Modification of QED cross-sections at large impact parameters

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Observation and main idea

- In 1980, at VEPP-4 (INP e-e+ collider), a cross-section of e-e+ bremsstrahlung was measured as ~ 30% lower than QED number (Yuriy Tikhonov).

- In the following discussions, Slava and Yuriy independently suggested a possible explanation:

- The QED cross-section contains a big logarithm:

$$d\sigma \propto \int \frac{dq_{\perp}}{q_{\perp}} = \ln(m_e / q_{\perp\min}) ; \quad q_{\perp\min} \simeq q_{\min} / \gamma = \omega / \gamma^3$$

- If the beam size a is small enough, $a q_{\perp\min} < 1$, then the lower limit of the integration over the momentum $q_{\perp}$ will be limited rather by a than $q_{\perp\min}$

Some history

- When Slava discussed this issue, Arkady Vainstein suggested an idea, how the beam size can be accounted in QED calculation. After that, Slava asked me, then his PhD student, make it as a useful exercise. We did that together, published as an INP preprint (Russian only), and did not ever return to that issue.

- Independently, this problem was considered by V. Bayer, V. Katkov and V. Strakhovenko (INP) ; they got the same result by a different method.

- Later on, the problem has been extensively studied by G. Kotkin and V. Serbo (Novosibirsk) with more details and applications.

- Below, I am essentially recollecting Slava’s and mine old preprint.

Some related papers

ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ
СО АН СССР

А.В.Буров, Я.С.Дербенёв

ЭФФЕКТ КОНЕЧНОСТИ ПОПЕРЕЧНЫХ РАЗМЕРОВ ВСТРЕЧНЫХ ПУЧКОВ В ТОРМОЗНОМ ИЗЛУЧЕНИИ

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Role of geometrical factors in bremsstrahlung in colliding e^-e^+ beams

V. N. Baĭer, V. M. Katkov, and V. M. Strakhovenko

Institute of Nuclear Physics, Siberian Division, USSR Academy of Sciences
(Submitted 20 November 1981)
Yad. Fiz. **36**, 163–168 (July 1982)

The process of single bremsstrahlung is discussed with allowance for the finite transverse dimensions of the colliding beams. The cross section for the process is calculated with relativistic accuracy for arbitrary beam shape. The case of a Gaussian distribution of the particles in the beam is analyzed in detail.

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Beam-size effect and particle losses at Super$B$ factory developed in Italy

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Abstract: In the colliders, the macroscopically large impact parameters give a substantial contribution to the standard cross section of the $e^+e^- \to e^+e^-\gamma$ process. These impact parameters may be much larger than the transverse sizes of the colliding bunches. It means that the standard cross section of this process has to be substantially modified. In the present paper such a beam-size effect is calculated for bremsstrahlung at Super$B$ factory developed in Italy. We find out that this effect reduces beam losses due to bremsstrahlung by about 40%. We perform a critical comparison of our result with that presented in the Conceptual Design Report of the Italian Super$B$ factory [3].

Beam-size reduction of the cross section

- This limitation relates to high impact parameters, and so can be considered quasi-classically.

- Indeed, in e-e+ collisions, a photon with frequency ω can be emitted only for impact parameters

$$\rho \le \rho_{\max} = \gamma^3 / \omega$$

- Beam-size limitation does not happen, if

$$\rho_{\max} \ll a$$

- Otherwise, intensity of bremsstrahlung at that frequency is exponentially suppressed.

Quasiclassical matching

- Let the entire space of impact parameters be split on 2 areas:

1) $\rho \le \rho_1$ and 2) $\rho \ge \rho_1$, with $\frac{\hbar}{mc} \ll \rho_1 \ll a$

- For infinite beam size, number of photons emitted by a single electron per revolution would be

$$dN = d\sigma_0 \, n_+$$

where n_+ is transverse density of positrons, and $d\sigma_0$ - conventional QED cross-section.

- From another side, $dN = dN_1 + dN_2$ with $dN_2 = n_+ \int_2 d^2\rho \, dW_0(\rho)$

$dW_0(\rho)$ - bremsstrahlung probability for 2-particle collision.

- Thus, the 1st area contribution to bremsstrahlung:

$$dN_1 = n_+ d\sigma_0 - n_+ \int_2 d^2\rho \, dW_0(\rho)$$

Effective cross-section

- Contribution of the 2nd area:

$$dN_2(\mathbf{r}) = \int_2 d^2\boldsymbol{\rho}\, n_+(\mathbf{r}+\boldsymbol{\rho})\, dW_0(\rho)$$

- So, the total number of emitted photons per a single electron is

$$dN(\mathbf{r}) = d\sigma_0 n_+(\mathbf{r}) + \int d^2\boldsymbol{\rho}\; dW_0(\boldsymbol{\rho})\left[n_+(\mathbf{r}+\boldsymbol{\rho}) - n_+(\mathbf{r})\right]$$

- Yielding a total number of e- emitted photons per beam-beam collision:

$$dN_\Sigma = \int dN(\mathbf{r}) n_-(\mathbf{r}) d^2\mathbf{r} \equiv L\, d\sigma_{\text{eff}}$$

$$L = \int n_+(\mathbf{r}) n_-(\mathbf{r}) d^2\mathbf{r}$$

$$d\sigma_{\text{eff}} = d\sigma_0 - \int d^2\boldsymbol{\rho}\; dW_0(\rho) g(\rho)$$

$$g(\rho) = 1 - \frac{\int d^2\mathbf{r}\; n_-(\mathbf{r}) n_+(\mathbf{r}+\boldsymbol{\rho})}{\int n_+(\mathbf{r}) n_-(\mathbf{r}) d^2\mathbf{r}}$$

Scattering probability

- The probability of Compton scattering of the equivalent photon is

$$dW_0(\rho,\omega) = \int_q dn_q(\rho)\, d\sigma_q(\omega)$$

$$q\, dn_q(\rho) = \frac{|E_q|^2}{4\pi^2} dq\,; \quad E_q = \int_{-\infty}^{\infty} E(\rho,t)\exp(iqt)dt$$

flux of effective photons

$$d\sigma_q(\omega) = \frac{\pi r_e^2}{2}\frac{m^2 d\omega}{\varepsilon^2 q}\left(v + 4\frac{q_{\min}^2}{q^2} - 4\frac{q_{\min}}{q}\right)$$

$$v = \frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon}; \quad \varepsilon' = \varepsilon - \omega; \quad q_{\min} = \frac{m^2\omega}{4\varepsilon\varepsilon'}$$

conventional radiation cross-section (Klein-Nishina)

From here:

$$dW(\rho,\omega) = \frac{\alpha r_e^2}{2\pi}\frac{d\omega}{\gamma^4}\int_{q_{\min}}^{\infty} dq\, K_1^2(q\rho/\gamma)\left(v + 4\frac{q_{\min}^2}{q^2} - 4\frac{q_{\min}}{q}\right)$$

Effective cross-section: results

- For $\ln\left(\frac{\gamma}{q_{\min} a}\right) \gg 1$, the reduction of the cross-section is independent on the details of the distribution function. On the log approximation,

$$\Delta d\sigma = d\sigma_0 - d\sigma_{\text{eff}} = 4\alpha r_e^2 \frac{d\omega}{\omega} \frac{\varepsilon'}{\varepsilon}\left(v - \frac{2}{3}\right) \ln\left(\frac{\gamma}{q_{\min} a}\right)$$

$$d\sigma_{\text{eff}} = 4\alpha r_e^2 \frac{d\omega}{\omega} \frac{\varepsilon'}{\varepsilon}\left(v - \frac{2}{3}\right) \ln(ma)$$

- For Gaussian profile, non-log correction is calculated as:

$$d\sigma_{\text{eff}} = 4\alpha r_e^2 \frac{d\omega}{\omega} \frac{\varepsilon'}{\varepsilon}\left(v - \frac{2}{3}\right)\left[\ln\left(\frac{m a_x a_y}{a_x + a_y}\right) + \ln 2 + \frac{0.557}{2} + \frac{v - 5/9}{v - 2/3}\right]$$

Effects of magnetic field

- For pure Coulomb field at impact parameter a, the length of formation is $l_C \sim \gamma a$

- For synchrotron radiation, the formation length is

$$l_{SR} \sim \frac{R_0}{\gamma}; \quad q \simeq \frac{\gamma^3}{R_0}$$

- Magnetic field may be important if $l_{SR} \ll l_C \Rightarrow R_0 \ll \gamma^2 a$
- Similar calculations show that in this case the effective cross-section is modified as

$$d\sigma = d\sigma_C + d\sigma_{SR}$$

$$d\sigma_C \approx d\Sigma \ln(mR_0/\gamma^2); \quad d\sigma_{SR} \approx d\Sigma \sqrt{\frac{\gamma^2 a}{R_0}}$$

$$d\Sigma = \frac{16}{3}\alpha r_e^2 \frac{d\omega}{\omega}$$

Collective effects

- Due to its final length $a_{\|}$, positron bunch as a whole contributes to the electron-emitted photons as (Kotkin & Serbo, 2004):

$$d\sigma_{\text{coll}} \simeq N d\Sigma \exp\left(-(q_{\min} a_{\|})^2\right)$$

- If the beam temperature is so low, that the Debye radius (beam frame) is smaller than the beam radius,

$$r_D \ll \min(a_x, a_y)\,,$$

than the formulae above modify as $a \to r_D$.

What is most important

Apart of its practical interest and theoretical elegance, this solution reminded me an important lesson I learned as a PhD student of Slava and since then try to follow all my life.

Mathematics, as it works in Physics, is not just a sequence of sophisticated operations with symbols. In fact, it gets its power from that primary background, which cannot be expressed in any formal way, - from our insight. Science is a response to a human quest for understanding the Universe - and realization of cosmic power of human mind.

In particular, this specific problem was essentially solved before I started doing any math. It was solved even before Arkady Vainstein suggested his idea of quasi-classical matching – with all respect to that. Essentially, the problem was solved, when Slava saw its solution – may be, with writing a couple of simple formulas. I am tremendously thankful to Slava for letting me see his multiple insights and participate in their extremely interesting discussions and implementations during these 30 years of our cooperation.