Single-energy fits
πN Photoproduction
πN scattering

CMB group has 3 papers – 20 page paper on amalgamation alone
PRD20,2782(1979)
2804
2839

See also Höhler `bible' and Koch & Pietarinen. NPA336,331(1980)

Main analyses:

Karlsruhe-Helsinki, Carnegie-Mellon-Berkeley, GWU/VT

Differ from previous in that resonance structure is not input [i.e. PWA and resonance extraction are separate steps.]

In the KH and CMB analyses, data are initially shifted to common energies/angles to determine the partial-wave amplitudes up to ambiguities.

Numerous dispersion relation constraints are imposed to hopefully choose a unique solution. Once consistent amplitudes are found, resonance info is extracted.

VT/GWU differs in fit strategy and choice of dispersion relation constraints.

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The GWU/VT approach differs in also parameterizing the global energy-dependence via a Chew-Mandelstam ($\mathcal{CM}$) K-matrix .

The method was developed by Basdevant & Berger, and has also been applied to meson-meson, NN, and K^+ p scattering.

ρπ scattering — J.-L. Basdevant and E.L Berger, PRD19, 239 (1979).

NN scattering — B.J. Edwards and G.H Thomas, PRD22, 2772 (1980).

KN scattering — R.A. Arndt and L.D. Roper, PRD31, 2230 (1981).

generating function — J.H. Reid and N.N. Trofimenkoff, J. Math Phys 25, 3540 (1984).

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CNS Center for Nuclear Studies Data Analysis Center

`contact with the lattice'

AN INVESTIGATION OF THE LOW EQUATION AND THE CHEW-MANDELSTAM EQUATIONS

Thesis by

Kenneth Geddes Wilson

In Partial Fulfillment of the Requirements

For the Degree of

Doctor of Philosophy

California Institute of Technology

Pasadena, California

1961

In the GWU/VT approach, a $\mathcal{CM}$ K-matrix was used in the fit

- real data for πN elastic and πN→ηN were fitted
- effective channels πN→πΔ , πN→ρN accounted for remaining inelasticity

$$
\begin{aligned}
T^{-1} &= K^{-1} - i\tilde{\rho} \\
&= (K^{-1} + \mathrm{Re}\, C) - (\mathrm{Re}\, C + i\tilde{\rho}) \\
&= \overline{K}^{-1} - C,
\end{aligned}
$$

where C is the $\mathcal{CM}$ function, fixed by Im C = ρ and a once subtracted DR. Compared to the usual K-matrix

$$
K = \frac{1}{1 - \overline{K}[\mathrm{Re}\, C]} \overline{K}
$$

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In the present fits we calculate the elements of the B^{ij}_α using one subtraction taken at the channel threshold $M_\alpha = (m_{1\alpha} + m_{2\alpha})$:

$$B^{ij}_\alpha(s) = B^{ij}_\alpha(M^2_\alpha) + (s - M^2_\alpha)\int\limits_{m^2_a}^{\infty} \frac{ds'}{\pi} \frac{g^{(R)i}_\alpha \rho_\alpha(s', m_{1\alpha}, m_{2\alpha}) g^{(L)j}_\alpha}{(s' - s - i0)(s' - M^2_\alpha)}.$$

In this case the expression for elements of the $\hat{B}$ matrix can be rewritten as:

$$B^{ij}_\alpha(s) = g^{(R)i}_a \left(b^\alpha + (s - M^2_\alpha)\int\limits_{m^2_a}^{\infty} \frac{ds'}{\pi} \frac{\rho_\alpha(s', m_{1\alpha}, m_{2\alpha})}{(s' - s - i0)(s' - M^2_\alpha)} \right) g^{(L)j}_\beta = g^{(R)i}_a B_\alpha g^{(L)j}_\beta$$

and D-matrix method equivalent to the K-matrix method with loop diagram with real part taken into account:

$$A = \hat{K}(I - \hat{B}\hat{K})^{-1} \qquad B_{\alpha\beta} = \delta_{\alpha\beta} B_\alpha$$

A. Sarantsev talk

Poles may appear in the usual K-matrix without explicit inclusion in the $\mathcal{CM}$ K-matrix.

Fits were done assuming a simple polynomial behavior for the $\mathcal{CM}$ K-matrix [except for the Δ(1232)] i.e. no resonances were assumed.

Poles were found in the T-matrix corresponding to the dominant 4-star states (but not all those seen by KH and CMB).

Breit-Wigner states added ‘by hand’ to see if fit would show significant improvement (found candidates in S_{11} and F_{15} but did not reproduce the KH and CMB sets).

Single-energy fits

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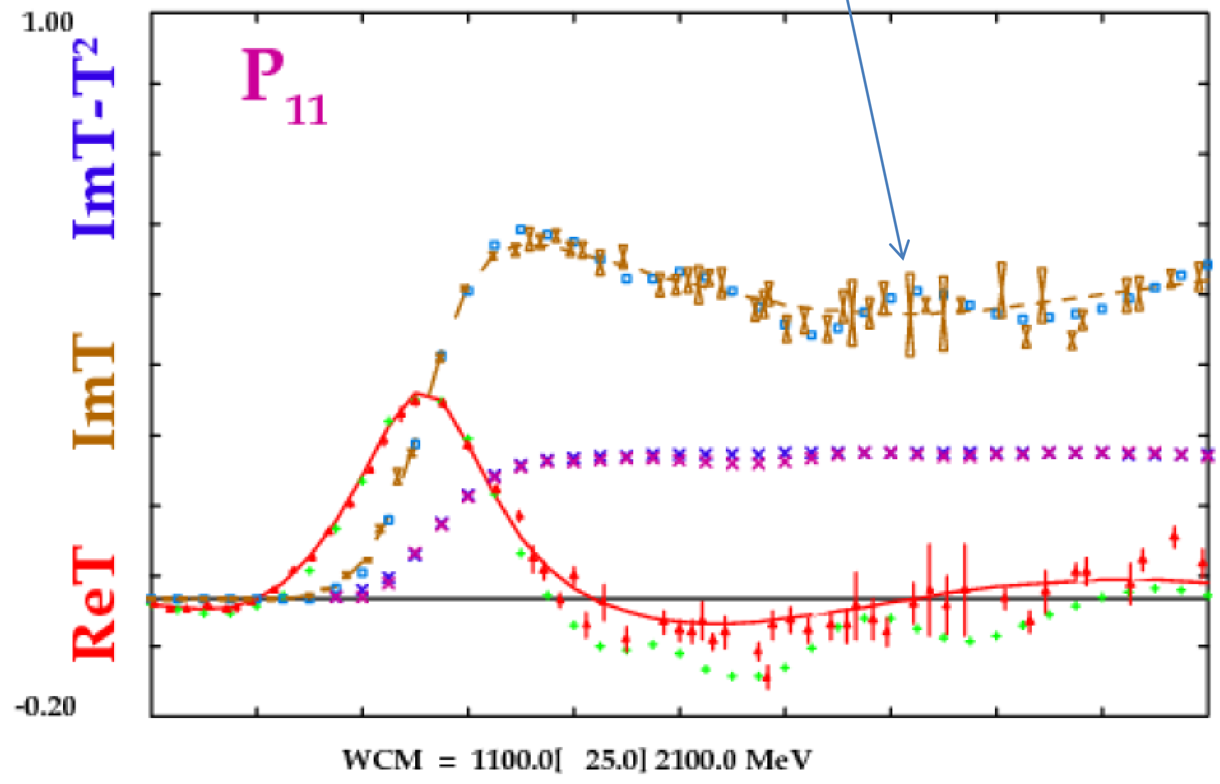
Compare GW (red/yellow) vs KH (green/blue)

extra wiggles near center of Argand plot



Single-energy fits
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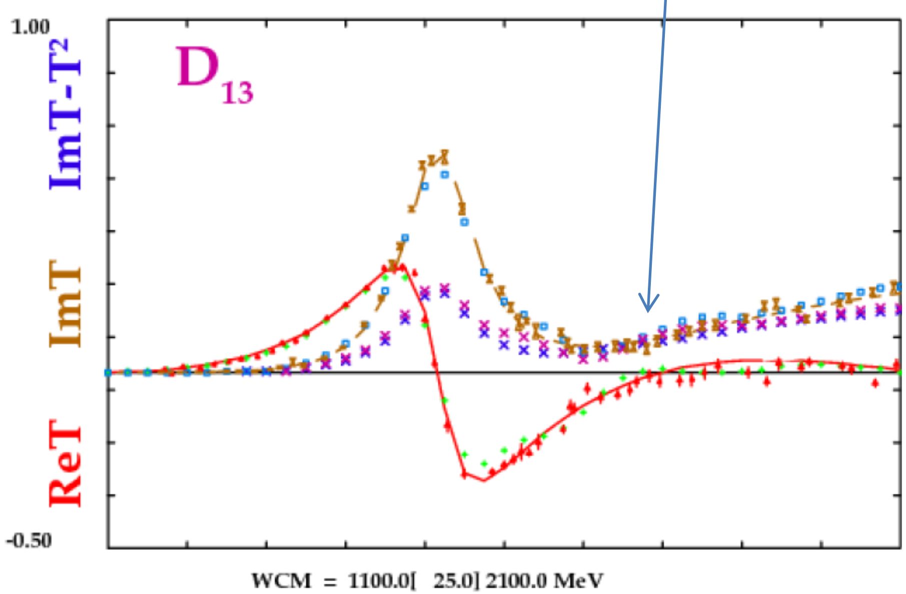
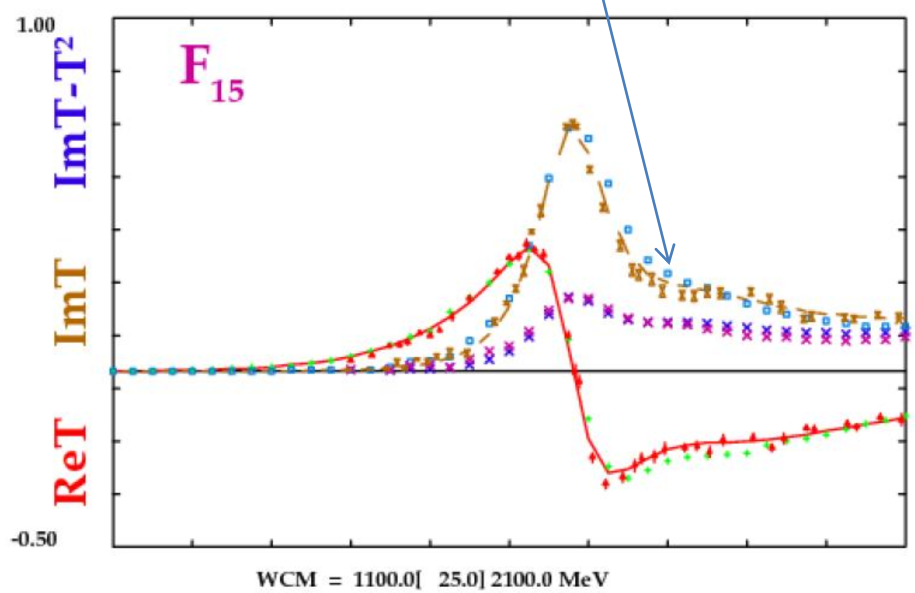
Compare GW (red/yellow) vs KH (green/blue)

KH state near
1730 MeV

extra structure may be
a second F_{15} state





Single-energy fits

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2008 PDG

change in mass and name

2012 PDG

$N(2000)\ F_{15}$

$I(J^P) = \frac{1}{2}(\frac{5}{2}^+)$ Status: ∗∗

OMITTED FROM SUMMARY TABLE

Older results have been retained simply because there is little information at all about this possible state.

N(2000) BREIT-WIGNER MASS

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
≈ 2000 OUR ESTIMATE				
1817.7	ARNDT	06	DPWA	$\pi N \to \pi N, \eta N$
1903 ±87	MANLEY	92	IPWA	$\pi N \to \pi N\ \&\ N\pi\pi$
1882 ±10	HOEHLER	79	IPWA	$\pi N \to \pi N$
2025	AYED	76	IPWA	$\pi N \to \pi N$
1970	[1] LANGBEIN	73	IPWA	$\pi N \to \Sigma K$ (sol. 2)
2175	ALMEHED	72	IPWA	$\pi N \to \pi N$
1930	DEANS	72	MPWA	$\gamma p \to \Lambda K$ (sol. D)
• • • We do not use the following data for averages, fits, limits, etc. • • •				
1814	ARNDT	95	DPWA	$\pi N \to N\pi$

$N(1860)\ 5/2^+$

$I(J^P) = \frac{1}{2}(\frac{5}{2}^+)$ Status: ∗∗

OMITTED FROM SUMMARY TABLE

Before the 2012 *Review*, all the evidence for a $J^P = 5/2^+$ state with a mass above 1800 MeV was filed under a two-star N(2000). There is now some evidence from ANISOVICH 12A for two $5/2^+$ states in this region, so we have split the older data (according to mass) between two two-star $5/2^+$ states, an N(1860) and an N(2000).

N(1860) BREIT-WIGNER MASS

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
1820 to 1960 (≈ 1860) OUR ESTIMATE				
1860^{+120}_{-60}	ANISOVICH	12A	DPWA	Multichannel
1817.7	ARNDT	06	DPWA	$\pi N \to \pi N, \eta N$
1903 ± 87	MANLEY	92	IPWA	$\pi N \to \pi N\ \&\ N\pi\pi$
1882 ± 10	HOEHLER	79	IPWA	$\pi N \to \pi N$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
1814	ARNDT	95	DPWA	$\pi N \to N\pi$

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New names and new states (mainly due to BoGa fits)

New headers are an attempt at disambiguation

$N(2000)\ 5/2^+$

$I(J^P) = \frac{1}{2}(\frac{5}{2}^+)$ Status: ∗∗

OMITTED FROM SUMMARY TABLE

Before the 2012 *Review*, all the evidence for a $J^P = 5/2^+$ state with a mass above 1800 MeV was filed under a two-star $N(2000)$. There is now some evidence from ANISOVICH 12A for two $5/2^+$ states in this region, so we have split the older data (according to mass) between two two-star $5/2^+$ states, an $N(1860)$ and an $N(2000)$.

N(2000) BREIT-WIGNER MASS

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
1950 to 2150 (≈ 2050) OUR ESTIMATE				
[1850 to 1950 MeV OUR 2011 ESTIMATE]				
2090±120	ANISOVICH	12A	DPWA	Multichannel
2025	AYED	76	IPWA	$\pi N \to \pi N$
1970	[1] LANGBEIN	73	IPWA	$\pi N \to \Sigma K$ (sol. 2)
2175	ALMEHED	72	IPWA	$\pi N \to \pi N$
1930	DEANS	72	MPWA	$\gamma p \to \Lambda K$ (sol. D)

Possible objection to GW/VT fit – parametrizing energy dependence results in ‘smoothing’.

Exercise: add explicit poles to the *CM* K-matrix

- may result in more T-matrix poles
- will test the model-dependence of the πN amplitudes.
- more consistent approach compared to the addition of BW structures by hand.

CM K-matrix pole → regular K-matrix pole → T-matrix pole

Single-energy fits
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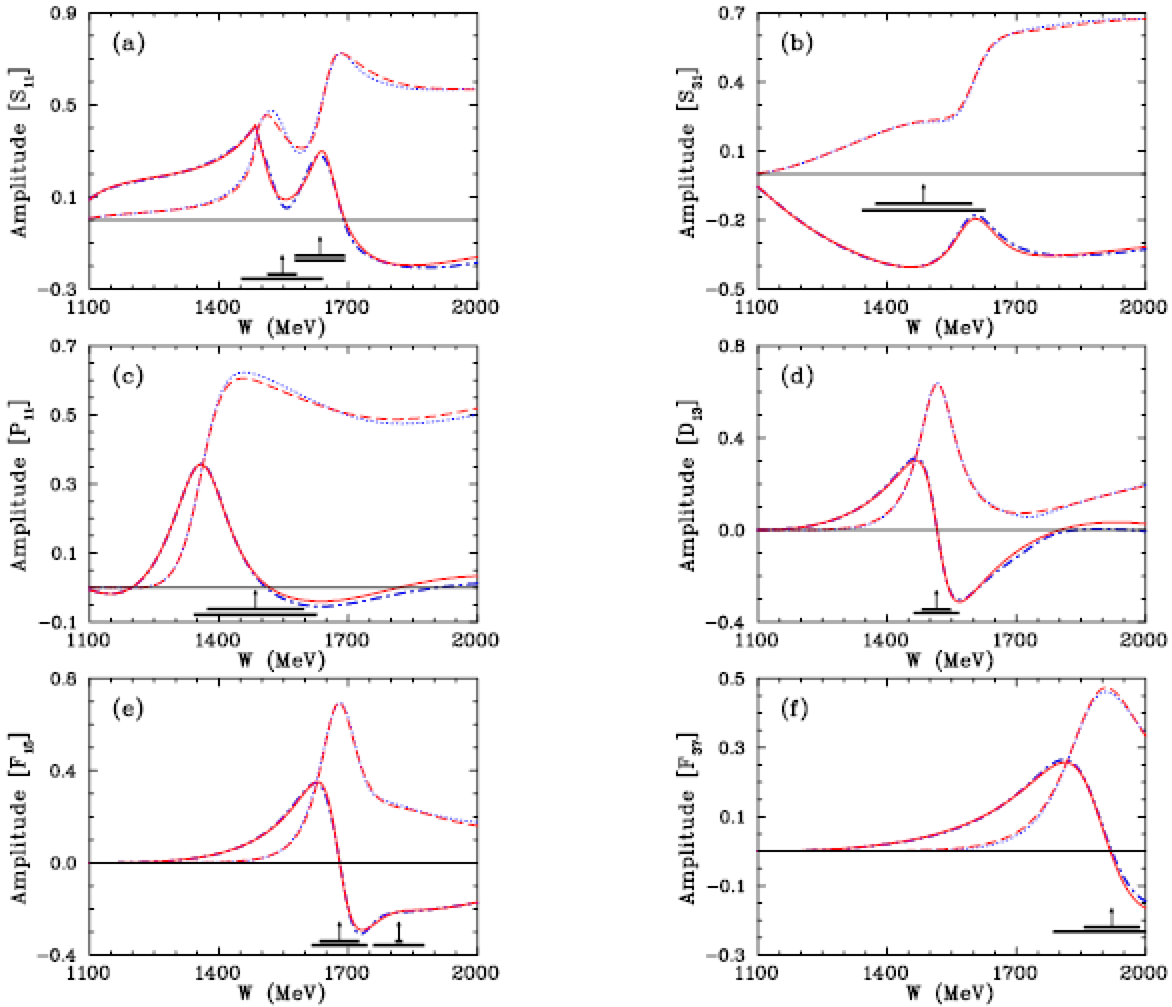
Fits with/without explicit poles are not very different (a good thing)

Where did the added poles go?



Eigenphases

$$T = K\,(1 - iK)^{-1}.$$

The real symmetric K matrix is diagonalized by an orthogonal transformation, U as

$$K_D = U^T\ K\ U.$$

This matrix also diagonalizes the T matrix, and therefore the S matrix, defined as $S = 1 + 2iT$. Since S is a unitary matrix,

$$(S_D)_{ij} = U^T S U = \delta_{ij}\, e^{2i\delta_i}$$

where the δ_i are eigenphases. Using the relation between K and T matrices above, we have

$$(K_D)_{ij} = i(1 - S_D)(1 + S_D)^{-1} = \delta_{ij} \tan \delta_i.$$

Single-energy fits

πN Photoproduction

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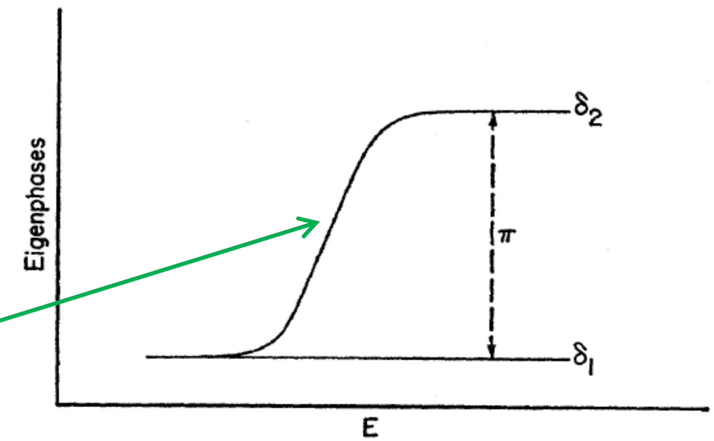
C. Goebel, K. McVoy, PR164,1932(1967)

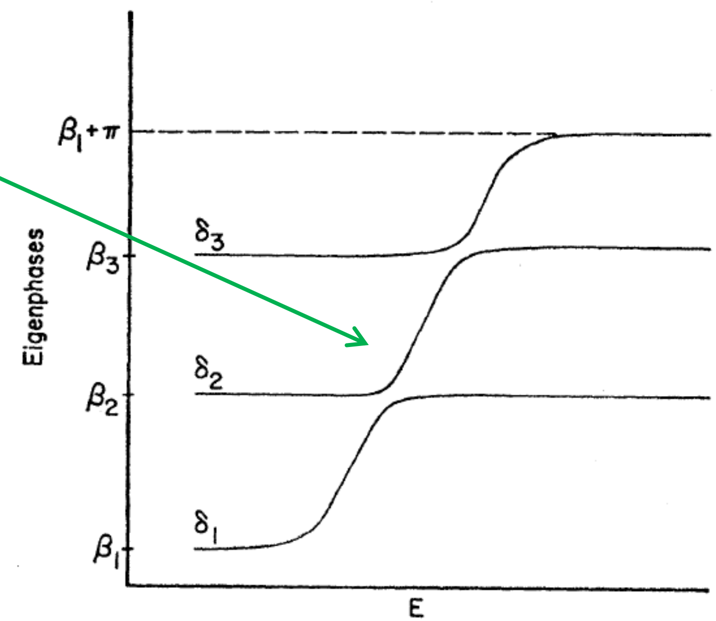
H. Weidenmuller, PLB24,441(1967)

Resonance behavior

Simple



Not so simple



Single-energy fits

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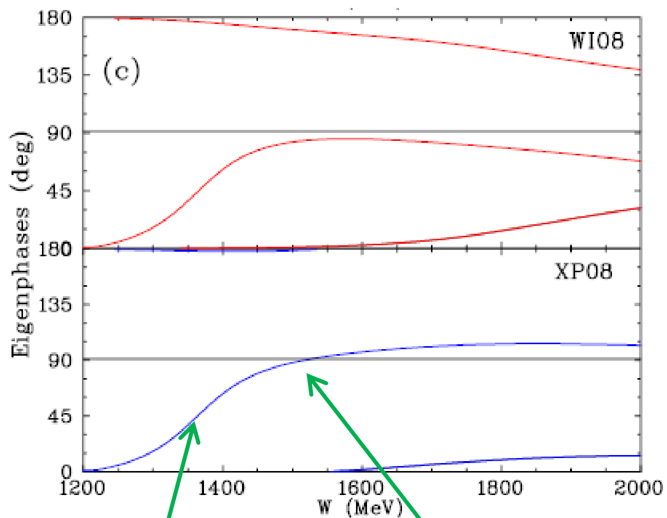
P_{11} (1440) [N(1440)1/2$^+$]

No CM pole

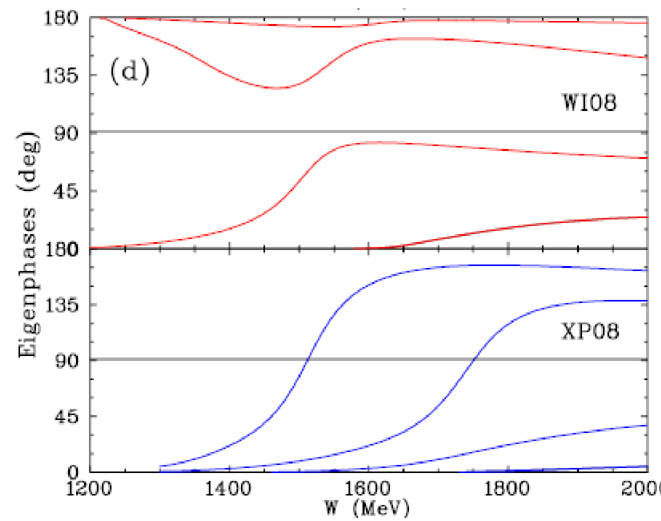
CM pole



Fastest variation
(T-matrix pole)

90 deg crossing
(K-matrix pole)

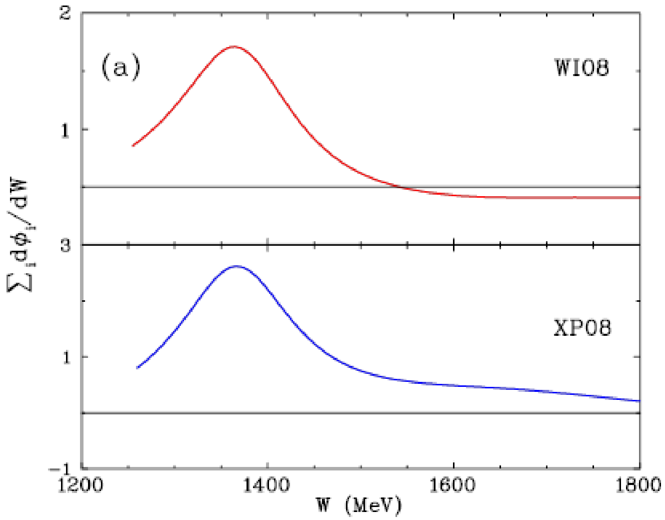
D_{13} (1520) [N(1520)3/2$^-$]



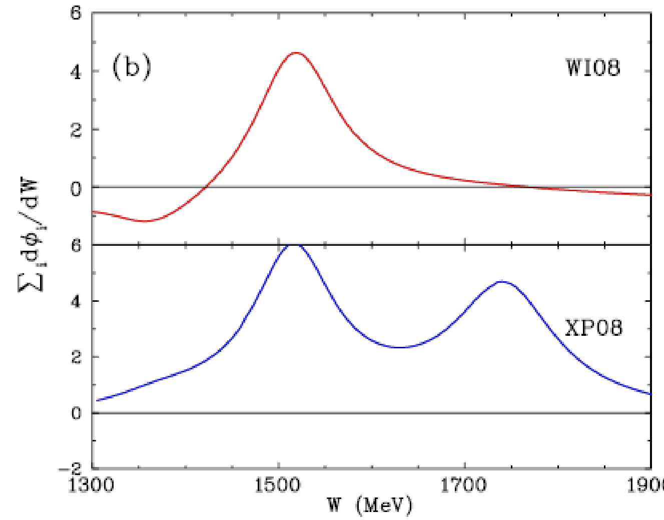
In XP08 solution (CM poles) must have
at least one 90 degree crossing

Single-energy fits
πN Photoproduction
πN scattering

P_{11} (1440) [N(1440)1/2^{+}]



D_{13} (1520) [N(1520)3/2^{-}]



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Turn off pole/non-pole contributions

added poles replaced states originally produced by the polynomial CM K-matrix

Better to find these `missing' states in other reactions.