Dalitz Plot Analysis of Heavy Quark Mesons Decays (4).
The question of $Z^+$ resonances.

Antimo Palano

INFN and University of Bari
Belle Experiment claims for the discovery of exotic charged charmonium states in B decays. $Z^+(4430) \rightarrow \psi(2S)\pi^+$ observed the decay $B \rightarrow \psi(2S)K\pi$ (Phys. Rev. Lett. 100, 142001, (2008)), (Phys. Rev. D 80, 031104(R) (2009))

Further $Z_1(4050)^+$ and $Z_2(4250)^+$ observed in the decay to $\chi_{c1}\pi^+$ in $B \rightarrow \chi_{c1}K\pi$ (Phys. Rev. D 78, 072004, (2008)).

BaBar published the search for $Z^+(4430) \rightarrow \psi(2S)\pi^+$ with negative results (Phys. Rev. D 79, 112001 (2009)).

BaBar published the search for $Z_1(4050)^+$ and $Z_2(4250)^+$ in $B \rightarrow \chi_{c1}K\pi$ with negative results (Phys. Rev. D).

No signal was also observed in the $J/\psi\pi$ system in the study of the $B \rightarrow J/\psi K\pi$ decay.

A lot of theoretical and experimental discussion. A charged charmonium state is not a simple $q\bar{q}$ meson.
Main points of discussion are:

- Interference effects between amplitudes in 3-body $B$ decay Dalitz plots produce peaks in quasi-two-body mass projections which may not be due to real states. A dramatic demonstration comes from charm decays. Dalitz plot of $D^0 \rightarrow \bar{K}^0 K^+ K^-$ and projection along the $\bar{K}^0 K^+$ axis: structures are not due to resonances.

- The angular structures in $B \rightarrow \psi(2S) K \pi$ and $B \rightarrow \chi_{c1} K \pi$ decays are very complex and cannot be described by only two variables as it is done in a simple Dalitz plot analysis. See BaBar analysis of $B \rightarrow J/\psi K \pi$ (arXiv:hep-ex/0411016).
Belle observation of $Z^+(4430)$.  

- They select events of the type $B \to K \pi^+ \psi'$, where the $\psi'$ decays either to $\ell^+ \ell^-$ or $\pi^+ \pi^- J/\psi$ with $J/\psi \to \ell^+ \ell^-$ ($\ell = e$ or $\mu$). Both charged and neutral ($K^0_S \to \pi^+ \pi^-$) kaons are used.
- Dalitz plot and $\pi^+ \psi'$ mass spectrum with $K^*$ veto.

- Some clustering of events in a horizontal band is evident in the upper half of the Dalitz plot near $M^2(\pi \psi') \sim 20 \text{ GeV}^2$.
- To study these events with the effects of the known $K \pi$ resonant states minimized, they restrict the analysis to the events with $|M(K\pi) - m_{K^*(890)}| \geq 0.1 \text{ GeV}$ and $|M(K\pi) - m_{K^*_2(1430)}| \geq 0.1 \text{ GeV} (K^* \text{ veto})$.
- Fitting the resulting $\pi^+ \psi'$ mass spectrum with a Breit-wigner they obtain the following parameters

\[ M = (4433 \pm 4(\text{stat}) \pm 2(\text{syst})) \text{ MeV}, \quad \Gamma = (45^{+18}_{-13}(\text{stat})^{+30}_{-13}(\text{syst})) \text{ MeV} \]

- commenting that $\Gamma$ is too narrow to be caused by interference effects in the $K \pi$ channel.
- The statistical significance of the observed peak is $6.5\sigma$. 

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Belle observation of $Z^+(4430) \rightarrow \psi(2S)\pi^+$. Dalitz analysis

- Belle re-analyzed the $B \rightarrow \psi(2S)K\pi$ data using a Dalitz analysis (arXiv:0905.2869).
- They confirm the signal for $Z^+(4430) \rightarrow \psi(2S)\pi^+$ with a mass:

$$M = (4443^{+15+19}_{-12-13})\text{ MeV}/c^2, \quad \Gamma = (107^{+86+74}_{-43-56})\text{ MeV}$$

- A somewhat larger width.
Belle observation of $Z_1(4050)^+$ and $Z_2(4250)^+$. Dalitz analysis

- Belle reconstructed the decay $B \rightarrow K^- \pi^+ \chi_{c1}$ with the $\chi_{c1}$ reconstructed in the $J/\psi \gamma$ decay mode and the $J/\psi$ reconstructed in the $\ell^+ \ell^-$ decay mode.
- Dalitz plot and $M(\pi^+ \chi_{c1})$ showing the new $Z$ resonances.

- In the Dalitz analysis the decay $B \rightarrow K^- \pi^+ \chi_{c1}$ is described by six variables.
- They take these to be $M(\pi^+ \chi_{c1})$, $M(K^- \pi^+)$, the $\chi_{c1}$ and $J/\psi$ helicity angles ($\theta_{\chi_{c1}}$ and $\theta_{J/\psi}$), and the angle between the $\chi_{c1}$ ($J/\psi$) production and decay planes $\phi_{\chi_{c1}}$ ($\phi_{J/\psi}$).
- Then they analyze the $B \rightarrow K^- \pi^+ \chi_{c1}$ decay process after integrating over the angular variables $\theta_{\chi_{c1}}$, $\theta_{J/\psi}$, $\phi_{\chi_{c1}}$ and $\phi_{J/\psi}$.
They perform a **binned likelihood fit** to the Dalitz plot distribution.

The angular function $T_\lambda$ is obtained using the helicity formalism.

For the $B \rightarrow K^*(\rightarrow K^-\pi^+)\chi_{c1}$ decay

$$T_\lambda = d^J_\lambda 0(\theta_{K^*}) ,$$

where $J$ is the spin of the $K^*$ resonance; $\theta_{K^*}$ is the helicity angle of the $K^*$ decay.

For the $B \rightarrow K^- Z^+ (\rightarrow \pi^+\chi_{c1})$ decay

$$T_\lambda = d^J_0 \lambda(\theta_{Z^+}) ,$$

where $J$ is the spin of the $Z^+$ resonance and $\theta_{Z^+}$ is the helicity angle of the $Z^+$ decay.

The resulting expression for the signal event density function is

$$S(s_x, s_y) =$$

$$\sum_{\lambda = -1, 0, 1} \left[ \sum_{K^*} a^K_\lambda e^{i\phi^K_\lambda} A^K_\lambda (s_x, s_y) + \right]$$

$$\sum_{\lambda' = -1, 0, 1} d^1_{\lambda'\lambda}(\theta) a^{Z^+}_{\lambda'} e^{i\phi^{Z^+}_{\lambda'}} A^{Z^+}_{\lambda'} (s_x, s_y) \right|^2 ,$$

where $a^R_\lambda$ and $\phi^R_\lambda$ are the normalizations and phases of the amplitudes for the intermediate resonance $R$ and $\chi_{c1}$ helicity $\lambda$. The phase $\phi^{K^*}_{0}(892)$ is fixed to zero.
Belle observation of $Z_1(4050)^+$ and $Z_2(4250)^+$. Dalitz analysis

- They fit with one or two resonances.

- The mass and width of the $Z^+$ found from the fit are
  \[ M = (4150^{+31}_{-16}) \text{ MeV/c}^2 \text{ and } \Gamma = (352^{+99}_{-43}) \text{ MeV}; \]

- When fitted with two Breit-Wigner resonance amplitudes, the resonance parameters are
  \[ M_1 = (4051 \pm 14^{+20}_{-41}) \text{ MeV/c}^2, \Gamma_1 = (82^{+21}_{-17}-22) \text{ MeV}, \]
  \[ M_2 = (4248^{+44}_{-29}+180) \text{ MeV/c}^2, \Gamma_2 = (177^{+54}_{-39}+316) \text{ MeV} \]
Babar made use of a different approach (arXiv:0811.0564).

First we observe that Babar and Belle data are consistent.
**BaBar analysis of** $B \to \psi(2S)K\pi$

- $K\pi$ mass spectra and $Y_L^0$ Legendre polynomials for $B \to \psi(2S)K\pi$ and $B \to \psi K\pi$.

- $K\pi$ mass spectra and $Y_L^0$ Legendre polynomials are similar between $B \to \psi K\pi$ and $B \to \psi(2S)K\pi$.
Binned $\chi^2$ fits to the background-subtracted and efficiency-corrected $K\pi$ mass spectra in terms of S, P, and D wave amplitudes.

Fitting function:

$$\frac{dN}{dm_{K\pi}} = N \times \left[ f_S \left( \frac{G_S}{\int G_S dm_{K\pi}} \right) + f_P \left( \frac{G_P}{\int G_P dm_{K\pi}} \right) + f_D \left( \frac{G_D}{\int G_D dm_{K\pi}} \right) \right]$$

where the fractions $f$ are such that: $f_S + f_P + f_D = 1$.

The $P$- and $D$-wave intensities are expressed in terms of relativistic Breit-Wigner with parameters fixed to the PDG values for $K^*(892)$ and $K^*_2(1430)$ respectively.

For S-wave contribution has been described by the LASS parametrization.

Notice the Log. scale. Notice also a discrepancy between the data and the LASS representation of the threshold region.
Description of the $\psi(2S)\pi$ mass spectrum.

- A localized structure in the $\psi(2S)\pi$ mass spectrum shows its effect in high $L$ Legendre polynomial moments $< Y^0_L >$.
- The BaBar analysis attempts to describe the $\psi(2S)\pi$ mass distribution using the information from the $K\pi$ system only using Legendre polynomials from $B \to \psi K\pi$ or the $B \to \psi(2S)K\pi$.
- They also limit $L$ to its minimum possible value.
- They generate a large number of MC events according to the following model.
  - $B \to \psi(2S)K\pi$ events are generated according to phase-space.
  - Label $w_{m(K\pi)}$ the weight corresponding to the fit to the $K\pi$ mass projection.
  - Incorporate the measured $K\pi$ angular structure by giving weight $w_L$ to each event according to the expression:
    \[
    w_L = \sum_{i=0}^{L_{\text{max}}} < Y^N_i > Y^0_i (\cos \theta)
    \]
    where $Y^N_i = Y^0_i / n$ are the normalized moments. The $Y^N_i$ are evaluated for the $m(K\pi)$ value by linear interpolation over consecutive $m(K\pi)$ mass intervals.
  - The total weight is thus:
    \[
    w = w_{m(K\pi)} \cdot w_L
    \]
The generated distributions, weighted by the total weight $w$, are then normalized to the number of data events after background-subtraction and efficiency-correction.

The simulation is performed using the $B \to J/\psi K \pi$ or $B \to J/\psi K \pi$ data.

Both simulations describe the data well.

No need for additional $Z$ resonances.

Areas in color describe the spread due to the statistical uncertainty on the Legendre polynomials.
Study of $B \to \chi_{c1}K\pi$.

- A slightly modified analysis was performed for the study of $B^0 \to \chi_{c1}K^-\pi^+$ and $B^+ \to \chi_{c1}K_S^0\pi^+$ (arXiv:1111.5919) by BABAR.
- The fit to the $K\pi$ mass distributions in this case require a small P-wave contribution from $K^*(1680)$ ($\approx 10\%$), not present in the $B \to J/\psi K\pi$ decays or $B \to \psi(2S)K\pi$.
- S-wave contribution larger than in $B \to J/\psi K\pi$ decays, where is $\approx 16\%$. 

![Graphs](image)
The $K\pi$ Legendre polynomial moments.

- Add $B^0$ and $B^+$ data. Weight the events by the $Y^0_L(cos\theta)$ Legendre polynomials.
- Efficiency-corrected and background-subtracted distributions.

- We observe the $S$-$P$ interference in the $<Y^0_1>$ moment.
- Significant enhancement in $Y^0_1$ at $\approx 1.7$ GeV indicating the presence of a $P$-wave.
- We observe the presence of the spin-1 $K^*(890)$ in the $<Y^0_2>$ moment.
- We have evidence for the spin-2 $K^*_2(1430)$ resonance in the $<Y^0_4>$ moment.
- $<Y^0_6>$ is consistent with zero.
MC simulations: $B \to J/\psi K\pi$

- We test the method on $B \to J/\psi \pi K$ where there is no evidence for narrow or broad $Z$ resonances.
- We vary $L_{max}$ between 4 and 6 and obtain the best description of the data with $L_{max} = 5$.

<table>
<thead>
<tr>
<th>$L_{max}$</th>
<th>$\chi^2/NDF$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>223/152</td>
</tr>
<tr>
<td>5</td>
<td>162/152</td>
</tr>
<tr>
<td>6</td>
<td>180/152</td>
</tr>
</tbody>
</table>

- MC/data comparison, the dotted line shows the effect of removing the angular $w_L$ weight.
Similar results are obtained for the $B \to \chi_{c1}K\pi$ channel.

<table>
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<th>$L_{max}$</th>
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<tr>
<td>4</td>
<td>53/58</td>
</tr>
<tr>
<td>5</td>
<td>46/58</td>
</tr>
<tr>
<td>6</td>
<td>49/58</td>
</tr>
<tr>
<td>“mixed”</td>
<td>63/58</td>
</tr>
</tbody>
</table>

$B \to J/\psi K\pi$ and $B \to \chi_{c1}K\pi$ data can be described using a similar approach. This indicates that there is no need for additional resonant structure in order to describe the $\chi_{c1}\pi$ mass distribution.

We also use a “mixed” Legendre polynomial composition, using $L_{max} = 3$ for $m(K\pi) < 1.2$ GeV and $L_{max} = 4$ above. This is justified by the fact that only spin 0 and spin 1 resonances are present in the low mass region.

This representation also gives an excellent description of the $\bar{B}^0 \to \chi_{c1}K\pi$ data.

We will use this “mixed” representation for computing upper limits on $Z$ production.
How would a Z resonance show up?

- We artificially add a \( \approx 25\% \) contribution of a scalar \( Z_2(4250)^+ \rightarrow \chi_{c1}\pi \) resonance in the \( \bar{B}^0 \rightarrow \pi^+K^-\chi_{c1} \) data.
- These MC toy events are obtained from MC data, weighted by a Breit-Wigner.
- We then compute Legendre polynomial moments for the whole sample and predict the \( \chi_{c1}\pi \) mass spectrum using the same algorithm as for real data.
- Using the “mixed” method, the resulting MC simulation does not describe the MC data well: \( \chi^2/NDF = 140/58 \)

- Black dots indicate the \( B^0 \rightarrow \pi^-K^+\chi_{c1} \) data, crosses indicate the total sample.
- In (a) The dashed curve shows a simulation with \( L_{\text{max}} = 15 \).
- In (b) the fit incorporates a Breit-Wigner lineshape describing the \( Z_2(4250)^+ \).
- The dashed curve represents the background model from the “mixed” simulation.
Search for $Z$ resonances.

- We now fit the $\chi_{c1}\pi$ mass spectrum using the following model:
  - Assume the prediction from the MC simulation (“mixed”) as background.
  - Include two scalar Breit-Wigner with parameters fixed to the Belle measurements.
  - Fit the full data set (Total).

<table>
<thead>
<tr>
<th>Data</th>
<th>Resonance</th>
<th>$N_\sigma$</th>
<th>Fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Total</td>
<td>$Z_1(4050)^+$</td>
<td>1.1</td>
<td>1.6 ± 1.4</td>
</tr>
<tr>
<td></td>
<td>$Z_2(4250)^+$</td>
<td>2.0</td>
<td>4.8 ± 2.4</td>
</tr>
<tr>
<td>b) Total</td>
<td>$Z(4150)^+$</td>
<td>1.1</td>
<td>4.0 ± 3.8</td>
</tr>
<tr>
<td>c) Window</td>
<td>$Z_1(4050)^+$</td>
<td>1.2</td>
<td>3.5 ± 3.0</td>
</tr>
<tr>
<td></td>
<td>$Z_2(4250)^+$</td>
<td>1.3</td>
<td>6.7 ± 5.1</td>
</tr>
<tr>
<td>d) Window</td>
<td>$Z(4150)^+$</td>
<td>1.7</td>
<td>13.7 ± 8.0</td>
</tr>
</tbody>
</table>
Search for $Z$ in $B \to J/\psi K\pi$.

- Belle experiment has recently performed a more complete Dalitz analysis of $B \to J/\psi K\pi$ (K. Chilikin, Talk at CHARM2012, 16 May 2012).

- No significant signal of $Z^+$ is found.

The variables considered are Dalitz variables $M^2(K,\pi)$, $M^2(J/\psi,\pi)$ and angles $\theta_{J/\psi}$, $\varphi_{J/\psi K}$. 

preliminary
Not the end of the story.

- New results from Belle:

  **Observation of two charged bottomonium-like resonances in \( \Upsilon(5S) \) decays.** [arXiv:1110.2251](http://arxiv.org/abs/1110.2251)

- As we have seen previously, confirmation of results is an essential ingredient of science and scientific method.

- However, the BaBar/Belle competition is broken here because BaBar has no data on \( \Upsilon(5S) \) decays.

- In April 2008 the *B factory* program at SLAC has been sharply interrupted and closed.

- Confirmation from LHCb?

- Much later in time: Super-B factories?