Single-energy fits
πN Photoproduction
πN scattering

Have considered single-energy fits in the context of amplitude reconstruction.

Here we will describe the single-energy fits displayed on SAID/MAID plots of amplitudes.

These have been used in multi-channel fits
[e.g. Geissen , Bonn-Gatchina]
in lieu of actual `data' for πN scattering and πN photoproduction.

Important to understand how they are determined, why they were produced, and what they say about the underlying database.

Single-energy fits
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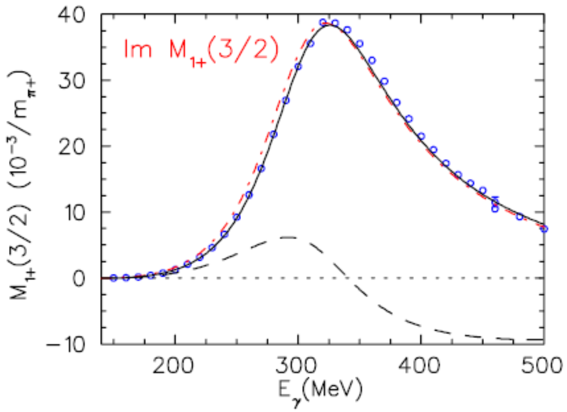
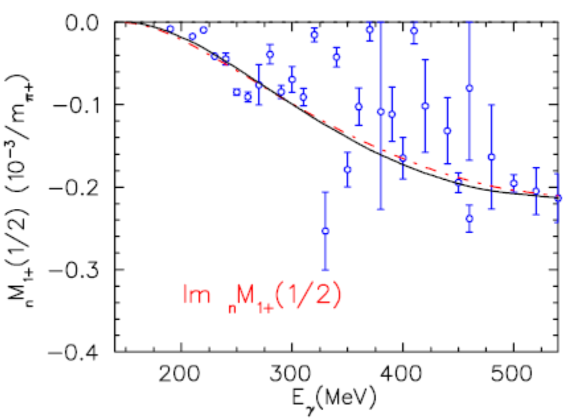
compare



to



Historical motivation:

Suppose you have a global (energy-dependent) fit to 25K data, over a 2 GeV energy range, varying 0.2K parameters.

Can the fit be improved?

Bin data over narrow energy ranges. Vary the most significant partial waves – assume the phase found from the global fit is correct. Fit the data along with partial-wave pseudo-data to keep the fit `close' to global solution – look for systematic deviations.

Consider what may be missing from your model to improve the global fit result.

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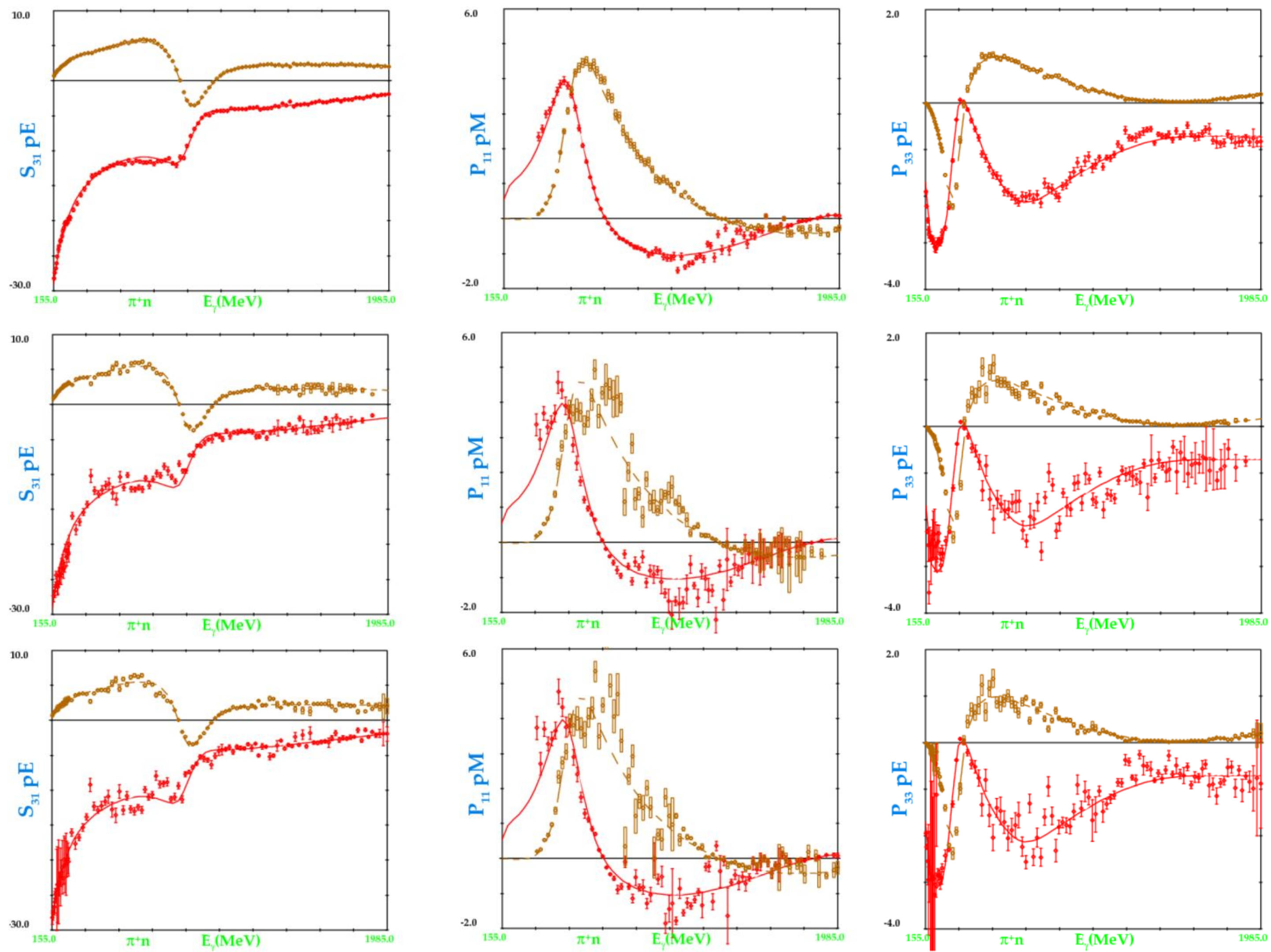
Single-energy vs global fit

Pseudo-data errors/10

Result on SAID

Pseudo-data errors * 10



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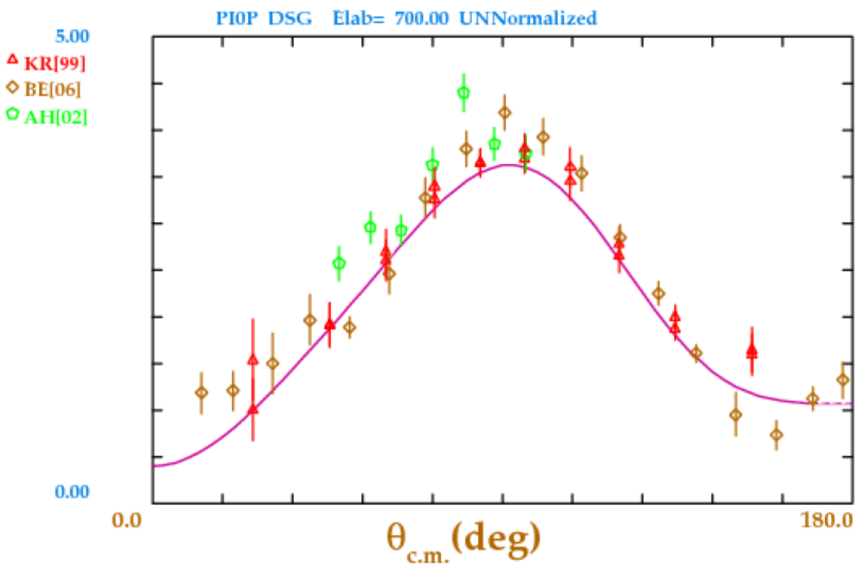
Results depend on (subjective) level of constraint from the global fit. Tighter constraints give back the global result.

Some partial-waves (with little constraint) scatter erratically.

Problem: bins contain old and new data (some contradictory) and may not contain sufficient observables to give a unique result.



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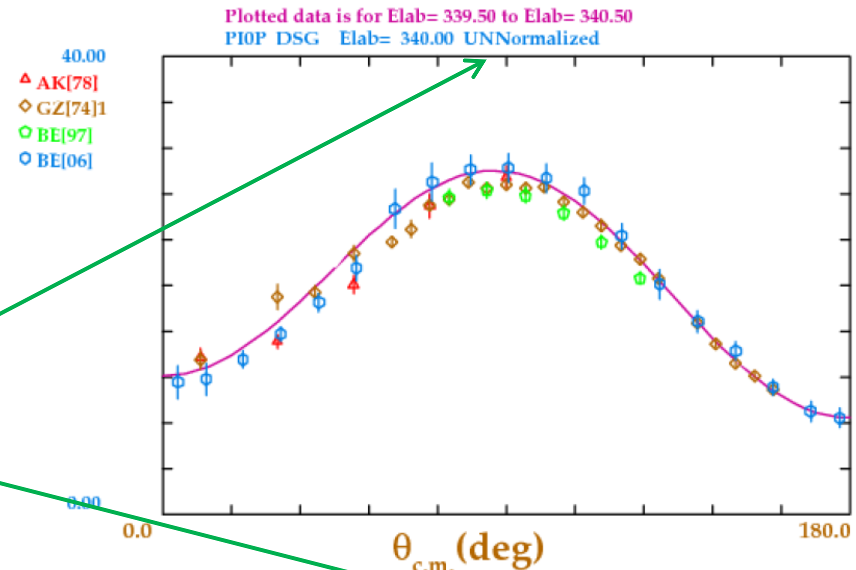
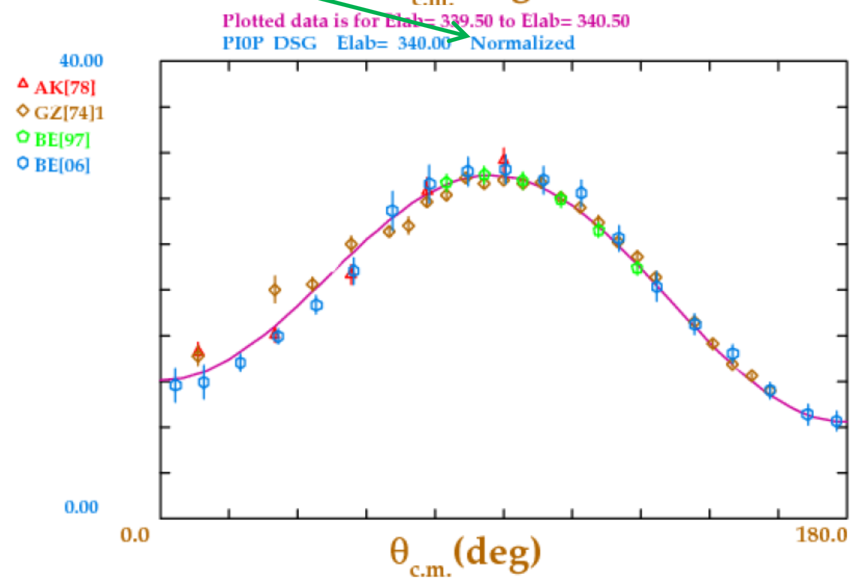


Some discrepancies can be reduced based on the given overall systematic error through a renormalization



This enters into a modified chi-squared definition

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For details see
R.A. Arndt, M.H. MacGregor
Methods in Computational Physics, Vol. 6 , 253 (1966).

$$\chi^2 = \sum_{i=1}^{N_D} \left(\frac{\alpha\theta^i(p) - \theta^i_{exp}}{\delta\theta^i_{exp}} \right)^2 + \sum_{j=1}^{N_\alpha} \left(\frac{\alpha_j - 1}{\delta\alpha_j} \right)^2$$

where α_j factors renormalize (N_α) angular distributions, including a total of N_D data. The α_j factors do not have to be included explicitly in the parameter (p) search.

This was used recently in a kaon photoproduction fit by Sandorfi et al.,
J. Phys. G38, 053001 (2011).

At the minimum,

$$p \to p \pm \Delta p \ , \ \chi^2 \to \chi^2 + 1 \ , \ Prob \to \frac{1}{e^{1/2}} Prob$$

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Try to generate single-energy fits based on a database generated from known amplitudes.

Can test effect of more (consistent) measurements on the fit.

Tiator talk at NSTAR 2011.

Exercise was done using MAID Monte Carlo data, then fitted using the SAID global fit.

[observables generated to be `reasonable' given existing beam/detector quality]

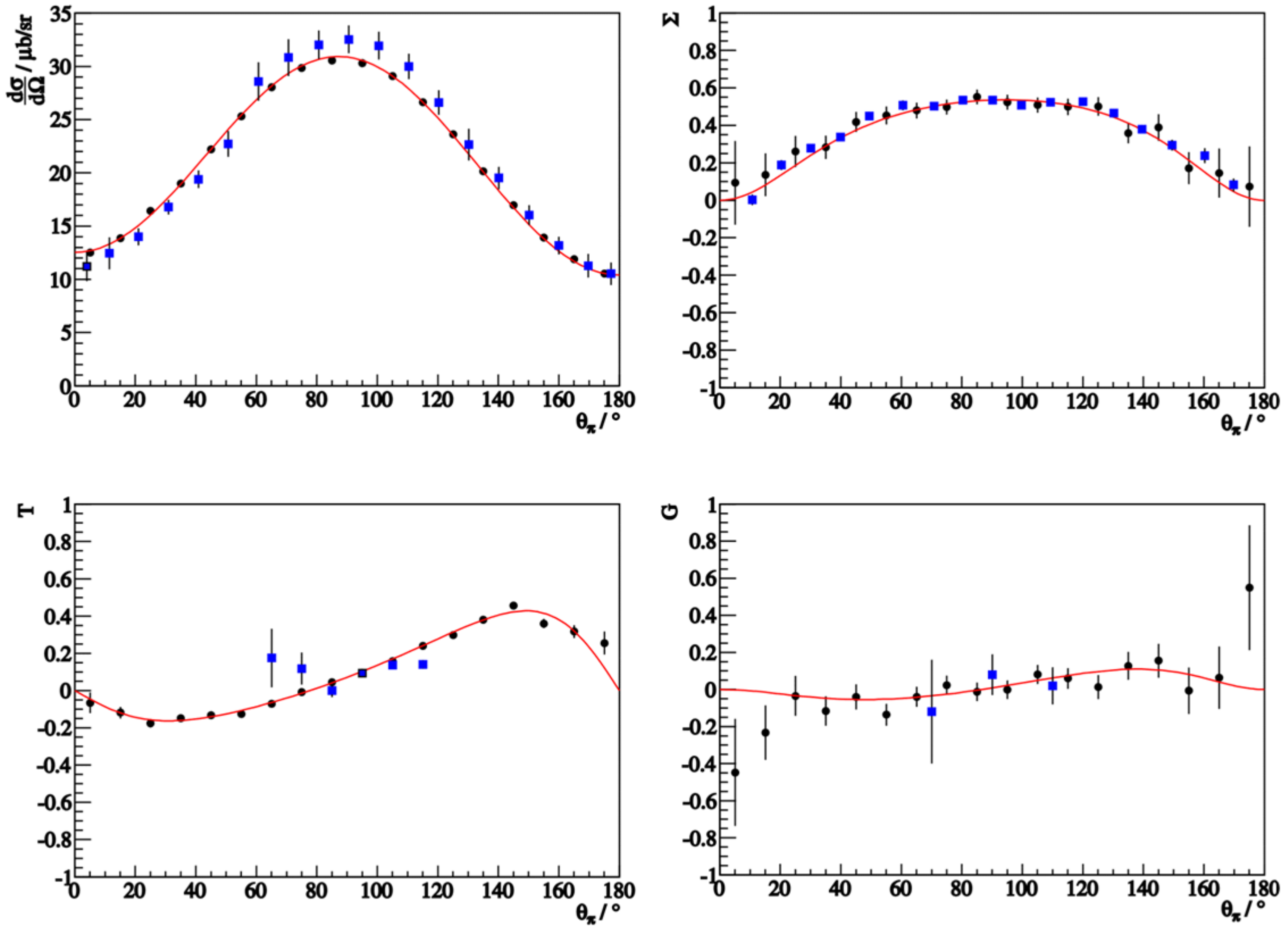
Single-energy fits

πN Photoproduction

πN scattering

MD07 (red curve) Pseudo-data (**black** points) Real data (blue points)

$\pi^0 p$ at 320-340 MeV and comparison with real data

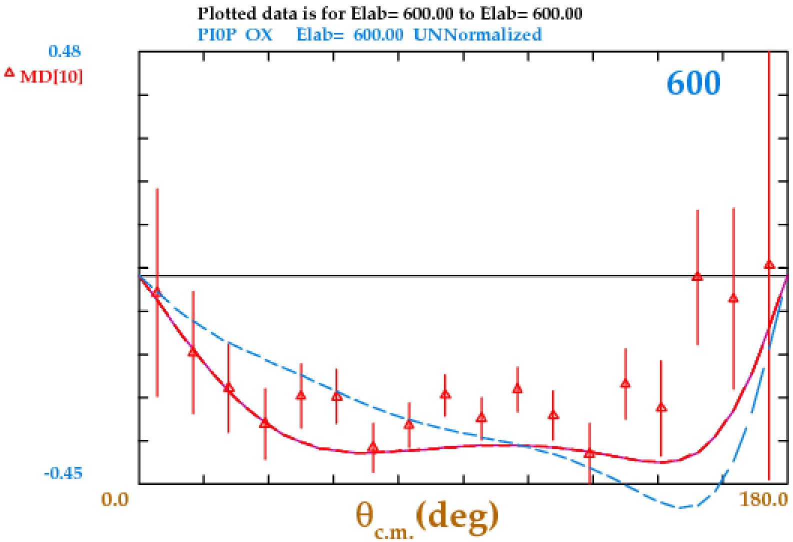


Single-energy fits

πN Photoproduction

πN scattering

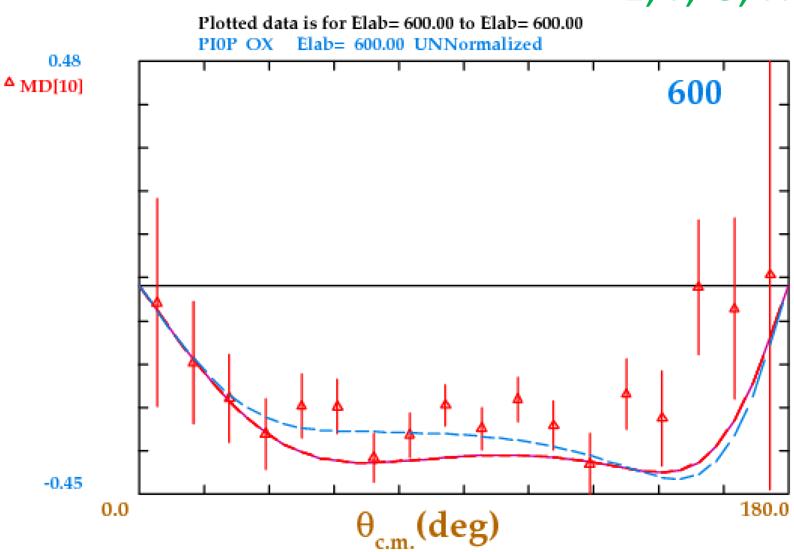
$d\sigma/d\Omega$, P, Σ, T

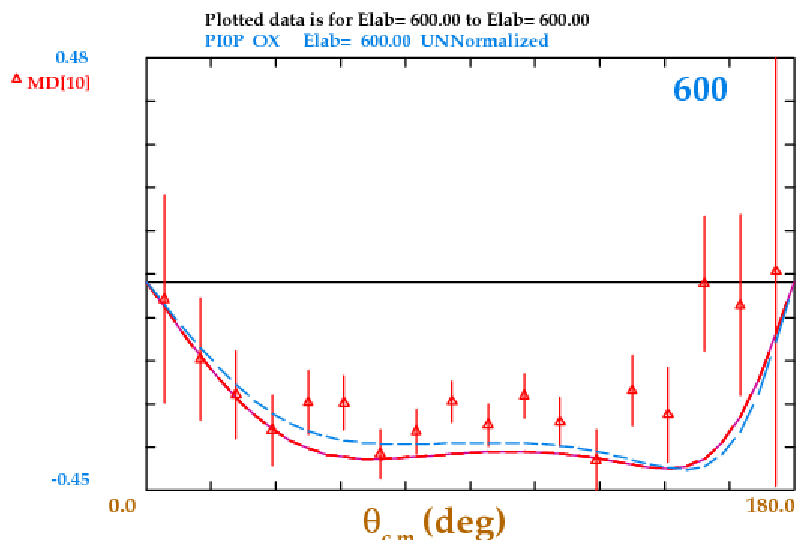


Prediction compared to a fit of double-polarization observable

$$O_x \frac{d\sigma}{dt} = -2\mathrm{Re}(b_1 b_3^* - b_2 b_4^*)$$

E, F, G, H



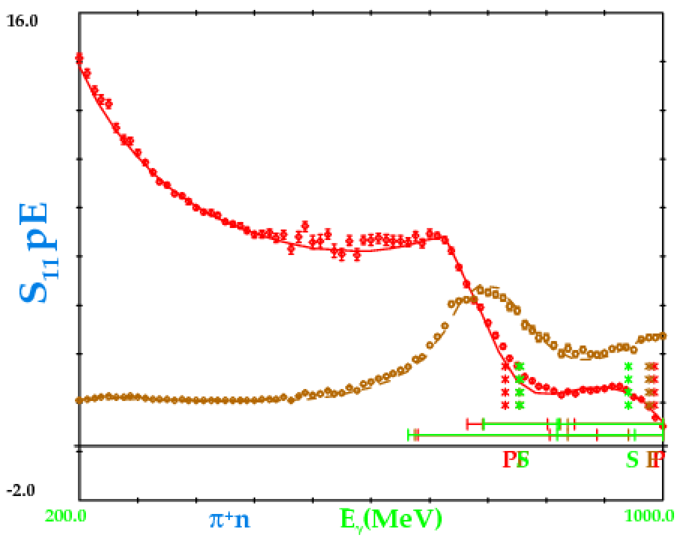


O_x , O_z , C_x , C_z

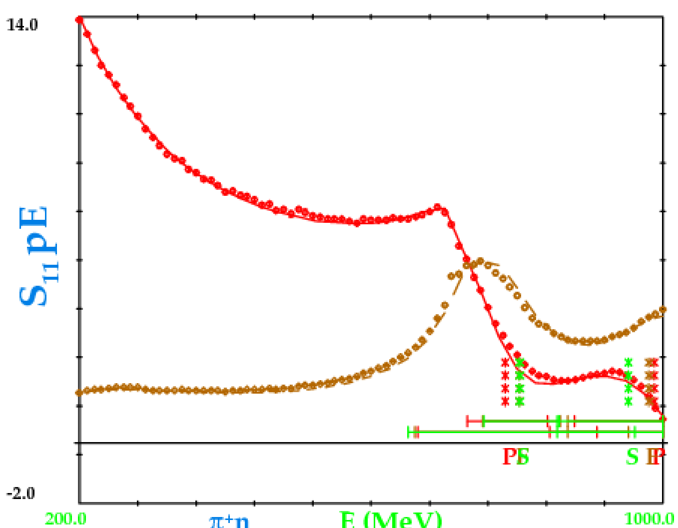
Single-energy fits

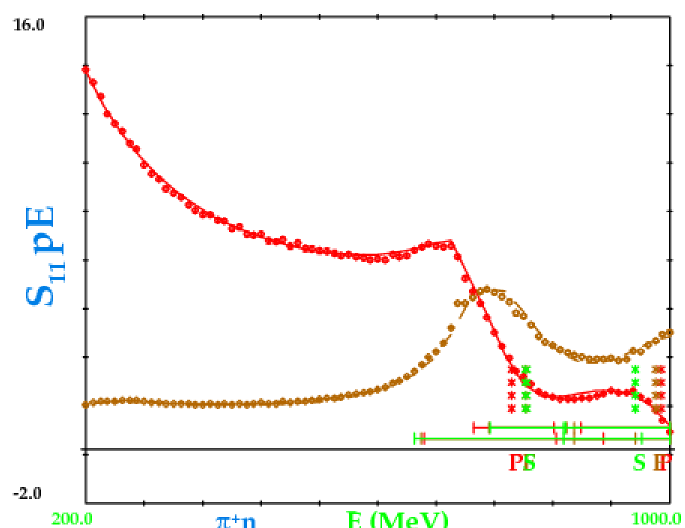
πN Photoproduction

πN scattering



Multipole: predicted vs input





Single-energy fits

πN Photoproduction

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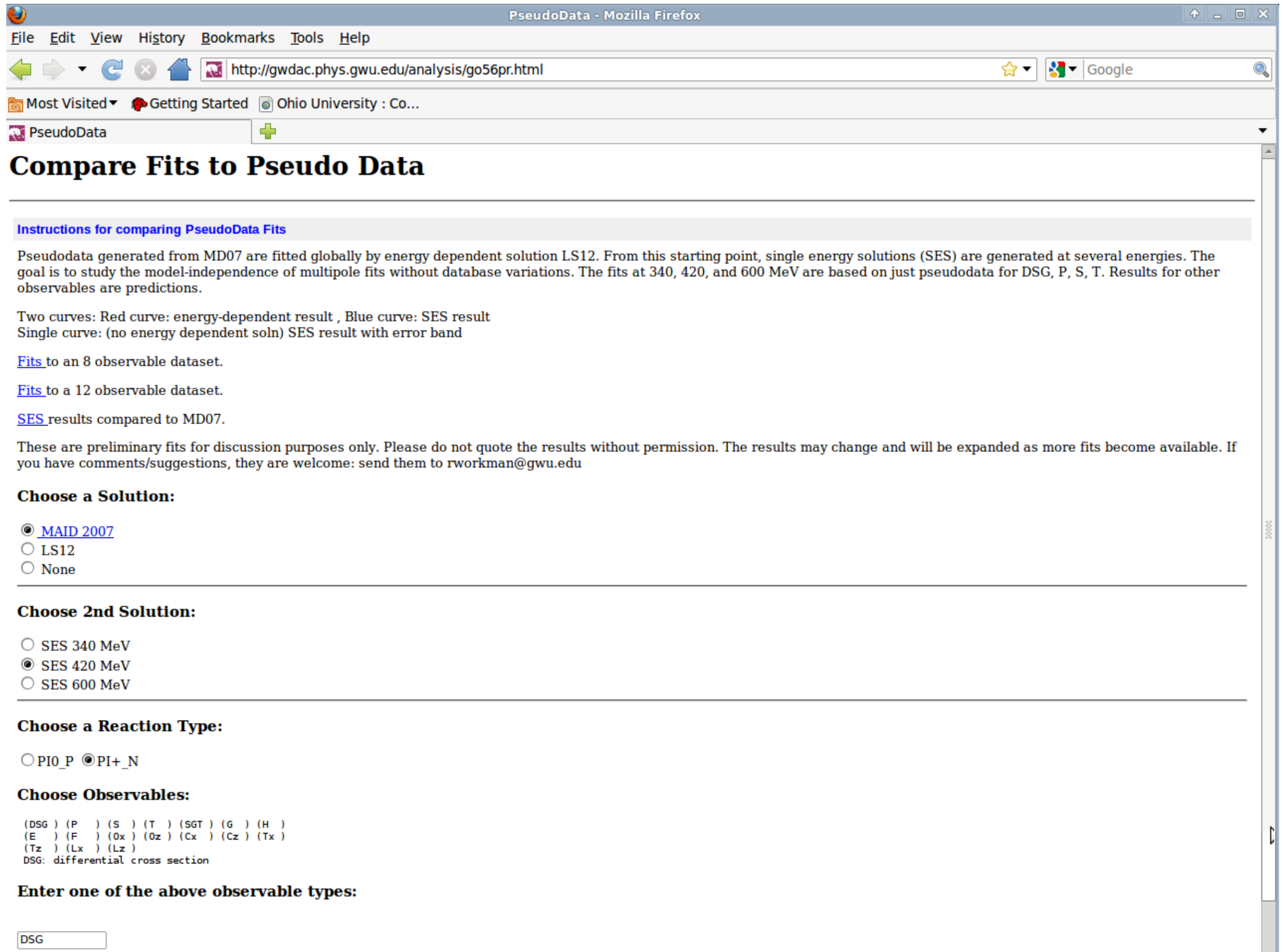

Compare Fits to Pseudo Data

Instructions for comparing PseudoData Fits

Pseudodata generated from MD07 are fitted globally by energy dependent solution LS12. From this starting point, single energy solutions (SES) are generated at several energies. The goal is to study the model-independence of multipole fits without database variations. The fits at 340, 420, and 600 MeV are based on just pseudodata for DSG, P, S, T. Results for other observables are predictions.

Two curves: Red curve: energy-dependent result , Blue curve: SES result
Single curve: (no energy dependent soln) SES result with error band

Fits to an 8 observable dataset.

Fits to a 12 observable dataset.

SES results compared to MD07.

These are preliminary fits for discussion purposes only. Please do not quote the results without permission. The results may change and will be expanded as more fits become available. If you have comments/suggestions, they are welcome: send them to rworkman@gwu.edu

Choose a Solution:

- (•) MAID 2007
- ( ) LS12
- ( ) None

Choose 2nd Solution:

- ( ) SES 340 MeV
- (•) SES 420 MeV
- ( ) SES 600 MeV

Choose a Reaction Type:

( ) PI0_P (•) PI+_N

Choose Observables:

```
(DSG ) (P   ) (S  ) (T  ) (SGT ) (G  ) (H  )
(E   ) (F   ) (Ox ) (Oz ) (Cx  ) (Cz ) (Tx )
(Tz  ) (Lx  ) (Lz )
DSG: differential cross section
```

Enter one of the above observable types:

DSG

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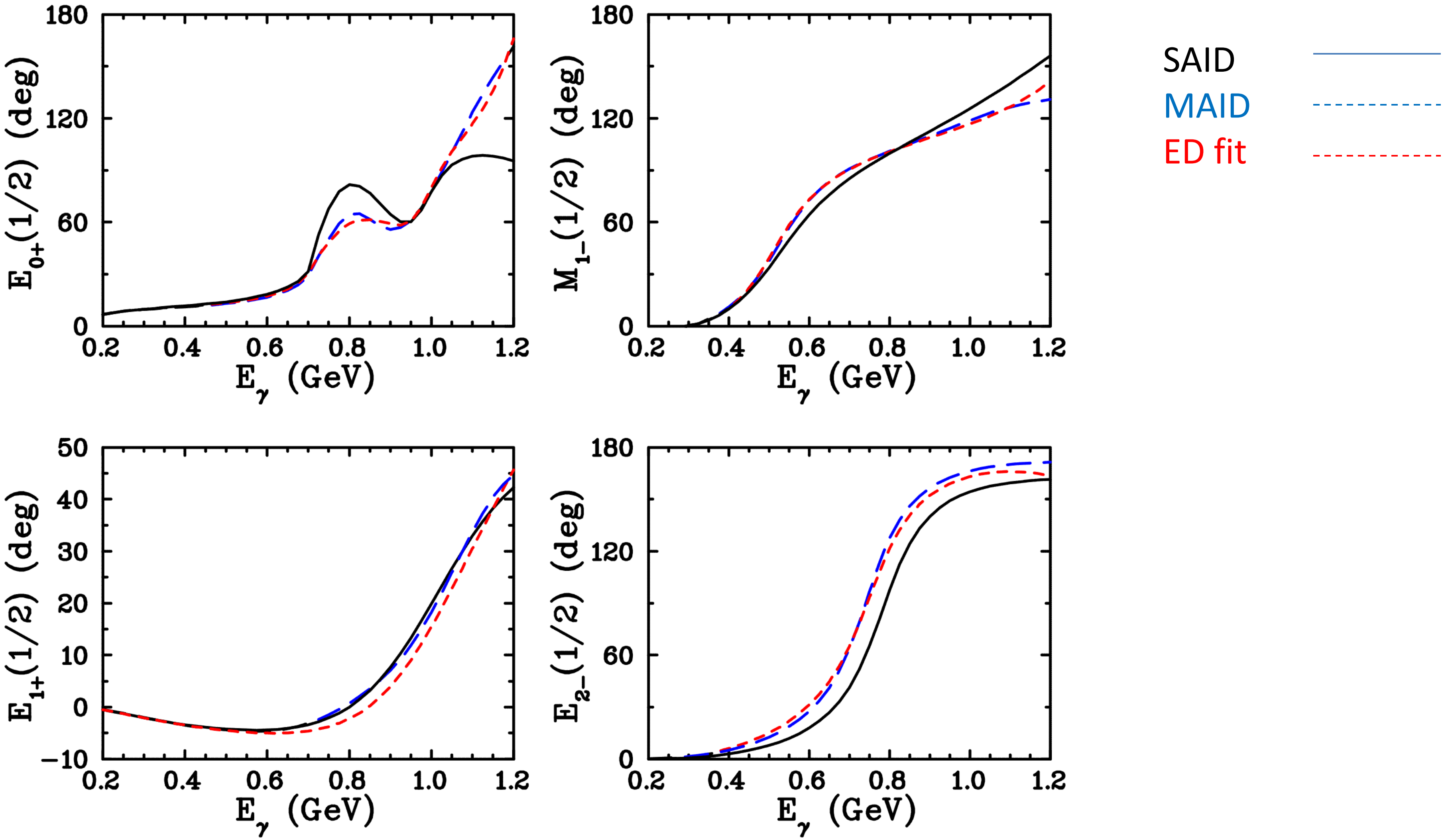
SAID vs MAID phase differences



Single-energy fits
πN Photoproduction
πN scattering

Effect of searching phases in single-energy fits

Multipole	SP09	LS12	SE4	SE8	MD07
$E_{0+}^{1/2}$	18.4	17.4	16.0(3.0)	16.2(0.9)	16.7
$M_{1-}^{1/2}$	64.4	73.2	68.2(2.4)	73.4(1.6)	72.7
$M_{1-}^{3/2}$	167.1	172.0	176.8(6.5)	167.5(1.8)	163.9
$E_{2-}^{1/2}$	17.9	31.4	27.5(3.1)	26.1(1.0)	27.6
$M_{2-}^{1/2}$	22.2	26.7	25.5(1.8)	26.7(1.0)	26.8

Multipole phases (degrees) from single-energy fits to 4 (SE4) and 8 (SE8) observables at 600 MeV

Overall fit quality: single-energy vs global fit

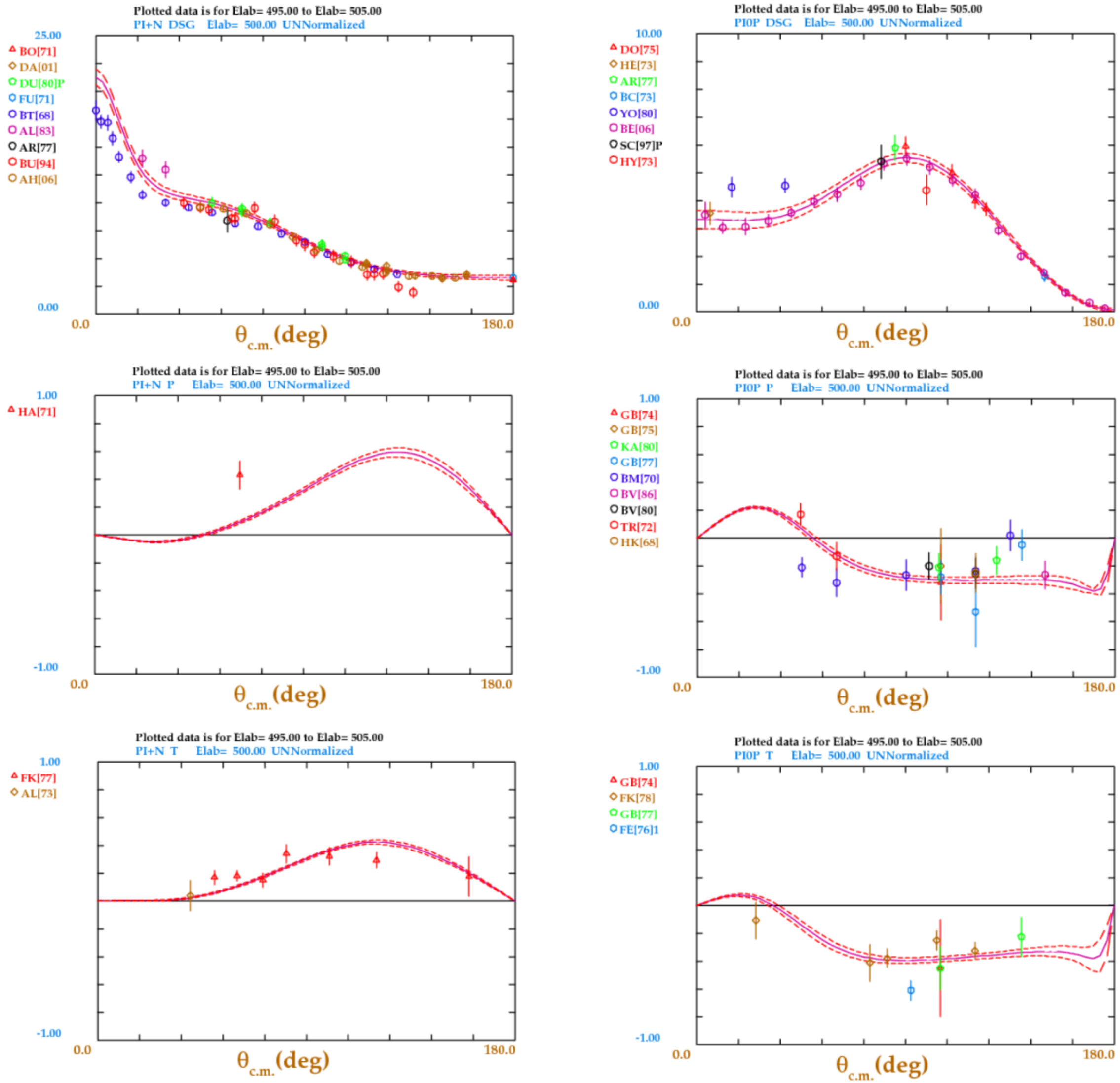
E_γ (lab)	global χ^2	single-energy χ^2	data/bin
294-316	1068	957	605
494-516	710	582	377
734-757	1450	1276	778
1494-1515	226	141	113

global χ^2 ~ 2
single-energy χ^2 ~ 1.5

Single-energy fits

πN Photoproduction

πN scattering



Some observations:

1. The real-world database is not self-consistent.

2. The single-energy amplitudes are not a representation of the data (which is not possible – see 1).

3. Some data could be pruned without changing the result, but lowering the overall χ^2 .

Question:

If we drop `really bad' data, can we get a fit χ^2 / data near unity?

Simple exercise:

1. prune data if $\chi^2 > 5$ for point or distribution

2. Re-fit remaining data

3. Repeat step 1

4. Stop if nothing left to prune

Single-energy fits

πN Photoproduction

πN scattering

Starting global fit:

π^0 p	π^+ n	π^- p	π^0 n
32751/13646	16833/8521	4749/2333	1108/364

Pruned global fit:

π^0 p	π^+ n	π^- p	π^0 n
11716/11338	7584/7496	1978/2022	241/251

Summary :

Pruned for $\chi^2 > 5$

3757 pruned data contributing 33234 to χ^2

Some of the pruned data are not old

Multipoles change very little

Need a consensus evaluation of database
(this was done for πN scattering)

Dangerous to toss data just because it is hard to fit.

Single-energy fits
πN Photoproduction
πN scattering

Single –energy results have been tied to global fits

Two global fit forms, discussed so far, are:

SAID (original form)

$$T_{\gamma\pi} = (\text{Born} + \text{A})(1 + iT_{\pi\pi}) + BT_{\pi\pi}$$

Automatically satisfies Watson's theorem

MAID

$$T_{\gamma\pi} = (\text{Born})(1 + iT_{\pi\pi}) + e^{i\phi}T_{\text{BW}}$$

Phase to satisfy Watson's theorem

where $T_{\pi\pi}$ is the associated πN partial-wave amplitude, and T_{BW} is a Breit-Wigner resonance term.

Motivation (simple K-matrix):

Watson's theorem

$$T_{\gamma\pi} = (1 + iT_{\pi\pi})K_{\gamma\pi}$$

Need pole, or is zero at resonance

$$K_{\gamma\pi} = \frac{A}{W - W_R} + B \;\; ; \;\; K_{\pi\pi} = \frac{C}{W - W_R} + D$$

$$T_{\gamma\pi} = \left(\frac{A}{W - W_R} + B\right) \frac{(W - W_R)}{W - W_R - i[C + (W - W_R)D]}$$

One term→zero at W_R
Other term has BW behavior

Expanding to include another hadronic channel,

$$K = \begin{pmatrix} K_{\gamma\gamma} & K_{\gamma\pi} & K_{\gamma\Delta} \\ K_{\gamma\pi} & K_{\pi\pi} & K_{\pi\Delta} \\ K_{\gamma\Delta} & K_{\pi\Delta} & K_{\Delta\Delta} \end{pmatrix}$$

The relation *T(1-iK)=K* gives,

$$T_{\gamma\pi}(1 - iK_{\gamma\gamma}) = (1 + iT_{\pi\pi})K_{\gamma\pi} + iK_{\gamma\Delta}T_{\pi\Delta}$$

Drop $K_{\gamma\gamma}$ as small

which can be massaged into the form,

$$T_{\gamma\pi} = (1 + iT_{\pi\pi})\left(K_{\gamma\pi} - \frac{K_{\gamma\Delta}K_{\pi\pi}}{K_{\pi\Delta}}\right) + \frac{K_{\gamma\Delta}}{K_{\pi\Delta}}T_{\pi\pi}$$

Aside:

CGLN
Schwela & Weizel,
Z. Phys 221, 71 (1969)

The form of the $M_{1+}^{3/2}$ multipole amplitude from CGLN in the static limit (neglect O(1/M) terms) is also

$$`A\,(1 + i\,T_{\pi\pi}) + B\,T_{\pi\pi}'$$

with the factors *A* and *B* fixed.

See, for example,
S.N. Yang,
J. Phys. G 11, L205 (1985).
L. Resnick,
Phys. Rev. D2, 1975 (1972).

More elaborate methods, dynamical models, use of the Muskhelishvili-Omnes result will give first term, but the second is generally replaced by a PV integral.

Bonn-Gatchina approach (multi-channel)

This `P-matrix' approach is similar – but the database includes other reactions.

$$A_a = \hat{P}_b\,(\hat{I} - i\hat{\rho}\hat{K})^{-1}_{ba}$$

Photoproduction amplitude

$$P_b = \sum_\alpha \frac{g^{(\alpha)}_{\gamma \mathrm{N}}\, g^{(\alpha)}_b}{M^2_\alpha - s} + \tilde{f}_b$$

P-vector

$$K_{ab} = \sum_\alpha \frac{g^{(\alpha)}_a\, g^{(\alpha)}_b}{M^2_\alpha - s} + f_{ab}$$

Hadronic K-matrix

A modified SAID approach, which incorporates more info from the πN elastic scattering analysis, is based on a Chew-Mandelstam K-matrix approach

$$T_{\alpha\beta} = \sum_{\sigma} [1 - \overline{K}C]^{-1}_{\alpha\sigma} \overline{K}_{\sigma\beta}$$

From πN scattering

leading to

$$T_{\alpha\gamma} = \sum_{\sigma} [1 - \overline{K}C]^{-1}_{\alpha\sigma} \overline{K}_{\sigma\gamma}$$

Fitted to photo data

C = Re C + i ρ

Single-energy fits
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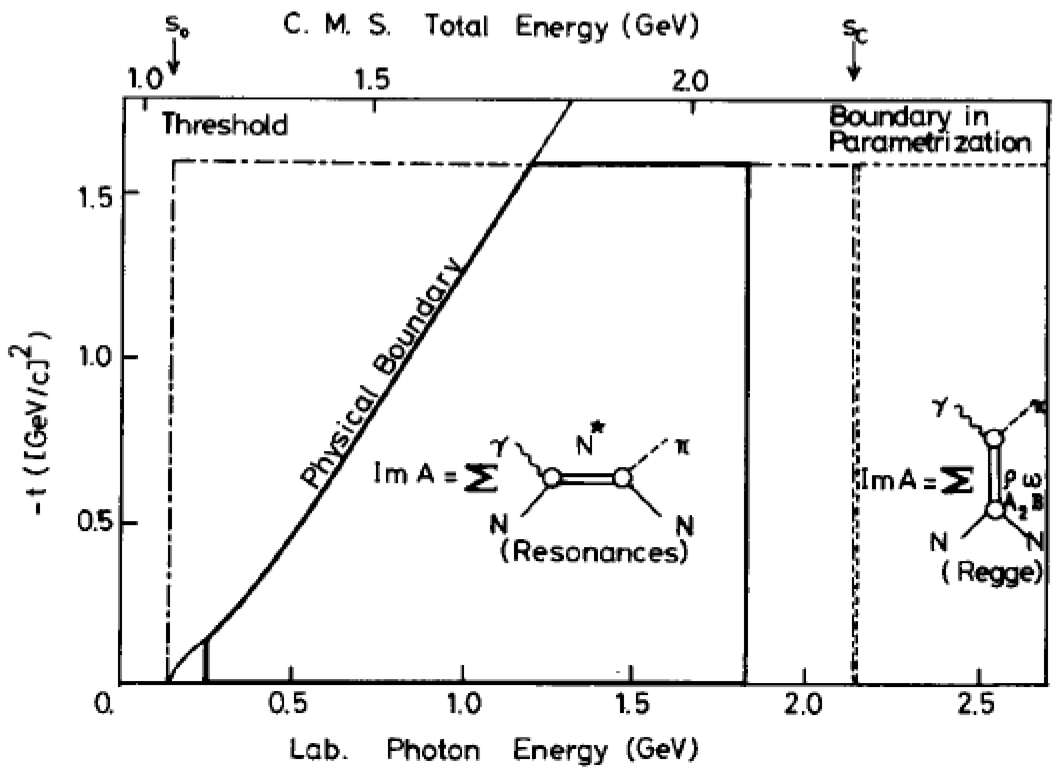
$$\mathrm{Re}\,A_{i,\mathrm{ch}}(s,t) = R_{i,\mathrm{ch}}(s,t) + \frac{1}{\pi} \times P\int_{(m+M)^2}^{\infty} \mathrm{d}s' \left\{ \frac{\mathrm{Im}\,A_{i,\mathrm{ch}}(s',t)}{(s'-s)} + \xi_i \frac{\mathrm{Im}\,A_{i,\overline{\mathrm{ch}}}(s',t)}{(s'-u)} \right\}$$

Arai/Fujii
Crawford
Aznauryan

Parameterize *Im A*
[A/F: 3-ch K-matrix]
calc. *Re A*



Single-energy fits
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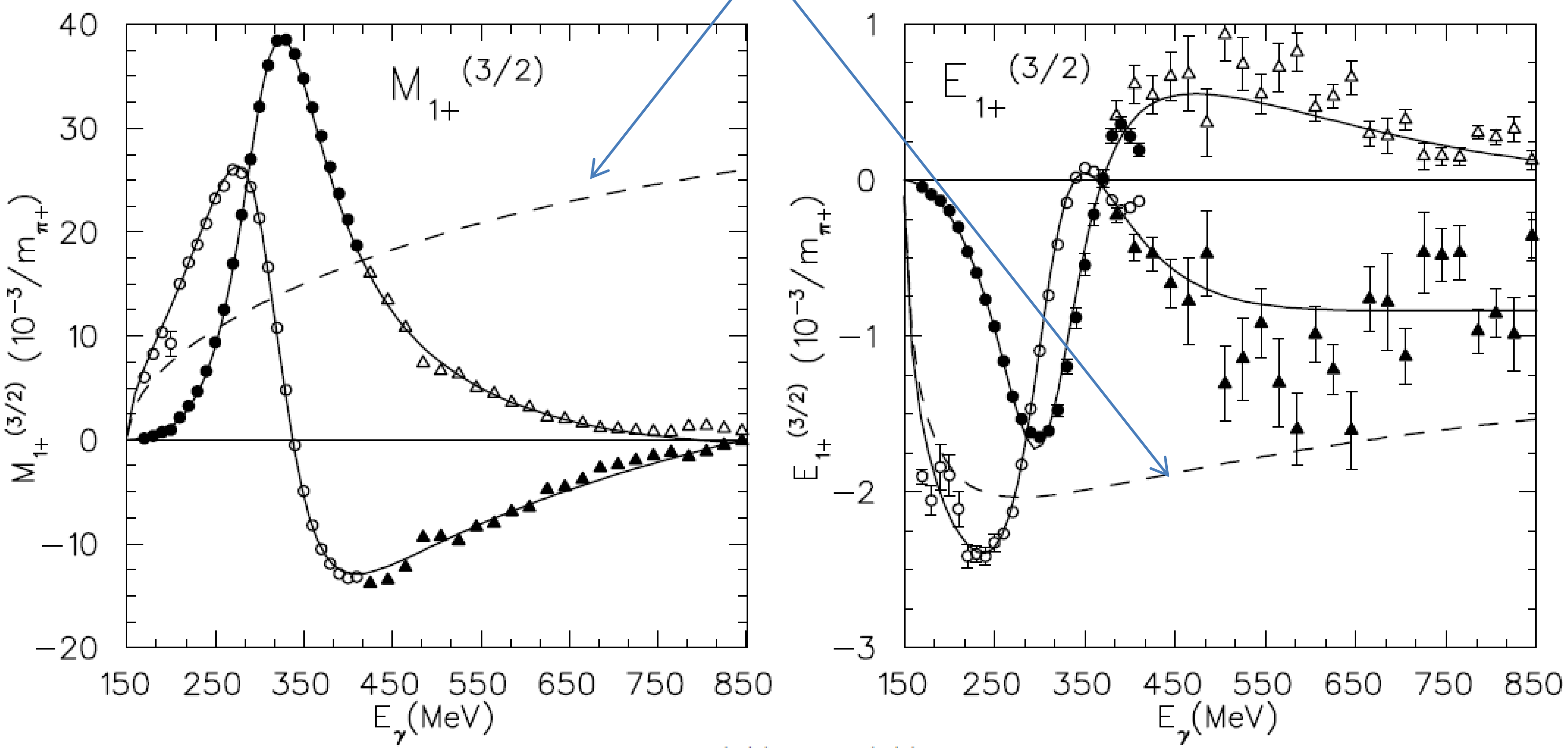
Born terms



Above 500 MeV
Born > largest multipole

Born*(1+ i$T_{\pi\pi}$) gives
almost everything here

Single-energy fits

πN Photoproduction

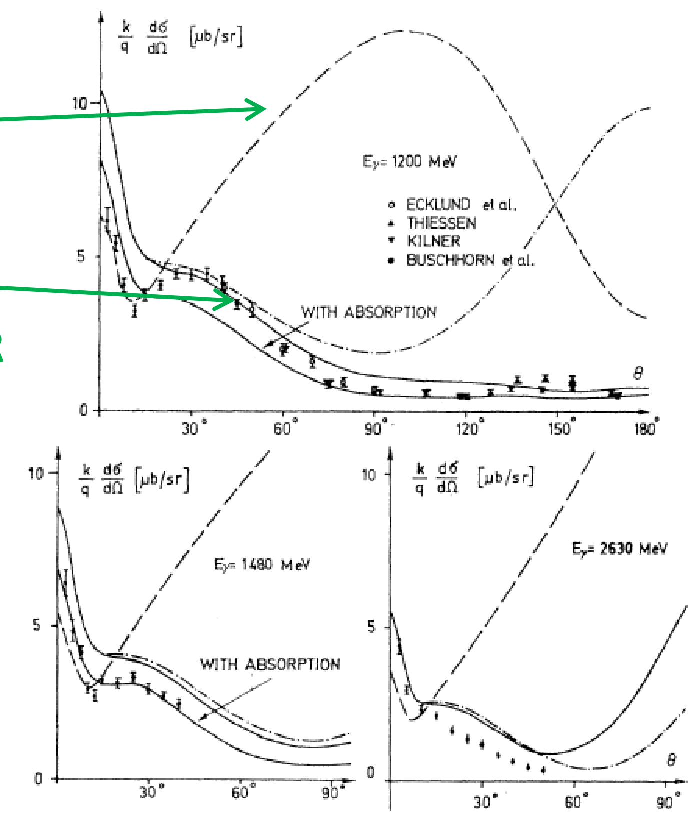
πN scattering

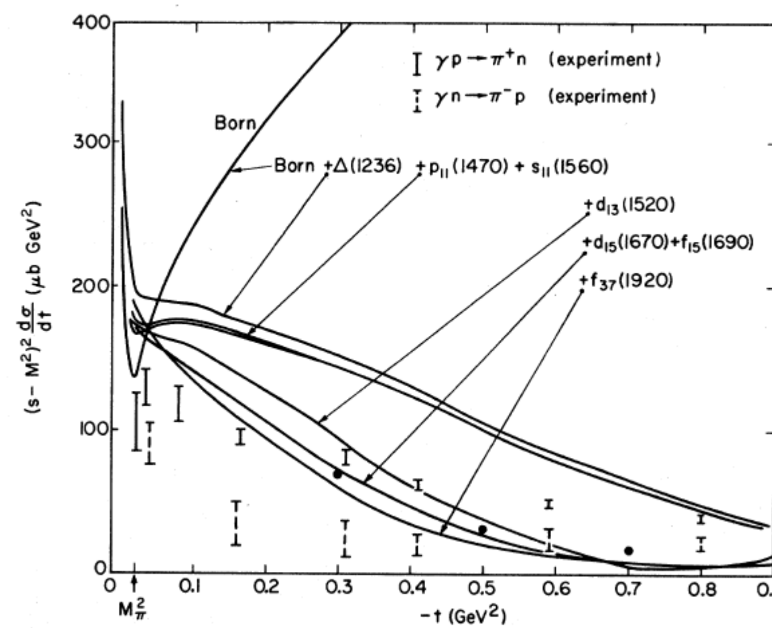
Born

add

Δ(1232)

to RHS of DR





16 GeV

Barbour, Malone, Moorhouse, PRD4,1521(1971)

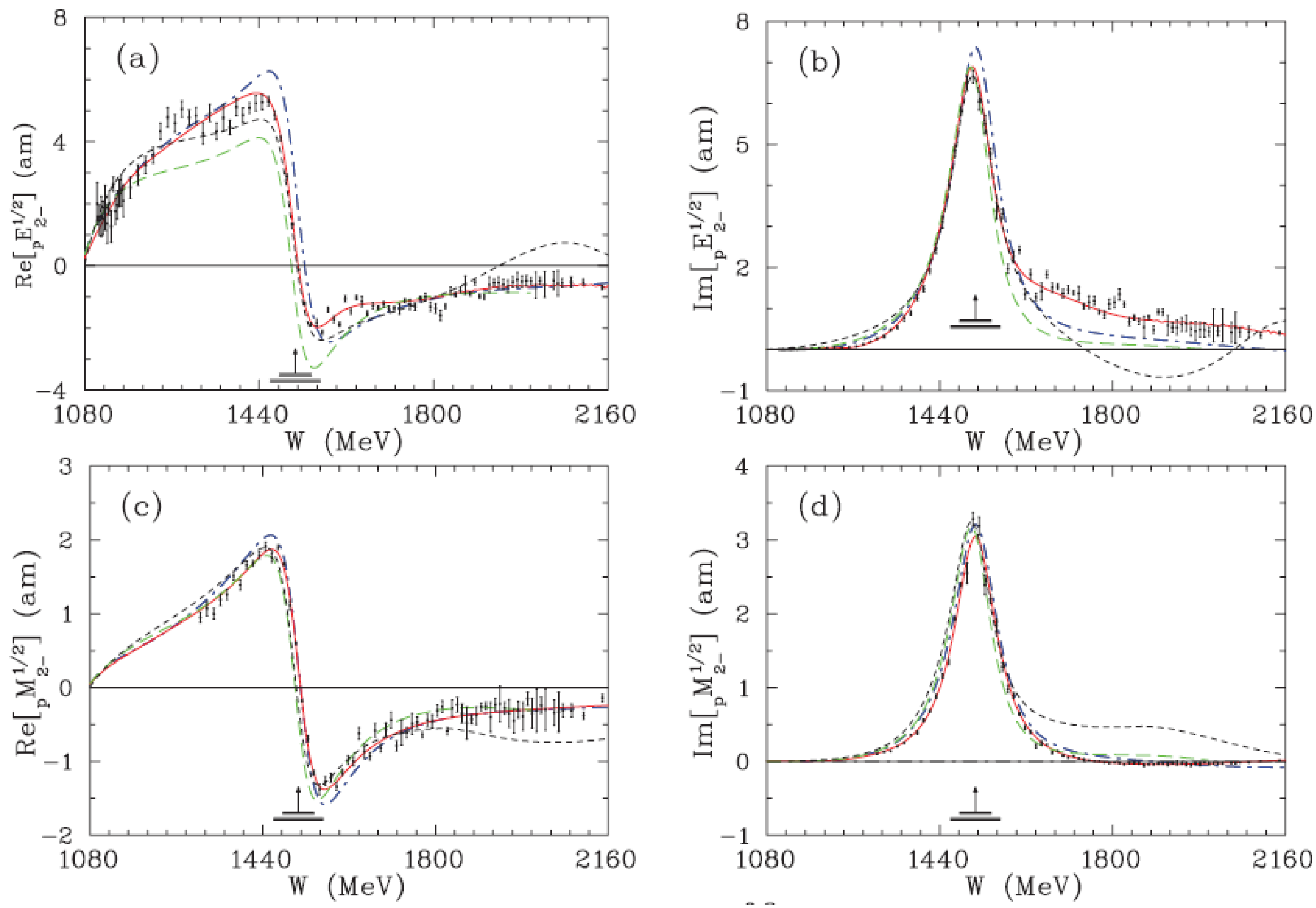
Engels, Schwiderski, Schmidt, PR166,1343(1968).

How well do the
SAID/MAID/BoGa
analyses agree on the multipoles?

Single-energy fits
πN Photoproduction
πN scattering

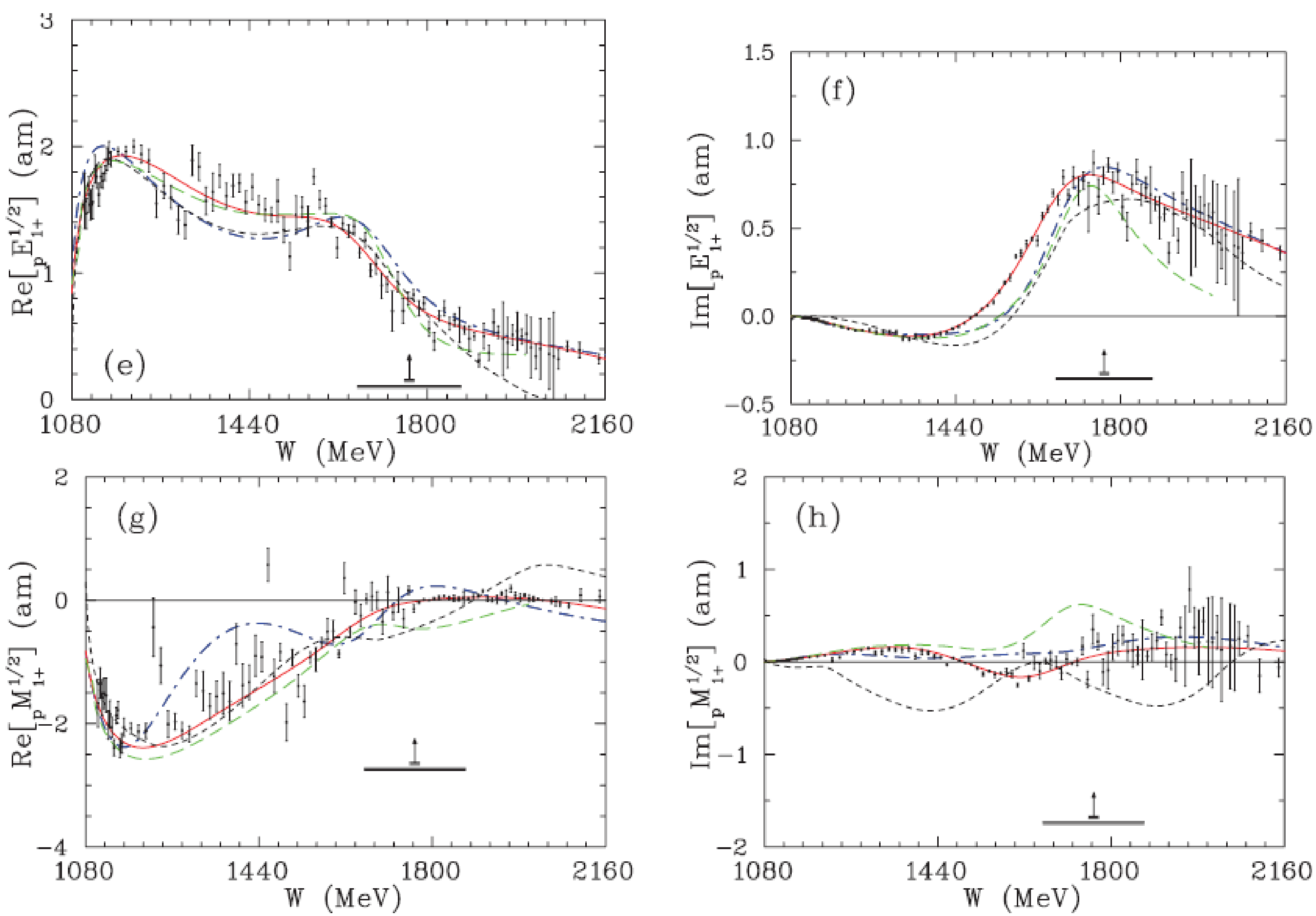
D_{13} (1520) [N(1520)3/2$^-$]

SAID *CM* ———
MAID - - - - - -
BoGa ----------



Single-energy fits
πN Photoproduction
πN scattering

P_{13}(1720) [N(1720)3/2$^+$]



The Baryon Summary Table gives $N\gamma$ branching fractions for those resonances whose couplings are considered to be reasonably well established. The $N\gamma$ partial width Γ_γ is given in terms of the helicity amplitudes $A_{1/2}$ and $A_{3/2}$ by

$$\Gamma_\gamma = \frac{k^2}{\pi} \frac{2M_N}{(2J+1)M_R} \left[|A_{1/2}|^2 + |A_{3/2}|^2 \right] \quad .$$

Here M_N and M_R are the nucleon and resonance masses, J is the resonance spin, and k is the photon c.m. decay momentum.

Must construct $A_{1/2}$, $A_{3/2}$
at the `resonance’ position
i.e. BW parameterization

Single-energy fits
πN Photoproduction
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$$A_{l+}^{1/2} = -\tfrac{1}{2}[(l+2)\bar{E}_{l+} + l\bar{M}_{l+}]\ ,$$
$$A_{l+}^{3/2} = \tfrac{1}{2}\sqrt{l(l+2)}[\bar{E}_{l+} - \bar{M}_{l+}]\ ,$$
$$A_{(l+1)-}^{1/2} = -\tfrac{1}{2}[l\bar{E}_{(l+1)-} - (l+2)\bar{M}_{(l+1)-}]\ ,$$
$$A_{(l+1)-}^{3/2} = -\tfrac{1}{2}\sqrt{l(l+2)}[\bar{E}_{(l+1)-} + \bar{M}_{(l+1)-}]\ ,$$

Resonance contribution to multipole

Resonance part straightforward in
MAID (BW form)
BoGa (K-matrix poles)
but not in SAID (resonances hidden in $T_{\pi\pi}$)

In practice, SAID results extracted assuming
a MAID-like background-resonance separation