Data
Amplitudes
Ambiguities
Simple Methods

Baryon Amplitude Analysis

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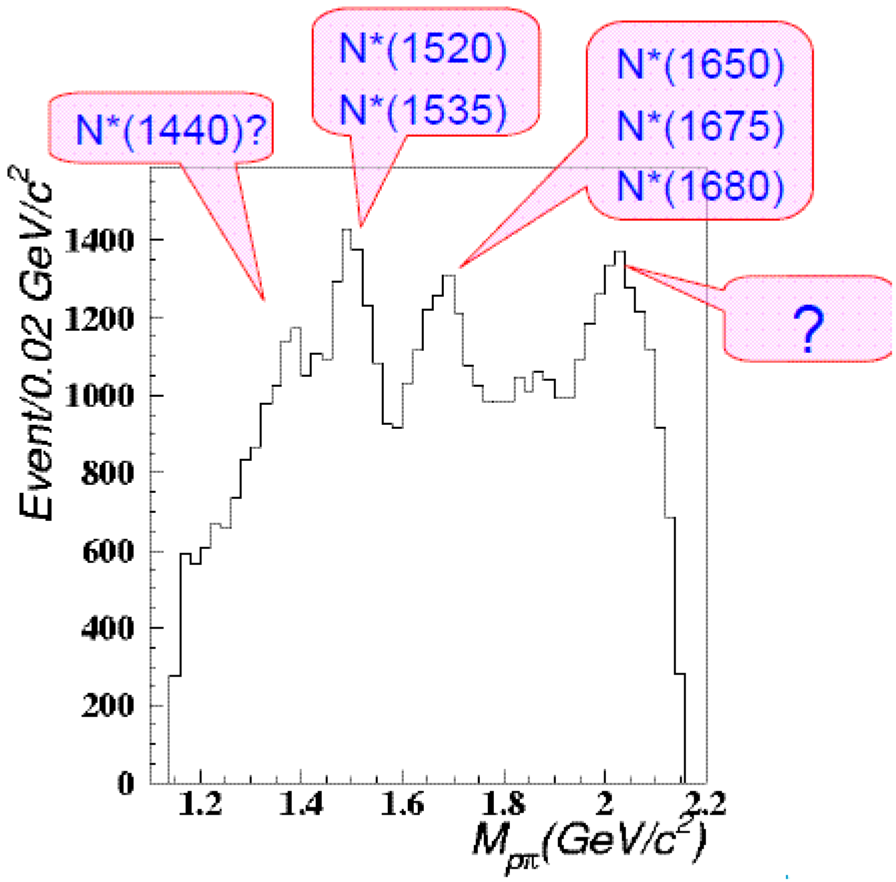
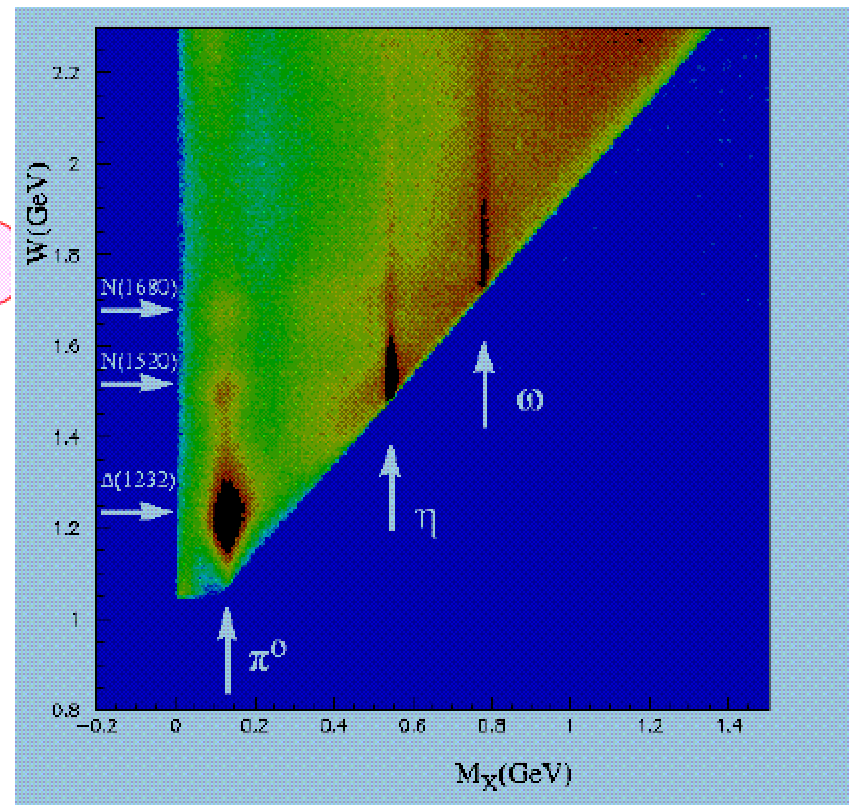
Techniques of Amplitude Analysis
Jefferson Lab ASI2012
Williamsburg, VA

Many possible reactions:

$$\pi N \to \pi N,\ \pi\pi N,\ \ldots$$
$$\gamma N \to \pi N,\ \pi\pi N,\ \ldots$$
$$\gamma^* N \to \pi N,\ \pi\pi N,\ \ldots$$
$$pp \to pp\pi^0\ ,\ pp\pi\pi,\ \ldots$$
$$J/\Psi \to p\bar{p}\pi^0,\ p\bar{n}\pi^-,\ \ldots$$



$J/\psi \to p\bar{n}\pi^-$



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PRL 97, 062001 (2006) PHYSICAL REVIEW LETTERS week ending 11 AUGUST 2006

Observation of Two New N^* Peaks in $J/\psi \to p\pi^-\bar{n}$ and $\bar{p}\pi^+n$ Decays

(BES Collaboration)

The decay $J/\psi \to \bar{N}N\pi$ provides an effective isospin 1/2 filter for the πN system due to isospin conservation. Using 58×10^6 J/ψ decays collected with the Beijing Electromagnetic Spectrometer at the Beijing Electron Positron Collider, more than 100 thousand $J/\psi \to p\pi^-\bar{n}$ + c.c. events are obtained. Besides the two well-known N^* peaks at around 1500 MeV/c^2 and 1670 MeV/c^2, there are two new, clear N^* peaks in the $p\pi$ invariant mass spectrum around 1360 MeV/c^2 and 2030 MeV/c^2 with statistical significance of 11σ and 13σ, respectively. We identify these as the first direct observation of the $N^*(1440)$ peak and a long-sought missing N^* peak above 2 GeV/c^2 in the πN invariant mass spectrum.

New state?

$N(2100)$ $1/2^+$ $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ Status: *

OMITTED FROM SUMMARY TABLE

The latest GWU analysis (ARNDT 06) finds no evidence for this resonance.

$N(2100)$ BREIT-WIGNER MASS

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
≈ 2100 OUR ESTIMATE				
2125±75	CUTKOSKY	80	IPWA	$\pi N \to \pi N$
2050±20	HOEHLER	79	IPWA	$\pi N \to \pi N$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
2157±42	BATINIC	10	DPWA	$\pi N \to N\pi, N\eta$
$2068 \pm 3^{+15}_{-40}$	ABLIKIM	06K	BES2	$J/\psi \to (p\pi^-)\bar{n}$
2084±93	VRANA	00	DPWA	Multichannel
$1986 \pm 26^{+10}_{-30}$	PLOETZKE	98	SPEC	$\gamma p \to p\eta'(958)$

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Another BES result suggested a large $K^+ \Lambda$ coupling for the N(1535) $1/2^-$

B.C. Liu and B.S. Zou, PRL 96, 042002 (2006)

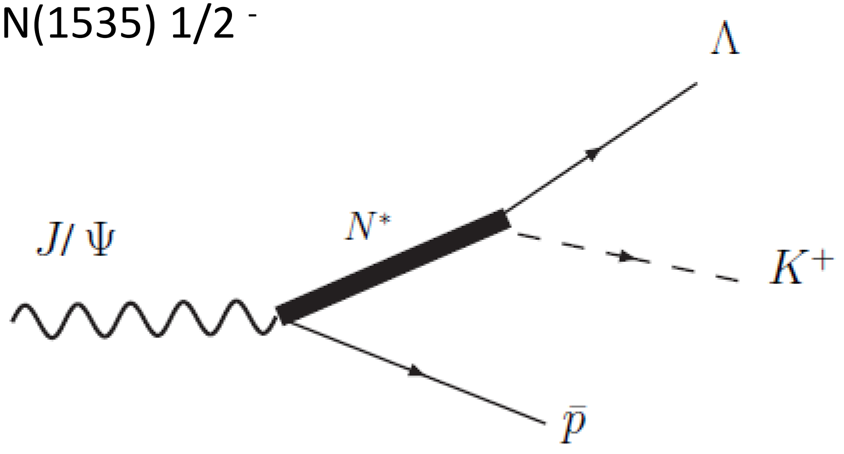

FIG. 1. Feynman diagram for $\psi \to \bar{p} K^+ \Lambda$ through N^*

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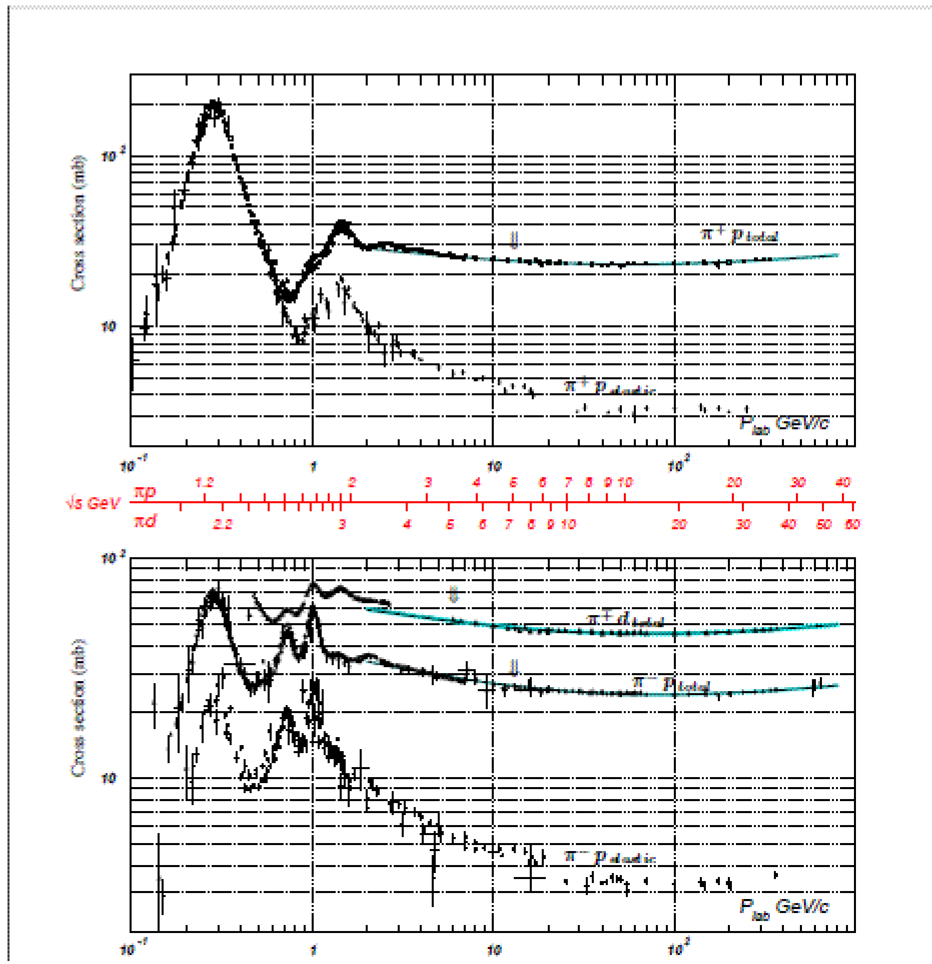

πN cross sections have only two or three distinct \`bumps’

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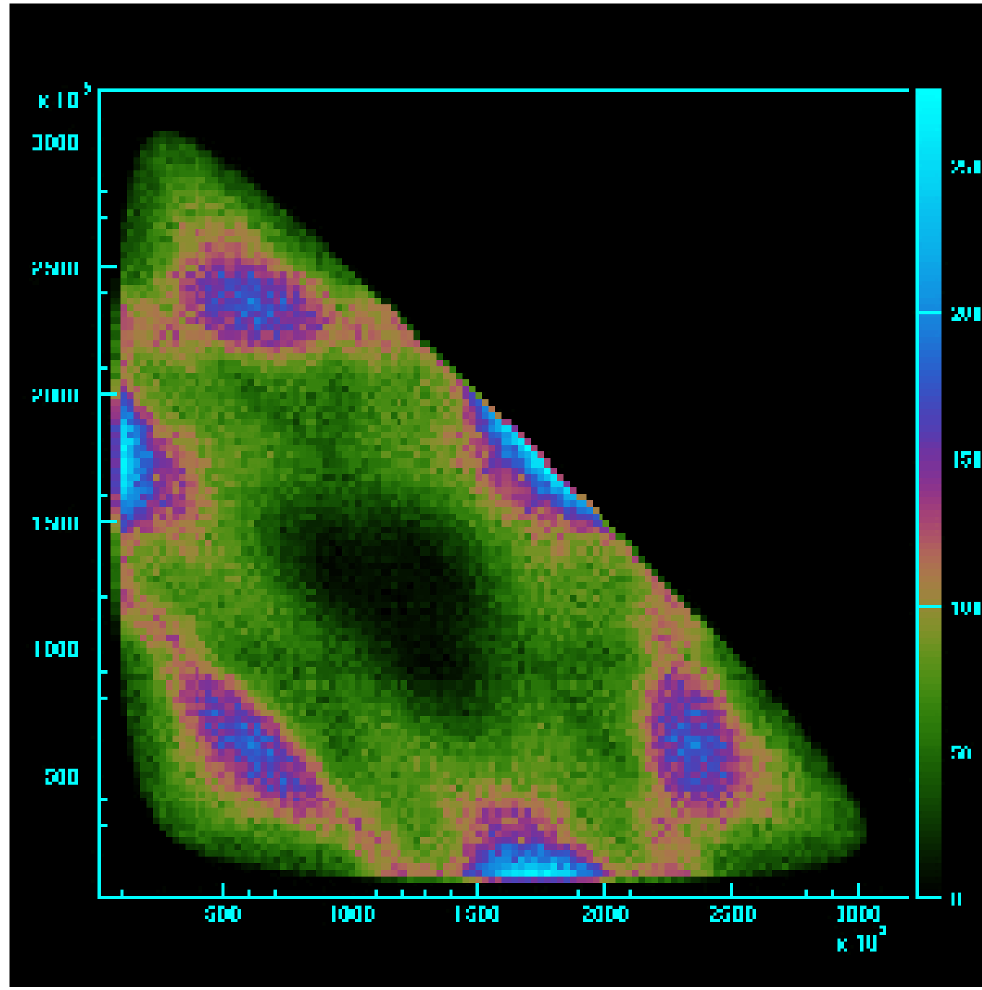
We will not be
working with
data so suggestive as
seen in Klaus Peters'
Dalitz plot



Focus on two reactions:

$$\pi N \rightarrow \pi N$$
$$\gamma N \rightarrow \pi N$$

- most PDG info from these sources (presently)
- πN scattering is highly constrained
- resonance structure is correlated
- 2-body final state, fewer amplitudes

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Isospin 1/2 N* states listed and rated by the PDG

Table 1. The status of the N and Δ resonances. Only those with an overall status of *** or **** are included in the main Baryon Summary Table.

			Status as seen in —						
Particle	$L_{2I\cdot 2J}$	Overall status	$N\pi$	$N\eta$	ΛK	ΣK	$\Delta\pi$	$N\rho$	$N\gamma$
$N(939)$	P_{11}	****							
$N(1440)$	P_{11}	****	****	*			***	*	***
$N(1520)$	D_{13}	****	****	***			****	****	****
$N(1535)$	S_{11}	****	****	****			*	**	***
$N(1650)$	S_{11}	****	****	*	***	**	***	**	***
$N(1675)$	D_{15}	****	****	*	*		****	*	****
$N(1680)$	F_{15}	****	****	*			****	****	****
$N(1700)$	D_{13}	***	***	*	**	*	**	*	**
$N(1710)$	P_{11}	***	***	**	**	*	**	*	***
$N(1720)$	P_{13}	****	****	*	**	*	*	**	**
$N(1900)$	P_{13}	**	**					*	
$N(1990)$	F_{17}	**	**	*	*	*			*
$N(2000)$	F_{15}	**	**	*	*	*	*	**	
$N(2080)$	D_{13}	**	**	*	*				*
$N(2090)$	S_{11}	*	*						
$N(2100)$	P_{11}	*	*	*					
$N(2190)$	G_{17}	****	****	*	*	*		*	*
$N(2200)$	D_{15}	**	**	*	*				
$N(2220)$	H_{19}	****	****	*					
$N(2250)$	G_{19}	****	****	*					
$N(2600)$	$I_{1\,11}$	***	***						
$N(2700)$	$K_{1\,13}$	**	**						

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Isospin 3/2 N* states listed and rated by the PDG

Particle	$L_{2I \cdot 2J}$	Overall status	Status as seen in — $N\pi$	$N\eta$	ΛK	ΣK	$\Delta\pi$	$N\rho$	$N\gamma$
$\Delta(1232)$	P_{33}	****	****	F					****
$\Delta(1600)$	P_{33}	***	***	o			***	*	**
$\Delta(1620)$	S_{31}	****	****	r			****	****	***
$\Delta(1700)$	D_{33}	****	****	b		*	***	**	***
$\Delta(1750)$	P_{31}	*	*	i					
$\Delta(1900)$	S_{31}	**	**	d		*	*	**	*
$\Delta(1905)$	F_{35}	****	****	d		*	**	**	***
$\Delta(1910)$	P_{31}	****	****	e		*	*	*	*
$\Delta(1920)$	P_{33}	***	***	n		*	**		*
$\Delta(1930)$	D_{35}	***	***			*			**
$\Delta(1940)$	D_{33}	*	*	F					
$\Delta(1950)$	F_{37}	****	****	o		*	****	*	****
$\Delta(2000)$	F_{35}	**		r			**		
$\Delta(2150)$	S_{31}	*	*	b					
$\Delta(2200)$	G_{37}	*	*	i					
$\Delta(2300)$	H_{39}	**	**	d					
$\Delta(2350)$	D_{35}	*	*	d					
$\Delta(2390)$	F_{37}	*	*	e					
$\Delta(2400)$	G_{39}	**	**	n					
$\Delta(2420)$	$H_{3\,11}$	****	****						*
$\Delta(2750)$	$I_{3\,13}$	**	**						
$\Delta(2950)$	$K_{3\,15}$	**	**						

****	Existence is certain, and properties are at least fairly well explored.
***	Existence ranges from very likely to certain, but further confirmation is desirable and/or quantum numbers, branching fractions, *etc.* are not well determined.
**	Evidence of existence is only fair.
*	Evidence of existence is poor.

July 30, 2010 14:34

Rating is subjective but only the ** | *** border is important

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A passage in the `Note on N and Δ resonances' adds a restriction:

Table 1 lists all the N and Δ entries in the Baryon Listings and gives our evaluation of the status of each, both overall and channel by channel. Only the "established" resonances (overall status 3 or 4 stars) appear in the Baryon Summary Table. We generally consider a resonance to be established only if it has been seen in at least two independent analyses of elastic scattering and if the relevant partial-wave amplitudes do not behave erratically or have large errors.

Could not see a new 4-star state – or un-see an existing one

(now changed)

$\Delta(1232)\ P_{33}$

$I(J^P) = \frac{3}{2}(\frac{3}{2}^+)$ Status: ∗∗∗∗

Most of the results published before 1975 were last included in our 1982 edition, Physics Letters **111B** 1 (1982). Some further obsolete results published before 1984 were last included in our 2006 edition, Journal of Physics, G **33** 1 (2006).

Δ(1232) BREIT-WIGNER MASSES

MIXED CHARGES

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
1231 to 1233 (≈ 1232) OUR ESTIMATE				
1230 ±2	ANISOVICH	10	DPWA	Multichannel
1233.4±0.4	ARNDT	06	DPWA	$\pi N \to \pi N$, ηN
1231 ±1	MANLEY	92	IPWA	$\pi N \to \pi N$ & $N\pi\pi$
1232 ±3	CUTKOSKY	80	IPWA	$\pi N \to \pi N$
1233 ±2	HOEHLER	79	IPWA	$\pi N \to \pi N$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
1232.9±1.2	ARNDT	04	DPWA	$\pi N \to \pi N$, ηN
1228 ±1	PENNER	02C	DPWA	Multichannel
1234 ±5	VRANA	00	DPWA	Multichannel
1233	ARNDT	95	DPWA	$\pi N \to N\pi$

Some values above and below `the line’.

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Mass shift given only for the Δ(1232)

$m_{\Delta^0} - m_{\Delta^{++}}$

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
• • • We do not use the following data for averages, fits, limits, etc. • • •				
2.86±0.30	GRIDNEV	06	DPWA	$\pi N \to \pi N$
2.25±0.68	BERNICHA	96		Fit to PEDRONI 78
2.6 ±0.4	ABAEV	95	IPWA	$\pi N \to \pi N$
2.7 ±0.3	[1] PEDRONI	78		See the masses

[1] Using $\pi^{\pm} d$ as well, PEDRONI 78 determine $(M^{-} - M^{++}) + (M^{0} - M^{+})/3 = 4.6 \pm 0.2$ MeV.

Δ(1232) BREIT-WIGNER WIDTHS

MIXED CHARGES

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
116 to 120 (≈ 118) OUR ESTIMATE				
112 ± 4	ANISOVICH	10	DPWA	Multichannel
118.7± 0.6	ARNDT	06	DPWA	$\pi N \to \pi N, \eta N$
118 ± 4	MANLEY	92	IPWA	$\pi N \to \pi N$ & $N \pi \pi$
120 ± 5	CUTKOSKY	80	IPWA	$\pi N \to \pi N$
116 ± 5	HOEHLER	79	IPWA	$\pi N \to \pi N$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
118.0± 2.2	ARNDT	04	DPWA	$\pi N \to \pi N, \eta N$
106 ± 1	PENNER	02C	DPWA	Multichannel
112 ±18	VRANA	00	DPWA	Multichannel
114	ARNDT	95	DPWA	$\pi N \to N \pi$

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Δ(1232) POLE POSITIONS

REAL PART, MIXED CHARGES

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
1209 to 1211 (≈ 1210) OUR ESTIMATE				
1211±1	ANISOVICH	10	DPWA	Multichannel
1211	ARNDT	06	DPWA	$\pi N \to \pi N, \eta N$
1209	[2] HOEHLER	93	ARGD	$\pi N \to \pi N$
1210±1	CUTKOSKY	80	IPWA	$\pi N \to \pi N$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
1210	ARNDT	04	DPWA	$\pi N \to \pi N, \eta N$
1217	VRANA	00	DPWA	Multichannel
1211	ARNDT	95	DPWA	$\pi N \to N\pi$
1210	ARNDT	91	DPWA	$\pi N \to \pi N$ Soln SM90

−2×IMAGINARY PART, MIXED CHARGES

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
98 to 102 (≈ 100) OUR ESTIMATE				
100±2	ANISOVICH	10	DPWA	Multichannel
99	ARNDT	06	DPWA	$\pi N \to \pi N, \eta N$
100	[2] HOEHLER	93	ARGD	$\pi N \to \pi N$
100±2	CUTKOSKY	80	IPWA	$\pi N \to \pi N$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
100	ARNDT	04	DPWA	$\pi N \to \pi N, \eta N$
96	VRANA	00	DPWA	Multichannel
100	ARNDT	95	DPWA	$\pi N \to N\pi$
100	ARNDT	91	DPWA	$\pi N \to \pi N$ Soln SM90

Pole parameters given in addition to BW mass/width (less model-dependence)

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$\Delta(1232)$ PHOTON DECAY AMPLITUDES

Papers on γN amplitudes predating 1981 may be found in our 2006 edition, Journal of Physics, G **33** 1 (2006).

$\Delta(1232) \rightarrow N\gamma$, helicity-1/2 amplitude $A_{1/2}$

VALUE ($GeV^{-1/2}$)	DOCUMENT ID		TECN	COMMENT
−0.135 ±0.006 OUR ESTIMATE				
−0.136 ±0.005	ANISOVICH	10	DPWA	Multichannel
−0.139 ±0.004	DUGGER	07	DPWA	$\gamma N \rightarrow \pi N$
−0.137 ±0.005	AHRENS	04A	DPWA	$\vec{\gamma}\vec{p} \rightarrow N\pi$
−0.129 ±0.001	ARNDT	02	DPWA	$\gamma p \rightarrow N\pi$
−0.1357 ±0.0013 ±0.0037	BLANPIED	01	LEGS	$\gamma p \rightarrow p\gamma, p\pi^0, n\pi^+$
−0.131 ±0.001	BECK	00	IPWA	$\vec{\gamma}p \rightarrow p\pi^0, n\pi^+$
−0.140 ±0.005	KAMALOV	99	DPWA	$\gamma N \rightarrow \pi N$
−0.1294 ±0.0013	HANSTEIN	98	IPWA	$\gamma N \rightarrow \pi N$
−0.135 ±0.005	ARNDT	97	IPWA	$\gamma N \rightarrow \pi N$
−0.1278 ±0.0012	DAVIDSON	97	DPWA	$\gamma N \rightarrow \pi N$
−0.141 ±0.005	ARNDT	96	IPWA	$\gamma N \rightarrow \pi N$
−0.135 ±0.016	DAVIDSON	91B	FIT	$\gamma N \rightarrow \pi N$
−0.145 ±0.015	CRAWFORD	83	IPWA	$\gamma N \rightarrow \pi N$
−0.138 ±0.004	AWAJI	81	DPWA	$\gamma N \rightarrow \pi N$

$A_{3/2}$ also given. Values for $p\gamma$ and $n\gamma$ given for isospin ½ states

Values at the pole for some states

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E2/M1 ratio given at resonance `mass' and pole

$\Delta(1232) \rightarrow N\gamma$, E_2/M_1 ratio

VALUE	DOCUMENT ID		TECN	COMMENT
−0.025 ±0.005 OUR ESTIMATE				
−0.0274±0.0003±0.0030	AHRENS	04A	DPWA	$\vec{\gamma}\vec{p} \rightarrow N\pi$
−0.020 ±0.002	ARNDT	02	DPWA	$\gamma p \rightarrow N\pi$
−0.0307±0.0026±0.0024	BLANPIED	01	LEGS	$\gamma p \rightarrow p\gamma$, $p\pi^0$, $n\pi^+$
−0.016 ±0.004 ±0.002	GALLER	01	DPWA	$\gamma p \rightarrow \gamma p$
−0.025 ±0.001 ±0.002	BECK	00	IPWA	$\vec{\gamma} p \rightarrow p\pi^0$, $n\pi^+$
−0.0233±0.0017	HANSTEIN	98	IPWA	$\gamma N \rightarrow \pi N$
−0.015 ±0.005	[5] ARNDT	97	IPWA	$\gamma N \rightarrow \pi N$
−0.0319±0.0024	DAVIDSON	97	DPWA	$\gamma N \rightarrow \pi N$

$\Delta(1232) \rightarrow N\gamma$, absolute value of E_2/M_1 ratio at pole

VALUE	DOCUMENT ID		TECN	COMMENT
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.065±0.007	ARNDT	97	DPWA	$\gamma N \rightarrow \pi N$
0.058	HANSTEIN	96	DPWA	$\gamma N \rightarrow \pi N$

$\Delta(1232) \rightarrow N\gamma$, phase of E_2/M_1 ratio at pole

VALUE	DOCUMENT ID		TECN	COMMENT
• • • We do not use the following data for averages, fits, limits, etc. • • •				
−122 ±5	ARNDT	97	DPWA	$\gamma N \rightarrow \pi N$
−127.2	HANSTEIN	96	DPWA	$\gamma N \rightarrow \pi N$

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Changes of format
and some new states
in 2012 edition

Table 1. The status of the N resonances. Only those with an overall status of *** or **** are included in the main Baryon Summary Table.

		Status			Status as seen in —						
Particle	J^P	overall	πN	γN	$N\eta$	$N\sigma$	$N\omega$	ΛK	ΣK	$N\rho$	$\Delta\pi$
N	$1/2^+$	****									
$N(1440)$	$1/2^+$	****	****	****		***				*	***
$N(1520)$	$3/2^-$	****	****	****	***					***	***
$N(1535)$	$1/2^-$	****	****	****	****					**	*
$N(1650)$	$1/2^-$	****	****	***	***			***	**	**	***
$N(1675)$	$5/2^-$	****	****	***	*			*		*	***
$N(1680)$	$5/2^+$	****	****	****	*	**				***	***
$N(1685)$	$?^?$	*									
$N(1700)$	$3/2^-$	***	***	**	*			*	*	*	***
$N(1710)$	$1/2^+$	***	***	***	***		**	***	**	*	**
$N(1720)$	$3/2^+$	****	****	***	***			**	**	**	*
$N(1860)$	$5/2^+$	**	**							*	*
$N(1875)$	$3/2^-$	***	*	***			**	***	**		***
$N(1880)$	$1/2^+$	**	*	*		**		*			
$N(1895)$	$1/2^-$	**	*	**	**			**	*		
$N(1900)$	$3/2^+$	***	**	***	**		**	***	**	*	**
$N(1990)$	$7/2^+$	**	**	**					*		
$N(2000)$	$5/2^+$	**	*	**	**			**	*	**	
$N(2040)$	$3/2^+$	*									
$N(2060)$	$5/2^-$	**	**	**	*				**		
$N(2100)$	$1/2^+$	*									
$N(2150)$	$3/2^-$	**	**	**				**			**
$N(2190)$	$7/2^-$	****	****	***			*	**		*	
$N(2220)$	$9/2^+$	****	****								
$N(2250)$	$9/2^-$	****	****								
$N(2600)$	$11/2^-$	***	***								
$N(2700)$	$13/2^+$	**	**								

****	Existence is certain, and properties are at least fairly well explored.
***	Existence is very likely but further confirmation of quantum numbers and branching fractions is required.
**	Evidence of existence is only fair.
*	Evidence of existence is poor.

Plan of the talks:

- Explain what data are measured
- `Look' at them
- Outline the amplitude structure
- Show some tools to explore data
- Try some simple amplitude reconstructions (ambiguities)
- Do a simple fit
- Consider a few simple methods applied to the Delta
- Do pion photoproduction overview
- Do pion-nucleon scattering overview

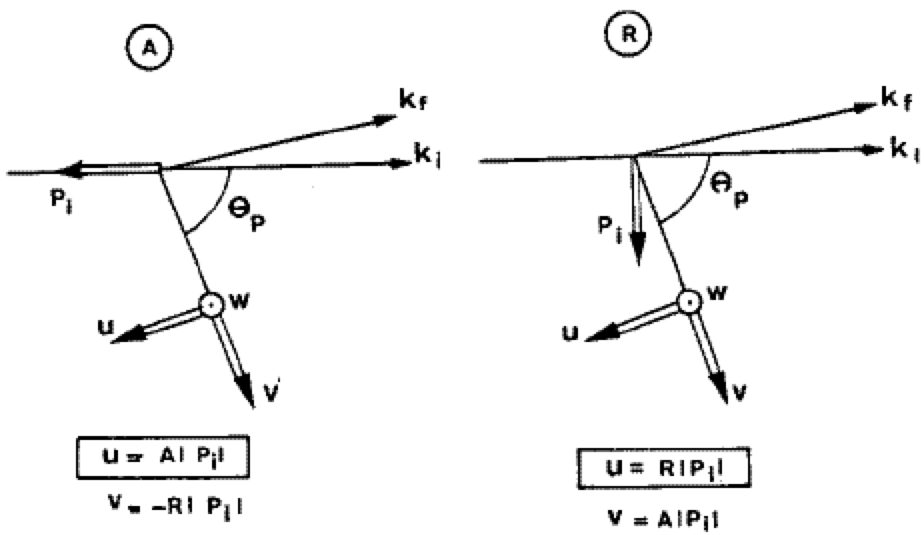
πN scattering data:

dσ/dΩ (unpolarized)
P (polarized target or recoil nucleon)
R and A (polarized target and recoil measured)

Not Independent: $P^2 + R^2 + A^2 = 1$

Abundant dσ/dΩ and P data
Very limited R and A data

Alekseev et al.,
EPJ C45,383(2006)
P_{beam} = 1.43 GeV/c
W_{cm} ~ 1.9 GeV/c^2



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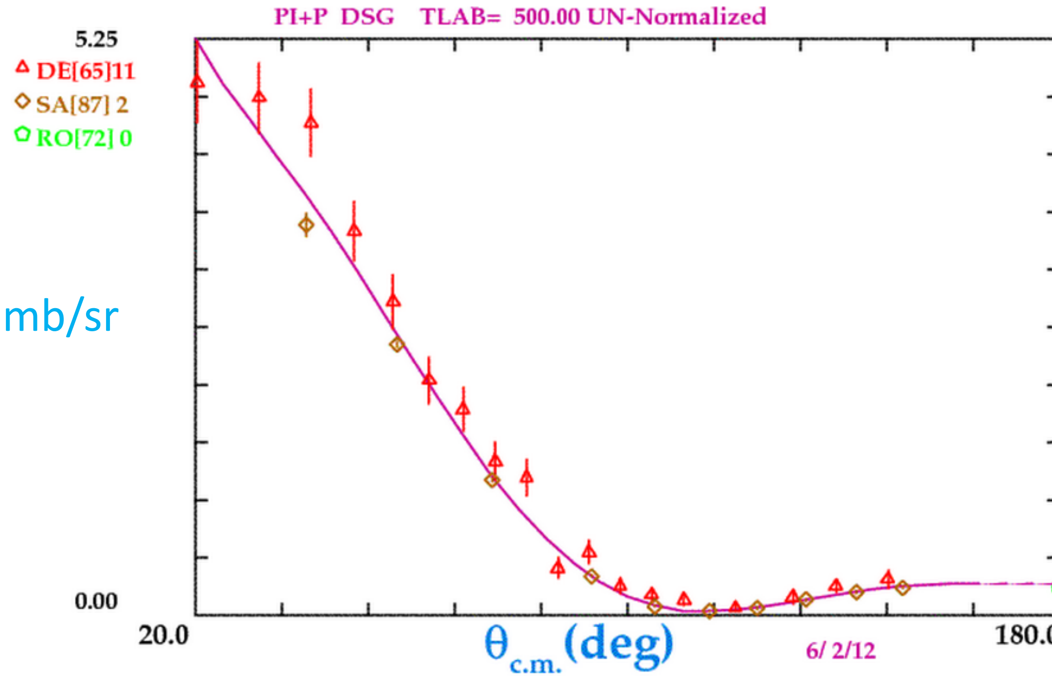
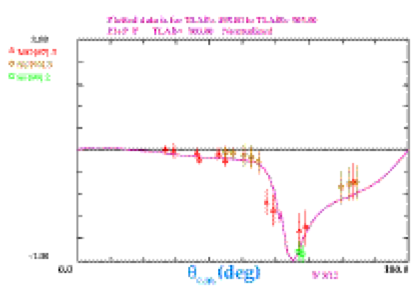
dσ/dΩ

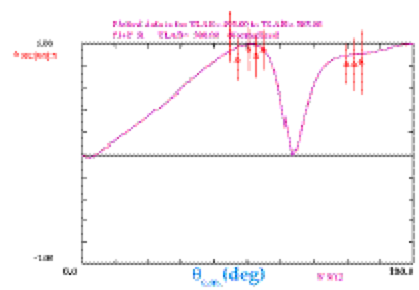
mb/sr



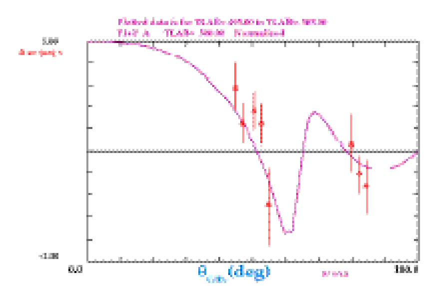
$\pi^+ p$ scattering at T_{Lab} = 500 MeV



P



R



A

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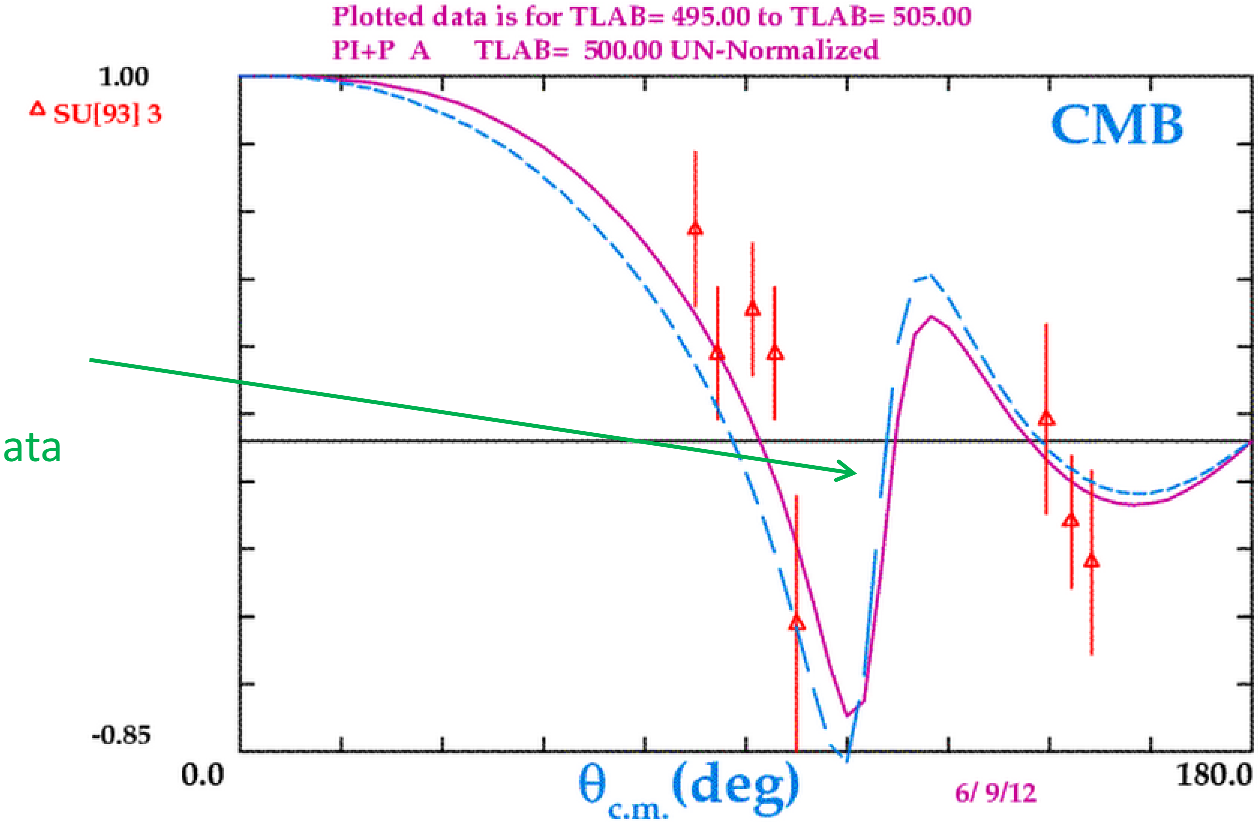
Different fits agree even where rapid variation is unconstrained by data



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πN photo-production data:

Barker, Donnachie, and Storrow coord. system

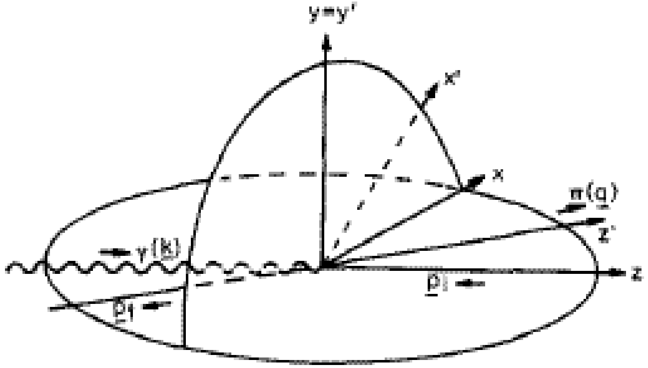

Fig. 1. Definition of axes. If $\boldsymbol{k}$ is the incoming photon momentum and $\boldsymbol{q}$ the outgoing meson momentum (both in the c.m. system) then the axes are defined by

$$z = \boldsymbol{k}/|\boldsymbol{k}|, \qquad y = \boldsymbol{k} \times \boldsymbol{q}/|\boldsymbol{k} \times \boldsymbol{q}|, \qquad x = y \times z,$$
$$z' = \boldsymbol{q}/|\boldsymbol{q}|, \qquad y' = y, \qquad x' = y \times z'.$$

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Un-polarized, single-, and double-polarization measurements

Not universally adopted

Usual symbol	Helicity representation	Transversity representation	Experiment required [a]	Type
$d\sigma/dt$	$\lvert N\rvert^2+\lvert S_1\rvert^2+\lvert S_2\rvert^2+\lvert D\rvert^2$	$\lvert b_1\rvert^2+\lvert b_2\rvert^2+\lvert b_3\rvert^2+\lvert b_4\rvert^2$	$\{-;-;-\}$	S
$\Sigma\, d\sigma/dt$	$2\mathrm{Re}(S_1^* S_2 - ND^*)$	$\lvert b_1\rvert^2+\lvert b_2\rvert^2-\lvert b_3\rvert^2-\lvert b_4\rvert^2$	$\{L(\frac{1}{2}\pi,0);-;-\}$ $\{-;y;y\}$	
$T\, d\sigma/dt$	$2\mathrm{Im}(S_1N^* - S_2D^*)$	$\lvert b_1\rvert^2-\lvert b_2\rvert^2-\lvert b_3\rvert^2+\lvert b_4\rvert^2$	$\{-;y;-\}$ $\{L(\frac{1}{2}\pi,0);0;y\}$	
$P\, d\sigma/dt$	$2\mathrm{Im}(S_2N^* - S_1D^*)$	$\lvert b_1\rvert^2-\lvert b_2\rvert^2+\lvert b_3\rvert^2-\lvert b_4\rvert^2$	$\{-;-;y\}$ $\{L(\frac{1}{2}\pi,0);y;-\}$	
$G\, d\sigma/dt$	$-2\mathrm{Im}(S_1S_2^* + ND^*)$	$2\mathrm{Im}(b_1b_3^* + b_2b_4^*)$	$\{L(\pm\frac{1}{4}\pi);z;-\}$	BT
$H\, d\sigma/dt$	$-2\mathrm{Im}(S_1D^* + S_2N^*)$	$-2\mathrm{Re}(b_1b_3^* - b_2b_4^*)$	$\{L(\pm\frac{1}{4}\pi);x;-\}$	
$E\, d\sigma/dt$	$\lvert S_2\rvert^2-\lvert S_1\rvert^2-\lvert D\rvert^2+\lvert N\rvert^2$	$-2\mathrm{Re}(b_1b_3^* + b_2b_4^*)$	$\{c;z;-\}$	
$F\, d\sigma/dt$	$2\mathrm{Re}(S_2D^* + S_1N^*)$	$2\mathrm{Im}(b_1b_3^* - b_2b_4^*)$	$\{c;x;-\}$	
$O_x\, d\sigma/dt$	$-2\mathrm{Im}(S_2D^* + S_1N^*)$	$-2\mathrm{Re}(b_1b_4^* - b_2b_3^*)$	$\{L(\pm\frac{1}{4}\pi);-;x'\}$	BR
$O_z\, d\sigma/dt$	$-2\mathrm{Im}(S_2S_1^* + ND^*)$	$-2\mathrm{Im}(b_1b_4^* + b_2b_3^*)$	$\{L(\pm\frac{1}{4}\pi);-;z'\}$	
$C_x\, d\sigma/dt$	$-2\mathrm{Re}(S_2N^* + S_1D^*)$	$2\mathrm{Im}(b_1b_4^* - b_2b_3^*)$	$\{c;-;x'\}$	
$C_z\, d\sigma/dt$	$\lvert S_2\rvert^2-\lvert S_1\rvert^2-\lvert N\rvert^2+\lvert D\rvert^2$	$-2\mathrm{Re}(b_1b_4^* + b_2b_3^*)$	$\{c;-;z'\}$	
$T_x\, d\sigma/dt$	$2\mathrm{Re}(S_1S_2^* + ND^*)$	$2\mathrm{Re}(b_1b_2^* - b_3b_4^*)$	$\{-;x;x'\}$	TR
$T_z\, d\sigma/dt$	$2\mathrm{Re}(S_1N^* - S_2D^*)$	$2\mathrm{Im}(b_1b_2^* - b_3b_4^*)$	$\{-;x;z'\}$	
$L_x\, d\sigma/dt$	$2\mathrm{Re}(S_2N^* - S_1D^*)$	$2\mathrm{Im}(b_1b_2^* + b_3b_4^*)$	$\{-;z;x'\}$	
$L_z\, d\sigma/dt$	$\lvert S_1\rvert^2+\lvert S_2\rvert^2-\lvert N\rvert^2-\lvert D\rvert^2$	$2\mathrm{Re}(b_1b_2^* + b_3b_4^*)$	$\{-;z;z'\}$	

no (-) sign

[a] Notation is $\{P_\gamma; P_T; P_R\}$ where:

P_γ = polarisation of beam, $L(\theta)$ = beam linearly polarised at angle θ to scattering plane,
C = circularly polarised beam;
P_T = direction of target polarisation;
P_R = component of recoil polarisation measured.

In the case of the single polarisation measurements we also give the equivalent double polarisation measurement.

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TABLE 1. Names for the coordinate-independent polarization ratios as used in the partial wave analyses of the groups from MAID [2], SAID [3], Bonn-Gatchina (BoGa) [4], Carnegie Mellon (CMU) [5] and JLab-EBAC [1].

	MAID, SAID	BoGa, CMU	SHKL-EBAC
R_S	Σ	Σ	Σ
R_T	T	T	T
R_P	P	P	P
R_E	E	–E	E
R_F	F	F	F
R_G	G	G	G
R_H	–H	H	H
$R_{Cx'}$	$-C_{x'}$	$C_{x'}$	$C_{x'}$
$R_{Cz'}$	$-C_{z'}$	$C_{z'}$	$C_{z'}$
$R_{Ox'}$	$-O_{x'}$	$O_{x'}$	$O_{x'}$
$R_{Oz'}$	$-O_{z'}$	$O_{z'}$	$O_{z'}$
$R_{Lx'}$	$-L_{x'}$	$L_{x'}$	$L_{x'}$
$R_{Lz'}$	$L_{z'}$	$L_{z'}$	$L_{z'}$
$R_{Tx'}$	$T_{x'}$	$T_{x'}$	$T_{x'}$
$R_{Tz'}$	$T_{z'}$	$T_{z'}$	$T_{z'}$

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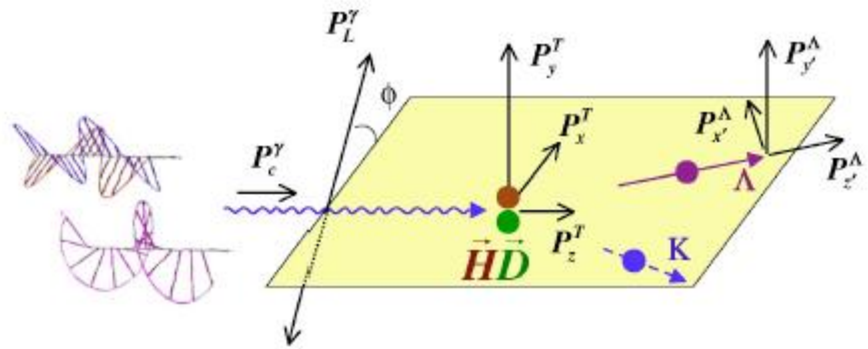

- Ratios defined with $d\sigma^{\mathrm{B,T,R}}(\vec{P}^\gamma, \vec{P}^T, \vec{P}^R)$ specified by $\vec{p}_\gamma$ (photon) & $\vec{p}_m$ (meson)
- construct $\hat{p}_1 = \dfrac{(\vec{p}_\gamma \times \vec{p}_m)\times \vec{p}_\gamma}{\left|(\vec{p}_\gamma \times \vec{p}_m)\times \vec{p}_\gamma\right|}$, $\vec{p}_2 = \dfrac{(\vec{p}_\gamma \times \vec{p}_m)}{\left|\vec{p}_\gamma \times \vec{p}_m\right|}$ and $\vec{p}_3 = \dfrac{(\vec{p}_\gamma \times \vec{p}_m)\times \vec{p}_m}{\left|(\vec{p}_\gamma \times \vec{p}_m)\times \vec{p}_m\right|}$

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(full list)

single-pol ratios:

$$R_S = \frac{\left[d\sigma_1^{B,T,R}\left(\phi_\gamma^L = +\pi/2,\ ave\ init,\ sum\ final\right) - d\sigma_2^{B,T,R}\left(\phi_\gamma^L = 0,\ ave\ init,\ sum\ final\right)\right]}{\left[d\sigma_1^{B,T,R} + d\sigma_2^{B,T,R}\right]}$$

$$R_T = \frac{\left[d\sigma_1^{B,T,R}\left(ave\ init,\ \vec{P}^T = +\hat{p}_2,\ sum\ final\right) - d\sigma_2^{B,T,R}\left(ave\ init,\ \vec{P}^T = -\hat{p}_2,\ sum\ final\right)\right]}{\left[d\sigma_1^{B,T,R} + d\sigma_2^{B,T,R}\right]}$$

$$R_P = \frac{\left[d\sigma_1^{B,T,R}\left(ave\ init,\ ave\ init,\ \vec{P}^R = +\hat{p}_2\right) - d\sigma_2^{B,T,R}\left(ave\ init,\ ave\ init,\ \vec{P}^R = -\hat{p}_2\right)\right]}{\left[d\sigma_1^{B,T,R} + d\sigma_2^{B,T,R}\right]}$$

B-T ratios:

$$R_E = \frac{\left[d\sigma_1^{B,T,R}\left(P_h^\gamma = +1,\ \vec{P}^T = -\hat{p}_\gamma,\ sum\ final\right) - d\sigma_2^{B,T,R}\left(P_h^\gamma = +1,\ \vec{P}^T = +\hat{p}_\gamma,\ sum\ final\right)\right]}{\left[d\sigma_1^{B,T,R} + d\sigma_2^{B,T,R}\right]}$$

$$R_F = \frac{\left[d\sigma_1^{B,T,R}\left(P_h^\gamma = +1,\ \vec{P}^T = +\hat{p}_1,\ sum\ final\right) - d\sigma_2^{B,T,R}\left(P_h^\gamma = -1,\ \vec{P}^T = +\hat{p}_1,\ sum\ final\right)\right]}{\left[d\sigma_1^{B,T,R} + d\sigma_2^{B,T,R}\right]}$$

Evolution of πN photoproduction observables:

- Low-energy region
  (low partial waves dominate)
- Δ(1232) resonance region
  (a single partial wave dominates)
- Upper resonance region
  (many partial waves interfere)

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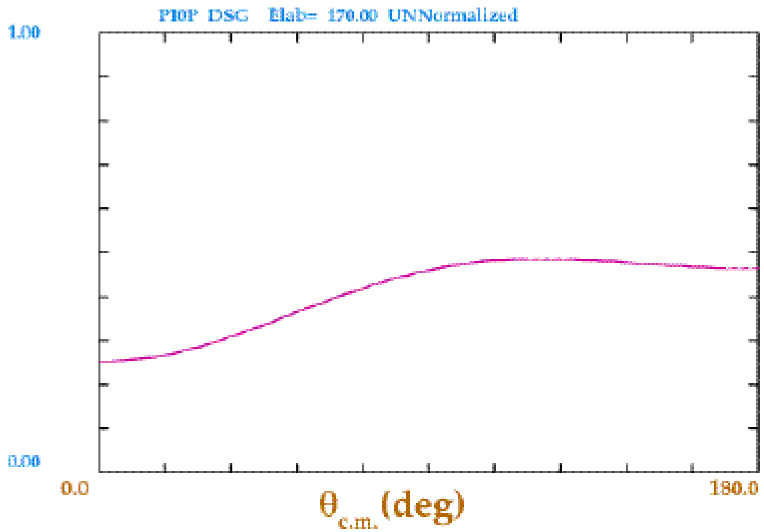
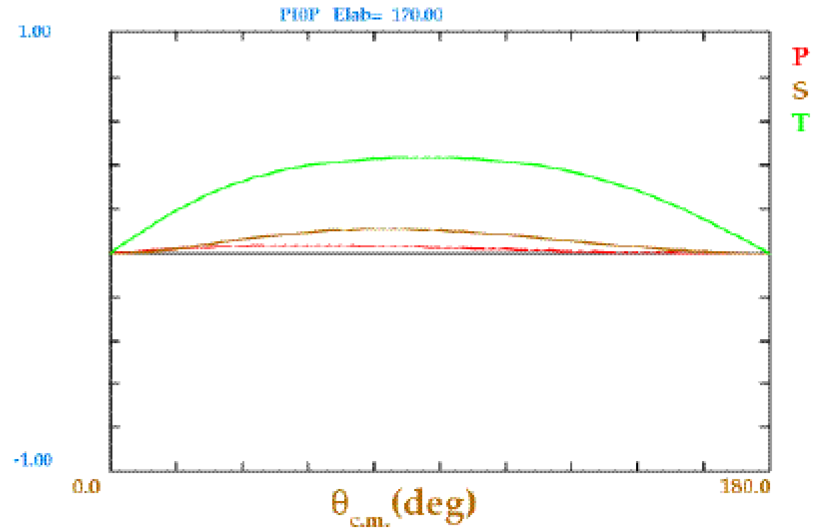
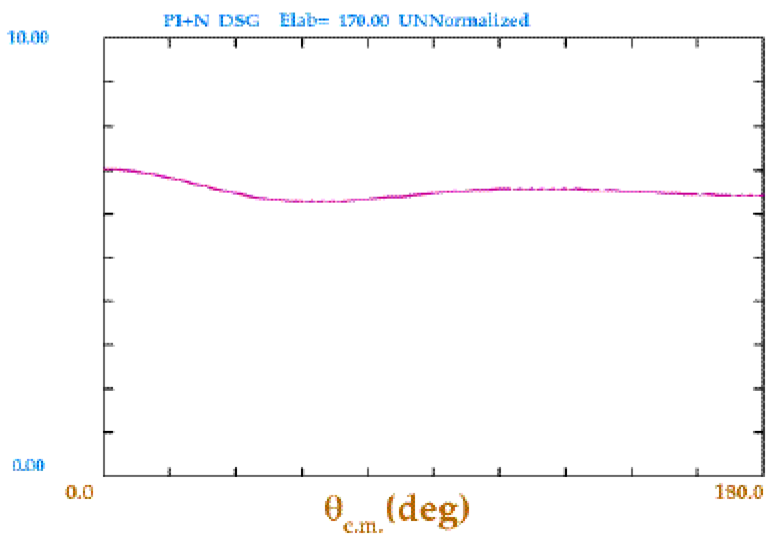
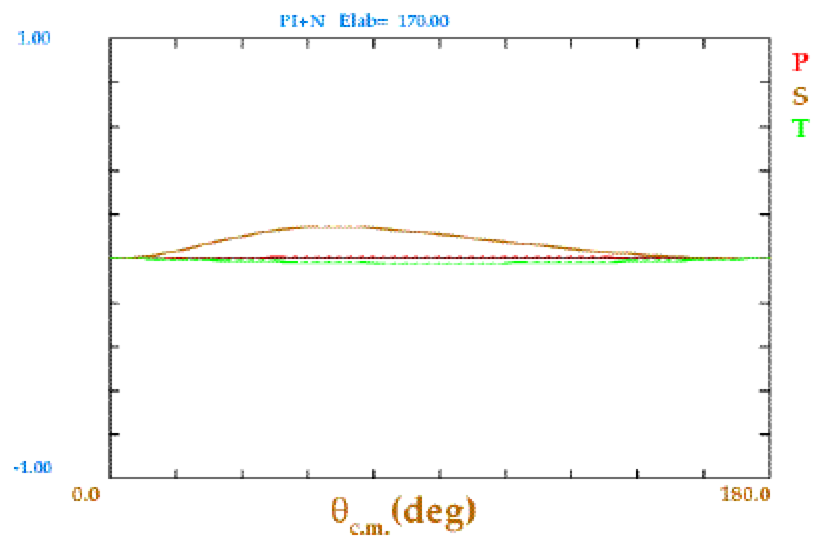
μb/sr





μb/sr





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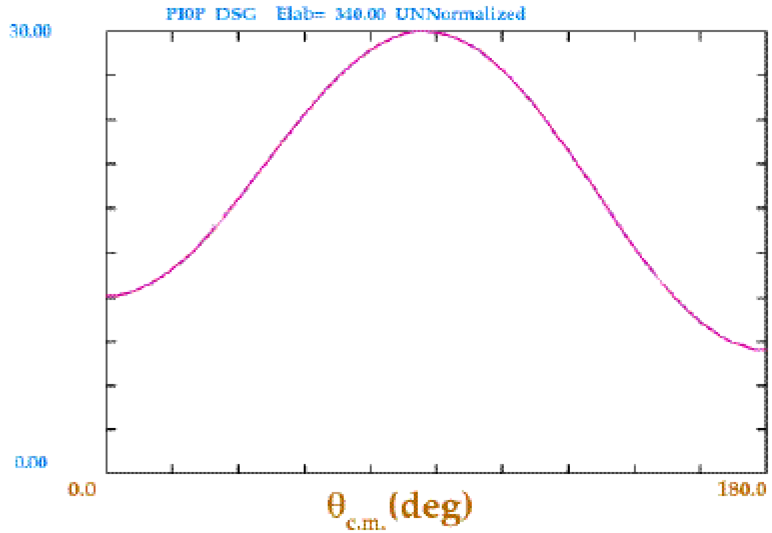
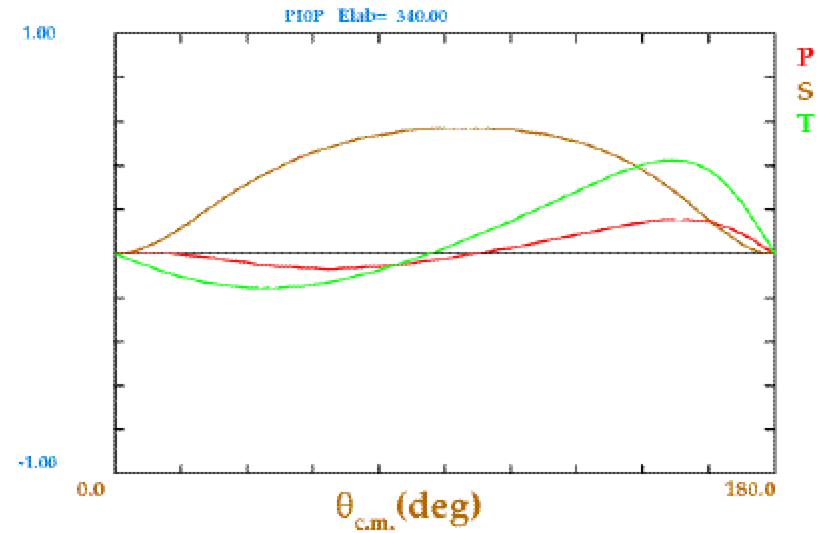
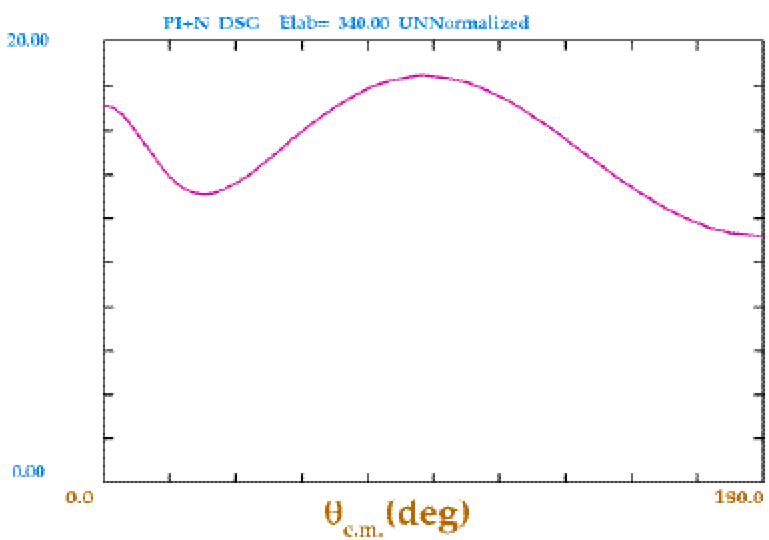
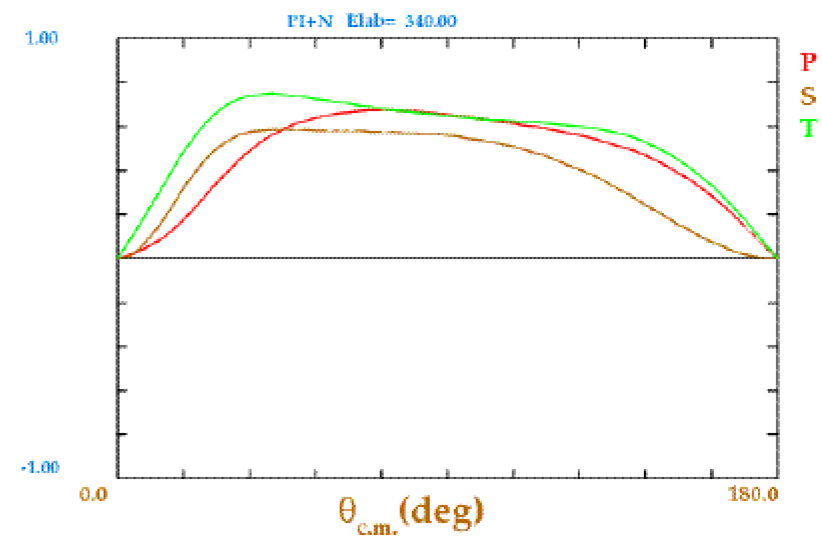
μb/sr





μb/sr





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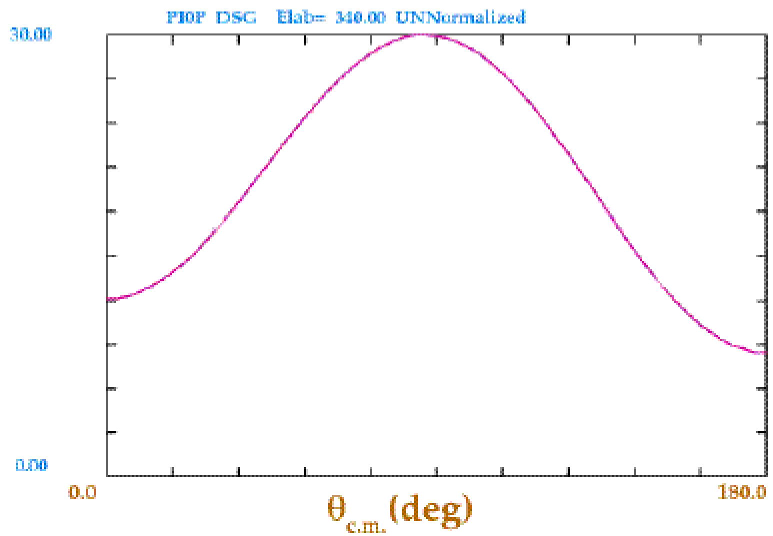
μb/sr



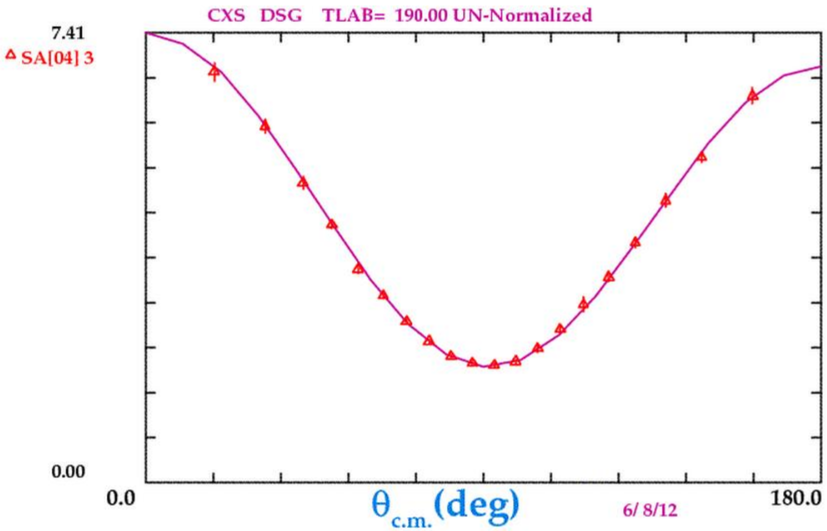
Compare

$\gamma p \rightarrow p\pi^0$

to

$\pi^- p \rightarrow n\pi^0$

mb/sr



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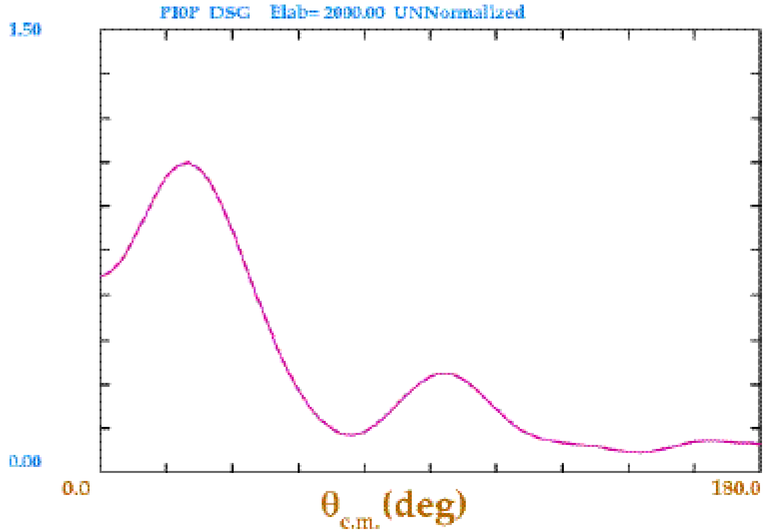
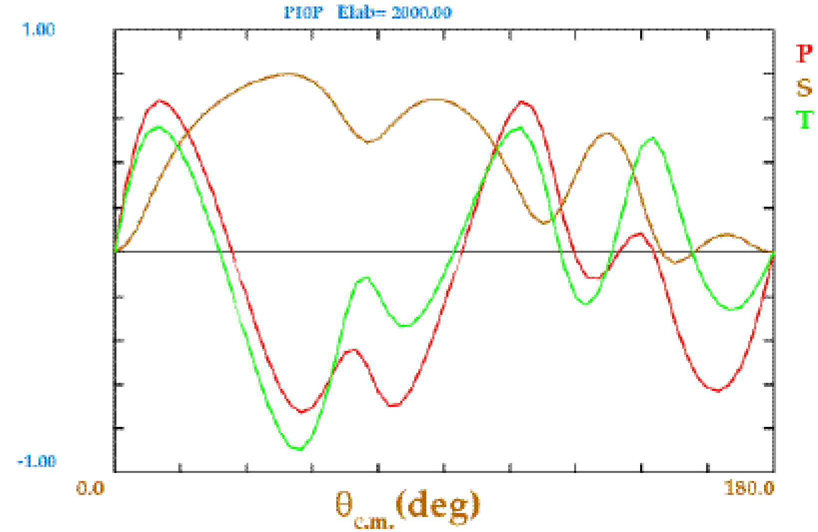
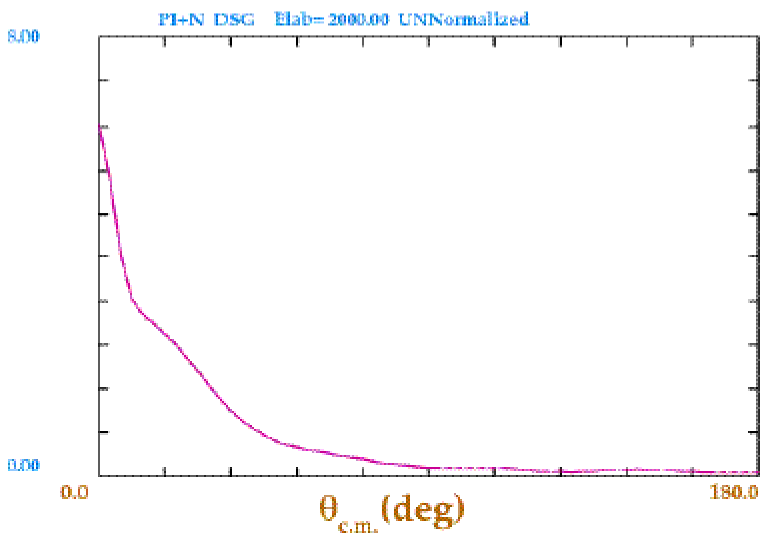
CNS Center for Nuclear Studies Data Analysis Center

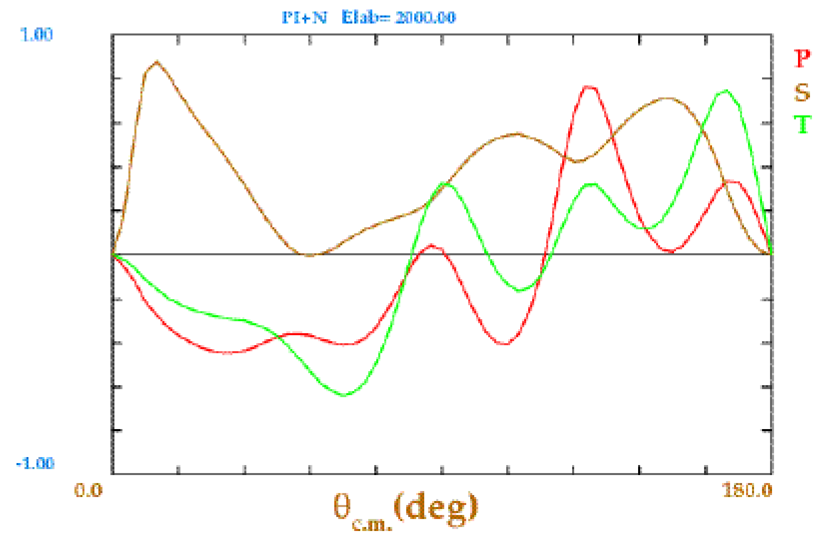
µb/sr





µb/sr





πN elastic scattering amplitudes

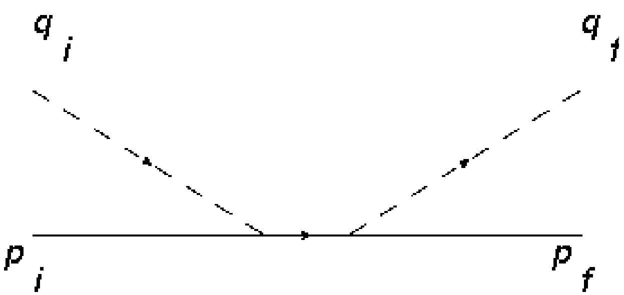
Some references*:*

B.H. Bransden and R.G. Moorhouse,
The Pion-Nucleon System

T. Ericson and W. Weise,
Pions and Nuclei

G. Höhler*,*
Pion Nucleon Scattering
Landolt-Börnstein Vol. I/9b2

CNS Center for Nuclear Studies Data Analysis Center



Pick 3 independent 4-vectors and combine with Dirac matrices

$$1\ ,\ \gamma_\mu\ ,\ \gamma_5\ ,\ \gamma_5\gamma_\mu,\quad \sigma_{\mu\nu}$$

then reduce to the simplest form

$$\overline{U}[\ A\ +\ BQ_\mu\gamma^\mu\ ]U$$

with $Q = (q_i + q_f)/2$. In terms of Pauli spinors, this can be written as

$$\chi^\dagger[\ f_1 + f_2\vec{\sigma}\cdot\vec{q_i}\ \vec{\sigma}\cdot\vec{q_f}\ ]\chi$$

Notice that $\vec{q_i}\times\vec{q_f}$ is normal to the scattering plane to re-write this as

$$\chi^\dagger[\ g + i\ h\ \vec{\sigma}\cdot\hat{n}\ ]\chi$$

Pick $\hat{n}$ along $\hat{y}$,

$$(g\ +\ i\ h\ \vec{\sigma}\cdot\hat{n})\,\chi \Rightarrow \begin{pmatrix} g & h \\ -h & g \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = g\begin{pmatrix} 0 \\ 1 \end{pmatrix} + h\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Hence, h is a 'spin flip' amplitude.

In terms of these amplitudes, the cross section is:

$$d\sigma/d\Omega = |g|^2 + |h|^2$$
$$P\, d\sigma/d\Omega = -2\, \mathrm{Im}\, g^* h$$

In terms of transversity amplitudes

$$F^+ = g + i\,h \;,\; F^- = g - i\,h$$

there is a more compact relation:

$$|F^+|^2 = d\sigma/d\Omega\,(1 + P) \quad , \quad |F^-|^2 = d\sigma/d\Omega\,(1 - P)$$

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(One way) to get the partial-wave decomposition:
Since the pion is spin 0 and the nucleon spin 1/2,
$J = \ell \pm 1/2$ (notation is $\ell_\pm$ for amplitudes).
Write down projection operators for $\ell_\pm$ and replace

$$\sum_\ell (2\ell+1) f_\ell P_\ell(\cos\theta)$$

with

$$\sum_\ell (2\ell+1)[f_{\ell+}P_+ + f_{\ell-}P_{\ell-}]P_\ell(\cos\theta)$$

The projection operators will generate P'_ℓ terms. Compare with

$$g + i\, h\, \vec{\sigma}\cdot\hat{n}$$

to find:

$$g \;=\; \sum_\ell [(\ell+1)\; f_{\ell+} + \ell f_{\ell-}]P_\ell(\cos\theta)$$

and

$$h \;=\; \sum_\ell [f_{\ell+} - f_{\ell-}]P'_\ell(\cos\theta)\;\sin\theta$$

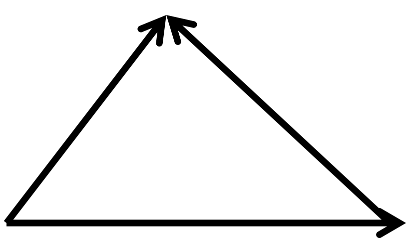
See, for example,
Levi Setti and Lasinski,
Strongly Interacting Particles

Helicity formalism:
Ch.5 of
Martin/Spearman

Since one goal of this analysis is the extraction of N and Δ resonances - we really want isospin amplitudes.

Must first account for electromagnetic corrections which add Coulomb scattering and Coulomb-nuclear interference terms (also mass-splitting).

$$f_{\ell\pm} = f_{\ell\pm}^{3/2} \quad (\pi^+ p \to \pi^+ p)$$



Isospin triangle

$$f_{\ell\pm} = \frac{1}{3}\left(f_{\ell\pm}^{3/2} + 2 f_{\ell\pm}^{1/2}\right) \quad (\pi^- p \to \pi^- p)$$

$$f_{\ell\pm} = \frac{\sqrt{2}}{3}\left(f_{\ell\pm}^{3/2} - f_{\ell\pm}^{1/2}\right) \quad (\pi^- p \to \pi^0 n)$$

πN photoproduction amplitudes

Some references:

CGLN, Phys Rev 106, 1345 (1957).

F.A. Berends, A. Donnachie, and D.L. Weaver, Nucl Phys B4, 1 (1967).

R.L. Walker, Phys Rev 182, 1729 (1969).

B.H. Bransden and R.G. Moorhouse, *The Pion-Nucleon System*

CGLN choose the 3 independent 4-vectors,

k, q, and $(p_1 + p_2)/2$. $P = (p_1 + p_2)/2$

With a spin 1 photon replacing the spin 0 pion,

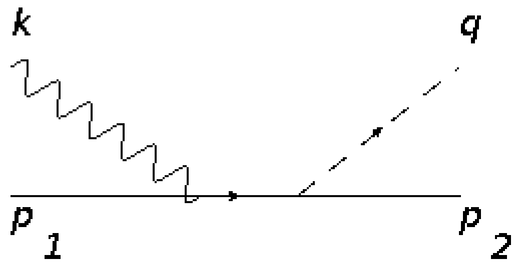
we have to construct invariants using both γ_μ and ϵ_μ.

The result must be linear in ϵ_μ,

contain a γ_5 factor (for the single pion),

and go to zero with $\epsilon_\mu \to k_\mu$.

There are four independent terms:



$$\gamma_5 \epsilon \cdot \gamma k \cdot \gamma$$

$$\gamma_5 \left(P \cdot \epsilon \gamma \cdot k - \epsilon \cdot \gamma P \cdot k \right)$$

$$\gamma_5 \left(q \cdot \epsilon \gamma \cdot k - \epsilon \cdot \gamma q \cdot k \right)$$

$$\gamma_5 \left(P \cdot \epsilon q \cdot k - q \cdot \epsilon P \cdot k \right)$$

CGLN choose a linear combination of these.

As with the πN amplitudes, there is a useful conversion from matrix elements involving Dirac to Pauli states, yielding the CGLN $\mathscr{F}_1$, $\mathscr{F}_2$, $\mathscr{F}_3$, $\mathscr{F}_4$

Expansion in terms of partial-wave amplitudes is given in CGLN and a conversion to helicity amplitudes H_1 , H_2 , H_3 , H_4 is given by Berends, Donnachie and Weaver.

As in the πN case, the transversity amplitudes simplify expressions for some of the observables.

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Norm issues:
BDS,
NPB79,431(1974)

Helicity Amplitudes (Walker)

$H_1 = A_{1/2\,,\,3/2}$ $H_2 = A_{1/2\,,\,1/2}$ $H_3 = A_{-1/2\,,\,3/2}$ $H_4 = A_{-1/2\,,\,1/2}$
$H_1 = S_1$, $H_2 = N$, $H_3 = D$, $H_4 = S_2$ (BDS notation)

$$H_1 = \frac{1}{\sqrt{2}} \cos\frac{\theta}{2} \sin\theta \sum_{\ell=1}^{\infty} \left[E_{\ell+} - M_{\ell+} - E_{(\ell+1)-} - M_{(\ell+1)-}\right] \left(P_\ell'' - P_{\ell+1}''\right),$$

$$H_2 = \frac{1}{\sqrt{2}} \cos\frac{\theta}{2} \sum_{\ell=0}^{\infty} \left[(\ell+2)E_{\ell+} + \ell M_{\ell+} + \ell E_{(\ell+1)-} - (\ell+2)M_{(\ell+1)-}\right] \left(P_\ell' - P_{\ell+1}'\right)$$

$$H_3 = \frac{1}{\sqrt{2}} \sin\frac{\theta}{2} \sin\theta \sum_{\ell=1}^{\infty} \left[(E_{\ell+} - M_{\ell+} + E_{(\ell+1)-} + M_{(\ell+1)-}\right] \left(P_\ell'' + P_{\ell+1}''\right),$$

$$H_4 = \frac{1}{\sqrt{2}} \sin\frac{\theta}{2} \sum_{\ell=0}^{\infty} \left[(\ell+2)E_{\ell+} + \ell M_{\ell+} - \ell E_{(\ell+1)-} + (\ell+2)M_{(\ell+1)-}\right] \left(P_\ell' + P_{\ell+1}'\right)$$

Transversity amplitudes

$$b_1 = \frac{1}{2}\left[(H_1 + H_4) + i\ (H_2 - H_3)\right],$$
$$b_2 = \frac{1}{2}\left[(H_1 + H_4) - i\ (H_2 - H_3)\right],$$
$$b_3 = \frac{1}{2}\left[(H_1 - H_4) - i\ (H_2 + H_3)\right],$$
$$b_4 = \frac{1}{2}\left[(H_1 - H_4) + i\ (H_2 + H_3)\right],$$

$$\frac{d\sigma}{dt} = |b_1|^2 + |b_2|^2 + |b_3|^2 + |b_4|^2,$$
$$P\frac{d\sigma}{dt} = |b_1|^2 - |b_2|^2 + |b_3|^2 - |b_4|^2,$$
$$\Sigma\frac{d\sigma}{dt} = |b_1|^2 + |b_2|^2 - |b_3|^2 - |b_4|^2,$$
$$T\frac{d\sigma}{dt} = |b_1|^2 - |b_2|^2 - |b_3|^2 + |b_4|^2.$$

For πN scattering, moduli of transversity amplitudes from: dσ/dΩ , P

For πN photoproduction moduli of transversity amplitudes from: dσ/dΩ , P , Σ , T

See Bransden & Moorhouse Ch.2

Isospin Decomposition of Multipoles and Amplitudes

4 different sets can be selected:

$$\left(A_p^{1/2}, A_n^{1/2}, A^{3/2}\right), \left(A^{1/2}, A^0, A^{3/2}\right), \left(A^0, A^+, A^-\right), \left(A_{\pi^+n}, A_{\pi^-p}, A_{\pi^0p}, A_{\pi^0n}\right)$$

relations among the different sets:

$$A_{\pi^+n} = \sqrt{2}\left(A_p^{1/2} - \tfrac{1}{3}A^{3/2}\right) = \sqrt{2}\left(A^0 + \tfrac{1}{3}A^{1/2} - \tfrac{1}{3}A^{3/2}\right)$$

$$A_{\pi^-p} = \sqrt{2}\left(A_n^{1/2} + \tfrac{1}{3}A^{3/2}\right) = \sqrt{2}\left(A^0 - \tfrac{1}{3}A^{1/2} + \tfrac{1}{3}A^{3/2}\right)$$

$$A_{\pi^0p} = A_p^{1/2} + \tfrac{2}{3}A^{3/2} = A^0 + \tfrac{1}{3}A^{1/2} + \tfrac{2}{3}A^{3/2}$$

$$A_{\pi^0n} = -A_n^{1/2} + \tfrac{2}{3}A^{3/2} = -A^0 + \tfrac{1}{3}A^{1/2} + \tfrac{2}{3}A^{3/2}$$

Notation and Conventions

Photon State	J	Parity	Final πN state
E1	$\frac{1}{2}$	$-$	$s_{1/2}$
	$\frac{3}{2}$	$-$	$d_{3/2}$
M1	$\frac{1}{2}$	$+$	$p_{1/2}$
	$\frac{3}{2}$	$+$	$p_{3/2}$
E2	$\frac{3}{2}$	$+$	$p_{3/2}$
	$\frac{5}{2}$	$+$	$f_{5/2}$
M2	$\frac{3}{2}$	$-$	$d_{3/2}$
	$\frac{5}{2}$	$-$	$d_{5/2}$

Multipole notation: ${}_{\mathrm{p,n}}(\mathrm{E,M})^{\mathrm{I}}_{\ell\pm}$ or $L_{2I,2J}(\mathrm{p,n})(\mathrm{E,M})$

e.g. ${}_{\mathrm{p}}\mathrm{M}^{1/2}_{2-}$ versus $D_{13}\mathrm{pM}$

Intermediate particle notation:

Either based on πN notation or $(N,\Delta)(mass)J^P$

e.g. $\mathrm{P}_{33}(1232) \Rightarrow \Delta(1232)3/2^+$

PDG notation 2010

$\Delta(1232)\ P_{33}$

$I(J^P) = \frac{3}{2}(\frac{3}{2}^+)$ Status: ∗∗∗∗

Most of the results published before 1975 were last included in our 1982 edition, Physics Letters **111B** 1 (1982). Some further obsolete results published before 1984 were last included in our 2006 edition, Journal of Physics, G **33** 1 (2006).

PDG notation 2012

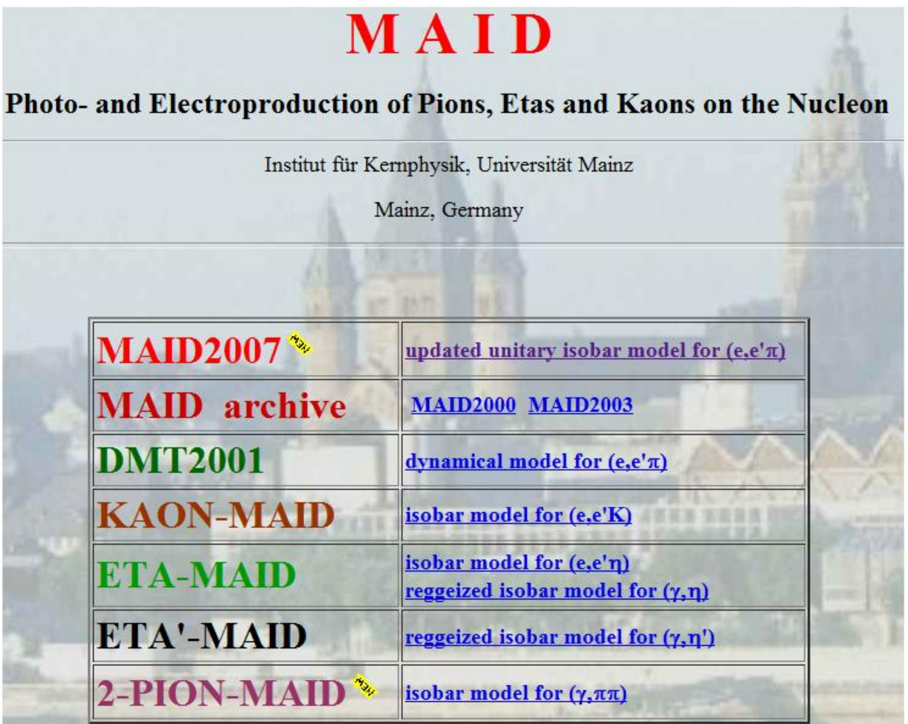
$\Delta(1232)\ 3/2^+$

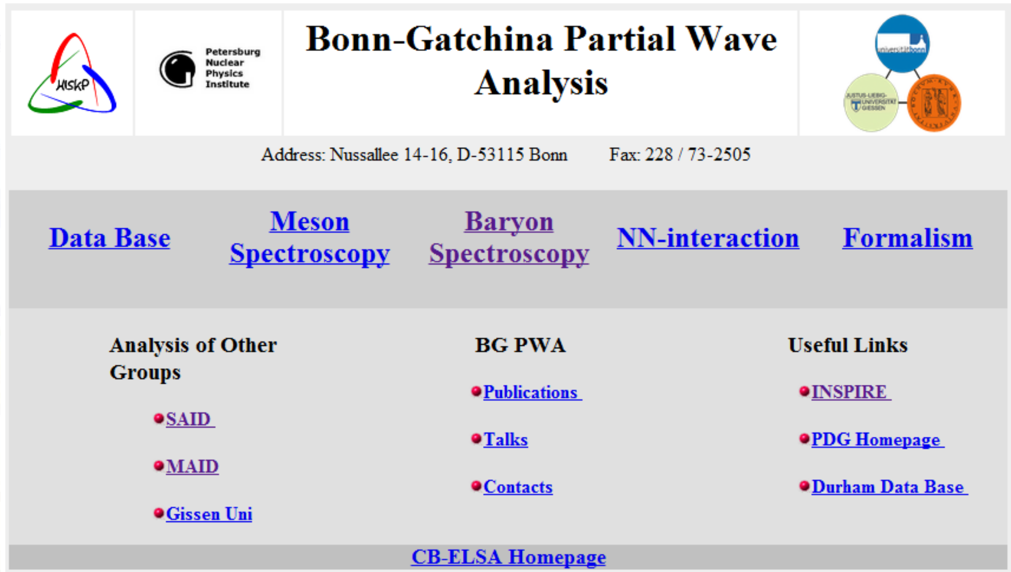
$I(J^P) = \frac{3}{2}(\frac{3}{2}^+)$ Status: ∗∗∗∗

Most of the results published before 1975 were last included in our 1982 edition, Physics Letters **111B** 1 (1982). Some further obsolete results published before 1984 were last included in our 2006 edition, Journal of Physics, G **33** 1 (2006).

Data
Amplitudes
Ambiguities
Simple Methods

Data and amplitudes are available on a number of sites:
http://gwdac.phys.gwu.edu
http://wwwkph.kph.uni-mainz.de/MAID//
http://pwa.hiskp.uni-bonn.de/





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The SAID site has the most interactive tools

CNS DAC Services [SAID Program]

- The SAID Partial-Wave Analysis Facility is based at GWU.
- New features are being added and will first appear at this site. Suggestions for improvements are always welcome.

CNS DAC Home
CNS DAC [SAID]
CNS Home

Partial-Wave Analyses at GW
[See Instructions]
Pion-Nucleon
Pi-Pi-N (under construction)
Kaon-Nucleon
Nucleon-Nucleon
Pion Photoproduction
Pion Electroproduction
Kaon Photoproduction
Eta Photoproduction
Eta-Prime Photoproduction
Pion-Deuteron (elastic)
Pion-Deuteron to Proton+Proton

Analyses From Other Sites
Mainz (MAID – Analyses)
Nijmegen (Nucleon-Nucleon OnLine)
Bonn-Gatchina (PWA)

Instructions for Using the Partial-Wave Analyses

The programs accessible with the left-hand side navigation bar allow the user to access a number of features available through the SAID program. Contact a member of our group if you are unfamiliar with the SSH version. If you enter choices which are unphysical, you may still get an answer (in accordance with the `garbage in, garbage out' rule). Please report unexpected garbage-out to the management.

Note: These programs use HTML forms to run the SAID code. If unfamiliar with the options, run the default setup first. The output is an (edited) echo of an interactive session which would have resulted had you used the SSH version. If the default example fails to clarify the specific task you have in mind, we can help (just send an e-mail message).

All programs expect energies in **MeV** units. All of the solutions and potentials have limited ranges of validity. Some are unstable beyond their upper energy limits. Extrapolated results may not make much sense.
Increments: The programs will not allow an arbitrary number of points to be generated. As a rule, stay below **100**.

ACKNOWLEDGMENTS

The **CNS Data Analysis Center** is partially funded by the U.S. Department of Energy, and the Research Enhancement Funds of The George Washington University, with strong support from the GW Northern Virginia Campus.

Not all of you know how this site works so …

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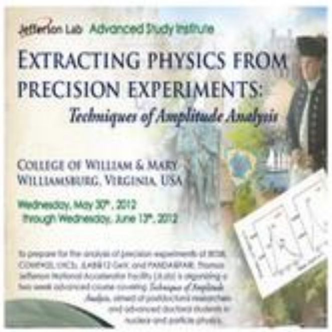

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- Webpage overview

SAID SSH Tutorials
- SSH Overview
- SSH Comparisons

Model Tutorial
- Model Fit
- Resonances

Media and Tutorials for the

Jefferson Lab Advanced Study Institute

EXTRACTING PHYSICS FROM PRECISION EXPERIMENTS:

Techniques of Amplitude Analysis

The George Washington University

(audio for these videos is a bit green and will be improved – and posted on the SAID site.)

Amplitude reconstruction

For πN scattering, dσ/dΩ and P
determine moduli of transversity amplitudes.
There is a relative and an overall angle remaining.
Measuring R or A gives sin/cos of relative angle
(leaves ambiguities – need to measure both).
The overall angle is not determined.

For πN photoproduction, there are 4 complex amplitudes. Measuring dσ/dΩ, P, Σ, T again determines moduli of transversity amplitudes.
Now have 3 relative angles.

The solution to this problem turns out to be much less obvious than was the case for πN elastic scattering.

See examples in:

Dean and Lee,
PRD 5, 2741 (1972)
(for πN elastic)

Chiang and Tabakin,
PRC 55, 2054 (1997)
(for πN photoproduction)

General approach:

Observables have the form:

$$O^i = M^i_{\alpha\beta} \; F_\alpha^* \; F_\beta$$

where the $M^i_{\alpha\beta}$ are Hermitian

Find transformations under which the O^i are invariant (or a subset are).

`Complete experiment' : determines the set of amplitudes up to an overall phase ambiguity.

Simple example:

In the BDS table of observables, change:

$$S_1 \rightarrow -S_1^*$$
$$S_2 \rightarrow -S_2^*$$
$$N \rightarrow N^*$$
$$D \rightarrow D^*$$

All type-S, half of BT, BR, TR observables are invariant.

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$8 = 4 \times 2$

$7 = 8 - 1$

all points connected by solid and/or dashed lines. By complete, we of course mean that the 7 measurements are sufficient to determine 7 independent bilinears, from which the amplitudes and phases can be extracted – up to the previously mentioned quadrant ambiguities. Thus, there will be a discrete set of solutions for the amplitudes, when the theorem is satisfied. In order to obtain a single solution, 3 more measurements (that only determine signs) are required to remove phase ambiguities.

$10 = 7 + 3$

Our results differ from those of Goldstein et al. [4] who claim that three measurements are necessary to solve the ambiguities in addition to the seven necessary to obtain the amplitudes up to the ambiguities. This is perhaps what one would naively expect as we have three twofold ambiguities. However, they do not give a proof or even an example of this, and, as we have seen, one measurement can resolve *two* twofold ambiguities.

$9 = 7 + 2$

In summary, the examination of ambiguity relations provides a simple and useful check of proposed complete sets of experiments. We have found that the rules for choosing observables are more complicated than those given in Ref. [3].

??

four transversity amplitudes without discrete ambiguities. That number of measurements is one less than previously believed. We approach this problem in two distinct ways: (1) solving for the amplitude magnitudes and phases directly, and (2) using a bilinear helicity product formulation to map an algebra of measurements over to the well-known algebra of the 4×4 gamma matrices. It is shown that the latter method leads to an alternate proof that eight carefully chosen experiments suffice for determining the transversity amplitudes

$8 = 7 + 1$

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W.-T. Chiang, F. Tabakin, PRC55, 2054 (1997).

TABLE III. Tables III–VIII enumerate all situations under which four double spin observables, along with the set $\mathcal{S}$, can completely determine the transversity amplitudes. In these tables, '**X**'s' indicate three initially selected measurements, and '**O**'s' indicate the possible choices for fourth observable that can resolve all the ambiguities.

G	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	
H	X	X	X	X	X	X	X	X																	$\mathcal{BT}$
E									X	X	X	X	X	X	X	X									
F																	X	X	X	X	X	X	X	X	
O_x	X		O		O	O	O	O	X	O	O	O	O	O	O	O	X		O		O			O	
O_z		X		O	O	O	O	O	O	X	O	O	O	O	O	O		X		O		O	O		$\mathcal{BR}$
C_x	O		X		O	O	O	O	O	O	X	O	O	O	O	O	O		X			O	O		
C_z		O		X	O	O	O	O	O	O	O	X	O	O	O	O		O		X	O			O	
T_x	O	O	O	O	X	O	O	O	O	O	O	O	X		O		O			O	X		O		
T_z	O	O	O	O	O	X	O	O	O	O	O	O		X		O		O	O			X		O	$\mathcal{TR}$
L_x	O	O	O	O	O	O	X	O	O	O	O	O	O		X			O	O		O		X		
L_z	O	O	O	O	O	O	O	X	O	O	O	O		O		X	O			O		O		X	

These relations and consistency relations between observables (like $P^2 + R^2 + A^2 = 1$ for πN) have been applied in kaon photoproduction. For example, the measured observable combinations

$$\Sigma P - C_{x'} O_{z'} + C_{z'} O_{x'} - T$$

$$O_{x'}^2 + O_{z'}^2 + C_{x'}^2 + C_{z'}^2 + \Sigma^2 - T^2 + P^2 - 1$$

should be zero – which provides a test for systematic errors.

Sandorfi et al.,
J. Phys G38, 053001 (2011)

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CNS Center for Nuclear Studies Data Analysis Center

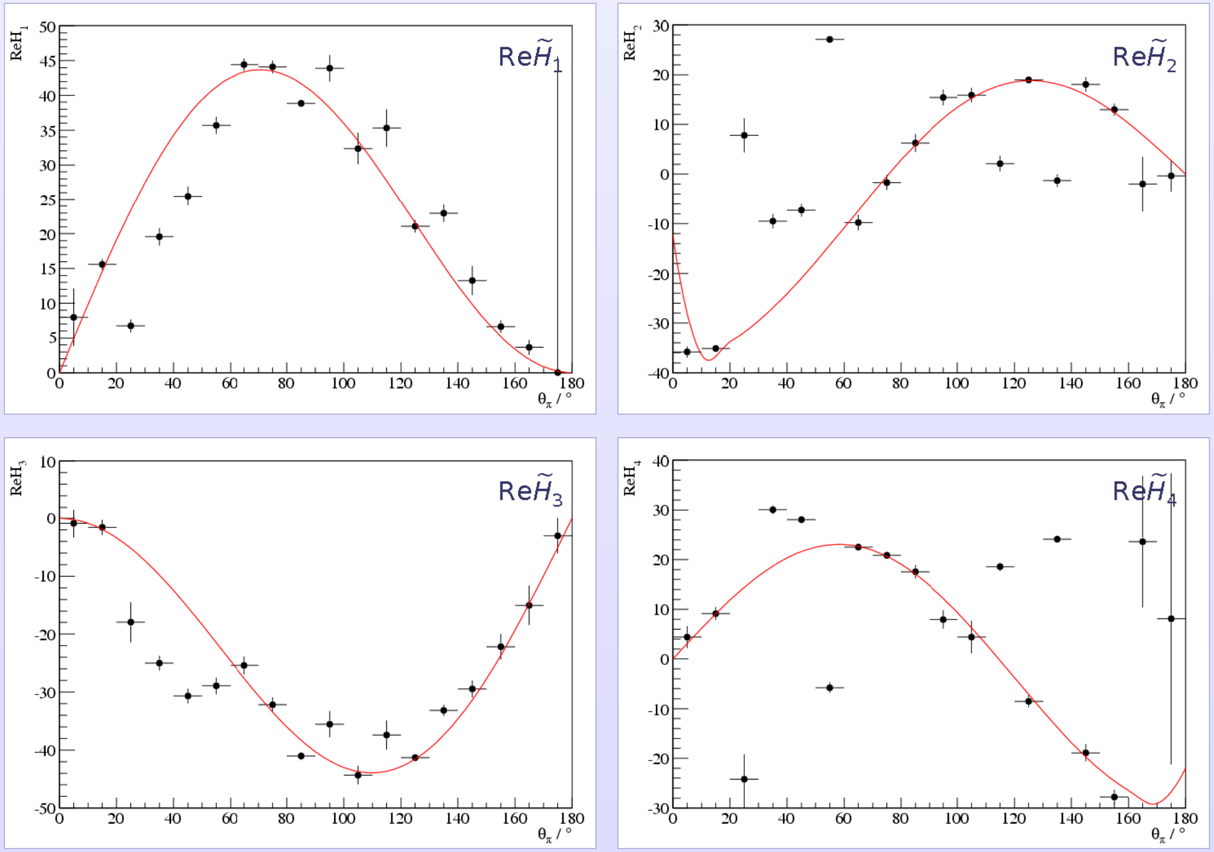
Lothar Tiator,
NSTAR 2011

Angular distribution of Re[H_1...H_4]

- Beam energy ω = 320 MeV, $\gamma p \to p \pi^0$
  W = 1217 MeV, Δ resonance region
  from observables σ_0, Σ, T, P, E, G, $C_{x'}$, $O_{x'}$

amplitude analysis with a

minimal complete set of 8 observables



Data
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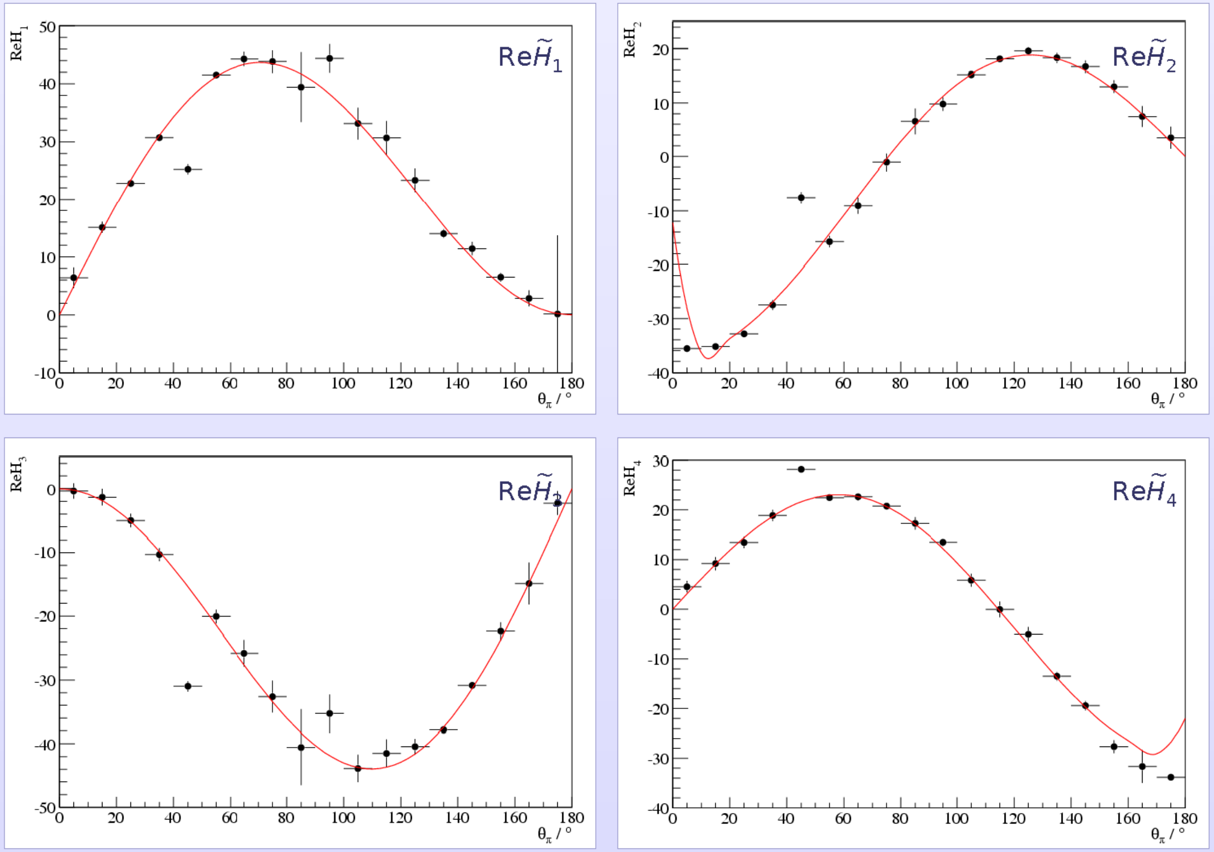
CNS Center for Nuclear Studies
Data Analysis Center

Angular distribution of Re[$H_1 \ldots H_4$]

- Beam energy ω = 320 MeV, $\gamma p \to p \pi^0$
  W = 1217 MeV, Δ resonance region
  from observables σ_0, Σ, T, P, E, F, G, H, $C_{x'}$, $O_{x'}$ ← overcomplete set



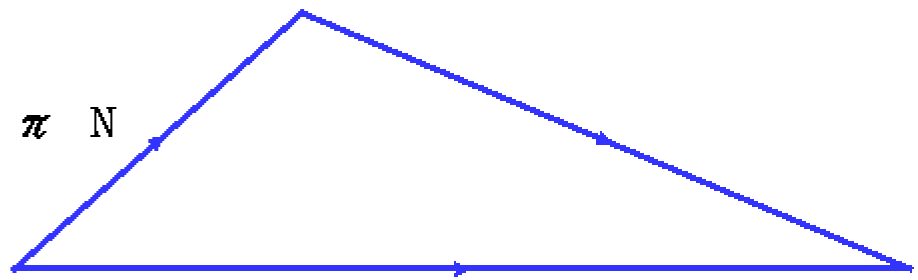
Lothar Tiator, NSTAR 2011

Isospin decomposition

$$\pi^- p + \sqrt{2}\,\pi^0 n = \pi^+ p$$

3 charge channels
2 isospin states
(triangle relation)

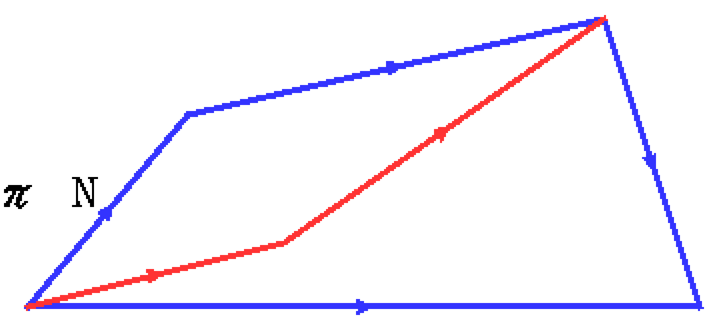
π N -> π N



$$\sqrt{2}\,\pi^0 n + \pi^+ n + \pi^- p = \sqrt{2}\,\pi^0 p$$

4 charge channels
3 isospin states
(quadrilateral relation)

γ N -> π N



Schematically, we start from amplitudes f at (E, θ) points

$$f = \sum_{\ell} (2\ell + 1) f_\ell(E) P_\ell(\cos\theta)$$

and have f only up to a phase $\phi(E, \theta)$, but want $f_\ell(E)$, which requires an integral we can't do (since ϕ is unknown).

Can try to determine the f_ℓ and sum to give f, but need to cut off (or estimate) the high-ℓ terms.

Now the ambiguities, and requirements for a solution, are different. Fit at single E and all θ.

Some references:

G. Höhler,
Pion Nucleon Scattering
Landolt-Börnstein, Vol. I/9b2

A. Gersten, NPB12, 537 (1969).

E. Barrelet, NCA8, 331 (1972).

N.W. Dean and P. Lee, PRD5, 2741 (1972).

A.S. Omelaenko, Sov. J. Nucl. Phys. 34, 406 (1981).

V.F. Grushin *et al.,* Sov. J. Nucl. Phys. 38, 881 (1983).

πN scattering example

Write the transversity amplitude as a product

$$F(w) = \frac{F(1)}{w^N} \prod_{i=1}^{2N} \frac{w - w_i}{1 - w_i}$$

where $w = e^{i\theta}$.
$|w| = 1$ corresponds to the physical region.
Since $F^+(-\theta) = F^-(\theta)$,
we have only a single function.
The change $w_i \to 1/w_i^*$
preserves $d\sigma/d\Omega$ and P.

Alternate trajectories branch at unit circle
where w_i and $1/w_i^*$ are equal.

Simple exercises:

Show that

$$F^+(-\theta) = F^-(\theta)$$

and

The change $w_i \to 1/w_i^*$
preserves $d\sigma/d\Omega$ and P

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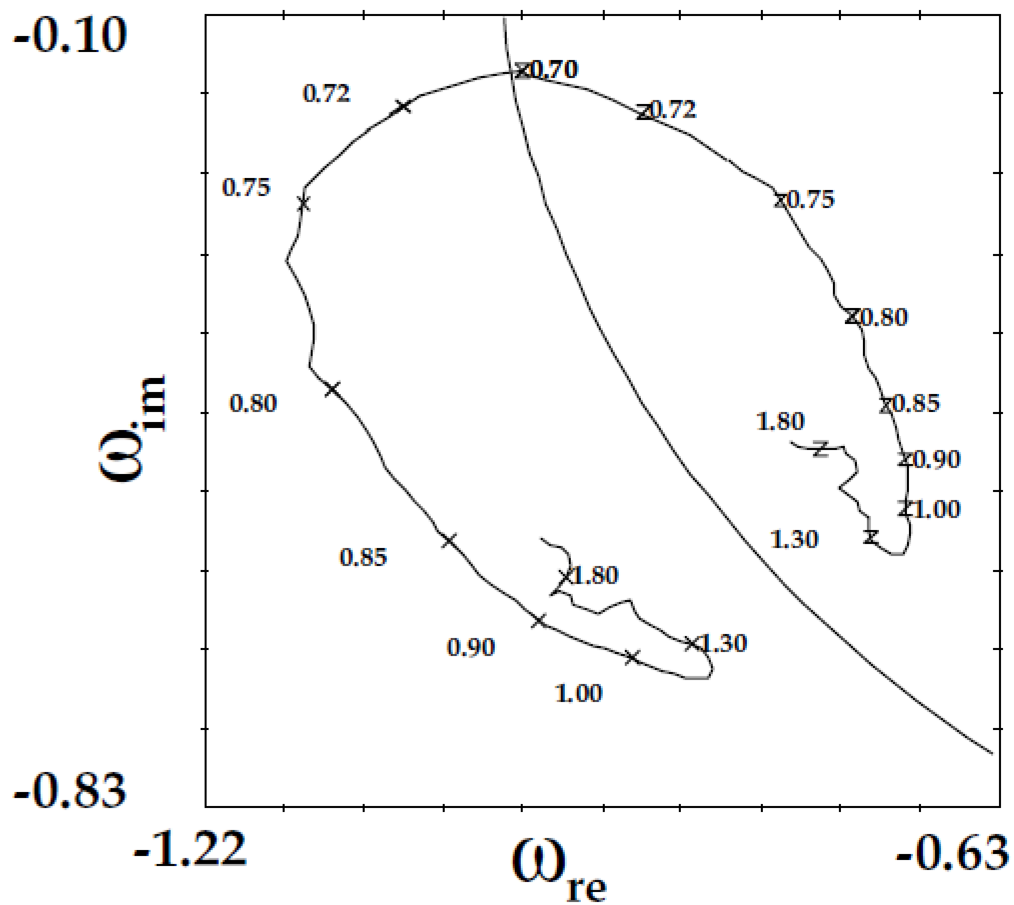
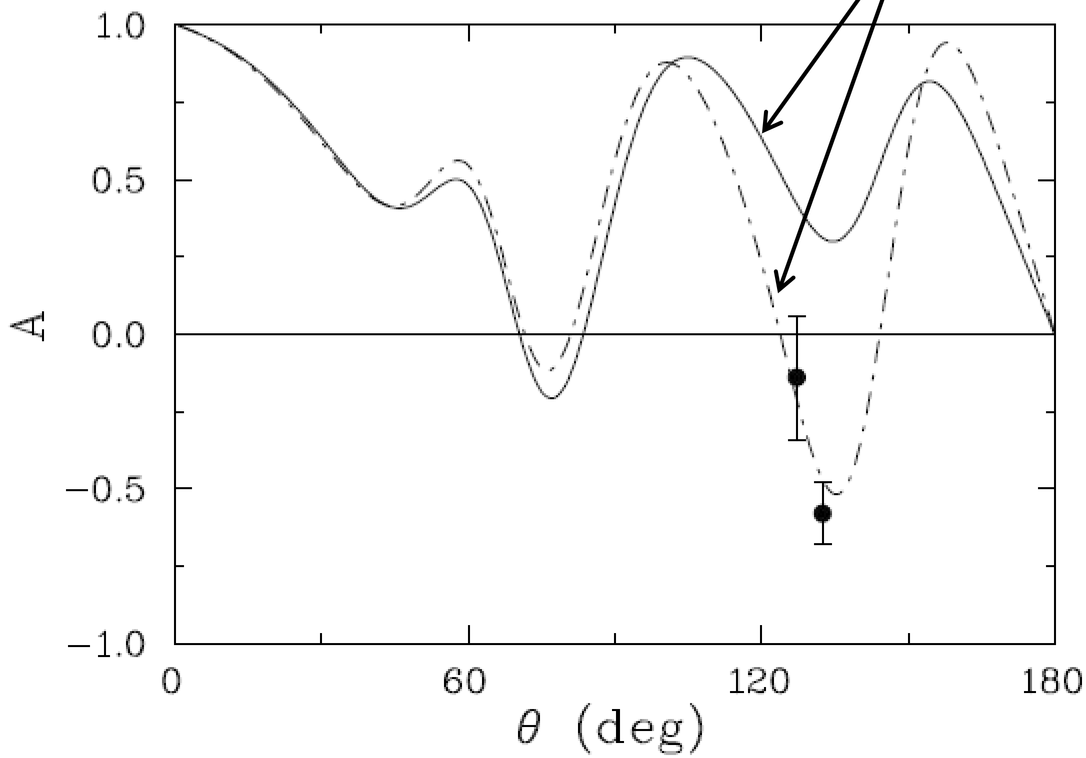
Alternate zero trajectories related by Barrelet conjugation



$\pi^+ p\ T_{lab} = 1.3$ GeV

Solutions related by Barrelet conjugation



R and A are not invariant under Barrelet conjugation

πN photoproduction (Omelaenko)

Gersten method (similar to Barrelet)
applied to transversity amplitudes:

$$b_1 = c a_{2L} \frac{e^{i\theta/2}}{(1+x^2)^L} \prod_{i=1}^{2L} (x - \alpha_i)$$

$$b_3 = -c a_{2L} \frac{e^{i\theta/2}}{(1+x^2)^L} \prod_{i=1}^{2L} (x - \beta_i)$$

$$\prod_{i=1}^{2L} \alpha_i = \prod_{i=1}^{2L} \beta_i$$

$$x = \tan \theta/2$$

$$b_1(\theta) = -b_2(-\theta) \text{ and } b_3(\theta) = -b_4(-\theta)$$

Applied to π^0 p photoproduction

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$$b_1 = c a_{2L} \frac{e^{i\theta/2}}{(1+x^2)^L} \prod_{i=1}^{2L} (x - \alpha_i) \qquad x = \tan\theta/2$$

More on the origin of this form in: S.U. Chung, PRD56, 7299 (1997)

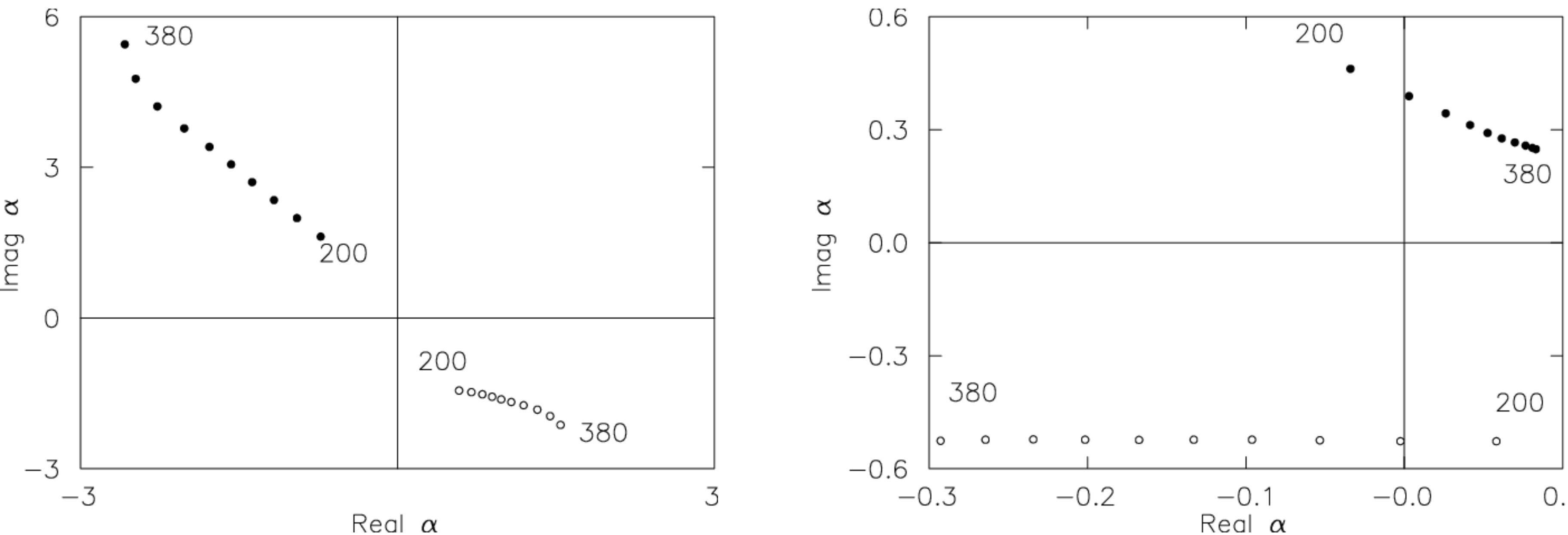
Why pick tan ϴ/2 ? (Gersten)

The b_i involve P_L and derivatives→ cos ϴ terms
with some factors of sin ϴ

Combined using:

$$\sin\theta = \frac{2\tan(\theta/2)}{1+[\tan(\theta/2)]^2} \;,\quad \cos\theta = \frac{1-[\tan(\theta/2)]^2}{1+[\tan(\theta/2)]^2}$$

Low-energy zero trajectories ($\pi^0 p$)



α : ●
β : o

(accidental symmetries give additional solutions)

$$\prod_{k=1}^{2L} \alpha_k = \prod_{k=1}^{2L} \beta_k$$

[see Omelaenko, Yad. Fiz. 34, 730 (1981)]

Problems:

Methods of Barrelet and Omelaenko may violate unitarity.

Cutting off the expansion at L_{max} may (at low energy) be okay for $\pi^0 p$ photoproduction but not okay for $\pi^+ n$ (due to t-channel pole).

Consider a fit to $\pi^0 p$ and $\pi^+ n$ photoproduction at low energies (above $\pi^+ n$ threshold) where Watson's can give the multipole phases in terms of the corresponding πN elastic scattering phase.

Method:

$M_L \rightarrow H_i \rightarrow$ Observables

Fit the individual multipoles.

Assume M_L phases known

or

(Grushin) assume only equality of phases for ($E^{1/2}_{1+}$, $M^{1/2}_{1+}$) and for ($E^{3/2}_{1+}$, $M^{3/2}_{1+}$)
[determine phases from the fit]

or

only assume the phases of ($E^{3/2}_{1+}$, $M^{3/2}_{1+}$) are given by πN scattering

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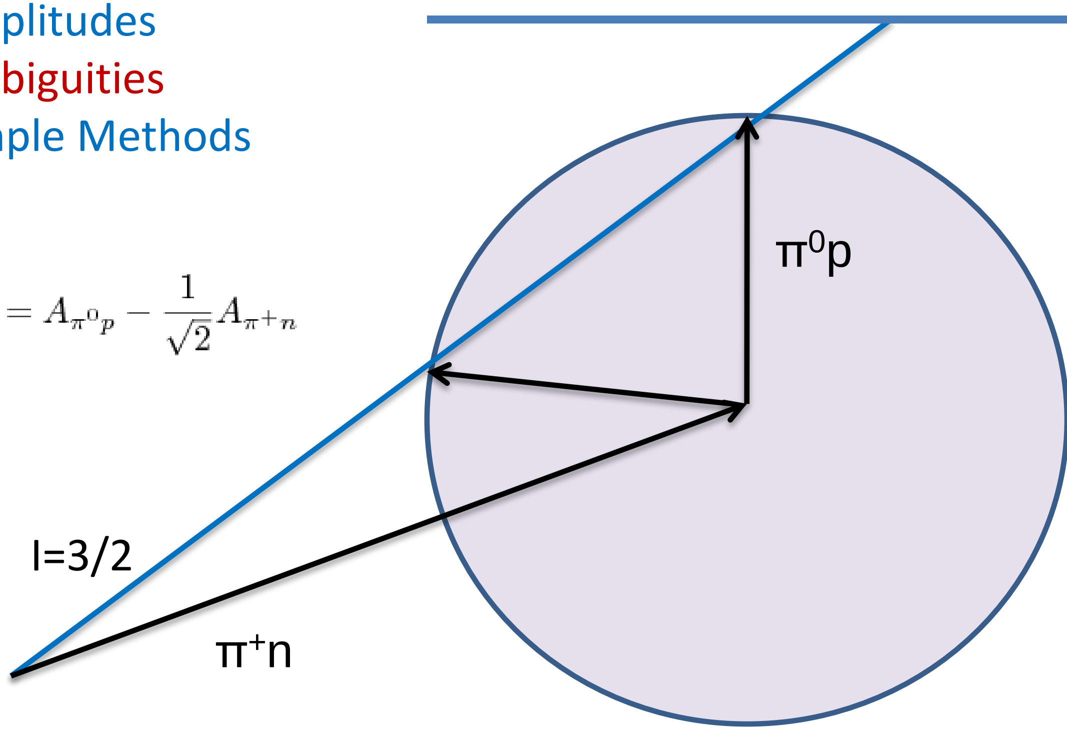
πN photoproduction

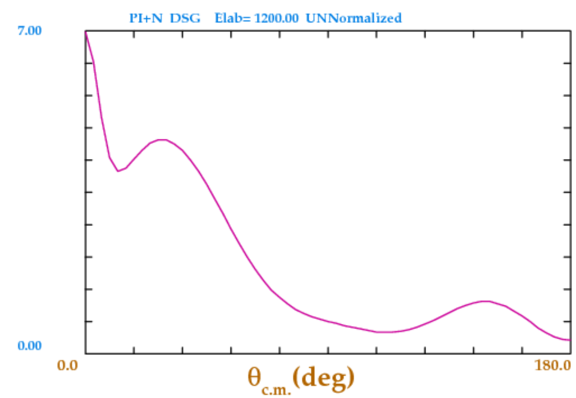
Fixing overall phases

$$A^{3/2} = A_{\pi^0 p} - \frac{1}{\sqrt{2}} A_{\pi^+ n}$$



$\pi^+ n$ phase fixed

[by ‘known’ high-L part (real)]



Grushin method:

$$\mathrm{Im} M_{1+}^{\pi^0 p} E_{1+}^{\pi^0 p\,*} + \mathrm{Im} M_{1+}^{\pi^+ n} E_{1+}^{\pi^+ n\,*} = 0$$

$$\mathrm{Im} M_{1+}^{\pi^0 p} E_{1+}^{\pi^+ n\,*} + \mathrm{Im} M_{1+}^{\pi^+ n} \left(E_{1+}^{\pi^0 p} + \frac{1}{\sqrt{2}} E_{1+}^{\pi^+ n} \right)^* = 0$$

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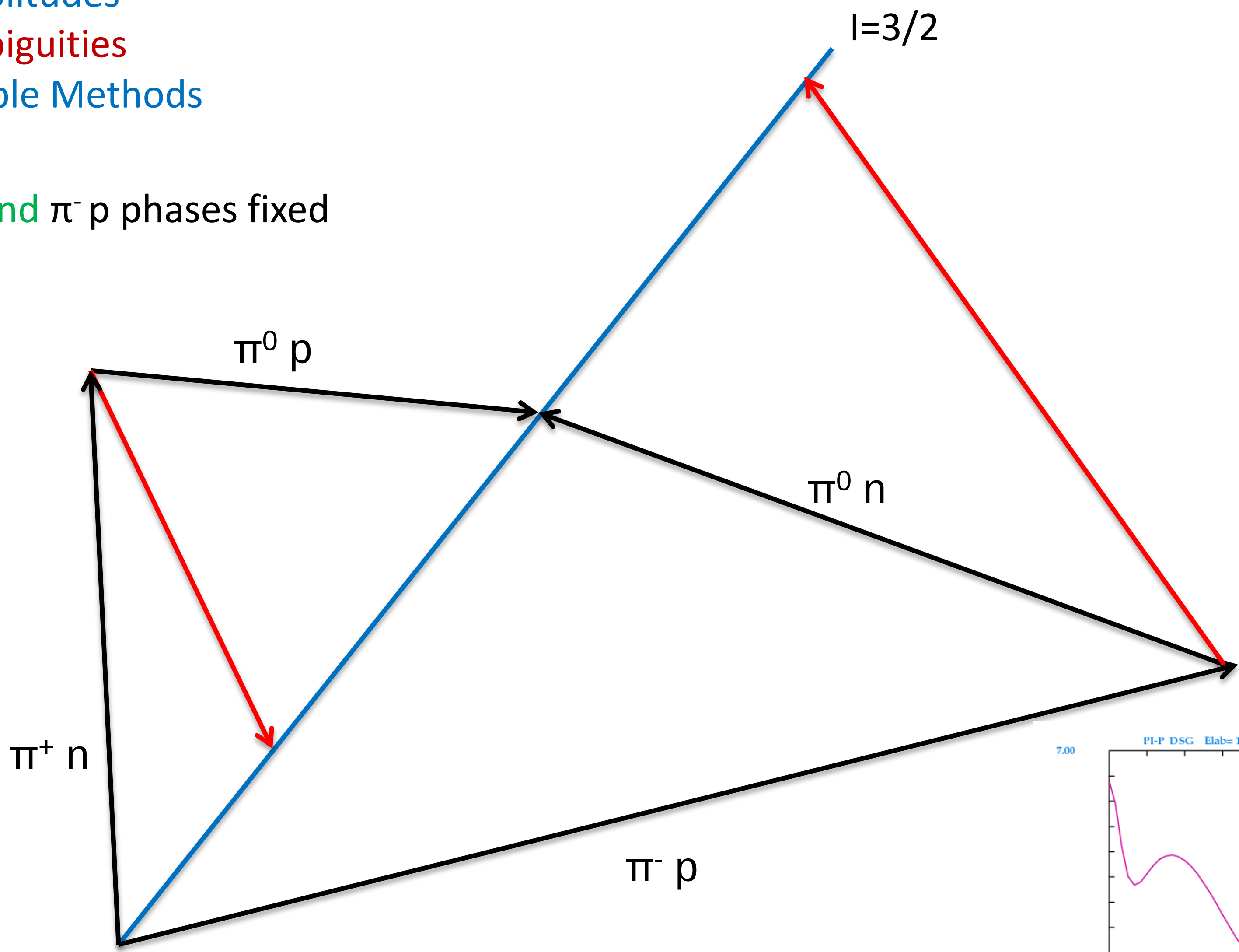
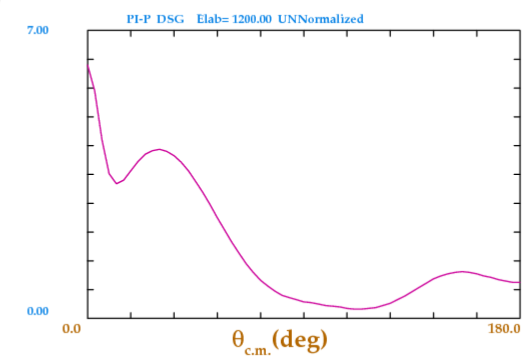
$\pi^+ n$ and $\pi^- p$ phases fixed





Grushin fit to $\pi^0 p$ and $\pi^+ n$ photoproduction data qualitatively gives the associated πN elastic scattering phase shifts

Simple exercise – re-do the Grushin fits.

(a) Fit E_{0+} , M_{1-} , E_{1+} , M_{1+} multipoles to data around the $\Delta(1232)$ resonance

(b) Compare to existing global fits

(c) Are there multiple solutions?

Details in
A.A. Komar, *Photoproduction of pions on nucleons and nuclei*

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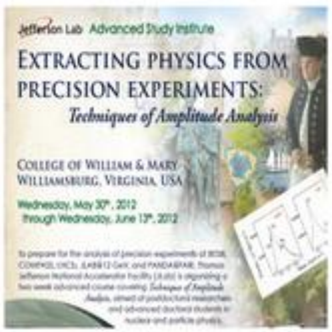
Here's an example of a simple fit using the 'Model' routine.



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EXTRACTING PHYSICS FROM PRECISION EXPERIMENTS:

Techniques of Amplitude Analysis

Fit to 280 MeV π^+n photo

Multipole	Grushin [1]	SES	Fit1	Fit2
Re E_{0+}	17.18(0.29)	16.2	16.72(0.18)	16.17(0.23)
Im E_{0+}	-3.10(0.98)	0.57	-3.41(0.87)	0.5
Re M_{1-}	3.84(0.19)	3.46	3.74(0.18)	3.75(0.29)
Im M_{1-}	-0.70(0.84)	-0.13	-2.02(0.87)	0.33(0.58)
Re E_{1+}	2.64(0.08)	2.96	2.99(0.06)	2.70(0.11)
Im E_{1+}	0.00(0.26)	0.70	-0.08(0.29)	0.78(0.19)
Re M_{1+}	-16.00(0.30)	-14.85	-16.24(0.24)	-14.76(0.18)
Im M_{1+}	-6.76(1.10)	-9.63	-5.96(0.98)	-10.06(0.35)

Quality of data fit?

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Difference mainly in lowest quality data

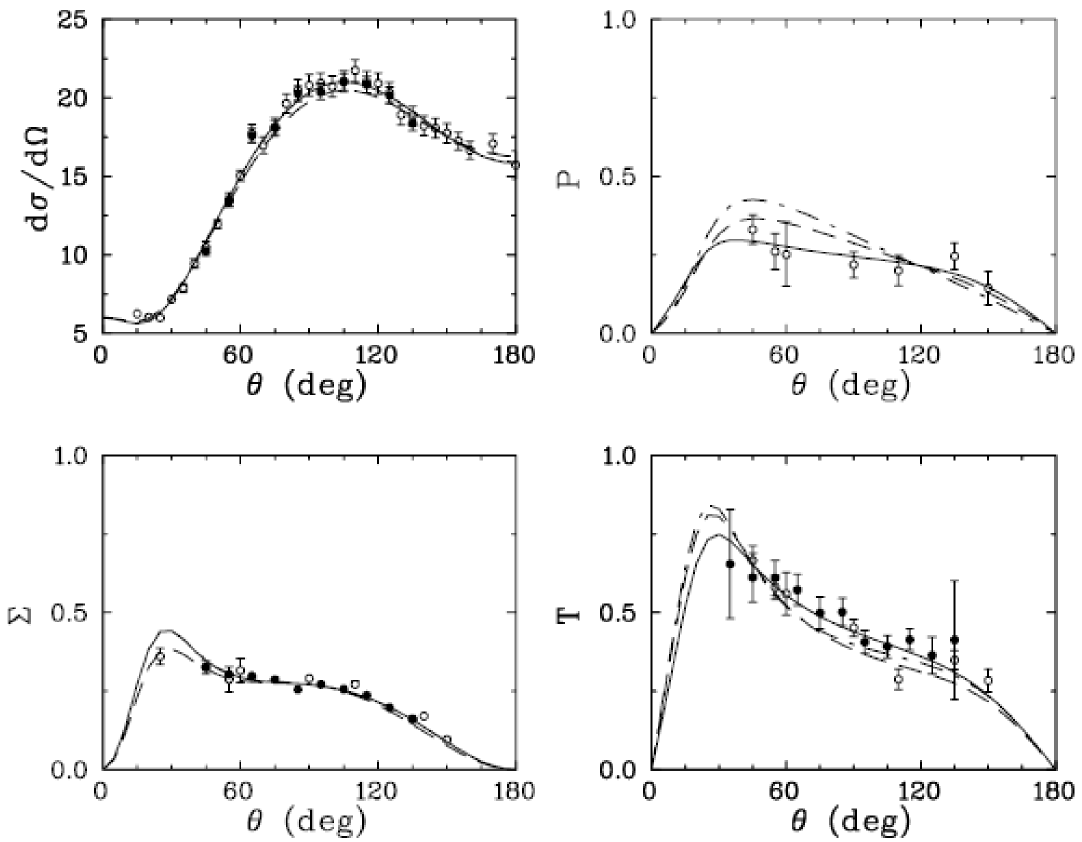

FIG. 1: Fits to $\pi^+ n$ type-S observables at 280 MeV. Fit1 (solid), SES (dashed), Fit2 (dot-dashed).

Fit to 340 MeV π^+n photo

Multipole	Grushin [1]	SES	Fit1	Fit2
Re E_{0+}	10.29(0.42)	11.36	11.19(0.41)	12.42(0.30)
Im E_{0+}	2.00(0.52)	-0.14	2.15(0.55)	0.0
Re M_{1-}	1.82(1.40)	4.53	2.89(1.45)	4.32(1.27)
Im M_{1-}	-0.11(0.22)	-0.17	1.17(0.30)	0.50(0.31)
Re E_{1+}	0.30(0.35)	1.79	0.69(0.38)	1.22(0.29)
Im E_{1+}	-0.41(0.10)	0.30	0.47(0.14)	0.18(0.16)
Re M_{1+}	1.34(0.98)	-1.82	1.11(0.24)	-1.66(0.61)
Im M_{1+}	-19.26(0.46)	-18.29	-18.84(0.22)	-18.31(0.21)

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Differences due to new Σ data and freedom of fit to P data

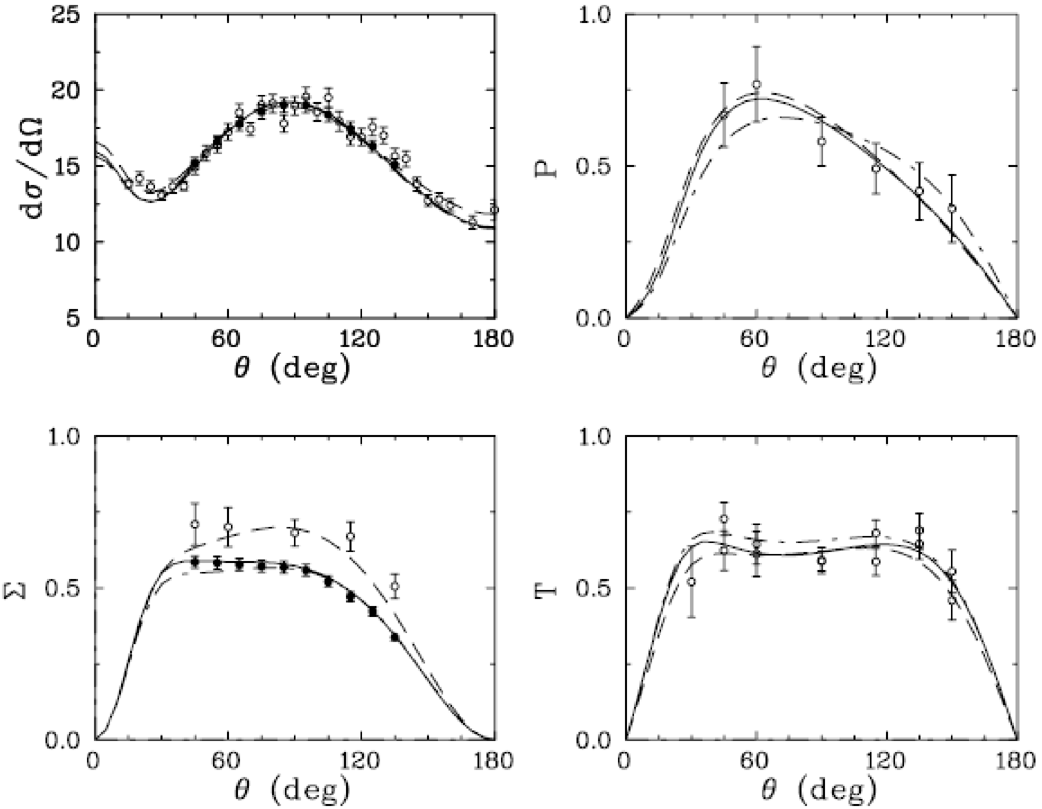


FIG. 2: Fits to $\pi^+ n$ type-S observables at 340 MeV. Fit1 (solid), Ref. [1] (dashed), Fit2 (dot-dashed). Data as in Fig. 1.

Fits to 350 MeV π^0p photo

Fit 1: fix one phase

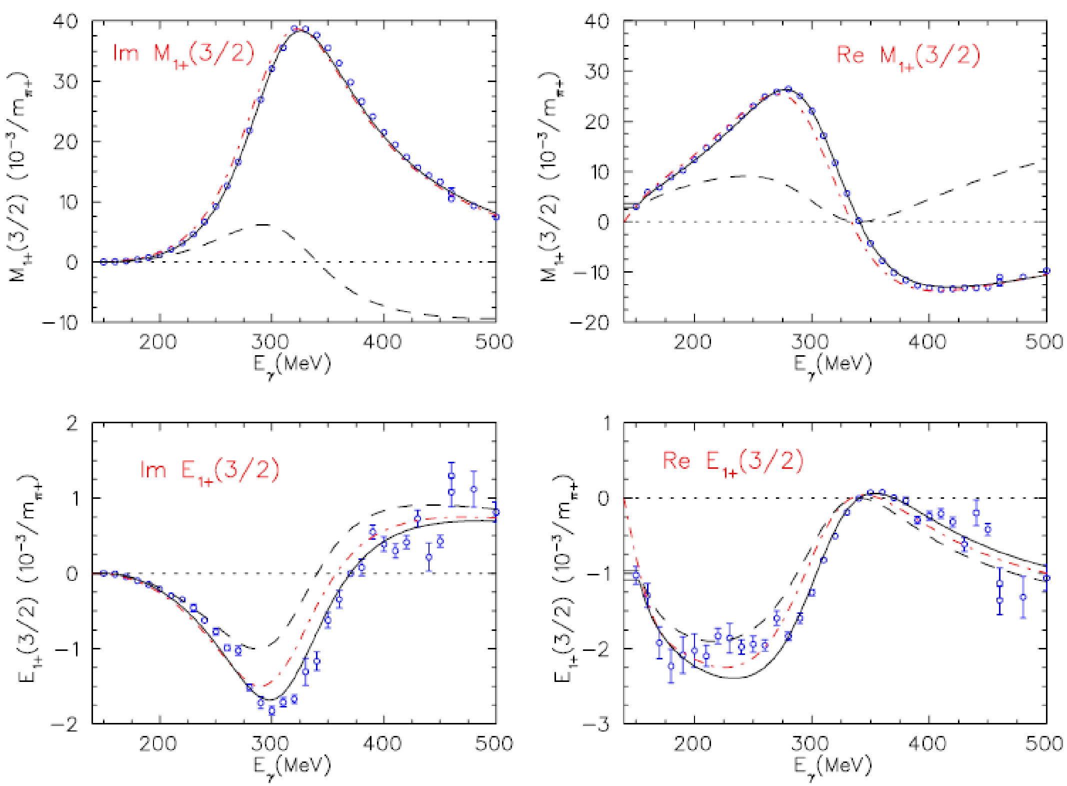
Fit 2: use $M_{1+}^{\pi^0 p} = \alpha e^{i\delta_{33}} + \frac{1}{\sqrt{2}} M_{1+}^{\pi^+ n}$

Fit 3: conjugate roots
(Omelaenko)

Multipole	Grushin [1]	SES	Fit1	Fit2	Fit3
Re E_{0+}	-1.64(0.46)	-2.69	-2.33(0.46)	-1.58(0.42)	-1.20
Im E_{0+}	1.03(0.24)	2.81	1.27(0.24)	2.14(0.31)	2.36
Re M_{1-}	-2.97(1.99)	-2.89	-2.84(1.84)	-2.73(1.85)	18.67
Im M_{1-}	0.57(0.17)	0.51	-0.33(0.45)	0.90(0.40)	-4.41
Re E_{1+}	0.70(0.62)	1.34	0.63(0.58)	0.38(0.57)	-7.74
Im E_{1+}	-0.78(0.08)	-0.30	-0.47(0.14)	-0.70(0.15)	1.44
Re M_{1+}	-1.3	-5.70	-6.41(0.40)	-4.13	15.50
Im M_{1+}	23.89(0.10)	22.81	23.0	23.56	-6.36

Fits 1-3 have the same chi-squared

At the Δ(1232) resonance energy, a single multipole dominates and some simple methods can be used to extract resonance properties



Simple methods in the Delta Resonance Region

$$\Sigma = \frac{d\sigma_\perp - d\sigma_\parallel}{d\sigma_\parallel + d\sigma_\perp}$$

Beck *et al*
PRL 78, 606 (1997)

$$\frac{d\sigma_j(\theta)}{d\Omega} = \frac{q}{k}[A_j + B_j \cos(\theta) + C_j \cos^2(\theta)]$$

$$A_\parallel = |E_{0^+}|^2 + |3E_{1^+} + M_{1^+} - M_{1^-}|^2,$$
$$B_\parallel = 2\,\mathrm{Re}[E_{0^+}(3E_{1^+} + M_{1^+} - M_{1^-})^*],$$
$$C_\parallel = 12\,\mathrm{Re}[E_{1^+}(M_{1^+} - M_{1^-})^*].$$

$$R = \frac{\mathrm{Re}(E_{1^+}M_{1^+}^*)}{|M_{1^+}|^2} \simeq \frac{1}{12}\frac{C_\parallel}{A_\parallel} = \frac{\mathrm{Re}[E_{1^+}(M_{1^+} - M_{1^-})^*]}{|M_{1^+} - M_{1^-}|^2}$$

Speed-plot method for the Delta

Hanstein *et al*
PL B385, 45 (1996)

$$SP(W) = \left| \frac{dT(W)}{dW} \right|$$

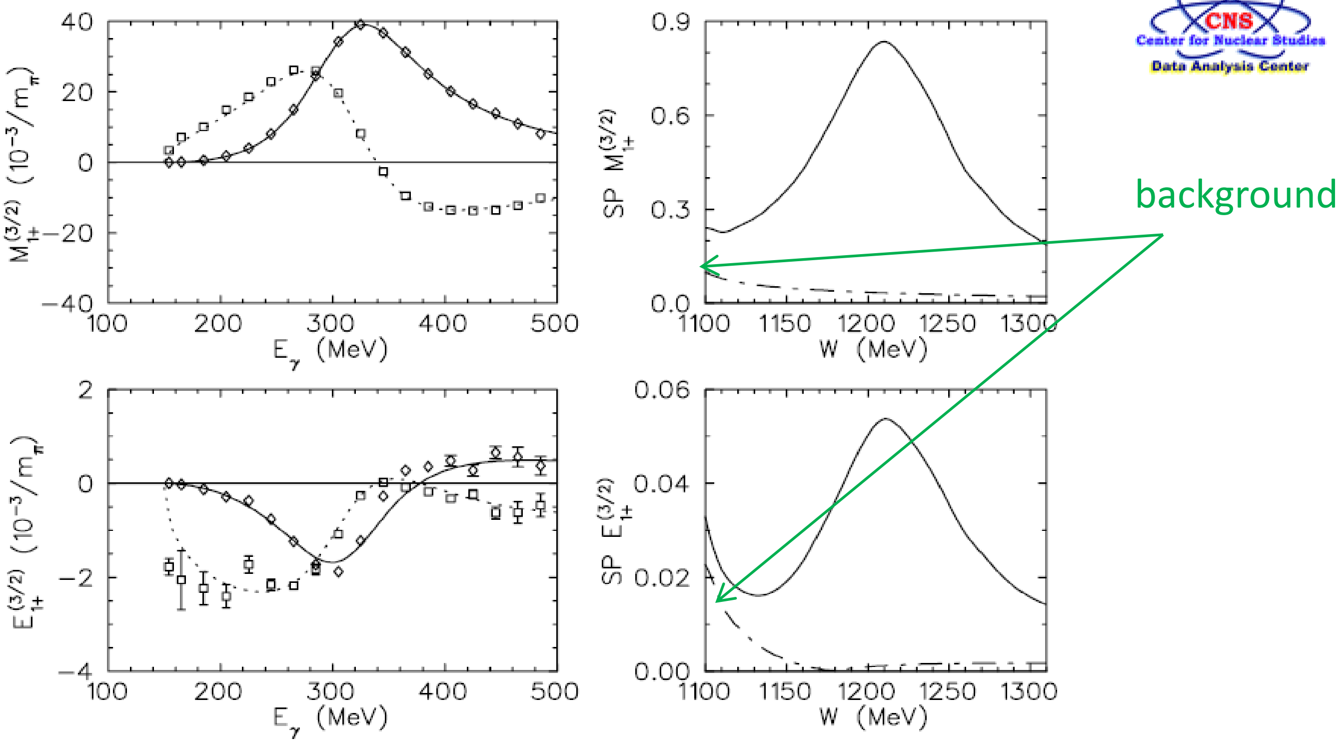
Background
`smooth'

$$T_R(W) = \frac{r\Gamma_R e^{i\phi}}{M_R - W - i\Gamma_R/2}$$

$$SP(W) = r\Gamma_R \frac{\{[(M_R - W)^2 - \Gamma_R^2/4]^2 + \Gamma_R^2(M_R - W)^2\}^{\frac{1}{2}}}{\{(M_R - W)^2 + \Gamma_R^2/4\}^2}$$

$$SP(M_R) = 4r/\Gamma_R = H \qquad SP(M_R \pm \Gamma_R/2) = H/2$$

Data
Amplitudes
Ambiguities
Simple Methods



	r $[10^{-3}\mathrm{MeV/m_\pi}]$	ϕ [°]	M_R [MeV]	Γ_R [MeV]
E	1.23	-154.7	1211±1	102±2
M	21.16	-27.5	1212±1	99±2

$$R_\Delta = \frac{r_E e^{i\phi^E}}{r_M e^{i\phi_M}} = -0.035 - 0.046i$$

Alternate approach at the pole

term ⟶ 0 at
BW mass

K-matrix approach
for E2 and M1

$$M = \alpha(1 + iT_{\pi N}) + \beta T_{\pi N}$$

term is pure
imaginary at
BW mass

	E2		
	α-term	β-term	Total
Fit A	$(1.12,\ -143^\circ)$	$(0.42,\ 157^\circ)$	$(1.38,\ -158^\circ)$
Fit B	$(0.98,\ -126^\circ)$	$(0.08,\ 157^\circ)$	$(1.01,\ -131^\circ)$
	M1		
	α-term	β-term	Total
Fit A	$(3.2,\ -136^\circ)$	$(21.9,\ -23^\circ)$	$(20.9,\ -31^\circ)$
Fit B	$(3.2,\ -136^\circ)$	$(21.7,\ -23^\circ)$	$(20.7,\ -31^\circ)$

E2/M1 ratio	K-matrix pole	T-matrix pole
Fit A	- 1.9 %	0.066, -127°
Fit B	- 0.4 %	0.049, -100°

Höhler parameterized the KH and CMB
πN elastic scattering solutions using a form

$$T(W) = T_{\rm B} + \frac{r\Gamma_R e^{i\phi}}{M_R - W - i\Gamma_R/2}$$

with $T_{\rm B} = (\eta_B e^{2i\delta_{\rm B}} - 1)/2i$
for elastic scattering $\eta_{\rm B} = 1$ and $\phi = 2\delta_{\rm B}$ ← Δ(1232)
[i.e. $S = S_{\rm B} S_{\rm R}$]

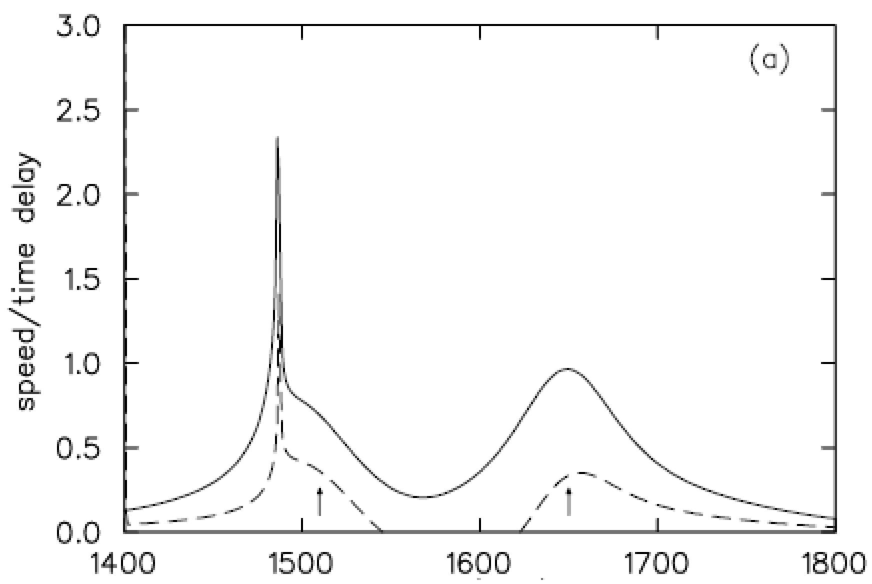
See
G. Höhler,
πN Newsletter
Vol. 9, 1 (1993).

Plotted dT/dW in an Argand diagram to
obtain phases of residues (listed in PDG)

Results: lots of bumps not → resonances

N(1535) particularly bad:

- pole position close to ηN threshold
- no width, residue reported



Pole vs BW parameters

$$T \approx \frac{f(W)}{M - W - i\Gamma(W)/2}$$

D.B. Lichtenberg, PRD10,3865(1974)

D.M. Manley, PRD51,4837(1995)

Lichtenberg/Manley took simple BW forms for the Delta, solved for the pole. If energy dependence ~ simple phase space or Blatt-Weiskopf factor, α is positive (~0.4)

$$D(W) = M - W - i\Gamma(W)/2$$

$$W_p \approx W_0 - \frac{D(W_0)}{D'(W_0)}$$

So, pole `mass' < BW mass
and
pole `width' < BW width

Γ' denotes $d\Gamma(W)/dW$ @ $W = M$

$$\alpha = \Gamma'/2$$

$$M_0 \approx M - \frac{\Gamma}{2}\left(\frac{\alpha}{1+\alpha^2}\right)$$

$$\Gamma_0 \approx \frac{\Gamma}{1+\alpha^2} .$$

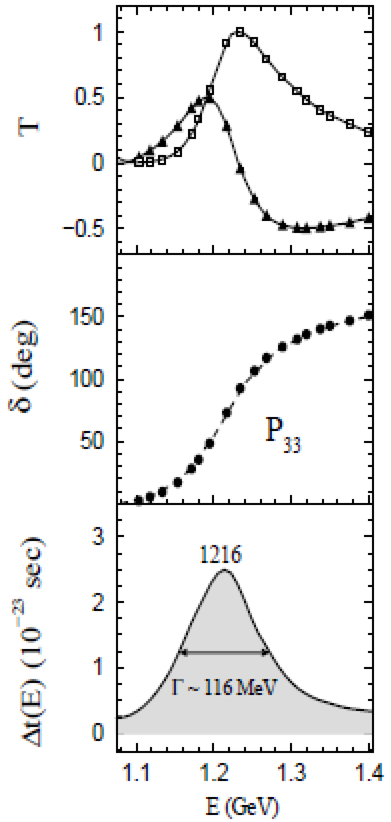
Pole parameters

Also see applications of `time delay' concept

$$\Delta t = 2\hbar \frac{d\delta}{dE}$$

Introduced by Wigner (elastic scattering) extended to multi-channel case.

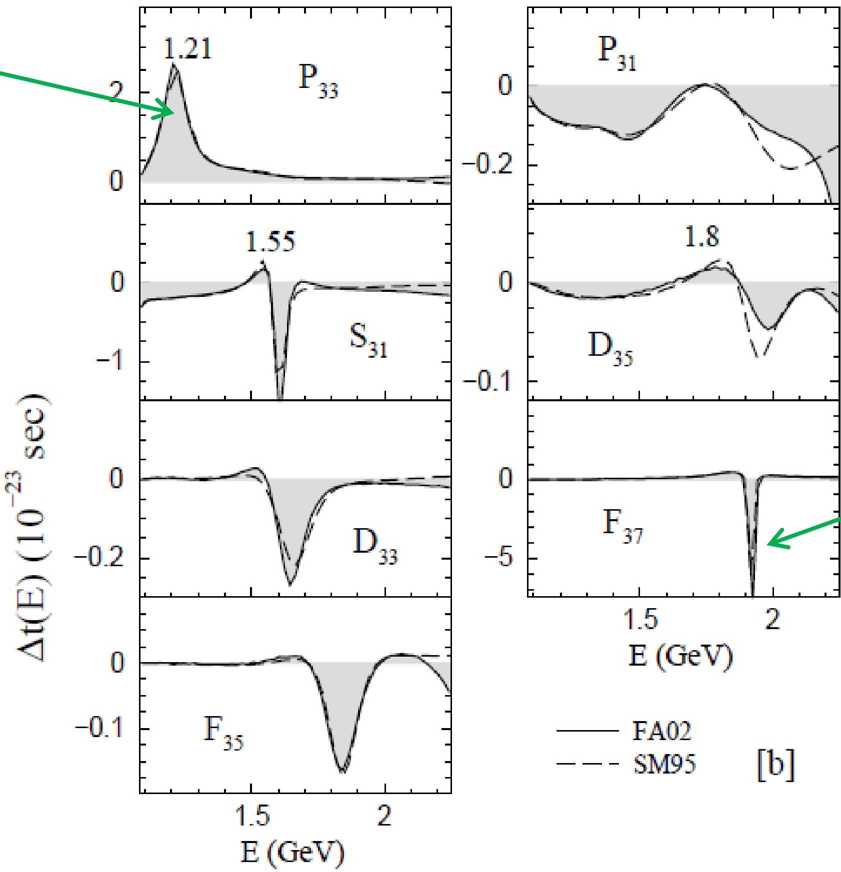
For the Δ(1232), the result is not really different from the speed plot.



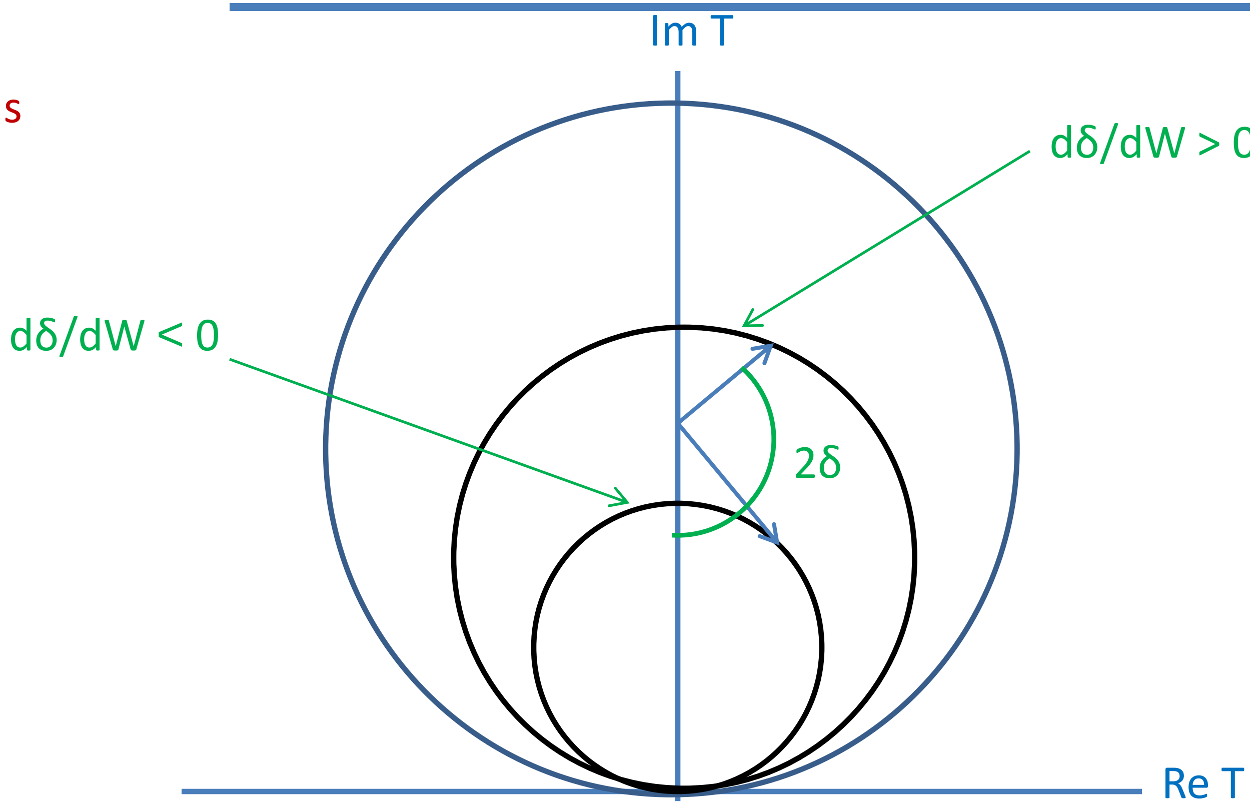
But applied to other partial-waves …

time delay

Kelkar *et al.*,
NPA730,121 (2004)



time `advance'


Im T
dδ/dW > 0
dδ/dW < 0
2δ
Re T