



# Amplitude Analysis

## An Experimentalists View

Lectures at the “*Extracting Physics from Precision Experiments Techniques of Amplitude Analysis*”

Jefferson Lab Advanced Study Institute

### EXTRACTING PHYSICS FROM PRECISION EXPERIMENTS: *Techniques of Amplitude Analysis*

COLLEGE OF WILLIAM & MARY  
WILLIAMSBURG, VIRGINIA, USA

Wednesday, May 30<sup>th</sup>, 2012  
through Wednesday, June 13<sup>th</sup>, 2012

To prepare for the analysis of precision experiments at BESIII, COMPASS, LHCb, JLAB@12 GeV, and PANDA@FAIR, Thomas Jefferson National Accelerator Facility (JLab) is organizing a two week advanced course covering *Techniques of Amplitude Analysis*, aimed at postdoctoral researchers and advanced doctoral students in nuclear and particle physics.

**LECTURERS:**

Suh-Urc Chung	(BNL/TUM)
Josef Dudek	(OCU)
Karlton Kubie	(Bonn)
T-S Harry Lee	(ANL)
Brian Meadows	(Cincinnati)
Arturo Palano	(Bari)
Klaus Peters	(GSI Darmstadt)
Michael Pennington	(JLab)
Ronald Workman	(GWU)

**CONTACT:**  
mfox@jlab.org

For application details and all other information see:  
<http://www.jlab.org/conferences/asi2012/>

Klaus Peters  
GSI Darmstadt and GU Frankfurt  
Williamsburg, June 2012



## 4 Lectures

### Introduction

Mission / Amplitude Analysis Concepts and Procedures / Use Cases

### Kinematics and more

Dalitz-Plots / Observables / Coordinate Systems / Examples

### K-Matrix

Derivation / Examples / Properties / Fitting / Interpretation

### Experiments

Selection / Background / Numerical Issues / Goodness-of-Fit / Computers

---



## 6 Lectures

### Introduction

Mission / Amplitude Analysis Concepts and Procedures / Use Cases

### Kinematics and more

Dalitz-Plots / Observables / Coordinate Systems / Examples

### Spin

...

### Dynamics

...

### K-Matrix

Derivation / Examples / Properties / Fitting / Interpretation

### Experiments

Selection / Background / Numerical Issues / Goodness-of-Fit / Computers

---



## Dynamics

J.M. Blatt & V.F. Weisskopf – *Theoretical Nuclear Physics*

J.R. Taylor – *Scattering Theory*

M.L. Goldberger & K.M. Watson – *Collision Theory*

J. Gillespie – *Final State Interactions*

H. Burkhardt – *Dispersion Relation Dynamics*

## Kinematics and more

M. Nikolic – *Kinematics and Multiparticle Systems*

E. Byckling & K. Kajantie – *Particle Kinematics*

## Spin

M.E. Rose – *Elementary Theory of Angular Momentum*

## Overview

D.V. Bugg (Editor) – Nato School on *Hadron Spectroscopy and the Confinement Problem*

---



## General

R.S. Longacre – *Techniques in Meson Spectroscopy*, BNL 49445

K. Peters – *A Primer on Partial Wave Analysis*, Int.J.Mod.Phys. A21 (2006) 5618-5624

## Spin

S.U. Chung – *Spin Formalisms*, CERN Yellow Report 71-8

V. Filippini et al. – *Covariant Spin Tensors in Meson Spectroscopy*, PRD 51(1995) 2247

## Dynamics

S.U. Chung et al. – *Partial wave analysis in K matrix formalism*, Annalen Phys. 4 (1995) 404-430

F.v. Hippel, C. Quigg – *Centrifugal-Barrier Effect in Resonance Partial Decay Widths, Shapes, and Production Amplitudes*, PRD 5 (1972) 624

I.J.R Aitchison – *K-matrix Formalism For Overlapping Resonances*, Nucl.Phys. A189 (1972) 417-423

---



# Amplitude Analysis

## *An Experimentalists View*

K. Peters

Jefferson Lab Advanced Study Institute

### EXTRACTING PHYSICS FROM PRECISION EXPERIMENTS: *Techniques of Amplitude Analysis*

COLLEGE OF WILLIAM & MARY  
WILLIAMSBURG, VIRGINIA, USA

Wednesday, May 30<sup>th</sup>, 2012  
through Wednesday, June 13<sup>th</sup>, 2012

To prepare for the analysis of precision experiments at BESIII, COMPASS, LHCb, JLAB@12 GeV, and PANDA@FAIR, Thomas Jefferson National Accelerator Facility (JLab) is organizing a two week advanced course covering *Techniques of Amplitude Analysis*, aimed at postdoctoral researchers and advanced doctoral students in nuclear and particle physics.

**LECTURERS:**

Suh-Urc Chung	(BNL/TUM)
Josef Dudek	(OCU)
Karlton Kubie	(Bonn)
T-S Harry Lee	(ANL)
Brian Meadows	(Cincinnati)
Arturo Palano	(Bari)
Klaus Petas	(GSI Darmstadt)
Michael Pennington	(JLab)
Ronald Workman	(GWU)

CONTACT:  
mfox@jlab.org

For application details and all other information see:  
<http://www.jlab.org/conferences/asi2012/>

# Part I

# Introduction



# Introduction



**Mission**

**Concepts**

**Procedures**

**Use Cases**

---

---

# What is the mission ?



8

---

Particle physics at small distances is quite well understood

One Boson Exchange, Heavy Quark Limits

This is not true at large distances

Hadronization, Light mesons

are barely understood compared to their abundance

Understanding interaction/dynamics of light hadrons will

improve our knowledge about non-perturbative QCD

parameterizations will give provide  
toolkit to analyze heavy quark processes

thus an important tool also for precise standard model tests

We need

Appropriate parameterizations for the multi-particle phase space

A translation from the parameterizations to effective degrees of  
freedom for a deeper understanding of QCD

---



# Intermediate State Mixing



Many states may contribute to a final state  
 not only ones with well defined (already measured) properties  
 not only expected ones

Many mixing parameters are poorly known  
 K-phases  
 SU(3) phases

In addition  
 also *D/S* mixing ( $b_1, a_1$  decays)

### Isoscalar Mixing:

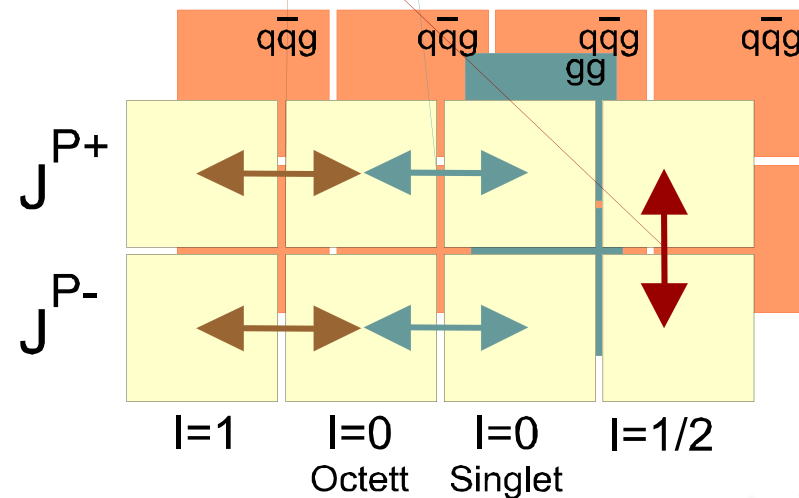
strong Int.:  $I^G$  und  $J^{PC}$  identical  
 $\eta$ - $\eta'$  or  $f_2$ - $f_2'$  and/or Glueballs

### I=0/I=1-Mixing:

elm. Int.:  $\Delta I=1$  :  $\rho$ - $\omega$

### Kaonmixing:

strong Int.: C undef.,  $I^G$  and  $J^P$  identical  
 $K_{1A}$ - $K_{1B}$

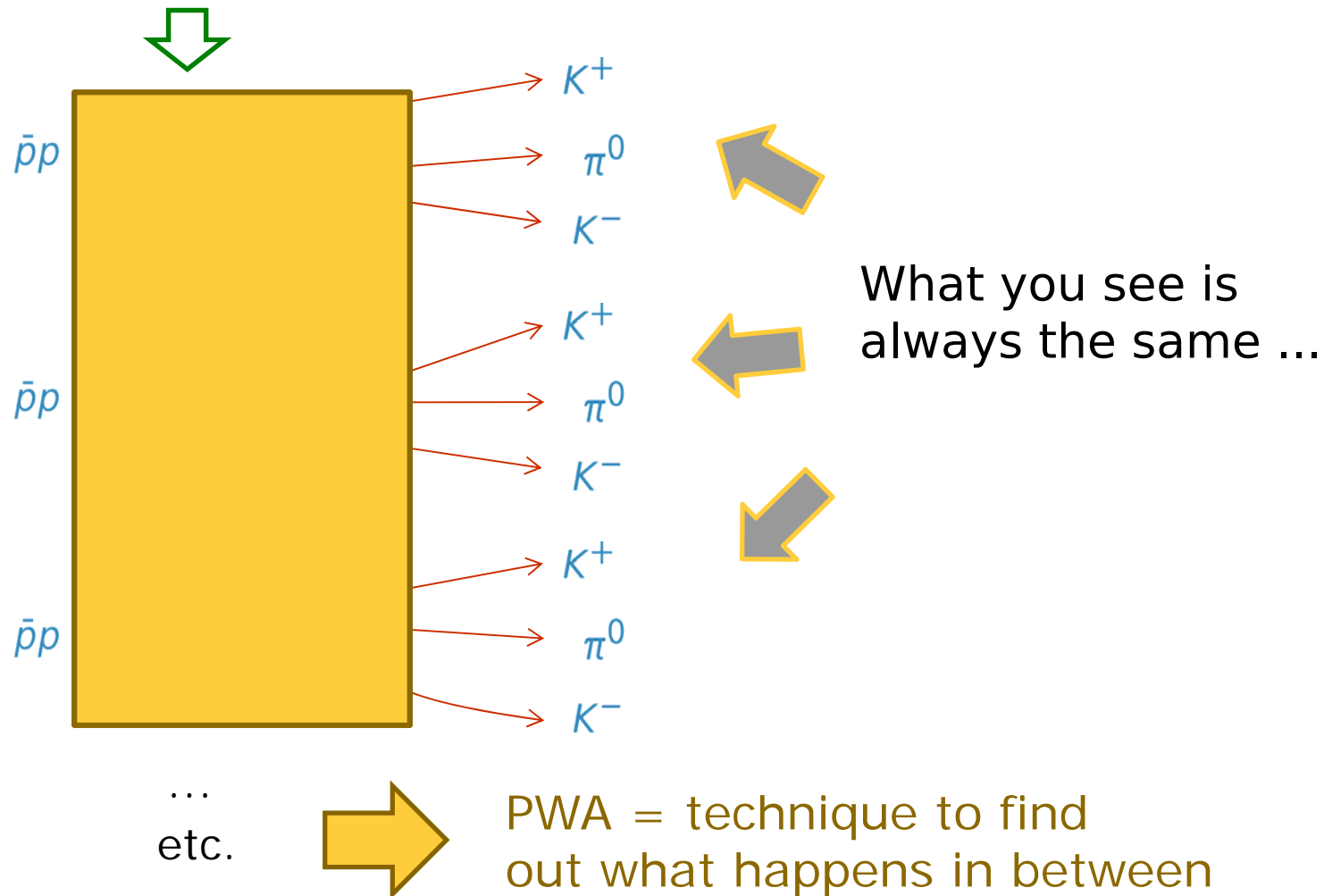


# The Need for Partial Wave Analysis



Example: Consider the reaction  $\bar{p}p \rightarrow K^+K^-\pi^0$

What *really* happened...





---

For whatever you need the parameterization  
of the  $n$ -Particle phase space

It contains the static properties of the unstable (resonant) particles  
within the decay chain like

mass

width

spin and parities

as well as properties of the **initial state**

and some constraints from the experimental **setup/measurement**

The main problem is, you don't need just a good description,  
you need the right one

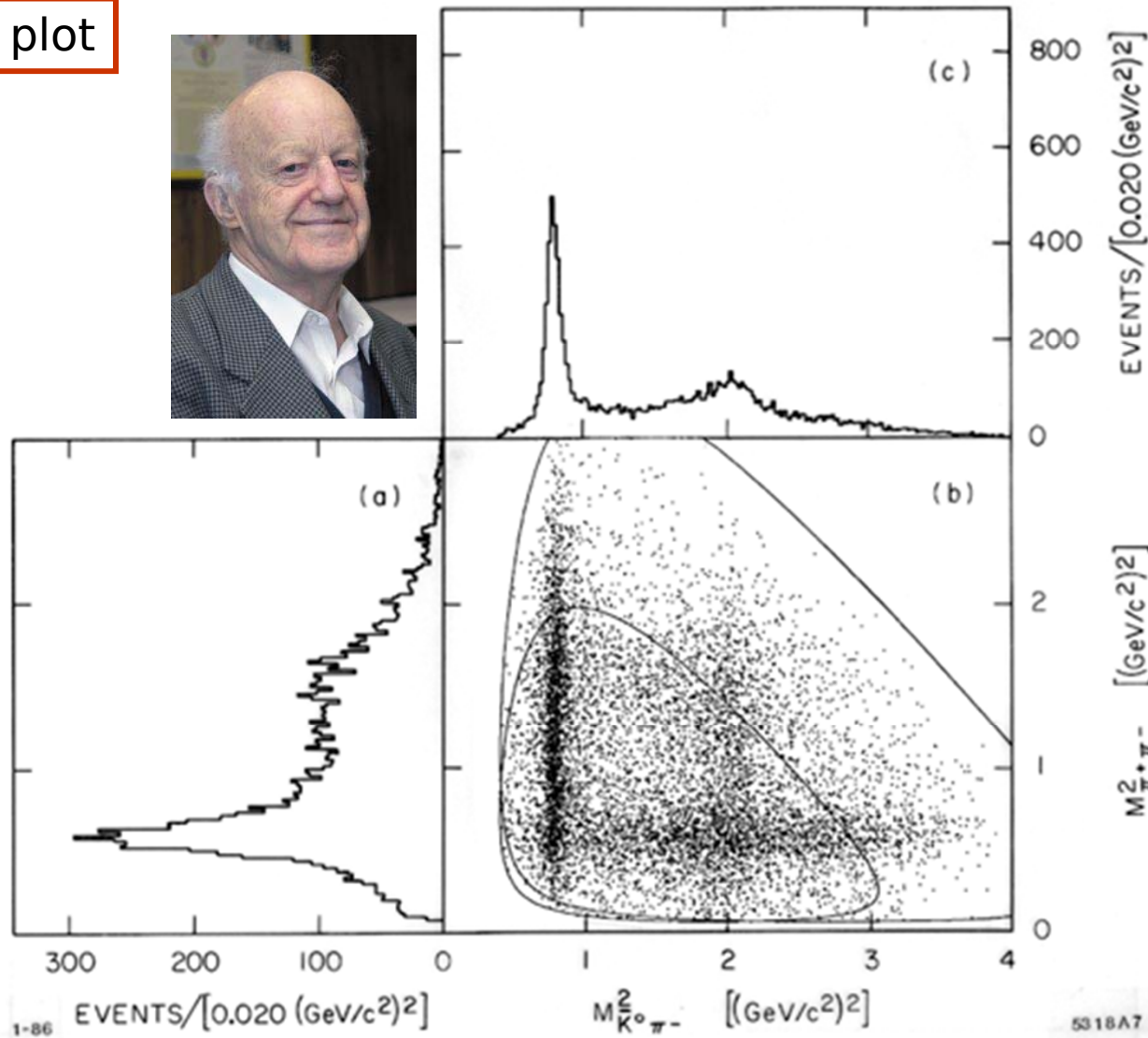
Many solutions may look alike, but **only one is right**

---

# $n$ -Particle Phase space, $n=3$



Dalitz plot



---

But...



13

---

the mission is way more general,

...there are many more questions, which can only be answered with a correct phase space description

whenever states mix and an need to be unambiguously disentangled

the focus then moves away from masses and line shapes to yields and phases

---

# example: $D^0\bar{D}^0$ -Mixing and CPV

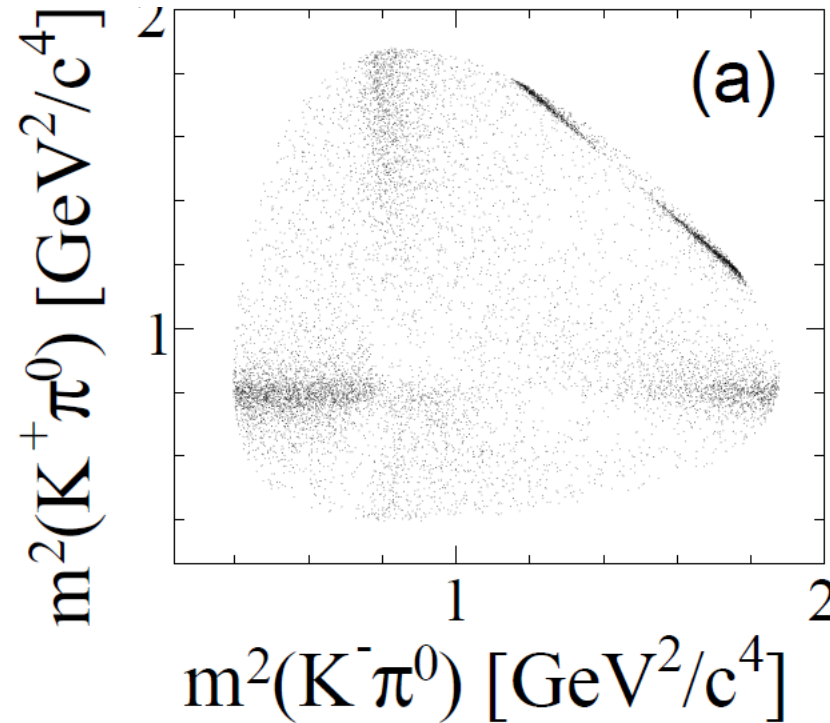
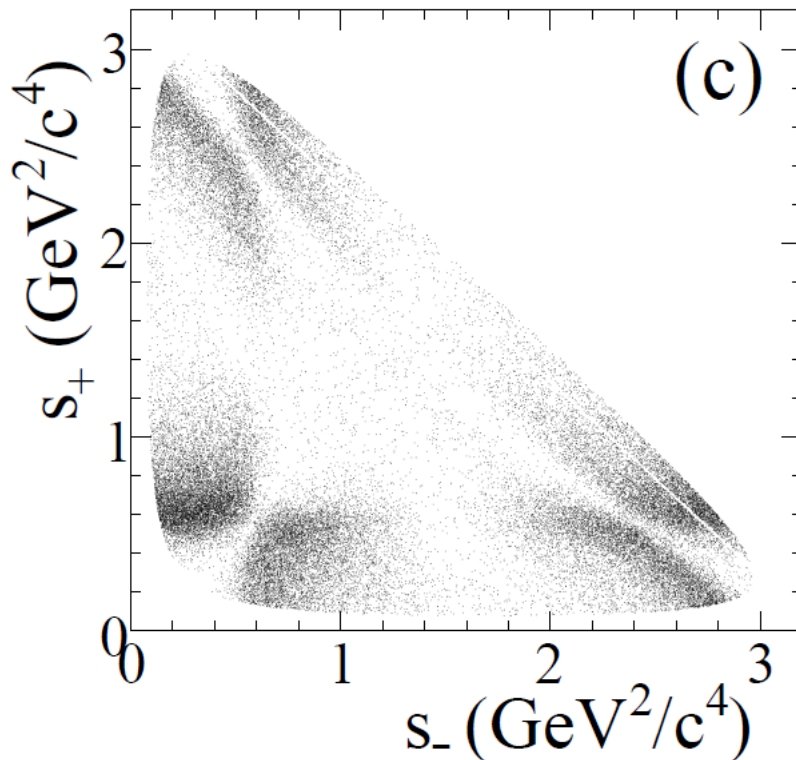


Data from BaBar

$$D^0 \rightarrow h^+h^-\pi^0, h=K,\pi$$

$$D^0 \rightarrow \pi^-\pi^+\pi^0$$

$$D^0 \rightarrow K^-K^+\pi^0$$



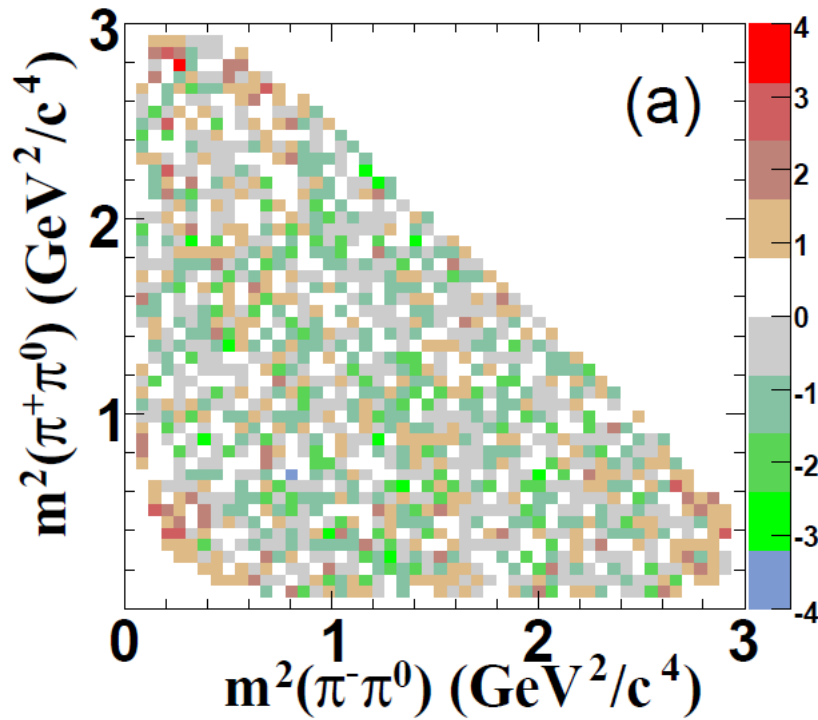
search for **asymmetry** in production cross section  
or in branching fractions

# example: $D^0\bar{D}^0$ -Mixing and CPV

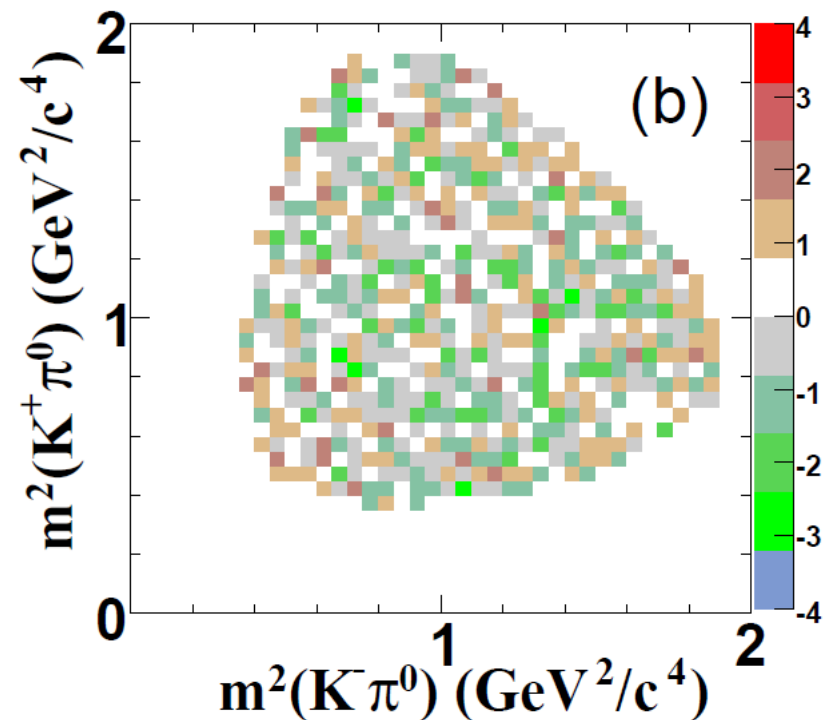


Data from BaBar

$$D^0 \rightarrow \pi^- \pi^+ \pi^0$$



$$D^0 \rightarrow K^- K^+ \pi^0$$



$\chi^2$ -distribution shows: no observed CP-violation

not enough statistics to verify SM prediction

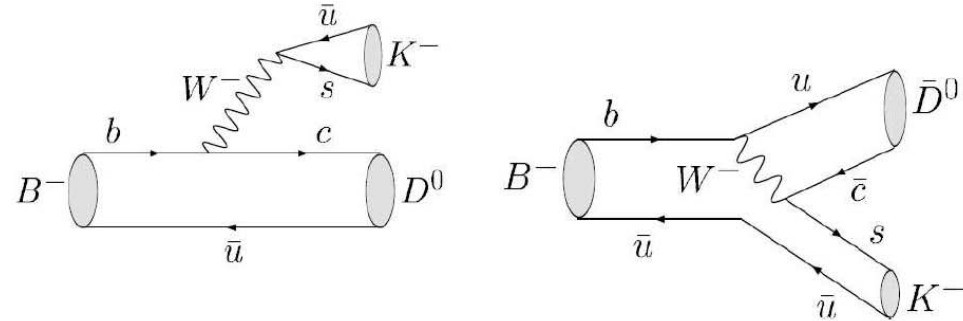
# example: CKM Angle $\gamma$ in $B^- \rightarrow D^0 K^-$ (+c.c.)



Direct  $CP$  violation in interference between  $b \rightarrow c\bar{u}s$ ,  $u\bar{c}s$

$$B^- \rightarrow \tilde{D}^{(*)0} K^{(*)-}$$

Interference, if  
 $D^0 \rightarrow f \leftarrow \bar{D}^0$



$$\frac{\mathcal{A}(B^- \rightarrow \bar{D}^0 K^-)}{\mathcal{A}(B^- \rightarrow D^0 K^-)} = r_B e^{i(\delta_B - \gamma)}, \quad \frac{\mathcal{A}(B^+ \rightarrow D^0 K^+)}{\mathcal{A}(B^+ \rightarrow \bar{D}^0 K^+)} = r_B e^{i(\delta_B + \gamma)}$$

$r_B$  Ratio of magnitudes of amplitudes, **small**

$\delta_B$   $CP$  invariant strong phase

Most sensitive channel to date:  $\tilde{D}^0 \rightarrow K_s^0 \pi^+ \pi^-$ :

GGSZ, Phys. Rev. D **68**, 054018 (2003), BP, Eur. Phys. Jour. **47**, 347 (2006)

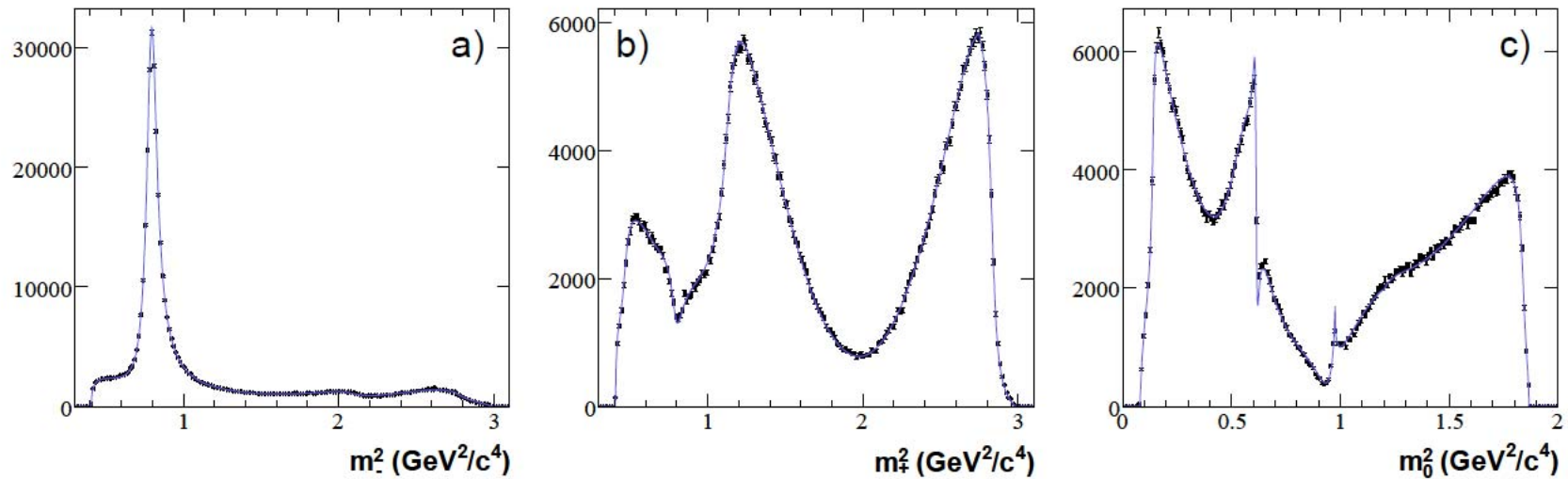
Requires a detailed understanding of the  $D^0$  decay as input





High Quality is needed

and achievable...



this lecture is basically about how to model the input for such fits

to reveal all the physics of a multi-particle reaction



## Important aspects...

General considerations

Course of action

Phase space

Observables

Hypotheses

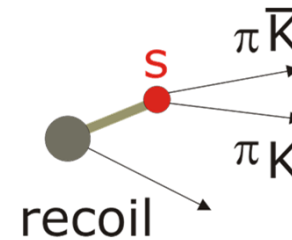
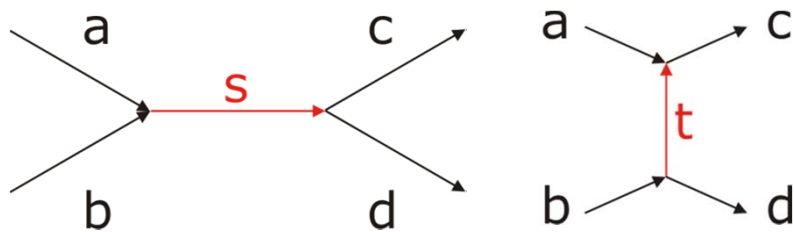
Background

Fitting

Mathematical problems

Quality Assurance

---



## Scattering Experiments

$\pi N - N^*$  measurement

$\pi N$  - meson spectroscopy

E818, E852 @ AGS, GAMS  
Compass, VES

$pp$  meson threshold production

WASA @ Celsius, COSY

$pp$  or  $\pi p$  in the central region

WA76, WA91, WA102

$\gamma N$  - photo production

Cebaf, Mami, Elsa, Graal

## "At-rest" Experiments

$\bar{p}N$  @ rest at LEAR

Asterix, Obelix, Crystal Barrel  
PANDA

$J/\psi$  decays

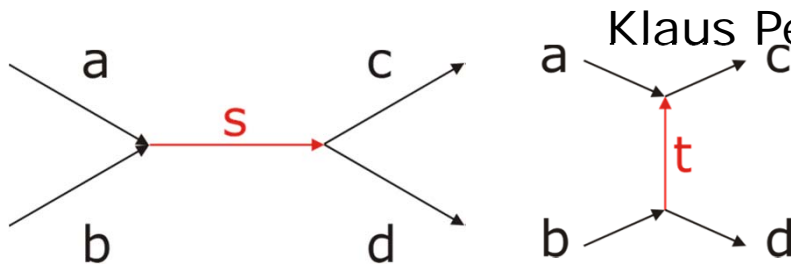
MarkIII, DM2, BES, CLEO-c

$\phi(1020)$  decays

Kloe @ Dafne, VEPP

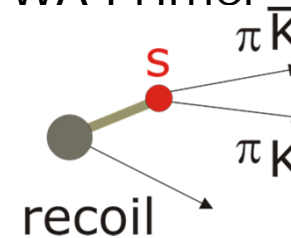
$D$  and  $D_s$  decays

FNAL, Babar, Belle, Belle-II



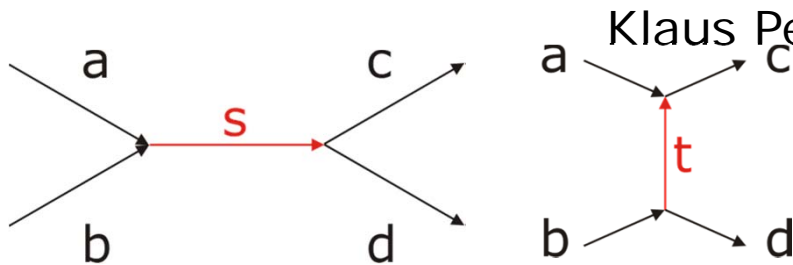
Scattering Experiments

partial waves decomposition  
→ via moment analysis  
systematic studies to limit  
#waves  
dynamics appear as amplitude  
variations  
resonance parameters from fits to  
amplitudes



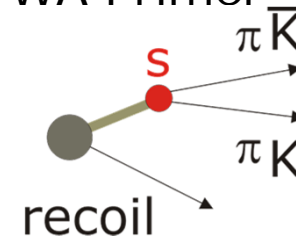
“At-rest” Experiments

ad-hoc introduction of waves  
ad-hoc introduction of dynamic  
amplitudes (“resonances”)  
systematic studies to limit  
#waves and #resonances  
resonance parameters appear as  
fit parameters



Scattering Experiments

exchange model needed  
ad-hoc intermediate resonances  
→ parameters fixed for wave  
decomposition



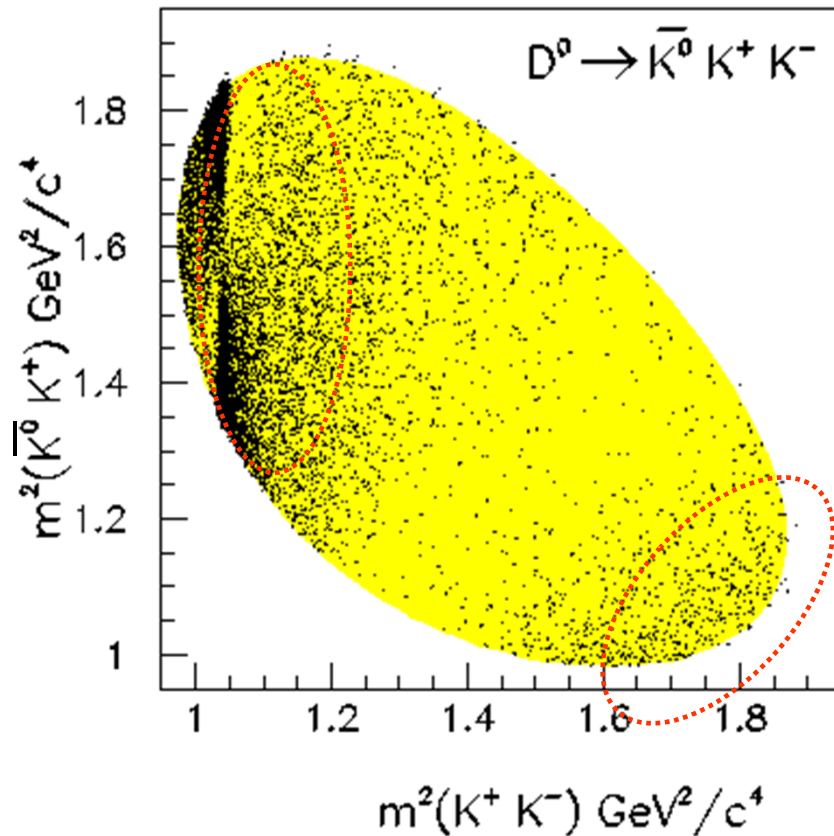
“At-rest” Experiments

independent of production model  
intermediate resonances treated  
→ identically to final state  
resonances  
crossing bands may provide high  
resolution interferometer

# Momentum Analysis in a Dalitz Plot

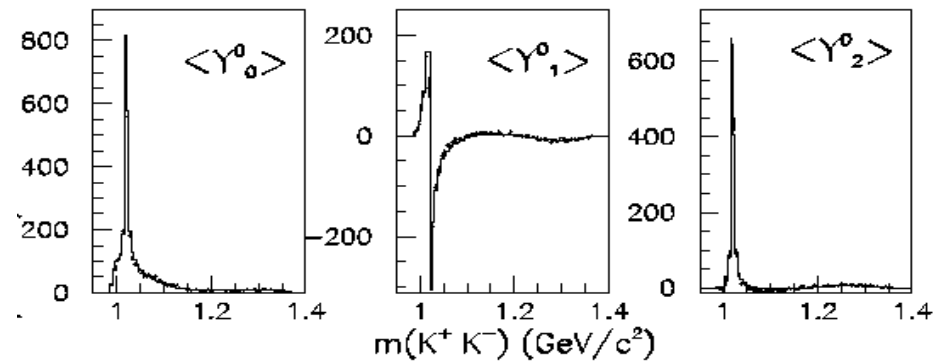


see M. Pappagallo, Charm06



$$\begin{cases} \sqrt{4\pi} \langle Y_0^0 \rangle = S^2 + P^2 \\ \sqrt{4\pi} \langle Y_1^0 \rangle = 2|S||P| \cos \Phi_{SP} \\ \sqrt{4\pi} \langle Y_2^0 \rangle = \frac{2}{\sqrt{5}} P^2 \end{cases}$$

In some cases it's possible if no sharp bands overlap





## Which processes take place?

### Interactions?

Basic processes – scattering vs. decay – which scattering  
(Physics of) Initial State – recoils – inclusive/exclusive

Physics background

Leading effects

### Scales?

Dynamics – range parameters

Approximations – low energy or threshold expansions

do scales differ for different sub-processes?

factorization of dynamics, like in open-charm decays

---



## What are conserved properties?

### kinematics

energy/momentum conservation

kinematically fitted data?

### quantum numbers

quark/isospin conservation/symmetries

good and bad quantum numbers (isospin, parity, CP)

impact on spin formalisms

interferences of Feynman graphs

phase space

full set of observables?

integrate over part of the phase-space

---



# Properties



	<b>Helicity</b>	<b>Transversity</b>	<b>Canonical</b>
property	possibility/simplicity		
partial wave expansion	simple	complicated	complicated
parity conservation	no	yes	yes
crossing relation	no	good	bad
specification of kinematical constraints	no	yes	yes

# Example: Isospin Dependence



$\bar{p}p$  initial states differ in isospin

$${}^1S_0 \quad I^G(J^{PC}) = 1^-(0^{-+})$$

$${}^3S_1 \quad I^G(J^{PC}) = 0^+(1^{--})$$

Calculate isospin Clebsch-Gordan

$$\rho^0\pi^0 \rightarrow (1010|00) = -\sqrt{\frac{1}{3}}$$

$$\rho^0\pi^0 \rightarrow (1010|10) = 0$$

$$\rho^\pm\pi^\mp \rightarrow (1(\pm 1) 1(\mp 1)|00) = \sqrt{\frac{1}{3}}$$

$$\rho^\pm\pi^\mp \rightarrow (1(\pm 1) 1(\mp 1)|10) = \pm\sqrt{\frac{1}{2}}$$

${}^1S_0$  destructive interferences

${}^3S_1$   $\rho^0\pi^0$  forbidden



## What are the relevant parameters?

### Order of magnitude

relevant for coding?

leading terms?

[[ Parameter too small, different formulation ]]

[[ Examples only smallest L ]]

### Relations

are the parameters related to each other? (D/S, phases, ...)

which one is the master and which the slave?

Normalization/Constraints

[[ Example relations]]

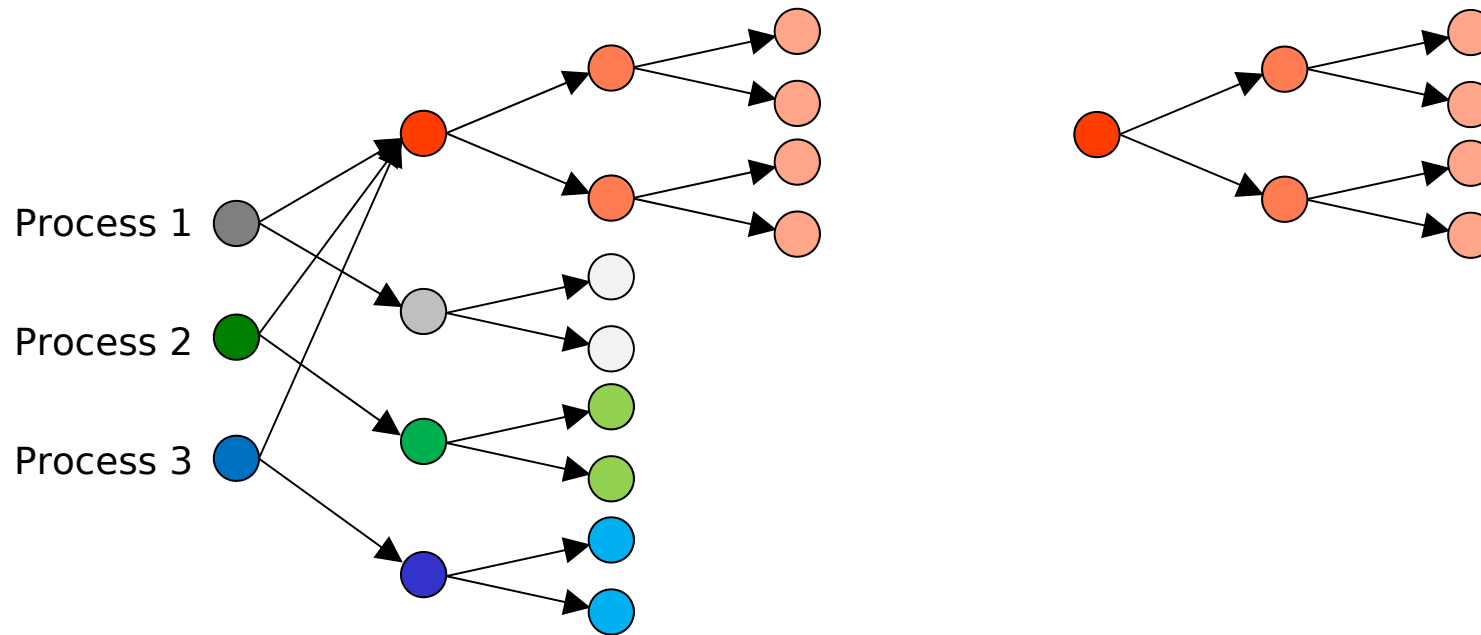
[[ Example couplings normalized to 1]]

---



Can the process be factorized or simplified?

Whole tree needed? or is a leave sufficient



## Rules

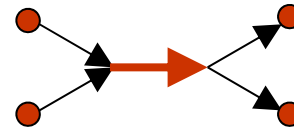
which rules/conditions can be used to formulate the model  
which rules/conditions have to be applied during the fit  
e.g. what is fixed by definitions



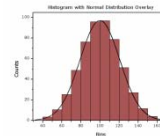
Data Analysis



Modelling



Fitting



Quality Assurance



Review and Publication





## Data analysis

### Data

extract relevant data set(s) with appropriate statistics, high purity and high efficiency



### MC

signal MC, may be mixed due to experimental conditions

### Background

extract from data and/or generate via Monte Carlo data sets from potential background channels

### Representation

represent the data in  $n$ -tupels of relevant (transformed?) observables for the fit and the visualization

---



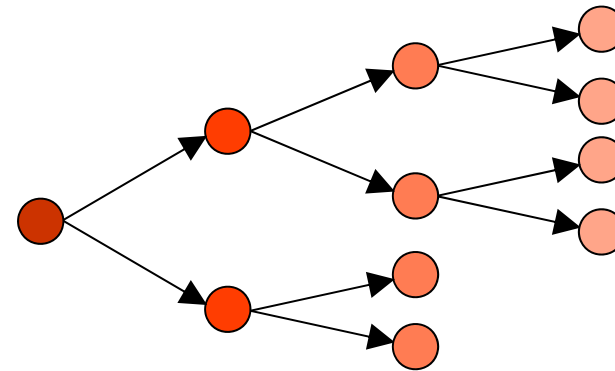
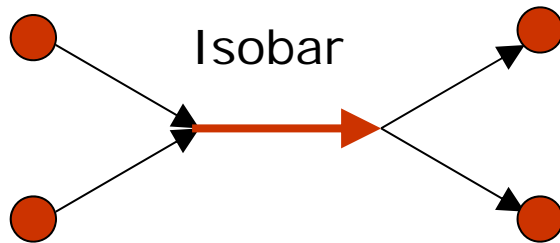
## Modelling

### Data

Visual inspection of the data !!

### Physics

create list of hypotheses (incl. production, spins, dynamics and if so, background)



### Mathematics

optimize the mathematical form

may improve speed and may reduce numerical instabilities

# Reduced Amplitudes (I)

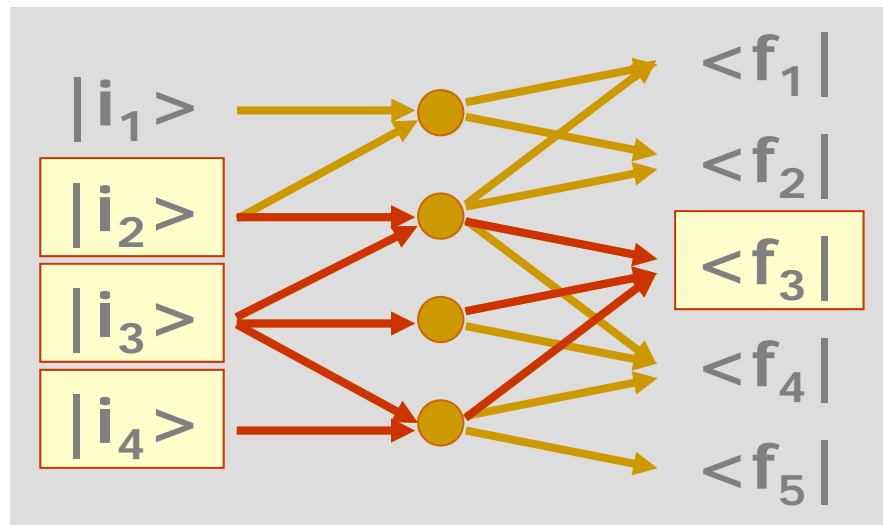


# of parameters explodes with increasing number of initial and final states

forget about the tree, reduce amplitude to the final state of interest

feed by many initial and several intermediate states

spin density matrix  $\rho_{mn}$





---

## Reduced Amplitudes (II)

---



### Caveat:

Two channels may interfere in one tree,  
but may not in another

thus measured rate I has different formalisms

Coherent

$$I = |A + e^{i\phi}B|^2 = |A|^2 + |B|^2 + 2[\operatorname{Re}(AB^*)\sin\phi + \operatorname{Im}(AB^*)\cos\phi]$$

Incoherent

$$I = |A|^2 + |B|^2$$

Effective coherence

$$I = |A|^2 + |B|^2 + C [\operatorname{Re}(AB^*)\sin\phi + \operatorname{Im}(AB^*)\cos\phi]$$

( $C = -2, \dots, 2$ )

---



## Fitting

fit model(s) to the data

likelihood definition, what is to be minimized  
(max. Likelihood, Chi2,...)

needs a strategy to find the best solution

systematic studies for a variety of hypotheses  
vary initial stats, resonances, parameterizations

need a strategy for each fit

optimizer (gradient/random/genetic)

sequence (different optimizers, fixation and release of parameters)

criteria for convergence and termination

---



## Quality Assurance

### Documentation

excellent documentation! is the key  
what was done? formulae!  
(intermediate) results!



### Validation

validation of the result (for example with toy MC)

### Significance

scrutinize the significance of new findings  
check various methods to investigate the goodness-of-fit

### Errors

determination of statistical and systematic errors

---



## Review and Publication

### review

process which may lead to a reanalysis at various entry points

### publication

publish only things you are confident about  
there is an undefined border where  
the experiment ends and the theoretical bias starts



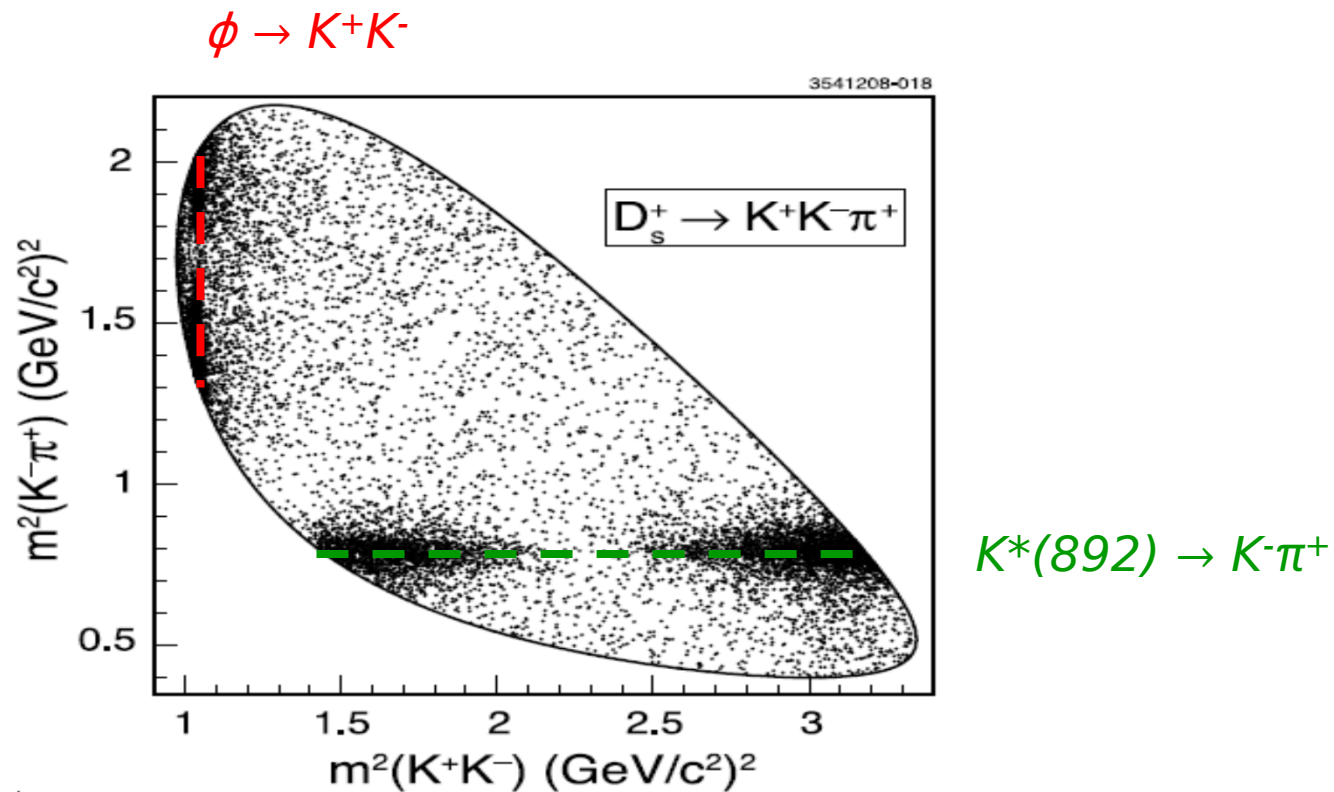


## visual inspection of the phase space distribution

are there structures?

structures from signal or background?

are there strong interferences, threshold effects, potential resonances?



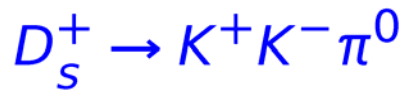
# Kinematical Reflections



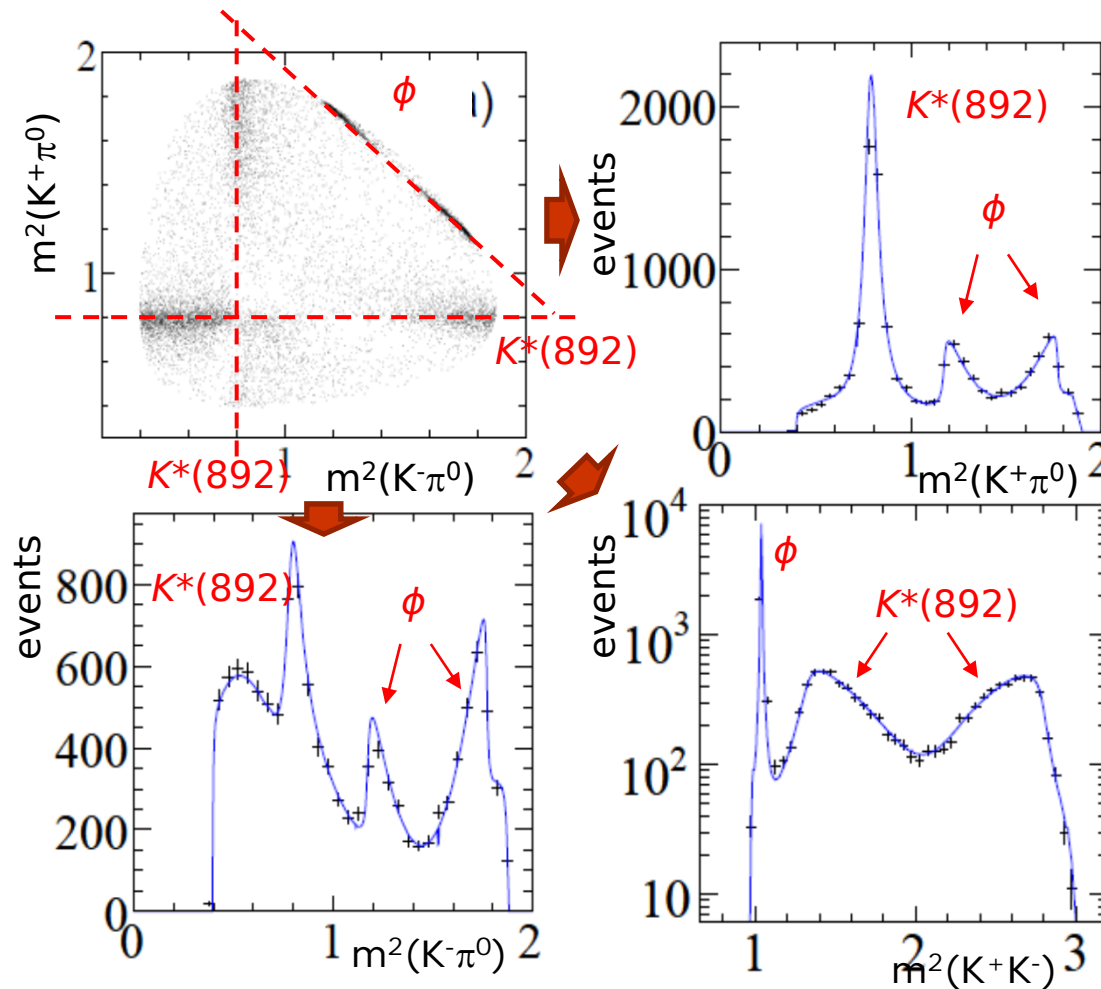
Kinematic situation can produce mass peaks not being true resonances → called Reflections

Example:

Dalitz plot of



in this case „fakes“ are simple to spot...

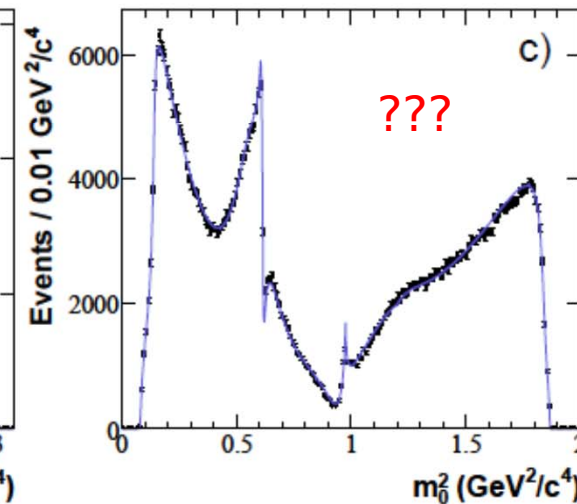
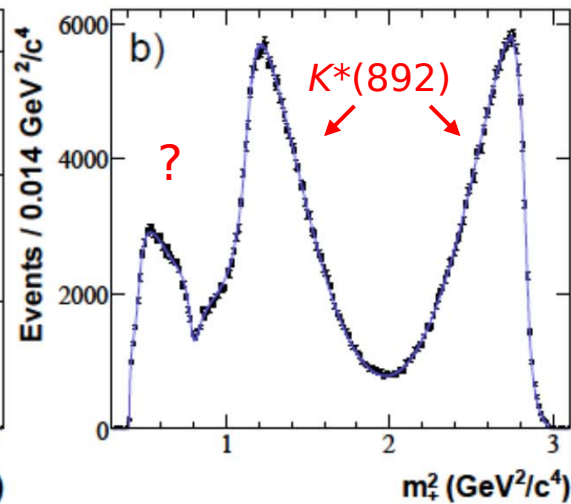
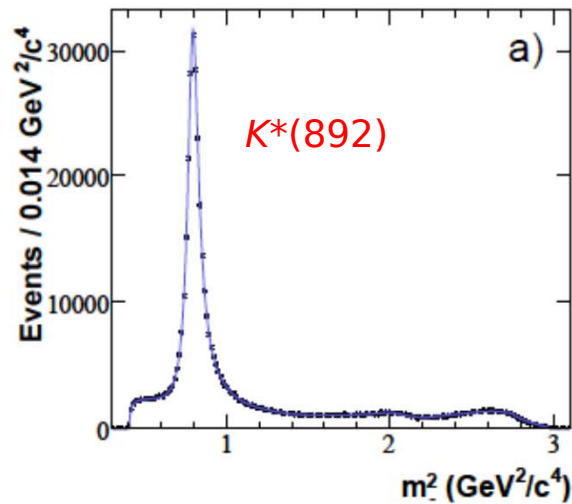
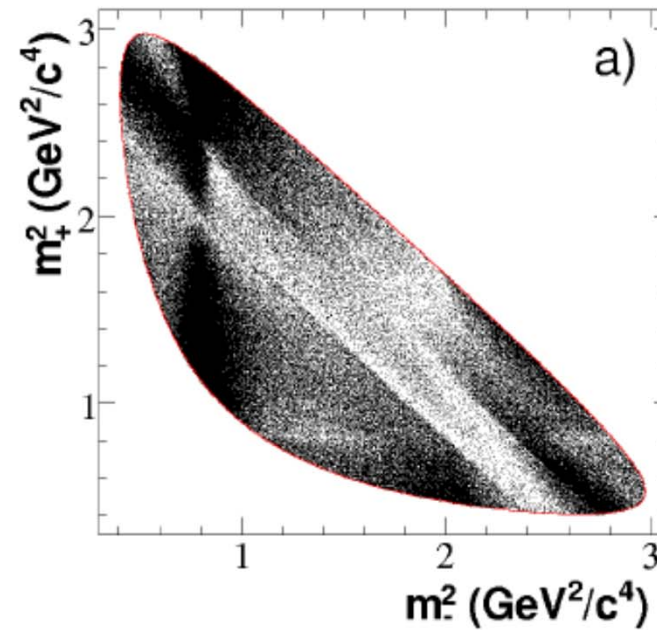


# Kinematical Reflections, cont'd



... but it can be much less obvious!

Example:  $D_s^+ \rightarrow K_S^0 \pi^+ \pi^-$





Observables should be aligned with the problem/process

is polarization relevant?

is dynamics present in all particle pairs?

are there isolated structures or regions with strong correlations?

Typical observables are

$m^2 (s)$

invariant mass square,  
Mandelstam  $s$

$T$

kinetic energy

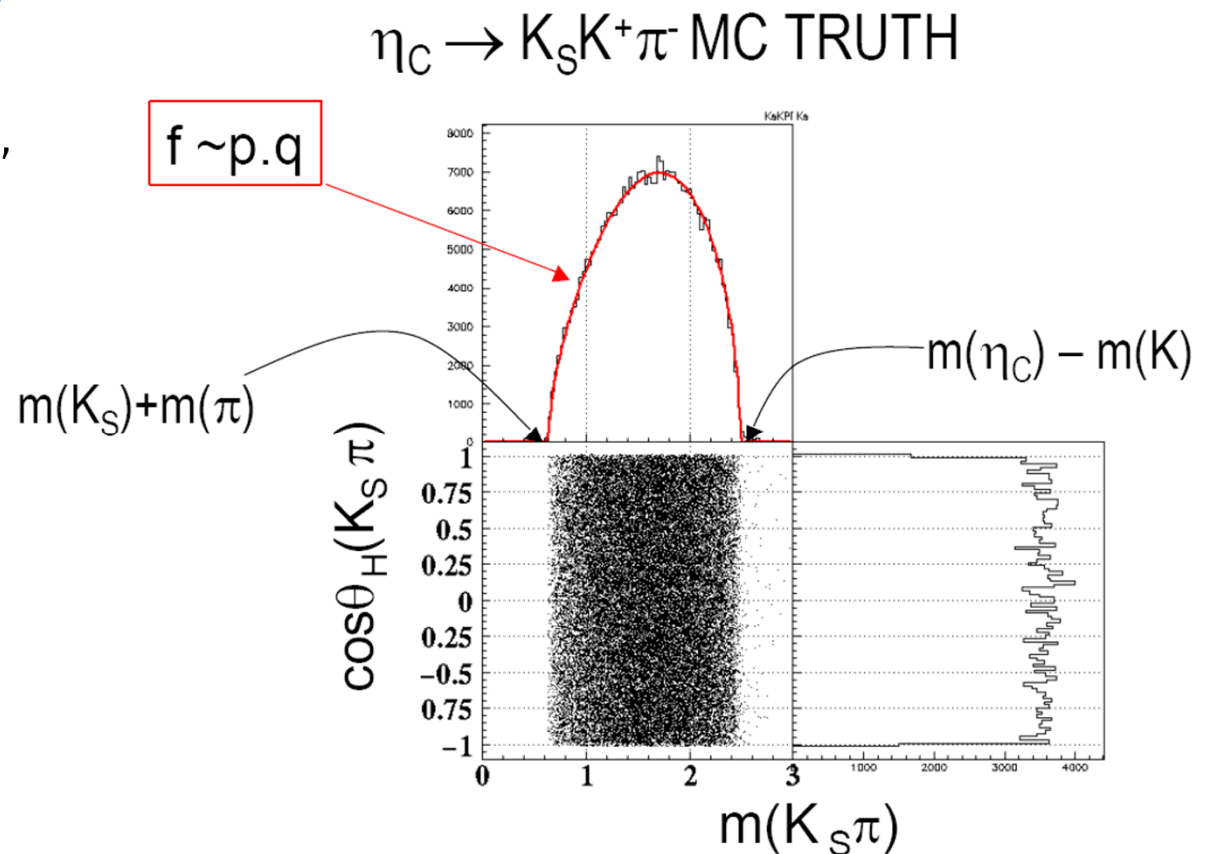
$\cos\theta$

decay angle of  
resonances

$\cos\psi$

angle between  
decay places,

..... a.m.o.m.







are there symmetries in the phase space?

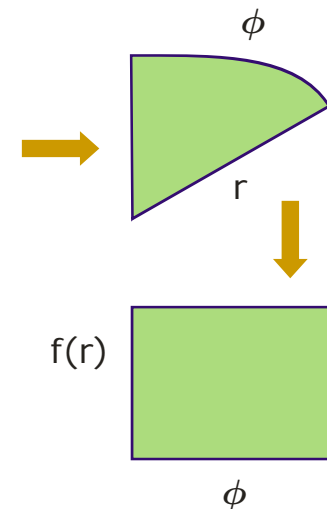
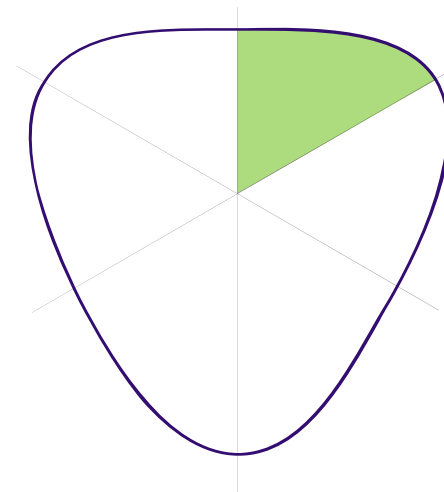
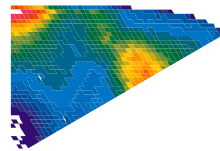
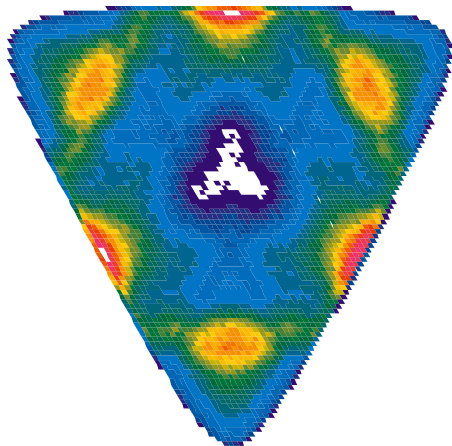
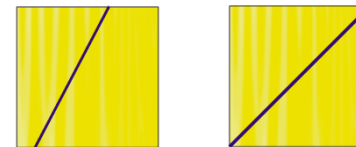
unique assignment of phase space coordinates  
is important to avoid double counting

transformation necessary?

Most Dalitz plots are symmetric:

Problem: sharing of events

Possible solution: transform DP





## Select basic model

usually isobar model, is not appropriate in all cases  
rescattering, t-channel and Deck effects may lead to artifacts

## Select formalism to handle the spin

select basis (helicity reflectivity, canonical....)  
or tensors (Zemach, covariant or Lorentz-invariant)  
depends on the process and the goals

## Select set of dynamical functions

which resonances and thresholds are known  
which do you guess from inspection  
how much freedom is needed,  
how well do I know the processes involved  
analysis of angular moments might be helpful as a start

## Selection of parameters and optimization

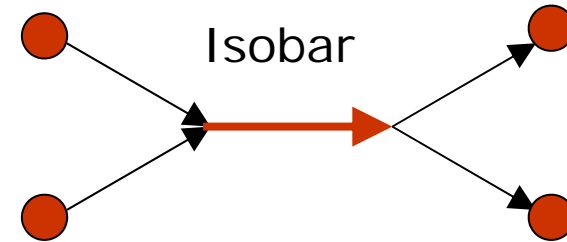
First results may indicate that the assumptions  
are wrong and one has to start over

---



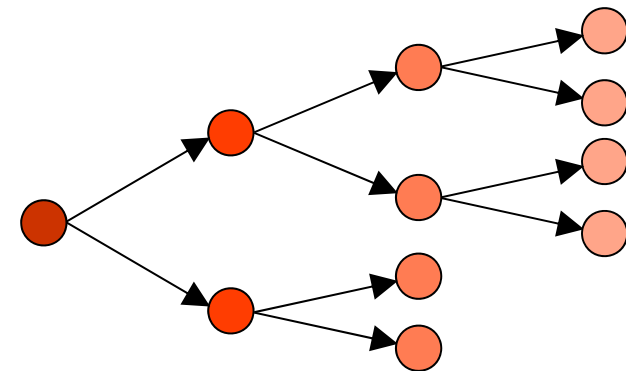
## Generalization

construct any many-body system  
as a tree of **subsequent two-body decays**  
the overall process is dominated  
by **two-body processes**  
the two-body systems behave  
**identical** in each reaction  
different initial states may interfere



## We need

need two-body “spin”-algebra  
**various formalisms**  
need two-body scattering formalism  
**final state interaction, e.g. Breit-Wigner**



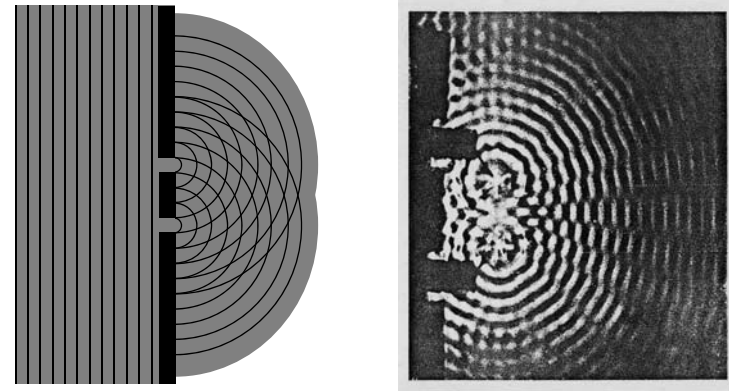
# Interference problem



44

## PWA

The phase space diagram in hadron physics shows a pattern due to interference and spin effects  
This is the unbiased measurement  
What has to be determined ?



## Analogy Optics $\Leftrightarrow$ PWA

# lamps  $\Leftrightarrow$  # level  
# slits  $\Leftrightarrow$  # resonances  
positions of slits  $\Leftrightarrow$  masses  
sizes of slits  $\Leftrightarrow$  widths

but only if spins  
are properly assigned

bias due to hypothetical  
spin-parity assumption

Optics

$$I(x) = |A_1(x) + A_2(x)e^{i\varphi}|^2$$

Dalitz plot

$$I(m) = |A_1(m) + A_2(m)e^{i\varphi}|^2$$



## Reactions (examples)

$J/\psi \rightarrow 3$  hadrons (e.g.  $\pi\pi\pi$ )

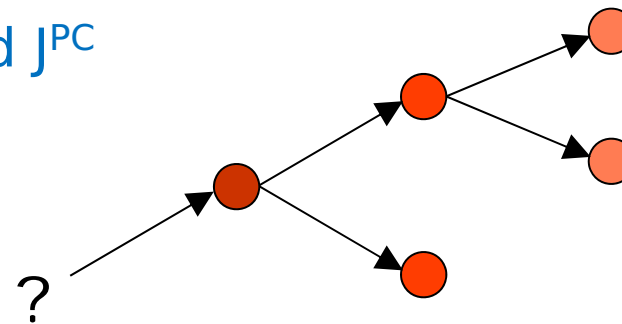
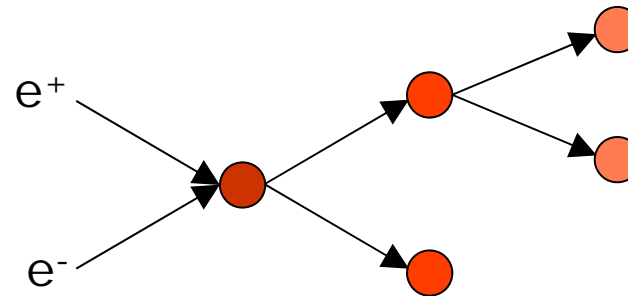
$D_s \rightarrow 3$  hadrons (e.g.  $K\bar{K}\pi$ )

Initial state has a well defined  $J^{PC}$

$J/\psi$       $J^{PC} = 1^{--}$

$D_s$          $J^{PC} = 0^{-+}$

## Typical Graphs



## Focus/Mission/Goal

Properties of intermediate resonances ( $J^{PC}$ , mass, width, decay ratios)

spin-parity of the decaying particle

## Typical Experiments

Tau-Charm Factories, B-Factories



### Procedure

Formulate spin-dependent amplitude

using all available constraints

hadronics decays conserve P, weak decays don't

Formulate dynamics (resonance cocktail)

with guesses for the properties (if not sufficiently well known)

Fit the parameters of the model to the data

Repeat for various hypothesis and identify the „best solution“

# Use Cases #2 of 4: $p\bar{p}$ annihilation



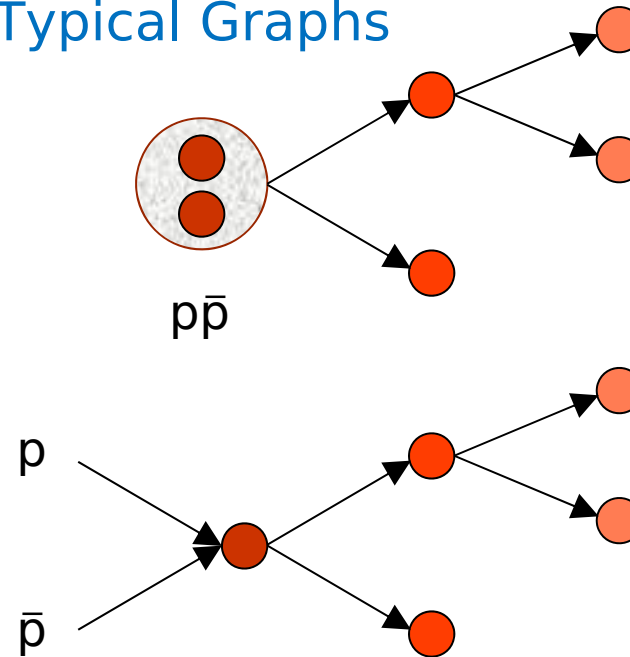
## Reactions (examples)

$p\bar{p} \rightarrow 3$  hadrons (at rest)

$p\bar{p} \rightarrow 3$  hadrons (in flight)

Initial state is a mixture of well defined  $J^{PC}$ 's

## Typical Graphs



## Focus/Mission/Goal

Properties of intermediate resonances ( $J^{PC}$ , mass, width, decay ratios)

## Typical Experiments

Asterix and Crystal Barrel @ LEAR

# Proton-Antiproton Annihilation @ Rest



## Atomic initial system

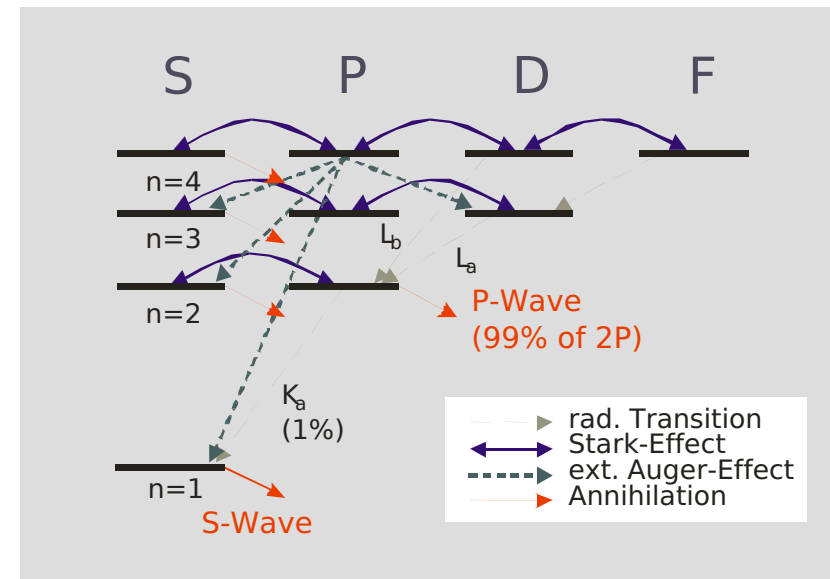
- formation at high  $n, l$  ( $n \sim 30$ )
- slow radiative transitions
- de-excitation through collisions (Auger effect)
- Stark mixing of  $l$ -levels (Day, Snow, Sucher, 1960)

## Advantages

- $J^{PC}$  varies with target density
- isospin varies with  $n$  (d) or  $p$  target
- incoherent initial states
- unambiguous PWA possible

## Disadvantages

- phase space very limited
- small kaon yield



	$J^{PC}$		$J^G$	$L$	$S$
$^1S_0$	$0^{++}$	pseudo scalar	$1^-; 0^+$	0	0
$^3S_1$	$1^{--}$	vector	$1^+; 0^-$	0	1
$^1P_1$	$1^{+-}$	axial vector	$1^+; 0^-$	1	0
$^3P_0$	$0^{++}$	scalar	$1^-; 0^+$	1	1
$^3P_1$	$1^{++}$	axial vector	$1^-; 0^+$	1	1
$^3P_2$	$2^{++}$	tensor	$1^-; 0^+$	1	1





Very similar to hadron decays for  $p\bar{p}$  at rest

### Procedure

Formulate spin-dependent amplitude  
using all available constraints

but for a couple of possible initial states !

hadronics decays conserve P, weak decays don't

Formulate dynamics (resonance cocktail)

with guesses for the properties (if not sufficiently well known)

Fit the parameters of the model to the data

Repeat for various hypothesis and identify the „best solution“

---

# Proton-Antiproton Annihilation in Flight



## Annihilation in flight

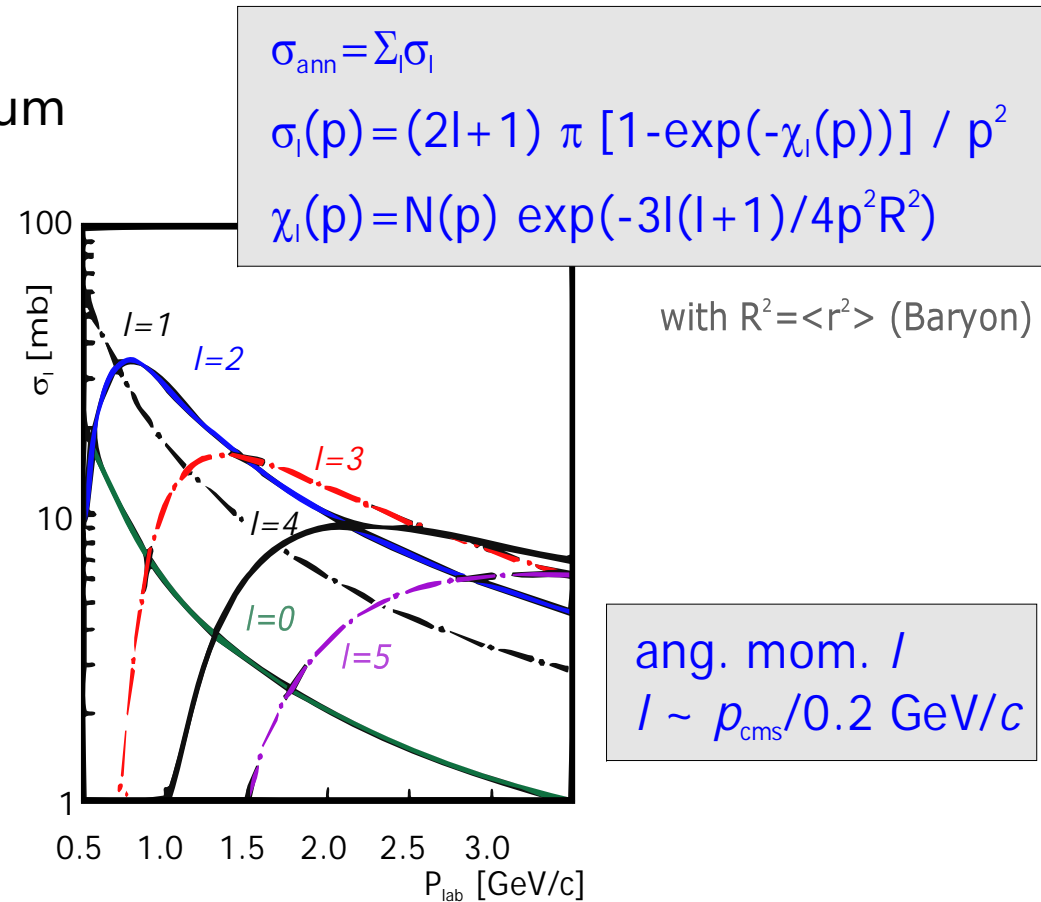
scattering process:  
no well defined initial state  
maximum angular momentum  
rises with energy

## Advantages

larger phase space  
formation experiments

## Disadvantages

many waves interfere  
with each other  
many waves due  
to large phase space



# Scattering Amplitudes in $\bar{p}p$ in Flight (II)



$$H_{\nu_1\nu_2}^J = \sum_{L,S} \frac{\sqrt{2L+1}}{\sqrt{2J+1}} (LOS\nu | J\nu) (s_1\nu_1s_2-\nu_2 | S\nu) (JMLS | M | JM)$$

using all constraints leads to 4 incoherent sets of coherent amplitudes

<i>Singlett even L</i>	$J^{PC}$	L	S	$H_{++}$	$H_{+-}$
$^1S_0$	$0^{-+}$	0	0	Yes	No
$^1D_2$	$2^{-+}$	2	0	Yes	No
$^1G_4$	$4^{-+}$	4	0	Yes	No

<i>Singlett odd L</i>	$J^{PC}$	L	S	$H_{++}$	$H_{+-}$
$^1P_1$	$1^{+-}$	1	0	Yes	No
$^1F_3$	$3^{+-}$	3	0	Yes	No
$^1G_5$	$5^{+-}$	5	0	Yes	No

<i>Triplet even L</i>	$J^{PC}$	L	S	$H_{++}$	$H_{+-}$
$^3S_1$ $\updownarrow$	$1^{-}$	0	1	Yes	Yes
$^3D_1$ $\downarrow$	$1^{-}$	2	1	Yes	Yes
$^3D_2$	$2^{-}$	2	1	Yes	Yes
$^3D_3$	$3^{-}$	2	1	Yes	Yes

<i>Triplet odd L</i>	$J^{PC}$	L	S	$H_{++}$	$H_{+-}$
$^3P_0$	$0^{++}$	1	1	Yes	No
$^3P_1$	$1^{++}$	1	1	No	Yes
$^3P_2$ $\updownarrow$	$2^{++}$	1	1	Yes	Yes
$^3F_2$ $\downarrow$	$2^{++}$	3	1	Yes	No
$^3F_3$	$3^{++}$	3	1	No	Yes
$^3F_4$	$4^{++}$	3	1	Yes	Yes

and only 2 of them contribute to a particular exclusive final state



## Reactions (examples)

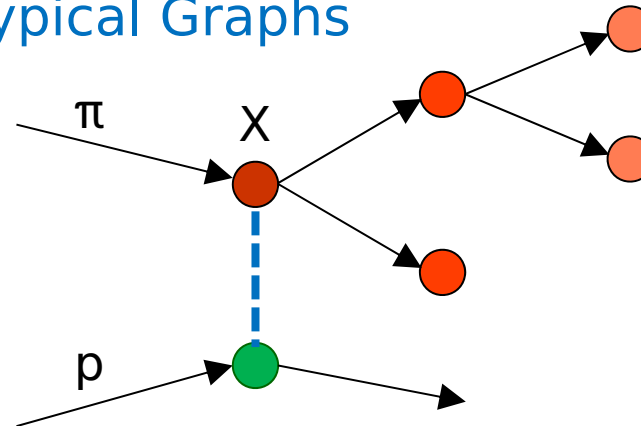
$\pi p \rightarrow 3 \text{ hadrons } p, n$

unknown initial state

scattering process

expressed in terms of moments

## Typical Graphs



## Focus/Mission/Goal

Properties of  $X$  ( $J^{PC}$ , mass, width, decay ratios)

## Typical Experiments

E852 at AGS (BNL) or COMPASS at CERN



## Procedure

$\pi^-$ -beam and t-exchange produce X

Partitioning in bins of  $m(3\pi)$

Potentially also binning in t.

Wave  $\sim t^m e^{-bt}$   $\rightarrow$  background as function of t

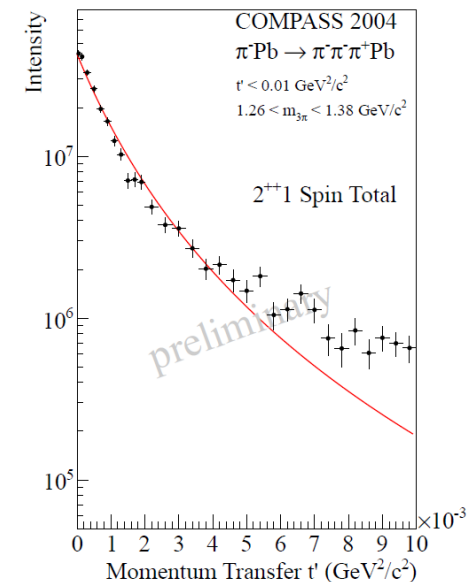
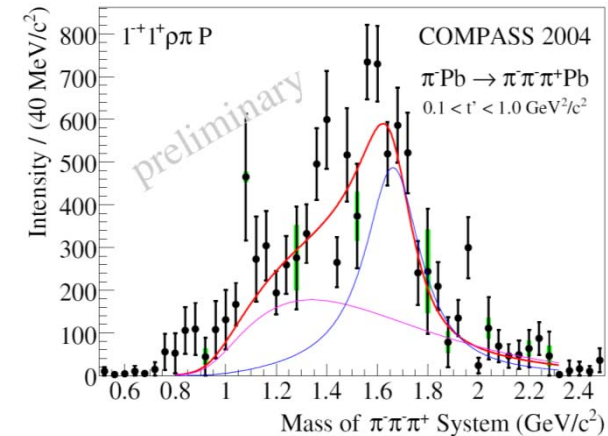
Formulate spin-dependent amplitude and dynamics for the intermediate resonance cocktail

with best guesses for the properties

## Mass dependent fits

performed for a fixed  $m(3\pi)$  (and may be t) and all fits result in moments (and errors) per bin, which are then translated into wave content

The result of one slice is then input (start value) for the next slice to be fitted.



# Use Cases #4 of 4: Photo-production



## Reactions (examples)

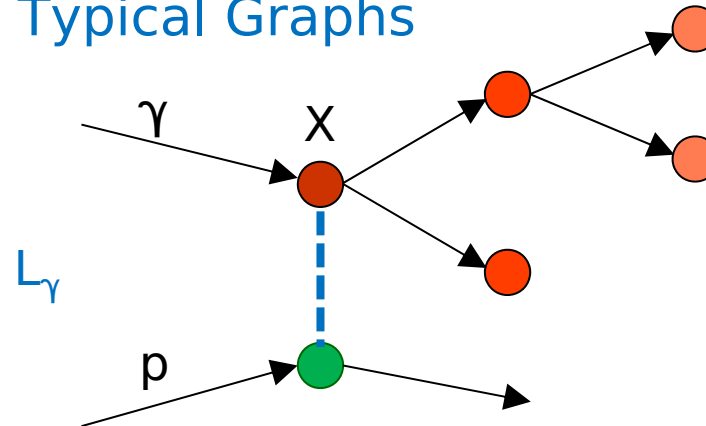
$\gamma p \rightarrow 3 \text{ hadrons } p, n$

known initial states  
scattering process

finally expressed in terms of moments

→ very similar to diffractive production for high energies and multi-hadron final states

## Typical Graphs

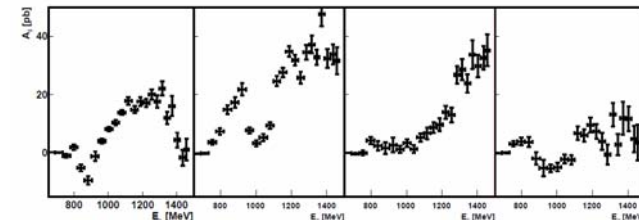
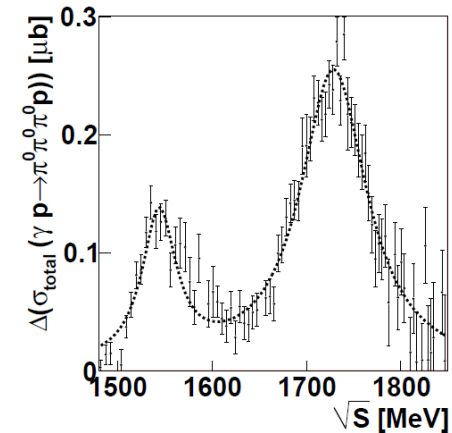


## Focus/Mission/Goal

Properties of X ( $J^{PC}$ , mass, width, decay ratios)

## Typical Experiments

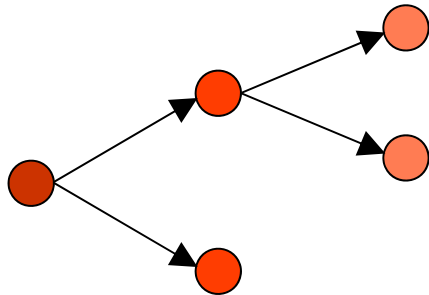
Crystal Barrel at ELSA or Gluex/Clas12 @ Jlab12





Use cases are different

There **IS** a common aspect



and for each decay we need a proper formulation in our model

and we always have to fit our model to the data

---



**THANK YOU**  
for today

---