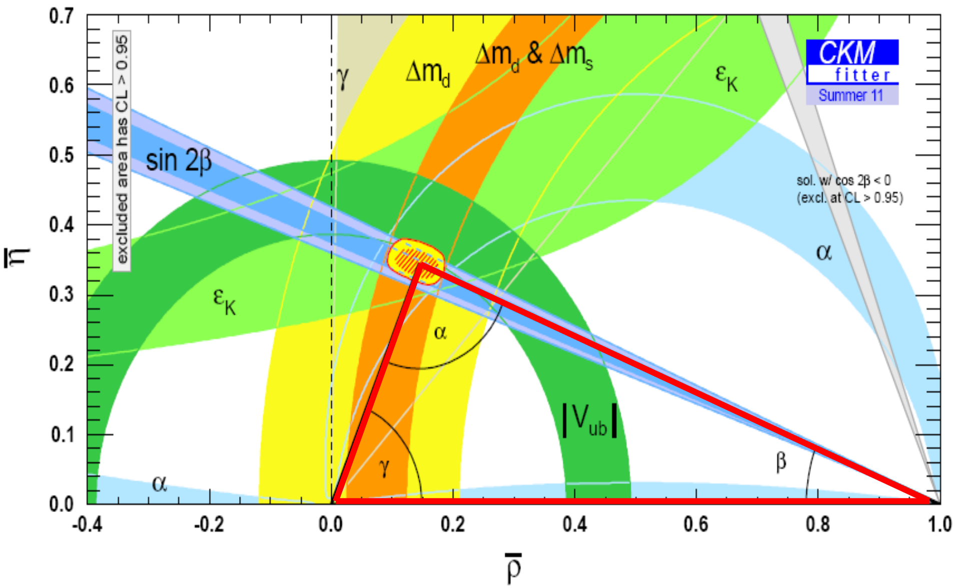
The *bd* Unitarity Triangle Today

- CKM parameters can be used to predict more observables

  ε_K, ΔM_d, ΔM_s, BF($B\to\tau\nu$), V_{ub}, $\sin 2\beta$, γ, V_{cb}, lattice, …



UTFit



CKMFitter

CKM Parameters

- CKM parameters from these fits:

	UTFit	CKM Fitter
λ	0.22545 ± 0.00065	0.22543 ± 0.00077
A	0.8095 ± 0.0095	$0.812^{+0.013}_{-0.027}$
ρ	0.135 ± 0.021	————————————
η	0.367 ± 0.013	————————————
$\overline{\rho}$	0.132 ± 0.020	0.144 ± 0.025
$\overline{\eta}$	0.358 ± 0.012	$0.342 + 0.016$

Significant discrepancies exist:

$\sin 2\beta$ ~3σ low

BF($B \to \tau\nu$) 2.7σ high

Is CKM model in question?

arXiv:1104.2117 [hep-ph]

- Some "tension" exists and it will be important to continue to check the *CKM* paradigm with more precise measurements.

Measurements of CKM elements

$\|V_{ud}\|$	0.974250	$\pm$0.0022	Super-allowed β decay. Best measurement
$\|V_{us}\|$	0.2253	$\pm$ 0.0008	K_{l2} & K_{l3} (need lattice) and $\tau \rightarrow K\nu_{\tau}$.
$\|V_{ub}\|$	0.00392	$\pm$ 0.00046	$B \rightarrow Xl\nu$, $B \rightarrow u$ decays. Some discrepancies
$\|V_{cd}\|$	0.230	$\pm$ 0.011	Charm prod by ν's. $D \rightarrow Kl\nu$ needs theory
$\|V_{cs}\|$	1.04	$\pm$ 0.06	$D \rightarrow Kl\nu$, $D_s \rightarrow l\nu$ (theory limited)
$\|V_{cb}\|$	0.0409	$\pm$ 0.0007	$B \rightarrow Dl\nu$, $B \rightarrow D^{*}l\nu$, *lattice*
$\|V_{td}\|$	0.0081	$\pm$ 0.0005	B_d *Mixing*, *lattice prediction for* $\|V_{ts}\|/\|V_{td}\|$
$\|V_{ts}\|$	0.0387	$\pm$ 0.0023	B_s *Mixing*
$\|V_{ts}\|$	0.88	$\pm$ 0.07	*Single top production (CDF,D0,Atlas,CMS)*

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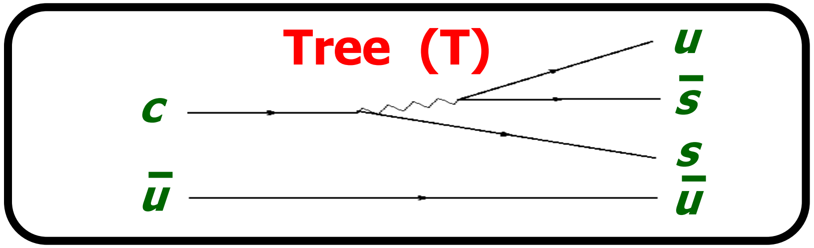
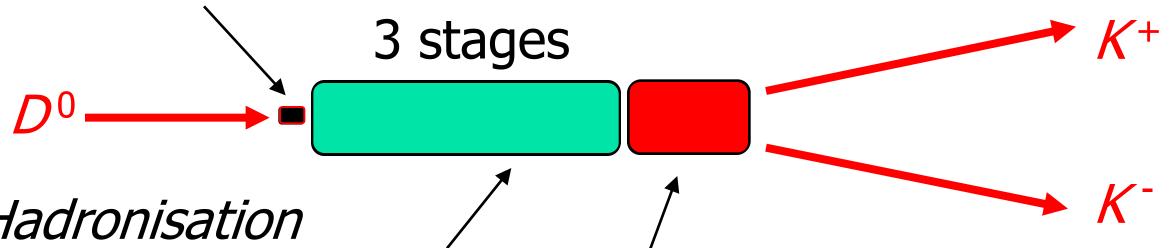
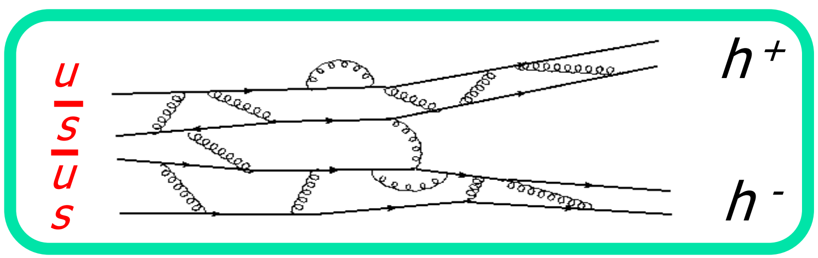
Anatomy of Weak Decays

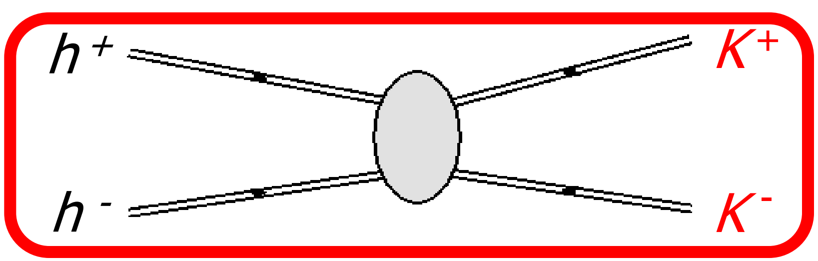




Hadronisation



Scattering



Weak (short range) **$|W|e^{i\phi}$**

Neither *I, U, V*-spin, *P* nor *CP* are (necessarily) conserved.

Strong (long range) **$|S|e^{i\delta}$**

Everything conserved.
I-spin, *P*, *CP*, ..

[Small E/M component** may not conserve I **]

Space-time regimes (2)

- The two space-time ranges differ greatly so that we can write overall decay amplitude $\mathcal{A}$ as a product

$$\mathcal{A} = A\, e^{i(\delta+\phi)} \qquad [\mathcal{A} \equiv W \times S]$$

Strong phase (δ) — Weak phase (φ)

- Under *CP* weak phase ϕ flips sign but δ does not

  so $$\bar{\mathcal{A}} = A\, e^{i(\delta-\phi)}$$

- This can give rise to *CPV*

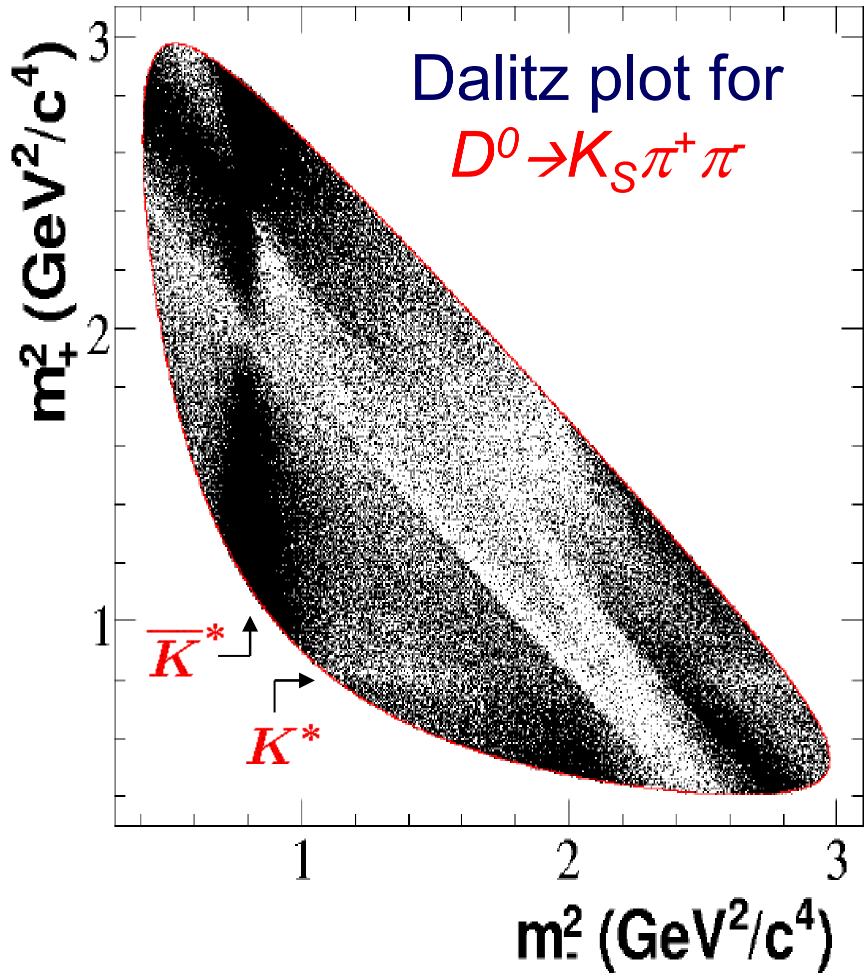
Similar considerations govern the behavior of all processes involving weak interactions.

Hadron – Friend or Foe!

- Hadrons were once thought to be a nuisance, obscuring the "more fundamental" aspects of the short-range weak interactions which lay at the heart of *CPV* !
- Interference between hadronic amplitudes, though, allows relative phases to be measured.
- The phases observed include both strong and weak components, so actually provide valuable information on the short range weak phases in the amplitudes governing the decay.

Weak phases through the hadron eye

- Dalitz plot for $D^0 \to K_S \pi^+ \pi^-$
- The flavour of the K_S (ie $K^0 \to \pi^+ \pi^-$) is undefined, so both K^{*0} and $\overline{K}^{*0}$ can be produced at the quark level:
  - $\overline{K}^{*0}$ ($c \to s\bar{u}d \propto \cos\theta_c^2$)
  - K^{*0} ($c \to d\bar{s}u \propto -\sin\theta_c^2$)
- NOTE change of sign of weak (ie "production" amplitude)
- Observe the K^{*0} (horizontal band) !



- Some things multi-hadron systems have done for “real physics”:
  - Provided evidence for CPV in B decays
  - Allowed measurement of weak phases
  - Exposed small parameters such as x and y, relating to D^0-D^0 mixing appearing linearly in the interference between the weak interactions involved.
  - Resolved ambiguities in CPV phases, B_s mixing parameters, …

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Mixing - another "useful nuisance"

- Mixing is a ubiquitous phenomenon readily observed in the K^0, D^0, B^0 and B_s neutral meson systems .
  - The T^0 could mix, but will decay before mixing can occur.
  - π^0, η^0, η' mesons do not mix since they are their own anti-particles.
  - The D^0 system is the only up-type meson that mixes. The SM greatly suppresses this, however.
- Mixing can lead to *CPV* and, in bringing meson and anti-meson decays into interference, it can allow the measurement of the weak decay phases involved.
- A brief desription follows

Neutral Meson (M^0) Systems

- Flavour eigenstates are not mass eigenstates so they mix:

$$\left.\begin{array}{rclcl} |M_1\rangle & = & p|M^0\rangle + q|\overline{M}^0\rangle & = & e^{i(m_1 - i\Gamma_1 t/2)} \\ |M_2\rangle & = & p|M^0\rangle - q|\overline{M}^0\rangle & = & e^{i(m_2 - i\Gamma_2 t/2)} \end{array}\right\} \quad p^2 + q^2 = 1$$

- We define four mixing parameters as

$$\left.\begin{array}{rclcrcl} x & = & (m_1 - m_2)/\Gamma & ; & r_m & = & |q/p| \\ y & = & (\Gamma_1 - \Gamma_2)/\Gamma & ; & \phi_M & = & \arg\{q/p\} \end{array}\right\} \quad \Gamma = \frac{\Gamma_1 + \Gamma_2}{2}$$

- *CP* is conserved only if $p=q$. In this case, then M_1 is *CP*-even and M_2 is *CP*-odd

Neutral Meson (M^0) Systems

- The flavour states oscillate in time differently

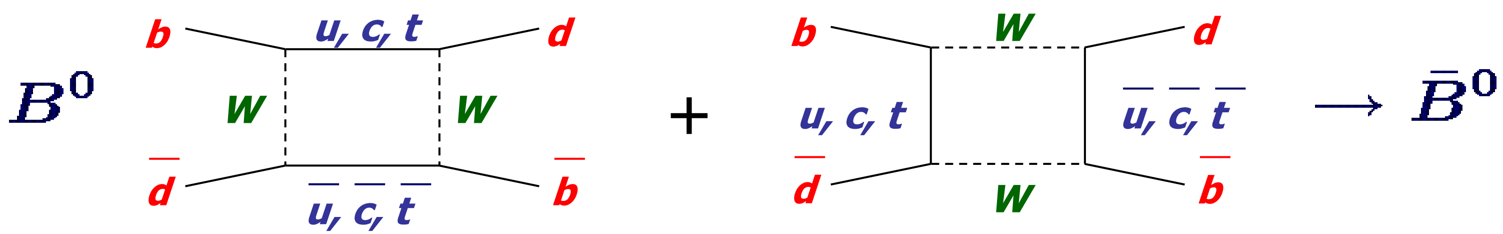
$$|M^0(t)\rangle = e^{-(\Gamma/2+im)t}\left\{\cosh[\Gamma t/2(y+ix)]|M^0\rangle + \left(\frac{q}{p}\right)\sinh[\Gamma t/2(y+ix)]|\bar{M}^0\rangle\right\}$$

$$|\overline{M}^0(t)\rangle = e^{-(\Gamma/2+im)t}\left\{\left(\frac{p}{q}\right)\sinh[\Gamma t/2(y+ix)]|M^0\rangle + \cosh[\Gamma t/2(y+ix)]|\bar{M}^0\rangle\right\}$$

- Unless $p = q$, these are in not phase and “mixing-induced”, time-dependent *CPV* asymmetry occurs.

Where does mixing come from ?

- Transitions that change flavour (ΔS, ΔC or $\Delta B=2$) are possible via box diagrams, illustrated for B_d^0 below:



- The weak phase ϕ_M=Arg{q/p} can be computed in the SM.

for B^0 system $\phi_M^B = \arg\left\{\frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}}\right\} \simeq 2\beta$

for K^0 system $\phi_M^K = \arg\left\{\frac{V_{td}V_{ts}^*}{V_{td}^*V_{ts}}\right\} \simeq 2\beta$

for B_s system $\phi_M^{B_s} = \arg\left\{\frac{V_{ts}V_{tb}^*}{V_{ts}^*V_{tb}}\right\} \simeq 0$

We return to D^0 system later

Observation of Mixing

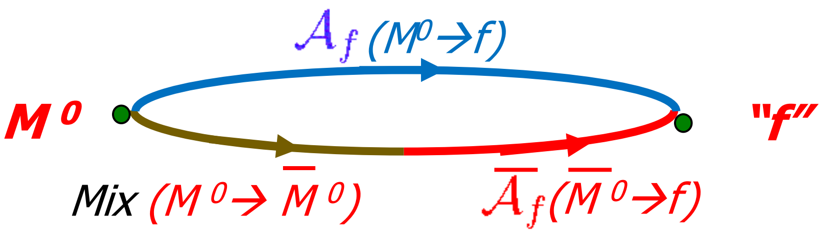
- For mixing to be observed, M^0 (or $\overline{M}^0$) must be flavour-tagged at time $t=0$ and then again when it decays
- Decays to final states accessible to both M^0 and $\overline{M}^0$ are possible, especially when the final state is comprised of hadrons.

  In such cases, interference between M^0 and $\overline{M}^0$ decays occurs and can be used to measure mixing and *CPV* parameters.

Decays to states accessible to both M^0 and $\overline{M}^0$

- We define amplitudes

$$\mathcal{A}_f = \langle f|H|M^0\rangle \;;\; \bar{\mathcal{A}}_f = \langle f|H|\overline{M}^0\rangle$$
$$\mathcal{A}_{\bar{f}} = \langle \bar{f}|H|M^0\rangle \;;\; \bar{\mathcal{A}}_{\bar{f}} = \langle \bar{f}|H|\overline{M}^0\rangle$$



Mixing brings decays from M and $\overline{M}$ into interference

Opens the possibility to measure the weak phase

- The important parameter is

$$\lambda_f = \frac{q}{p}\frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f}$$

The phase of λ_f includes ϕ_M and the weak and strong phases in the decay.

and its measurement is crucial to *CPV* studies.

Time-dependent "Dalitz plot fits"

- The effects of mixing can be included in a fit to a multi-body hadronic final state *f* arising from decays of neutral mesons, such as M^0.
- The *t-dependent* decay amplitude for a state M^0 at $t = 0$ is:

$$\langle f|H|M^0(t)\rangle \equiv \mathcal{A}_f(t) =$$
$$e^{-(\Gamma/2+im)t}\left\{\cosh[\Gamma t/2(y+ix)]\langle f|H|M^0\rangle + \left(\frac{q}{p}\right)\sinh[\Gamma t/2(y+ix)]\langle f|H|\bar{M}^0\rangle\right\}$$

  where $\langle f|H|M^0\rangle \equiv \mathcal{A}_f$ and $\langle f|H|\overline{M}^0\rangle \equiv \bar{\mathcal{A}}_f$ are linear combinations of amplitudes (eg "isobar", etc.) normally present in a fit at $t = 0$. They depend upon the position $\vec{s}$ in the phase space

- This can be written in the form:

$$\mathcal{A}_f(\vec{s},t) =$$
$$\mathcal{A}_f(\vec{s},0)e^{-(\frac{\Gamma}{2}+im)t}\left\{\cosh[\frac{\Gamma}{2}(y+ix)t] + \lambda_f(\vec{s})\sinh[\frac{\Gamma}{2}(y+ix)t]\right\}$$

- The dependences on point in phase space $\vec{s}$ and time *t factorize.*

Time-dependent "Dalitz plot fits"

- The Dalitz plot density is then $\propto |\mathcal{A}_f(\vec{s},t)|^2$
- The Dalitz plot density can be written, similarly, for $\overline{M}^0$.
  - It is not, in general, the same as that for the M^0.
- Note that
  - mixing brings both M^0 and $\overline{M}^0$ into interference in the decay of $\overline{M}^0$;
  - The phase space $\vec{s}$ time t dependences factorize.
  - *The parameter* λ_f *becomes* $\lambda_f(\vec{s})e^{i\phi_W}$

The weak phase, ϕ_W, of λ_f is assumed not to depend on $\vec{s}$

Three types of *CPV*

1. Origin is in the mixing ("indirect *CPV*")

$$\frac{q}{p} = r_M e^{i\phi_M} \neq 1 \qquad \left(r_M \equiv \left|\frac{q}{p}\right|\right).$$

2. Origin in the decay ("direct *CPV*")

$$|\bar{\mathcal{A}}_f| \neq |\mathcal{A}_f|$$

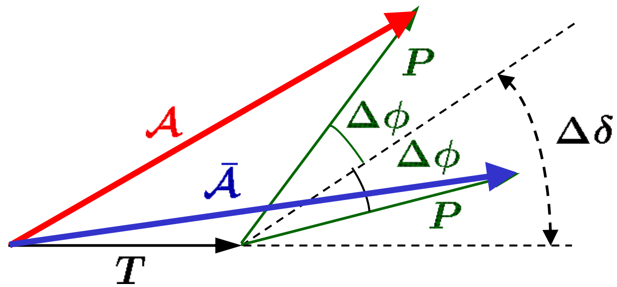
3. Coming from interference between mixing and decay ("indirect *CPV*" – a.k.a. "mixing-induced *CPV*")

$$\lambda_f = \frac{q}{p}\frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f} \neq 1$$

Direct *CPV*

- When two (or more) amplitudes, *T* and *P* for example, mediate a decay process, then decay amplitudes are

$$\left.\begin{array}{rcl}\mathcal{A} &=& T + Pe^{i(\Delta\delta_P+\Delta\phi_P)} \\ \bar{\mathcal{A}} &=& T + Pe^{i(\Delta\delta_P-\Delta\phi_P)}\end{array}\right\} \quad \left\{\begin{array}{rcl}\Delta\phi &\equiv& (\phi_T-\phi_P) \\ \Delta\delta &\equiv& (\delta_T-\delta_P)\end{array}\right.$$



$$|\bar{\mathcal{A}}| \neq |\mathcal{A}|$$

This leads to a *CP* asymmetry in decay rates

$$A^{CP} = \frac{|\mathcal{A}_f|^2 - |\bar{\mathcal{A}}_f|^2}{|\mathcal{A}_f|^2 + |\bar{\mathcal{A}}_f|^2} = \frac{2r\sin(\Delta\phi)\sin(\Delta\delta)}{1+r^2+2r\cos\Delta\delta\cos\Delta\phi} \qquad (r \equiv P/T)$$

NOTE A^{CP} = 0 unless

$$\Delta\phi \neq 0 \quad \text{AND} \quad \Delta\delta \neq 0$$

A^{CP} is largest when $P=T$.

Mixing-induced *CPV* in *M⁰* Decays

- Since M^0 and M^0 states oscillate differently in time, so do the rates for their decays $\Gamma(\overline{\Gamma})$ for $M^0(\overline{M}^0)$ to f :

$$\Gamma \propto e^{-\Gamma\Delta t}\left[\cosh(y\Gamma\Delta t) + \frac{2Re(\lambda_f)}{1+|\lambda_f|^2}\sinh(y\Gamma\Delta t) + \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}\cos(x\Gamma\Delta t) - \frac{2Im(\lambda_f)}{1+|\lambda_f|^2}\sin(x\Gamma\Delta t)\right]$$

$$\overline{\Gamma} \propto e^{-\Gamma\Delta t}\left[\cosh(y\Gamma t) + \frac{2Re(\lambda_f)}{1+|\lambda_f|^2}\sinh(y\Gamma\Delta t) - \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}\cos(x\Gamma\Delta t) + \frac{2Im(\lambda_f)}{1+|\lambda_f|^2}\sin(x\Gamma\Delta t)\right]$$

The result is a <u>time-dependent</u> *CP* asymmetry

$$A_{CP}(t) = \frac{\overline{\Gamma} - \Gamma}{\overline{\Gamma} + \Gamma} = \frac{(1-|\lambda_f|^2)\cos(x\Gamma t) - 2Im(\lambda_f)\sin(x\Gamma t)}{(1+|\lambda_f|^2)\cosh(y\Gamma t) + 2Re(\lambda_f)\sinh(y\Gamma t)}$$

Time-dependent *CPV* analysis *(TDCPV)*

- The parameter λ_f encodes the weak and strong phases into this asymmetry

$$\lambda_f = \frac{q\bar{A}_f}{pA_f} \propto e^{i(\phi_M - 2\phi_f + \delta_f)} \quad ; \quad \bar{\lambda}_f = \lambda_f^{-1}$$

Mixing (weak) — Decay (weak) — Decay (strong)

So measuring this M^0-$\bar{M}^0$ asymmetry as a function of time allows measurement of the weak phase $\phi_W = \phi_M - 2\phi_f$

BUT only if we know the strong phase δ_f

Time-dependent *CPV* analysis

- Three possibilities for measuring δ_f in a *TDCPV* analysis:

1. If *f* is a *CP* eigenstate f_{CP} (e.g. $\pi^+\pi^-$, ϕK_s, $J/\psi K_L$, etc.).
   strong phase of $\mathcal{A}_f$ same as that of $\bar{\mathcal{A}}_f$ so $\delta_f = 0$

2. Similarly, if *f* is *CP* self conjugate (sum of *CP-even* and *CP-odd* states) *e.g.* $K_s\pi^+\pi^-$, $\pi^+\pi^-\pi^0$, *etc.*
   strong phases of $\mathcal{A}$ are linked to those of $\bar{\mathcal{A}}$ so $\delta_f = 0, \pi$

3. If *f* is a <u>multi-hadron</u> system -
   Amplitude analysis of hadrons allows measurement of δ_f but there may also be an unknown phase offset too.

   From Dalitz Plot, etc.

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Application to B^0 Decays at BaBar/Belle

- For B^0 mesons, $y \approx 0$ so B^0 decay rates are:

$$d\Gamma/d\Delta t \;\propto\; e^{-\Gamma\Delta t}\,[1 + C\cos(x\Gamma\Delta t) - S\sin(x\Gamma\Delta t)]$$

$$d\bar{\Gamma}/d\Delta t \;\propto\; e^{-\Gamma\Delta t}\,[1 - C\cos(x\Gamma\Delta t) + S\sin(x\Gamma\Delta t)]$$

  with time-dependent *(TD) CP* asymmetry

$$A_{CP}(t) = S\sin(x\Gamma\Delta t) - C\cos(x\Gamma\Delta t)$$

  where

$$S = \frac{2Im(\lambda_f)}{1 + |\lambda_f|^2} \;\; ; \;\; C = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}$$

(Belle chose to use *"A"=-C*)

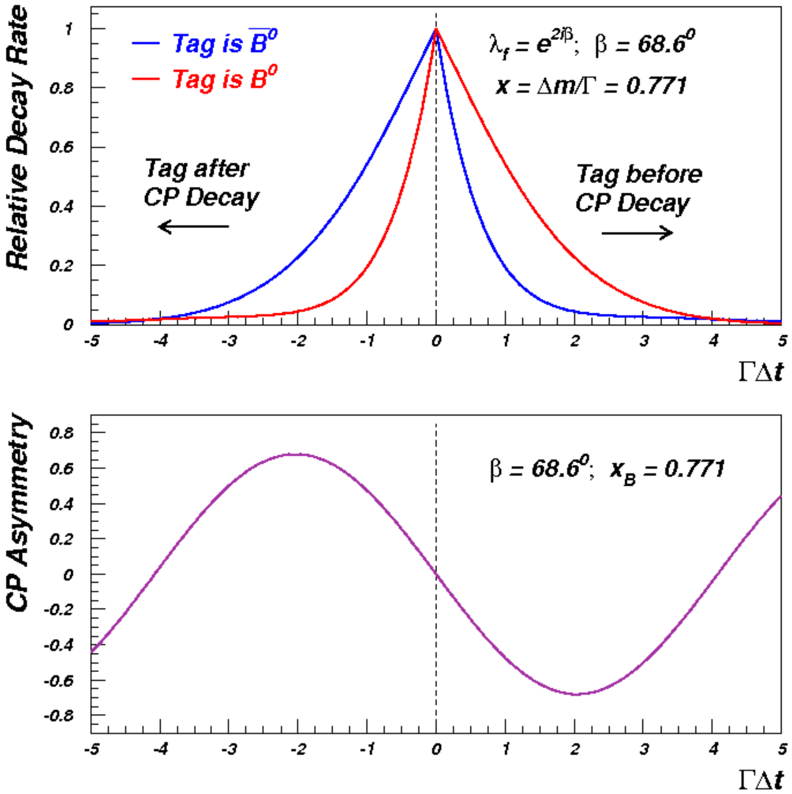
S measurable Only in *TD CPV*

C=0 in absence of direct *CPV*

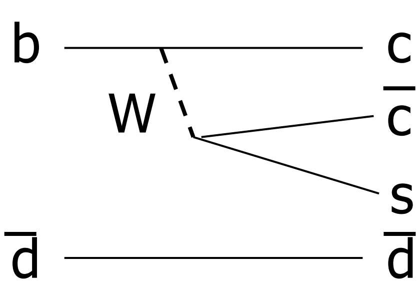
- *CPV* analyses at the *B factories* focus on measuring *C* (direct *CPV*) and *S* (to extract the *weak* phase of λ_f).

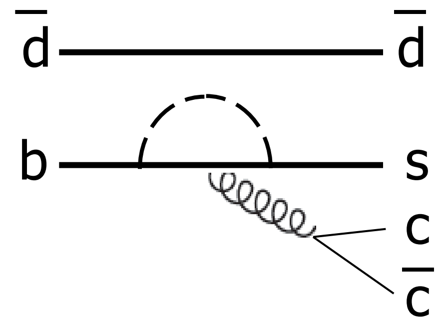
Original *B* Factory goals

- Use *Υ(4S)* decays to B^0-$\overline{B}^0$ pairs – one is “flavour tagged” as either B^0 or $\overline{B}^0$ and other B^0 decays in a *CP* eigenstate.
- Measure Δt between the two decays and the *CP* asymmetry at each Δt.
- Figure shows ideal case for $J/\psi K_s$
  - no experimental effects.
  - Asymmetry is large.



Sin 2β: $B^0 \rightarrow J/\psi K_s$ ($b \rightarrow c\bar{c}s$)





B mix phase: $\phi_M^B = \arg\left\{\frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}}\right\}$

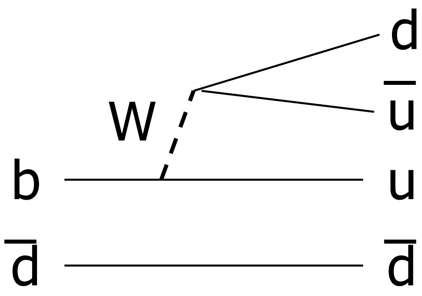
Tree phase: $2\phi_T = \arg\left\{\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}\right\}$ ← Penguin phase ϕ_P is the SAME!

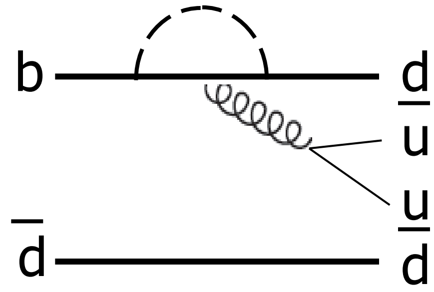
K_s mix phase: $\phi_M^K = \arg\left\{\frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}\right\}$ ← Comes with K^0

$V_{tb} \approx 1$; Arg{ V_{td}} = $\pi - \beta$; Arg{$V_{cb}V_{cd}^*$} ≈ 0

→ Weak phase: $\phi_W = -2\beta$

Sin 2α: $B^0 \rightarrow \pi^+\pi^-$ ($b \rightarrow u\bar{u}d$)





B mix phase: $\phi_M^B = \arg\left\{\frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}}\right\}$

Tree phase: $2\phi_T = \arg\left\{\frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}}\right\}$

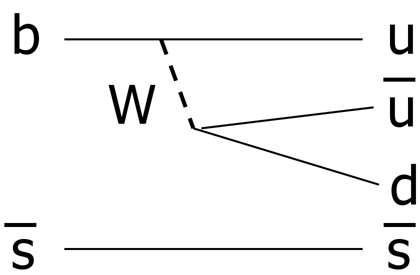
Penguin phase ϕ_P is NOT the same!

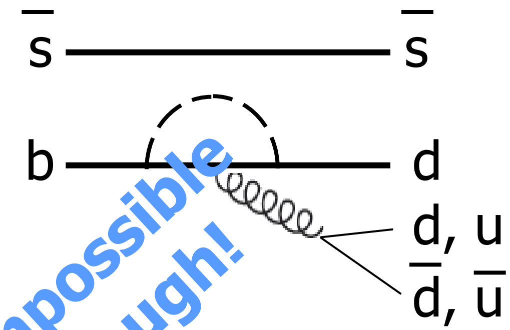
Needs to be measured

$V_{tb} \approx V_{ud} \approx 1$; Arg{ V_{td}} = π - β ; Arg{ V_{ub}} = γ

$\longrightarrow$ Weak phase: $\phi_W = 2\alpha$

Sin 2γ: $B_s \rightarrow \rho K_s$ ($b \rightarrow u\bar{u}d$)





B_s mix phase: $\phi_M^B = \arg\left\{\frac{V_{ts}V_{tb}^*}{V_{ts}^*V_{tb}}\right\}$

Tree phase: $2\phi_T = \arg\left\{\frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}}\right\}$ ← Penguin phase ϕ_P is NOT the same!

K_s mix phase: $\phi_M^K = \arg\left\{\frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}\right\}$ ← Comes with K^0

Experimentally impossible at B factories, though!

$V_{tb} \approx V_{ud} \approx 1$; Arg{ $V_{ts} V_{cs}^*$} ≈ 0 ; Arg{ V_{ub}} ≈ γ ; Arg{ V_{cd}} ≈ 0

→ Weak phase: $\phi_W = 2\gamma$

- Many other modes were actually used, including:
  - those where penguin contributions were important or even dominant
  - some requiring the invocation of *SU(2)* symmetry (*I*, *U* and *V*-spin)
  - *non CP-eigenstates*
  - multi-hadron hadron systems requiring *TD* Dalitz plot amplitude analyses.
  - various vector-vector modes (separating out the longitudinal helicity components (*CP=+1*)

- A credible set of methods to determine γ that did not involve *TD* studies of B_s mesons were also developed.

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We might expect …

- Even more modes *(LHCb and SuperB / Belle2)*
- Multi-hadron modes with 4-body *(or higher dimension)* amplitude analyses separating out the *CP-odd or CP-even* helicity components *(LHCb and SuperB/Belle2)*
- Full use of more precise strong phase measurements from charm threshold data *(BES3 and SuperB)*
- *TD* studies of B_s mesons *(LHCb)*.
- Much information on direct or time-integrated *CPV* *(mostly BES3/Panda but also LHCb/SuperB/Belle2)*.
- *CPV* studies of the charm triangle and further understanding of the origin for LHCb evidence of *direct CPV* in $D^0 \rightarrow \pi^+\pi^-$ and $D^0 \rightarrow K^+K^-$ decays *(LHCb and SuperB/Belle2)*.

Next time, we will cover some of the B factory methods and results in these CPV measurements.