Phase space - some recent experiments in beam physics

Beam Adapters or Phase Space Converters
Round to flat beam transformer FBT
& transverse to longitudinal emittance exchange EEX

H. Edwards

Magic

to transform or produce by or as if by magic

- The Breakaway from conventional uncoupled systems (x,x’); (y, y’); (z, z’)
- Round magnetized beam -> flat beam
  - For $\sigma^2=<x^2>=<y^2>$, A transformer ratio
  - $\varepsilon(x, x')$; $\varepsilon(y,y')$ -> $A\varepsilon(\sigma,\sigma)$; $(1/A)\ \varepsilon(\sigma',\sigma')$;
- Longitudinal <->transverse emittance exchange (EEX)
  - $\varepsilon(x, x')$ -> $\varepsilon(z,z')$ and $\varepsilon(z,z')$ -> $\varepsilon(x, x')$

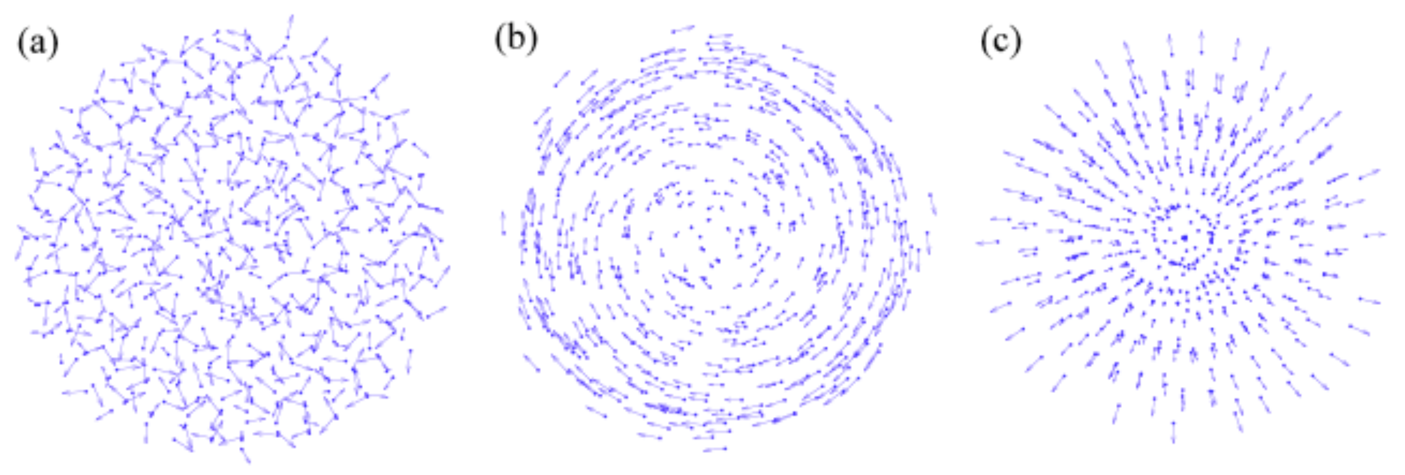
Round to Flat Beam Transform (FBT)

- Conventional RF gun wisdom- The Bz field on the cathode must be ~zero or it will contribute to beam emittance

$$\varepsilon_n = \frac{eB_c r_c^{\,2}}{8mc^2}$$

- Derbenev wisdom- Make the Bz field large on the cathode, solenoid field- Make an angular momentum dominated beam, vortex. It is a coherent motion

Emittance   angular momentum   space charge



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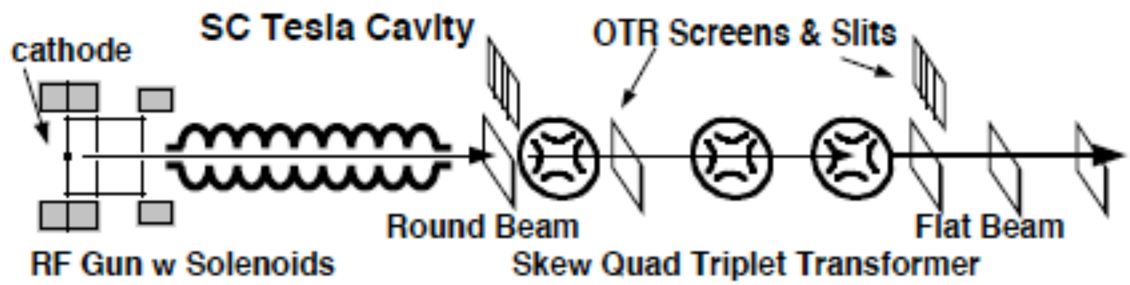
Special Thanks to Yin E Sun for many view graphs & illustrations for this talk

A simple representation of the idea



$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_0 = \begin{pmatrix} x_0 \\ -ky_0 \\ y_0 \\ kx_0 \end{pmatrix}$$

$$k \equiv \frac{1}{2}\frac{B_z}{(p_0/e)}.$$

Solenoid end field provides twist

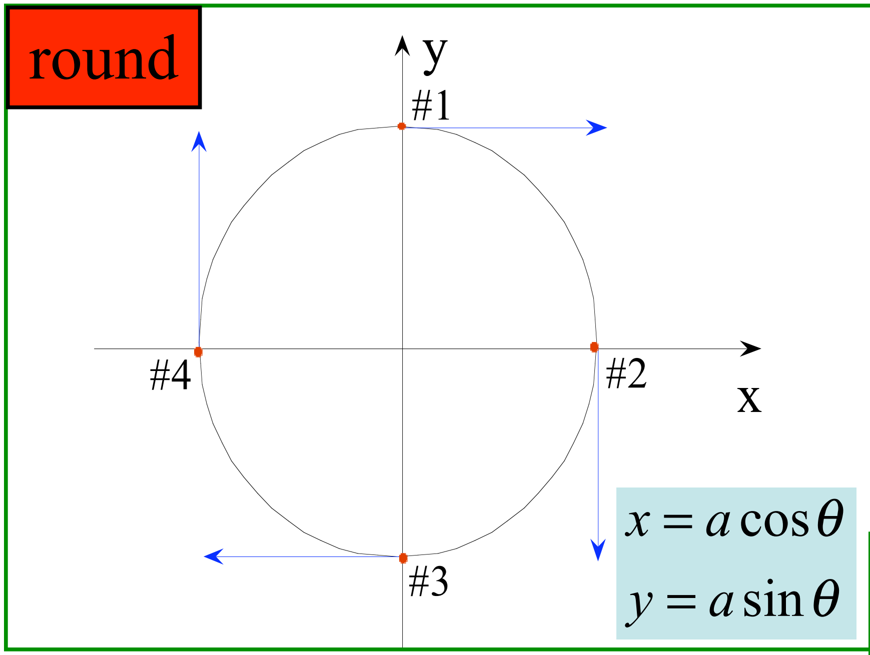
$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \beta \\ 0 & 0 & -\frac{1}{\beta} & 0 \end{pmatrix} \begin{pmatrix} x_0 \\ -ky_0 \\ y_0 \\ kx_0 \end{pmatrix}$$

$$= \begin{pmatrix} x_0 \\ -ky_0 \\ k\beta x_0 \\ -\frac{1}{\beta}y_0 \end{pmatrix} \overset{\beta=1/k}{\longrightarrow} \begin{pmatrix} x_0 \\ -ky_0 \\ x_0 \\ -ky_0 \end{pmatrix}.$$

Quad transformer with phase advance in y $\pi/2$ different from x

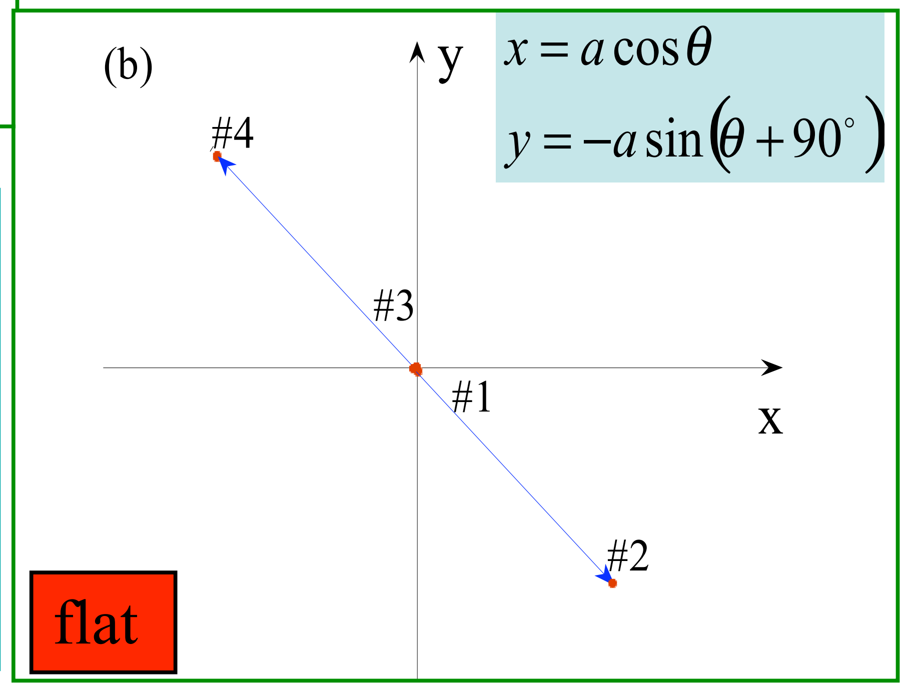
> line at 45 deg, does not rotate

Round-to-flat beam transformation: simple-minded model



90 deg rotation between x and y

$$\begin{bmatrix} x \\ p_x \\ y \\ p_y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ p_{0x} \\ y_0 \\ p_{0y} \end{bmatrix}$$



Particles are aligned diagonally; an additional 45º rotation (*skew*) for both x and y will align the particles along x or y-axis.

Derbenev's more complete transformation

$$S = R_{45}^{-1}\begin{pmatrix} M & 0 \\ 0 & N \end{pmatrix} R_{45}, \quad where\ R\ is\ 45\deg\ rotation, \quad M, N\ 2x2\ matries$$

$$M = -N \bullet F, \quad F = \begin{pmatrix} 0 & \frac{1}{k_1} \\ -k_1 & 0 \end{pmatrix}, \quad k_1 = \sqrt{k^2 + \frac{\sigma'^2_0}{\sigma_0^2}}$$

The betatron phase advance between M and N must be $\pi/2$
The β's, α's same at both ends

The resulting emittance

$$for\ \varepsilon_0=\sigma_0\sigma'_0,\quad \sigma_0{}^2=\left\langle x_0{}^2\right\rangle=\left\langle y_0{}^2\right\rangle,\quad etc\ for\quad \sigma'_0$$

$$\varepsilon_x=\sqrt{\varepsilon_0{}^2+(k\sigma_0{}^2)^2}+k\sigma_0{}^2=\sigma_0{}^2\left[\sqrt{\frac{\sigma'^2_0}{\sigma_0{}^2}+k^2}+k\right]\approx 2k\sigma_0{}^2$$

$$\varepsilon_y=\sqrt{\varepsilon_0{}^2+(k\sigma_0{}^2)^2}-k\sigma_0{}^2=\sigma_0{}^2\left[\sqrt{\frac{\sigma'^2_0}{\sigma_0{}^2}+k^2}-k\right]\approx \frac{\varepsilon_0{}^2}{2k\sigma_0{}^2}=\frac{\sigma'_0{}^2}{2k}$$

$$k\ the\ matching\ condition\quad k=\frac{eB_{z,c}}{2p_z}=\frac{1}{\beta},$$

$$"\approx"\ for\ k\sigma_0{}^2\gg\varepsilon_0,\quad k\gg\frac{\sigma'_0}{\sigma_0}$$

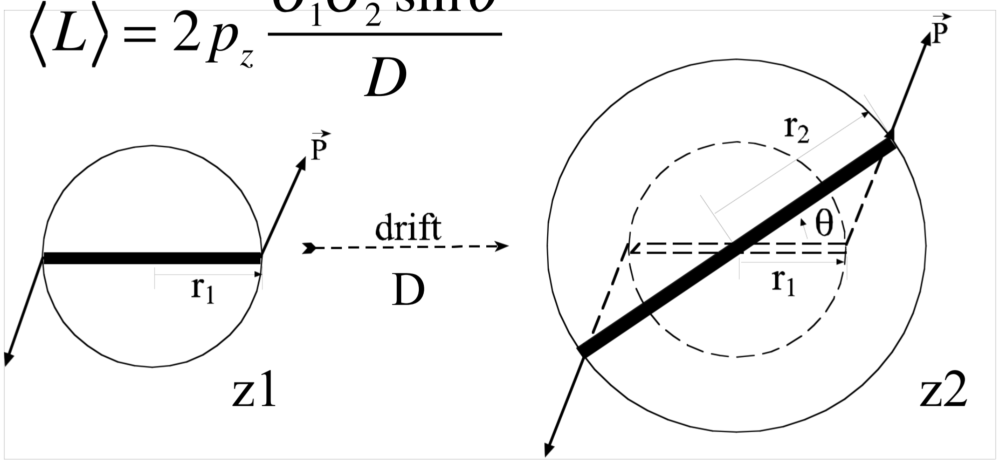
$$\frac{\varepsilon_x}{\varepsilon_y}\approx\frac{(2k\sigma_0{}^2)^2}{\varepsilon_0{}^2}=\frac{4k^2\sigma_0{}^2}{\sigma'_0{}^2},\quad and\quad \varepsilon_x\varepsilon_y=\varepsilon_0{}^2$$

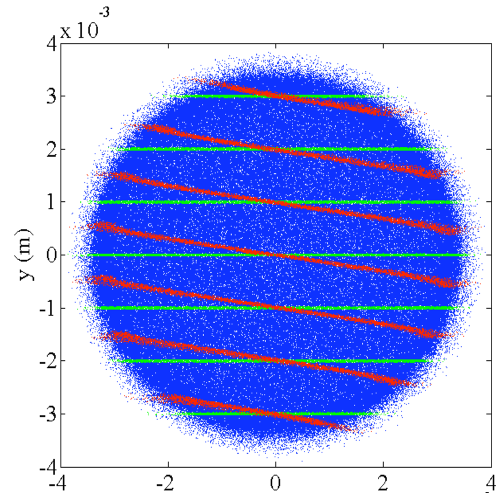
$$canonical\ angular\ momentum\ L=\gamma mr^2\dot{\phi}+\frac{1}{2}eB_zr^2,\quad \left\langle L\right\rangle=eB_c\sigma_0^2\rightarrow 2k\sigma_0{}^2p_z$$

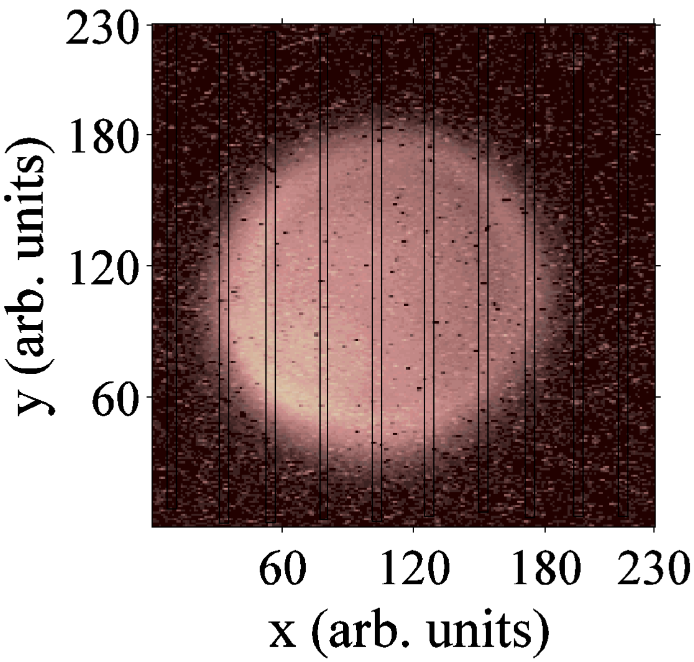
$$\left(k=\frac{eB_{z,c}}{2p_{z2}}\frac{\sigma_w^2}{\sigma_c^2}\quad if\ beam\ momentum\ at\ transformer\ is\ different\ from\ that\ at\ solenoid\ end\right)$$

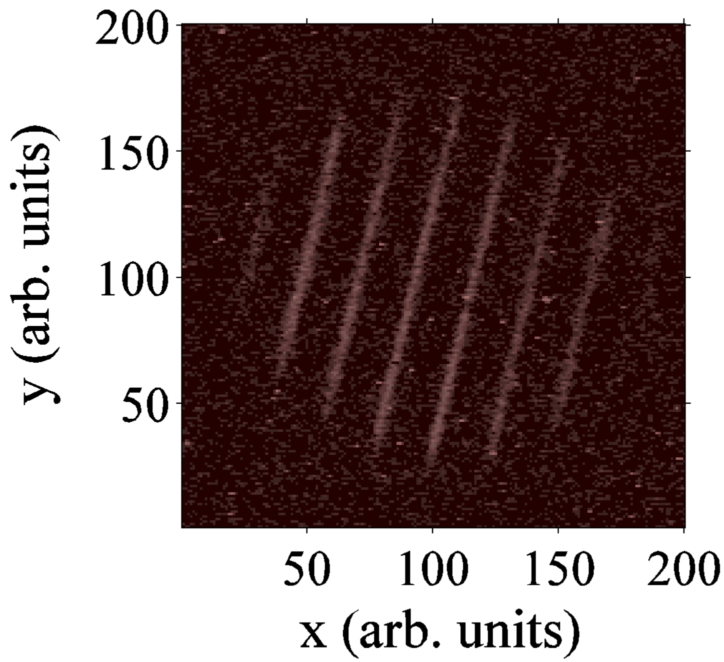
Measurement of mechanical angular momentum in a drift space

$$\langle L \rangle = 2 p_z \frac{\sigma_1 \sigma_2 \sin\theta}{D}$$



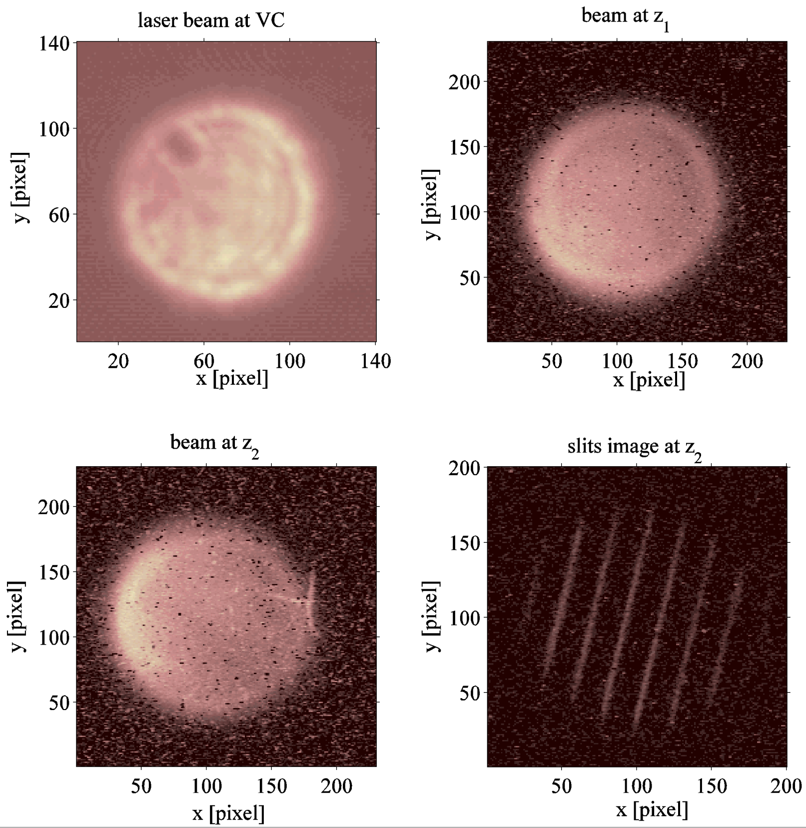


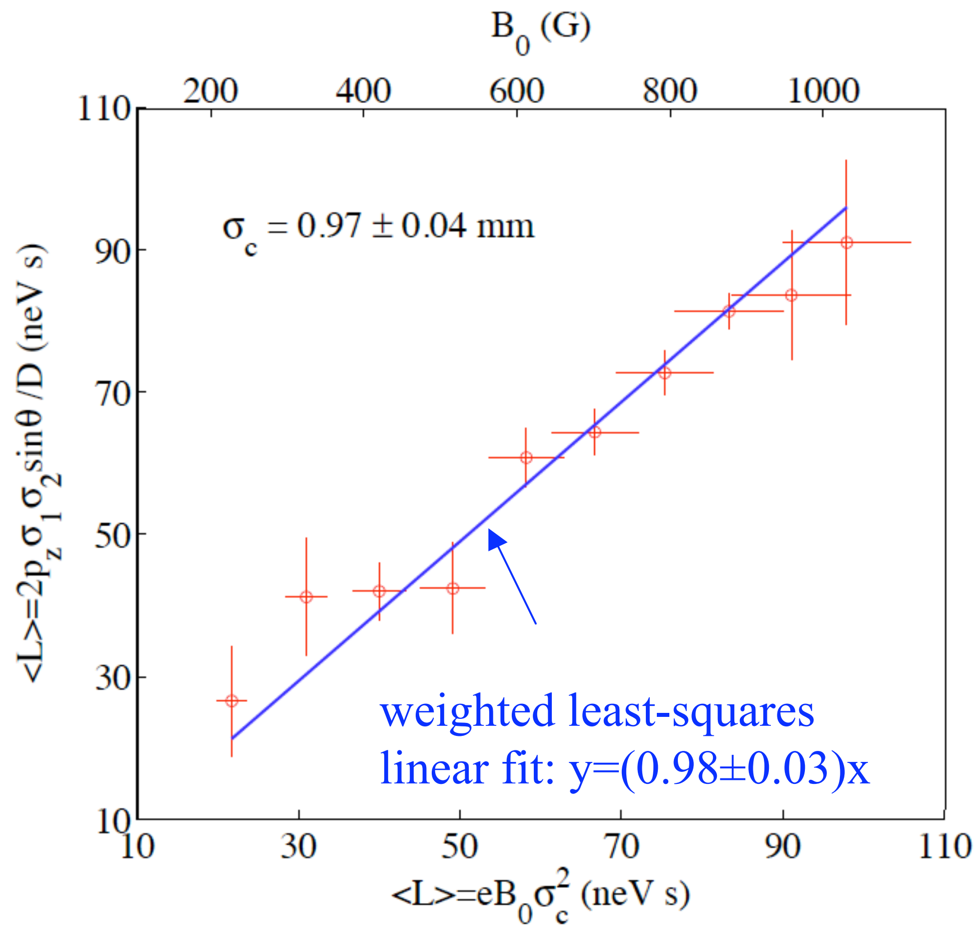




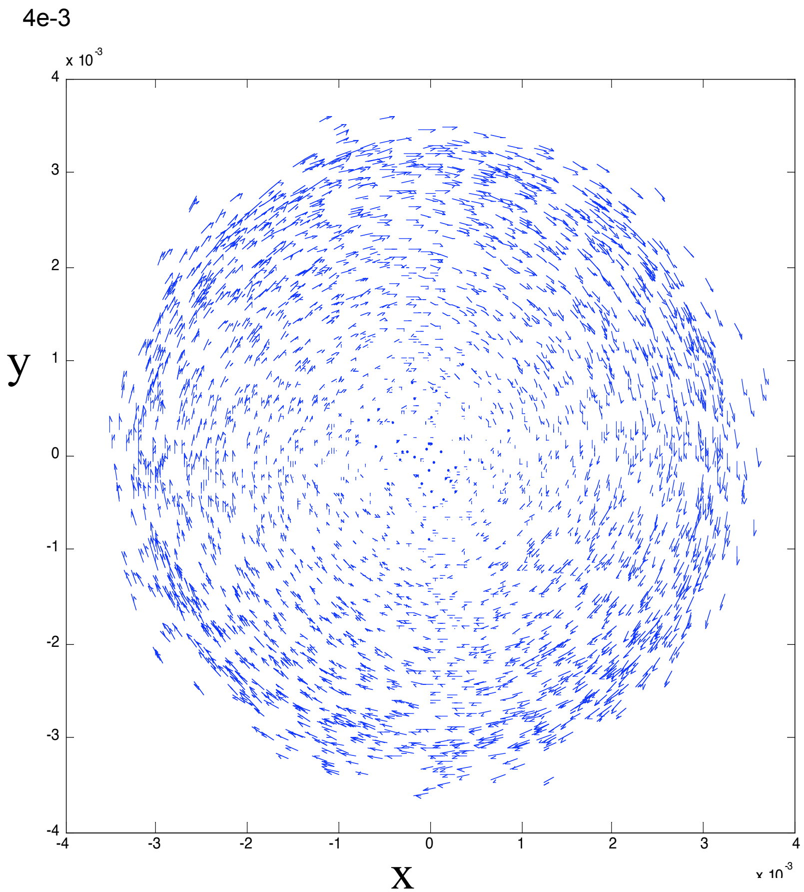
Demonstration of conservation of canonical angular momentum

as a function of magnetic field on cathode

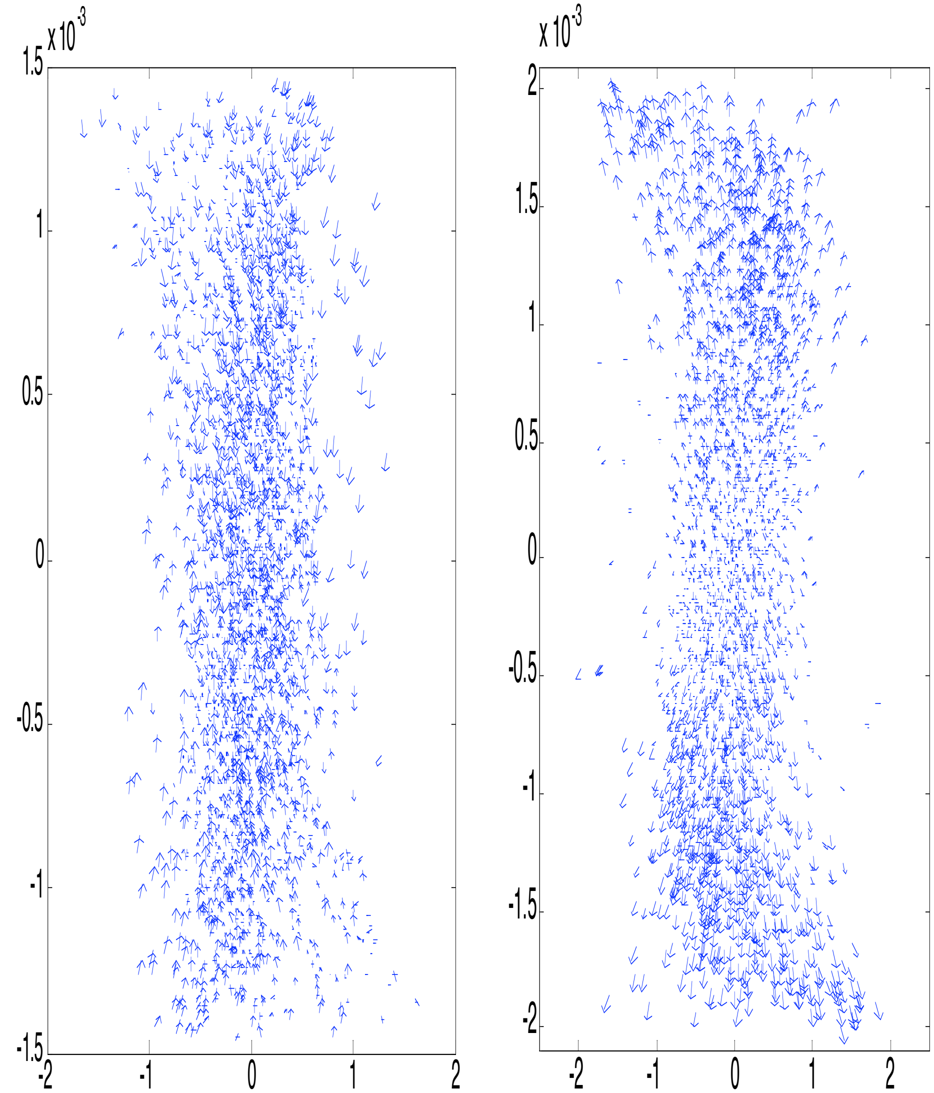




Position and velocity snap shots at the entrance/exit of the transformer



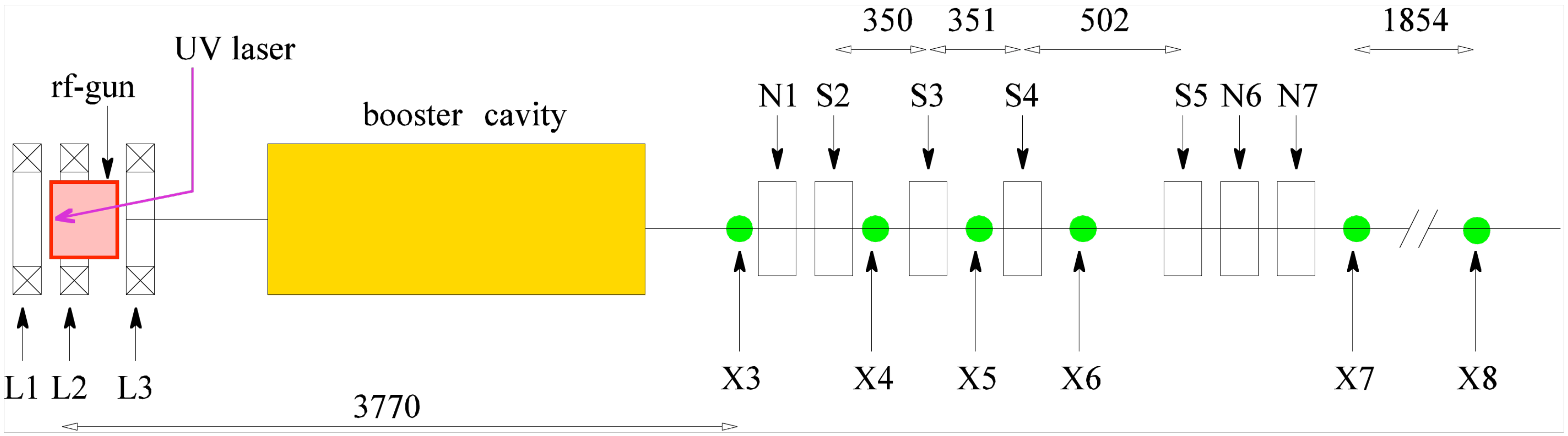
Round beam

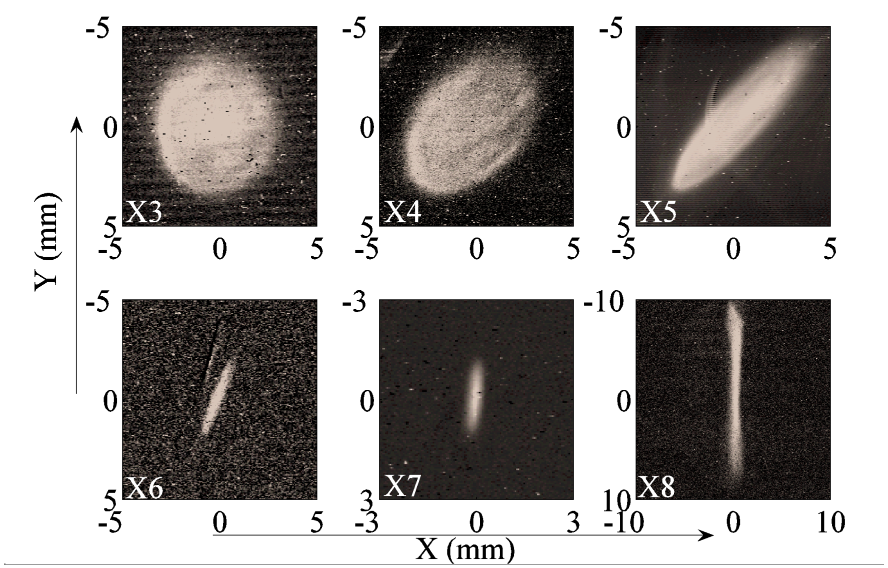


flat beam

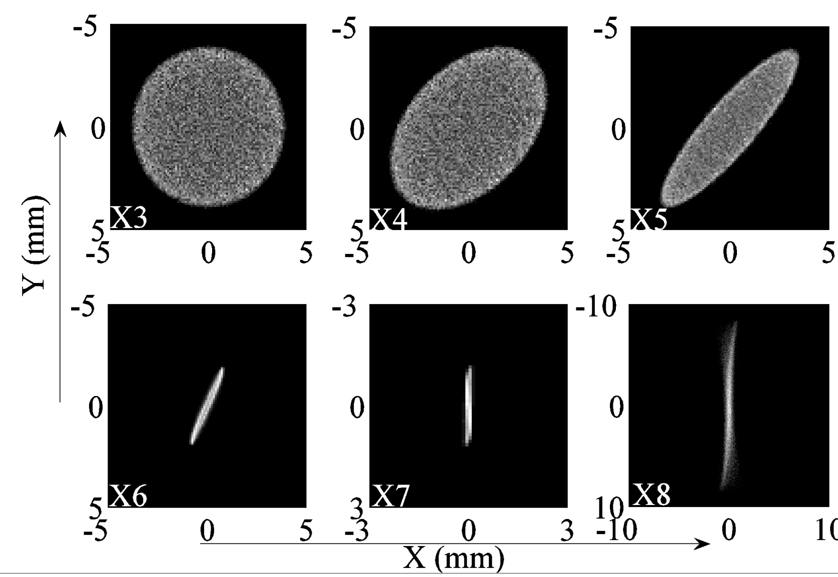
2 different possible quad solutions
converging & diverging

Removal of angular momentum and generating a flat beam





experiment



simulation

Compare measurement with simulation

	Experiment 90%	Experiment 95%	Simulation (ASTRA)
rms_cathode(mm)	0.97		0.97
B_cathode(Gauss)	898		898
I_Quad1 (A)	-1.97		-1.98
I_Quad2 (A)	2.56		2.58
I_Quad3 (A)	-4.55		-5.08
rms_X7y (mm)	0.58±0.01	0.63±0.01	0.77
rms_X7x (mm)	0.084±0.001	0.095±0.001	0.058
rms_X8_hslit (mm)	1.57±0.01	1.68±0.01	1.50
rms_X8_vslit (mm)	0.12±0.01	0.13±0.01	0.11
Lcath (mm mrad)		24.5±0.7	
Lmech (mm mrad)		26.6±0.5	
Emit-uncorrelated (mm mrad)		5.1±0.7	
ε_+ (mm mrad)	53.8±0.9		
ε_- (mm mrad)	0.49±0.13		
εx (mm mrad)	**0.39±0.02 (0.32)**	**0.49±0.02 (0.41)**	**0.27**
εy (mm mrad)	35.2±0.5	41.0±0.5	53
εy/εx	90±5 **(110+-7)**	83±4 **(100+-5)**	196
$(\varepsilon x \cdot \varepsilon y)^{0.5}$	3.7 (3.35)	4.5 (4.1)	3.8 mm mrad

(…) camera resolution corrected

EEX

not a Derbenev idea but akin in spirit

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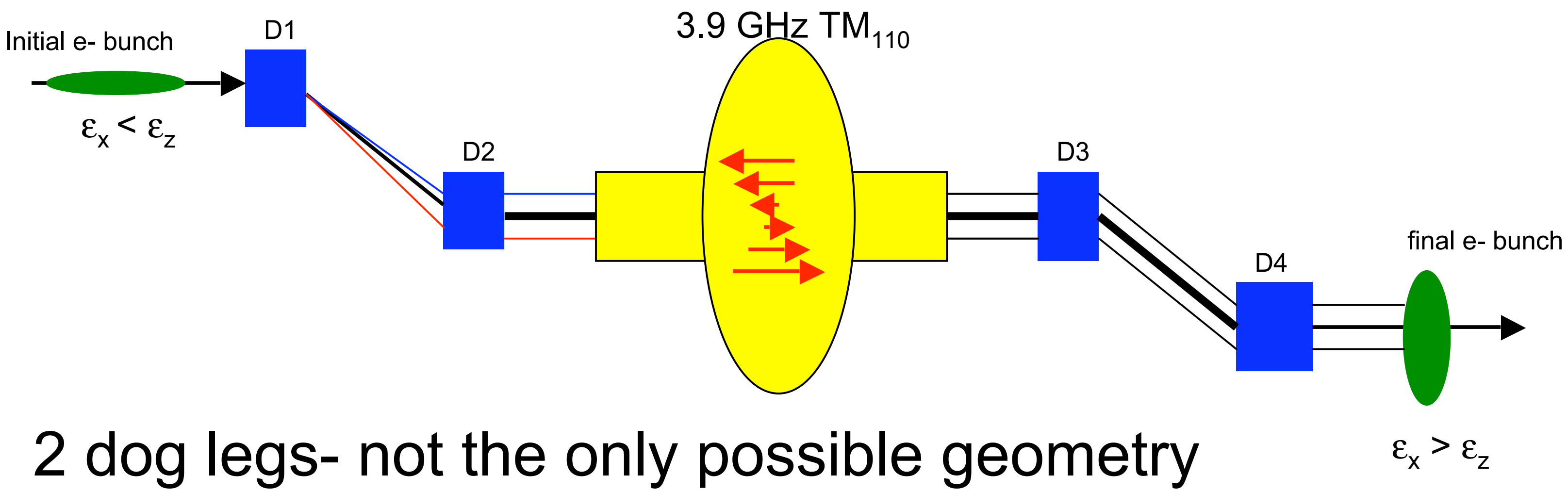
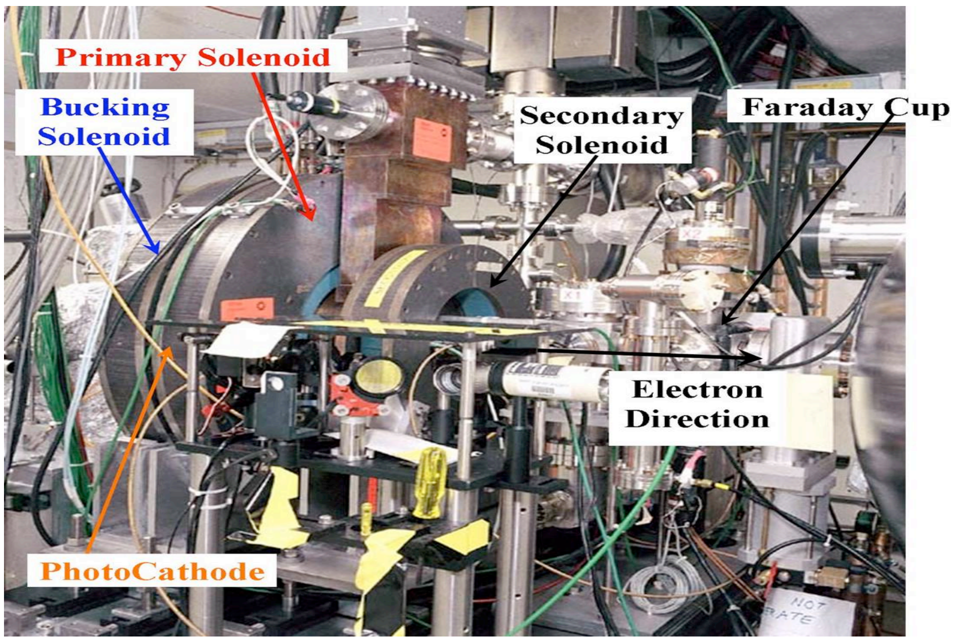
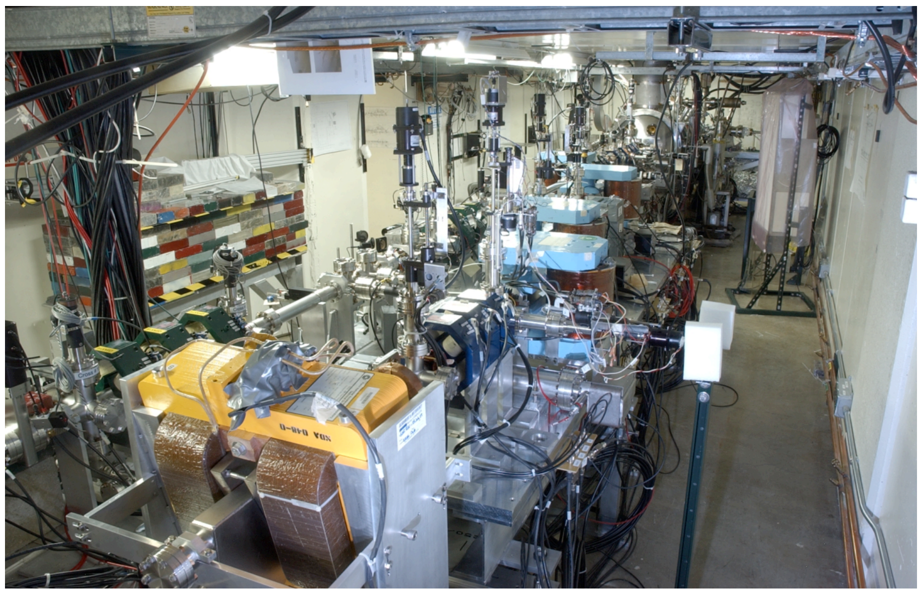
Thanks to T. Koeth, A. Johnson, Y. Sun for many view graphs

Emittance Exchange EEX



2 dog legs- not the only possible geometry





EEX Thin Lens Approx (1)

Emittance exchange thin lens approximation.

Following the notation of D. Edwards (Ref), let α be the bend of each magnet in a dogleg and L_1 the distance between bends, then the dog leg matrix is given by

$$M_{dog} = \begin{pmatrix} 1 & L_1 & 0 & \alpha L_1 \\ 0 & 1 & 0 & 0 \\ 0 & \alpha L_1 & 1 & \alpha^2 L_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & D/\alpha & 0 & D \\ 0 & 1 & 0 & 0 \\ 0 & D & 1 & \alpha D \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where D is the dispersion. Let this be followed by a drift, L_2 to a thin lens deflection mode cavity. The cavity matrix is given by

$$M_{cav} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & T & 0 \\ 0 & 0 & 1 & 0 \\ T & 0 & 0 & 1 \end{pmatrix}$$

EEX Thin Lens Approx (2)

Where for exchange T= -1/D = $\frac{-1}{\alpha L_1} = \frac{\omega}{c}\frac{eV_{cav}}{E_{beam}}$ and V_{cav} is the deflection strength of the cavity and E_{beam} is the beam energy.

The total exchange

$$M_{dog} L_2 M_{cav} L_2 M_{dog} = \begin{pmatrix} 0 & 0 & -\frac{1}{\alpha} - \frac{L_2}{D} & -L_2\alpha \\ 0 & 0 & \frac{-1}{D} & -\alpha \\ -\alpha & -\alpha L_2 & 0 & 0 \\ \frac{-1}{D} & \frac{-1}{\alpha} - \frac{L_2}{D} & 0 & 0 \end{pmatrix}$$

In our geometry D=0.33m and α = 22.5 deg.

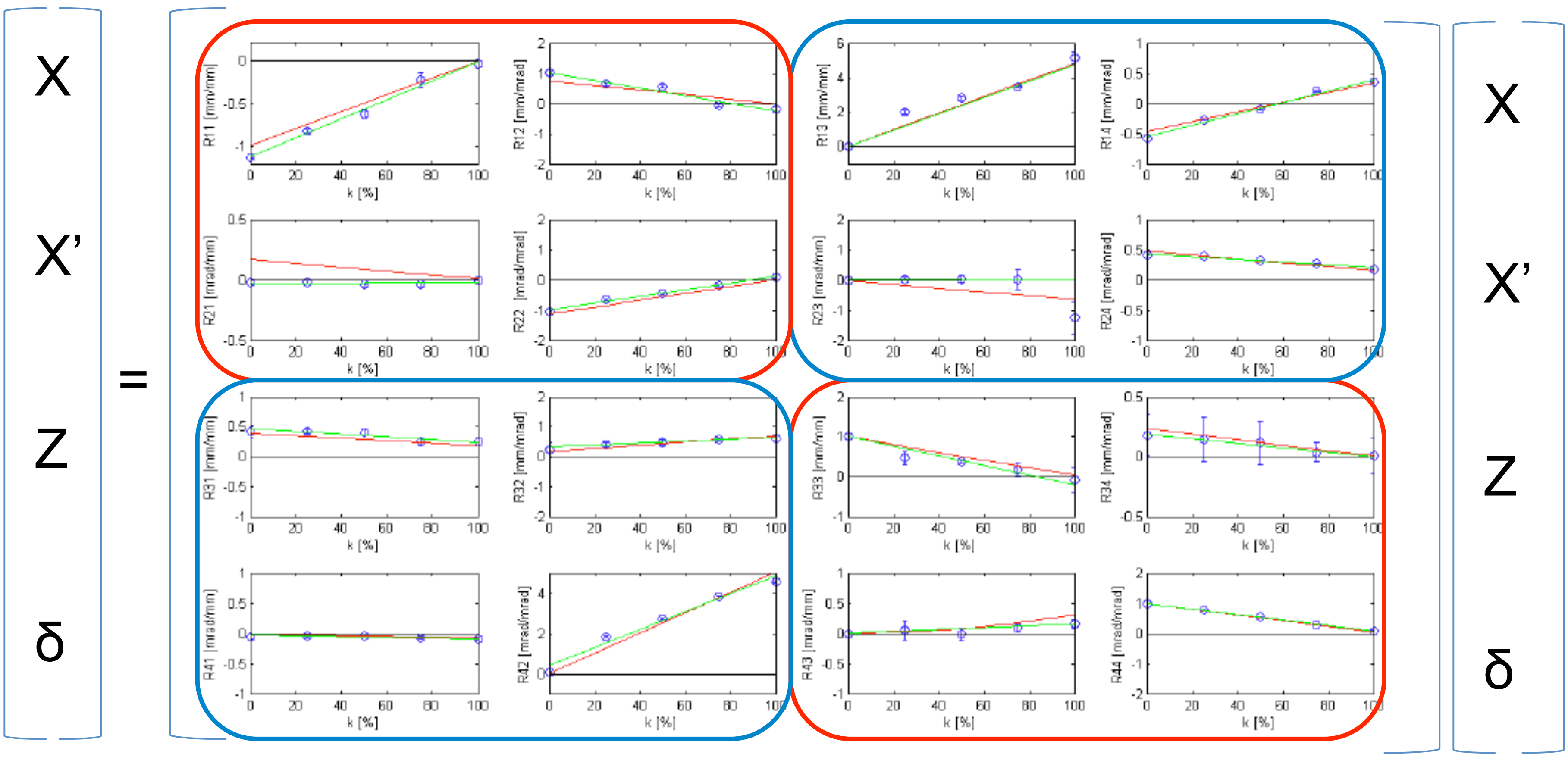
For a finite length cavity, the (4,3) element of the M_{cav} enters and the on diagonal (coupling) blocks start to show non zero values and will dilute the 2D emittances, especially the smaller one. The finite length cavity also causes the equilibrium orbit to follow a stair case trajectory through the cavity. These effects can be compensated to some extent by suitable choices of beam and cavity parameters.

For a finite length cavity 43 element does not vanish

Measured EEX Transport Matrix

TRANSPORT MATRIX AS A FUNCTION OF TM_{110} CAVITY STRENGTH

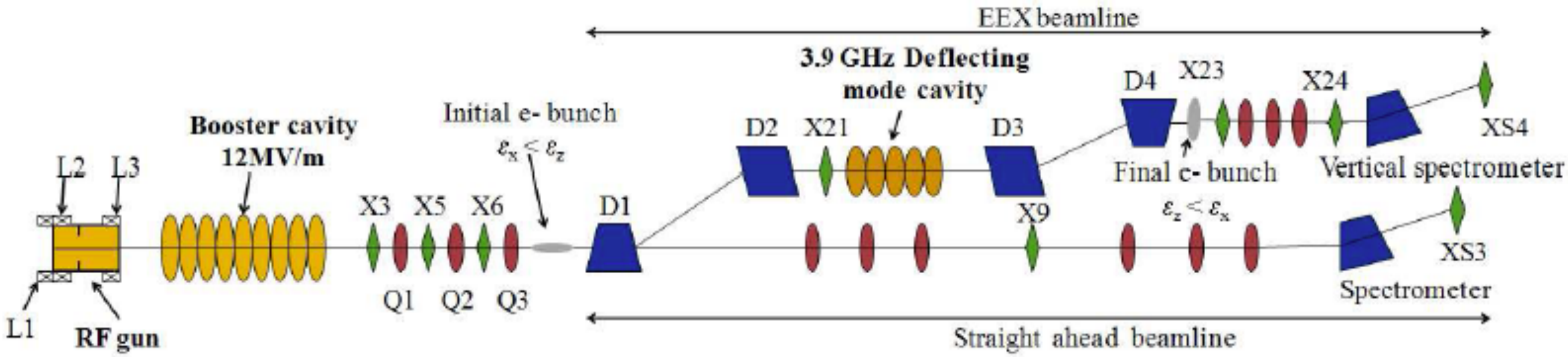
OUT 0-100%, red model, green data fit. IN



Red on-diagonal 2x2's ->0, except R43 finite length cavity

The vertical plane is unaffected by the cavity status

EEX Layout & Recent Results

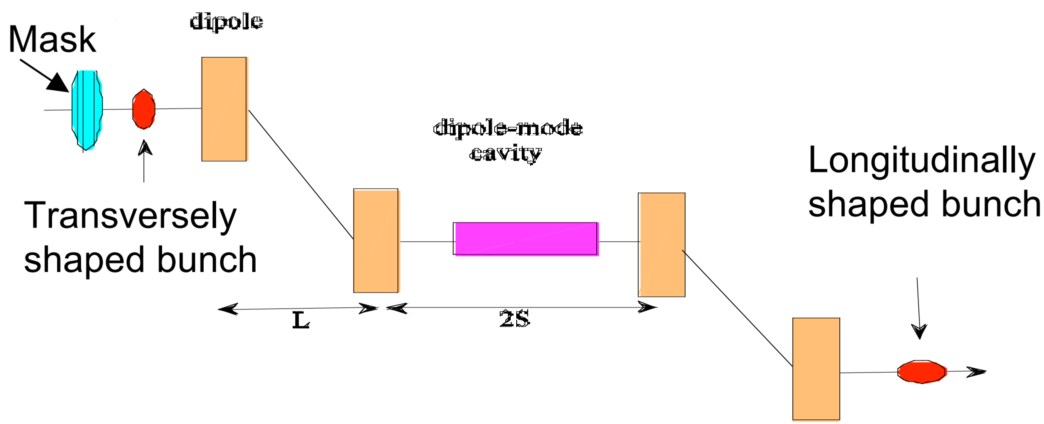


Results

Measured In	Measured Out	Simulated In	Simulated Out
ε_x = 3.71 mm·mrad	ε_x = 13.9 mm·mrad	ε_x = 3.71 mm·mrad	ε_x = 18.57 mm·mrad
ε_z = 16.23 mm·mrad	ε_z = 7.73 mm·mrad	ε_z = 16.23 mm·mrad	ε_z = 9.54 mm·mrad
ε_y = 3.3 mm·mrad	ε_y = 4.7 mm·mrad	ε_y = 3.3 mm·mrad	ε_y = 3.3 mm·mrad

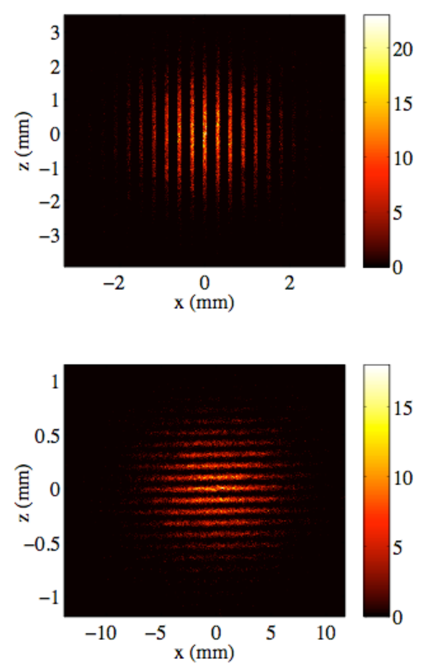
We determine the transverse emittances using the slit emittance technique [3]. Input x and y measurements are made using the X3-X6 diagnostic cross pair. Output x and y values are determined from the X23-X24 cross pair. Longitudinal emittance measurements are made from the minimum energy spread as measured in the spectrometer magnets and the bunch length as determined at X9 and X24. The measured emittance growth is partially explained by the present longitudinal diagnostic's inability to account for longitudinal position-energy correlation. Other possible contributions to a greater than 1:1 emittance exchange include the non-ideal emittance exchange matrix, i.e. effects of the thick lens cavity, space charge forces and potential coherent effects, such as coherent synchrotron radiation.

Piot & Sun, Bunch trains w EEX

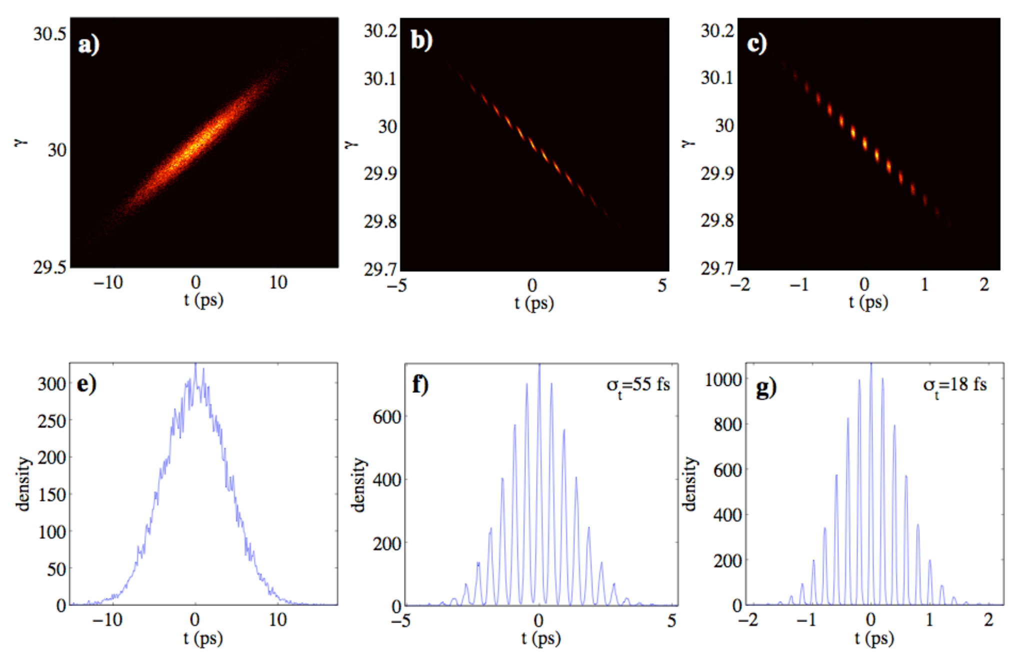


simulation

Z vs x after mask
Before EEX
After EEX

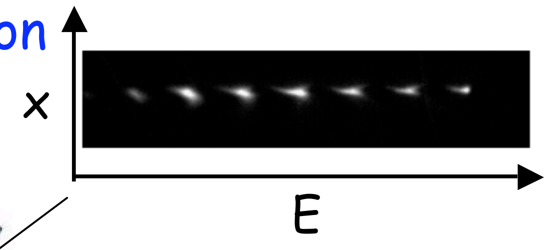


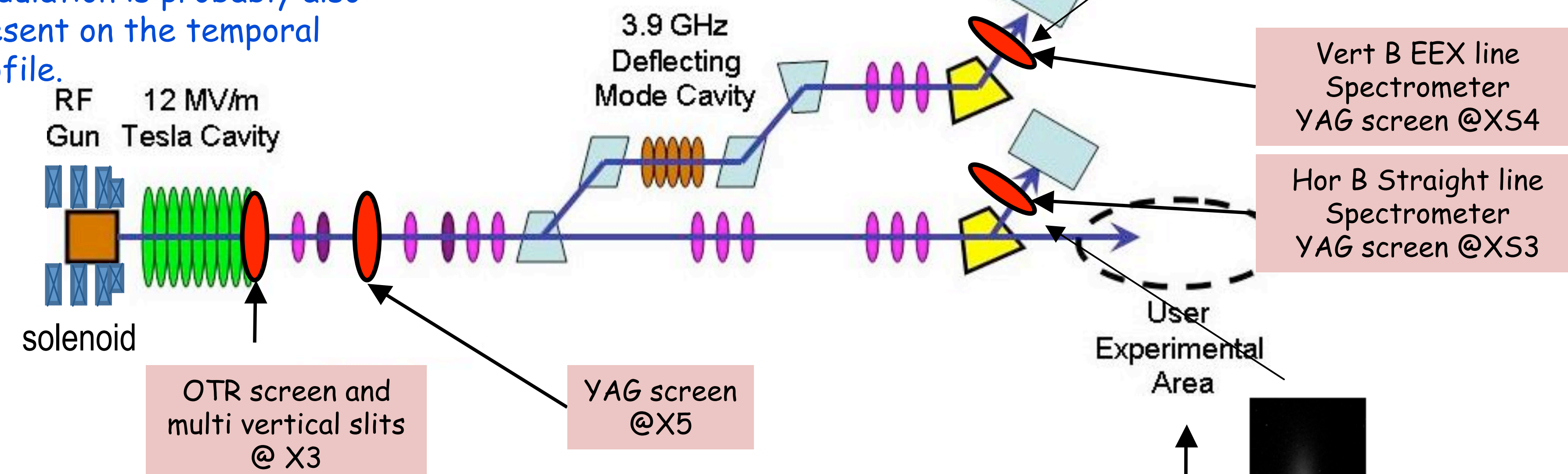
Longitudinal phase space
After mask After EEX After dog compress

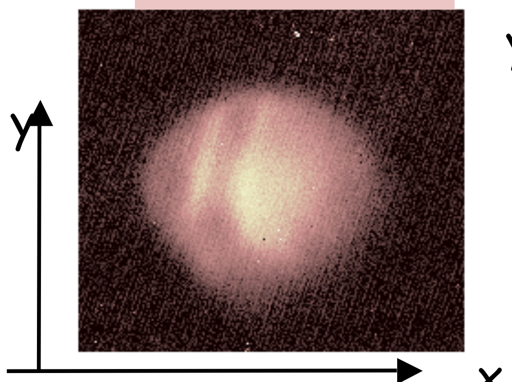


Longitudinal pulse shaping using EEX:
transverse density to energy modulation

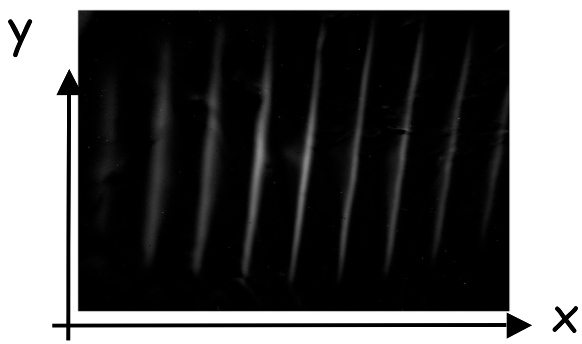
From simulations we know the longitudinal phase space is chirped so the modulation is probably also present on the temporal profile.



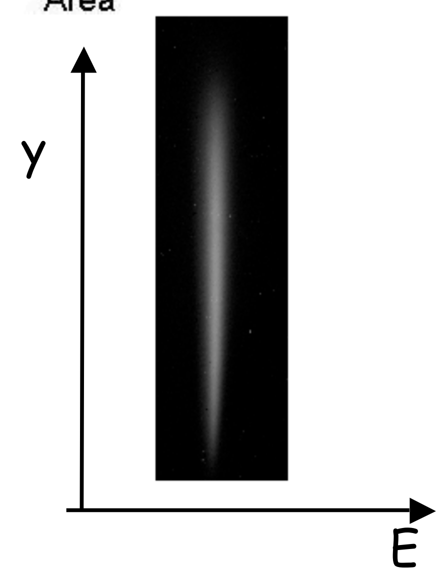




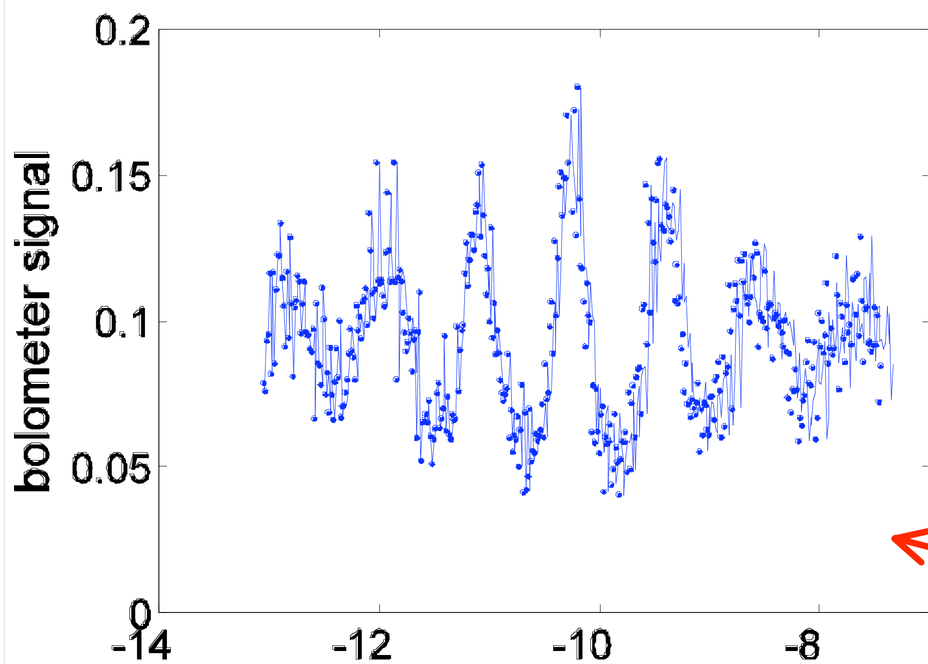
Beam at X3 (OTR)



Vertical beam slit image at X5 (single crystal YAG)



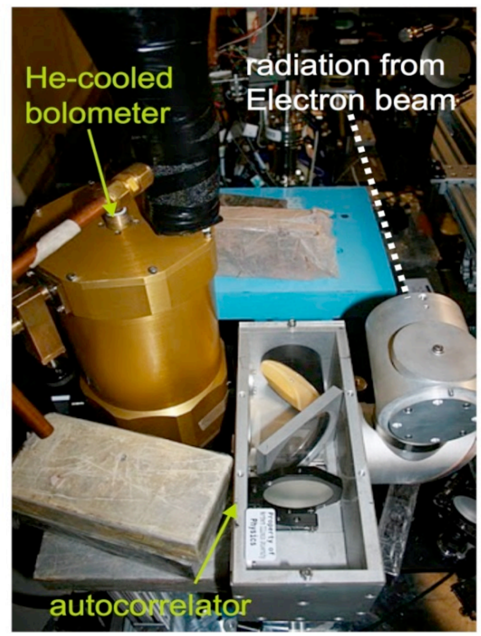
Longitudinal pulse shaping using EEX: direct measurement of temporal modulation

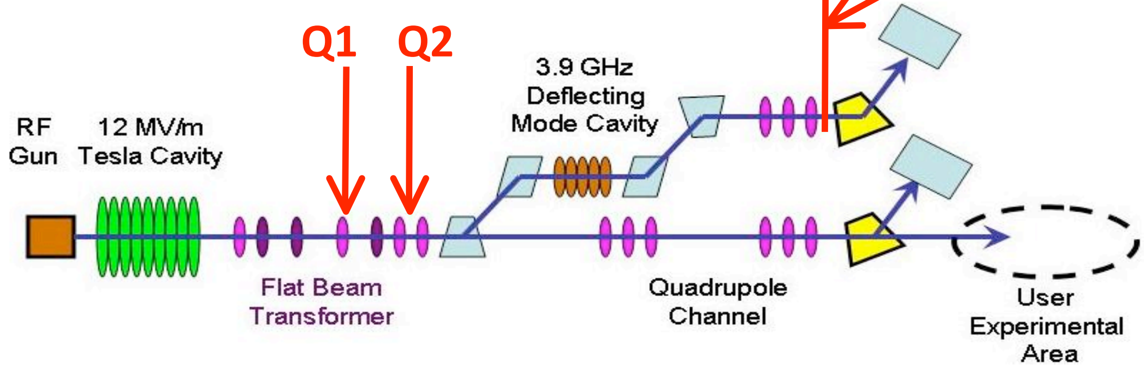


Autocorrelator/Bolometer

Left figure: signal measured by the He-cooled bolometer as a detector of the autocorrelator.
The bunch separation can be cotrolled by quads upstream of the EEX line.

Q1 (A)	Q2 (A)	multibunch separation
1.0	-0.5	425µm (1.4 ps)
1.5	-0.5	645 µm (2.2 ps)
1.8	-0.6	840 µm (2.8 ps)





Some Possible Applications

- ILC damping ring- flat to round in long straights (tune shift reduction) FBT
- ILC injector with Flat $\varepsilon x/ \varepsilon y$ ratio (do away with electron damping ring) FBT
- FEL emittance repartition for lower energy linac, shorter gain length FBT & EEF
- Bunch compression w/o energy chirp EEX
- Micro bunching EEX

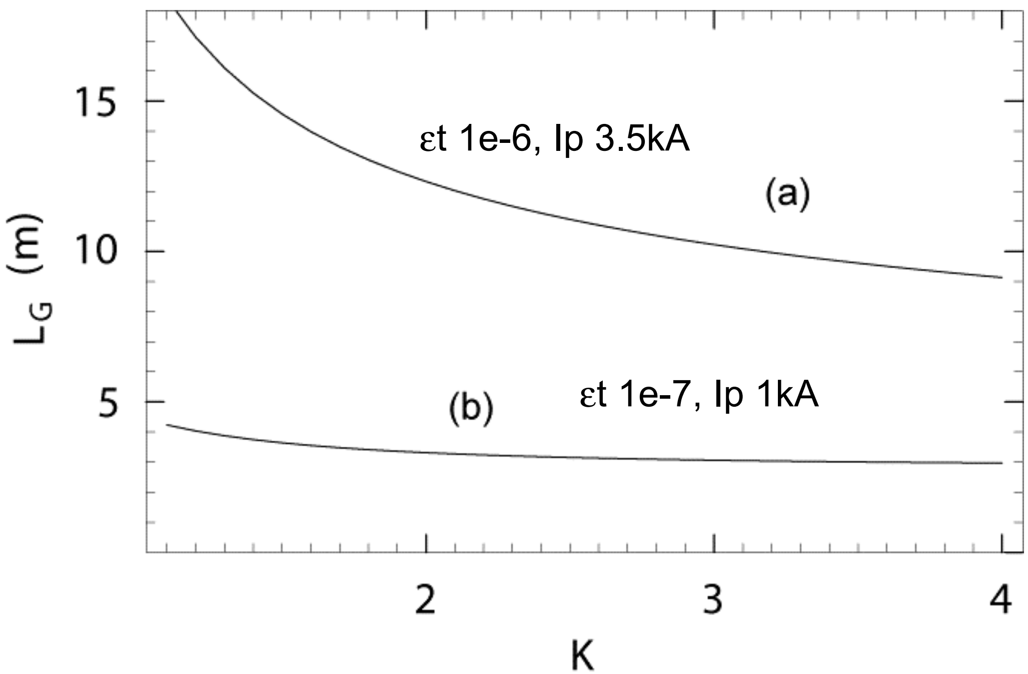
Combinations of FBT and EEX to change emittance

partition in x,y,z

Examples: Kim, Carlsten, Zholents

K. J. Kim Producing Matched e-Beams for X-Ray HG FEL

- Electron beam emittance should be matched to the radiation emittance: $\varepsilon_{xn} \sim \gamma\lambda/4\pi$
- For 1-Å with E=5 GeV, the matched emittance is $\varepsilon_x^n \sim 0.1\,\mu$m, *which is smaller by an order of magnitude than the current state-of-the-art*
- In current HGFEL projects, the mismatch is dealt with by a high E (>15 GeV), high K (3.7), and high current (a few kA)
- Noting that $\Delta E/E_{slice} < 10^{-6}$, two orders of magnitudes smaller than required, the FBT and EEX can be employed to produce a matched beam



Improved FEL Performance with matched beams

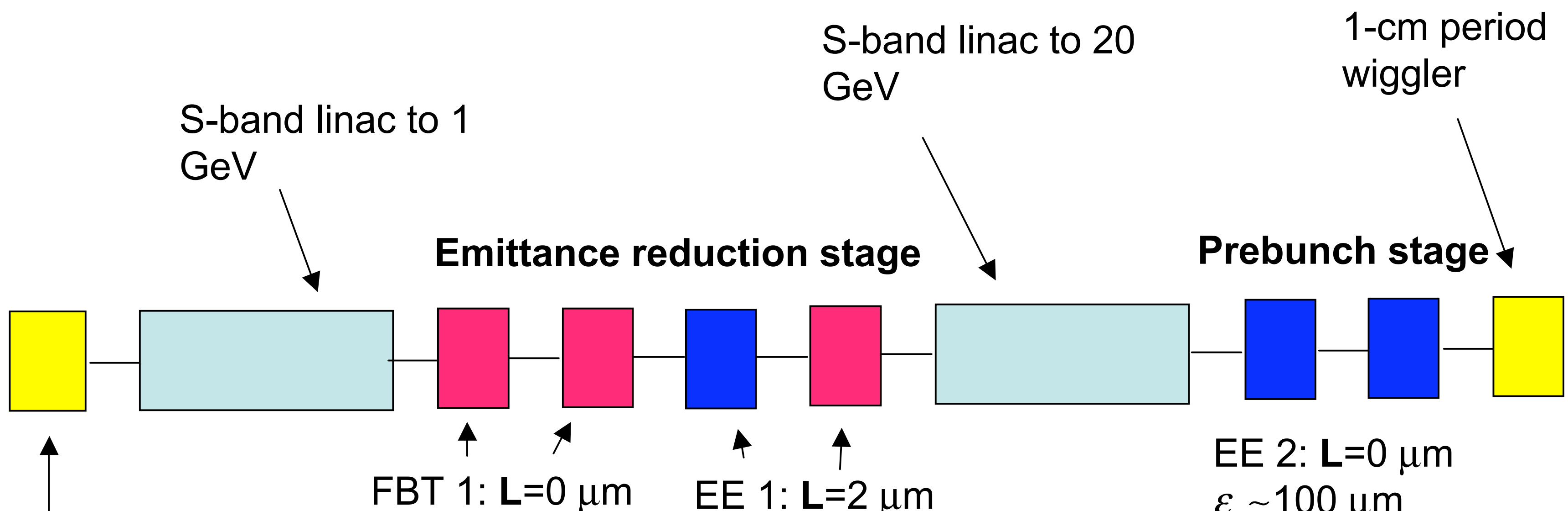
- **Higher gain**
- **The same gain with smaller K**
  - **Less magnets**
  - **Lower energy**

(a)

Power gain length *LG* of an x-ray FEL at 0.4 Å versus the undulator parameter *K* for (a) a beam with a normalized transverse emittance 1 × 10^{-6} m-r and a peak current 3.5 kA and (b) a beam with a normalized transverse emittance 1 × 10^{-7} m-r and a peak current 1 kA. The relative rms energy spread in both cases is 1 × 10^{-4} (courtesy of Z. Huang).

(b)

B. Carlsten: What We Are Thinking For New Baseline Design for MaRIE 50 keV XFEL



500 kV DC injector
or AFEL copy
or CTF3 copy
0.50 nC, **L**=35 μm
$\varepsilon_x \sim 0.7$ μm
$\varepsilon_y \sim 0.7$ μm
$\varepsilon_z \sim 1.4$ μm

FBT 1: **L**=0 μm
$\varepsilon_x \sim 70$ μm
$\varepsilon_y \sim 0.007$ μm
$\varepsilon_z \sim 1.4$ μm

FBT 2: **L**=2 μm
$\varepsilon_x \sim 35$ μm
$\varepsilon_y \sim 0.014$ μm
$\varepsilon_z \sim 1.4$ μm

EE 1: **L**=2 μm
$\varepsilon_x \sim 1.4$ μm
$\varepsilon_y \sim 0.014$ μm
$\varepsilon_z \sim 35$ μm

FBT 3: **L**=0 μm
$\varepsilon_x \sim 0.14$ μm
$\varepsilon_y \sim 0.14$ μm
$\varepsilon_z \sim 35$ μm

EE 2: **L**=0 μm
$\varepsilon_x \sim 100$ μm
$\varepsilon_y \sim 0.14$ μm
$\varepsilon_z \sim 0.14$ μm

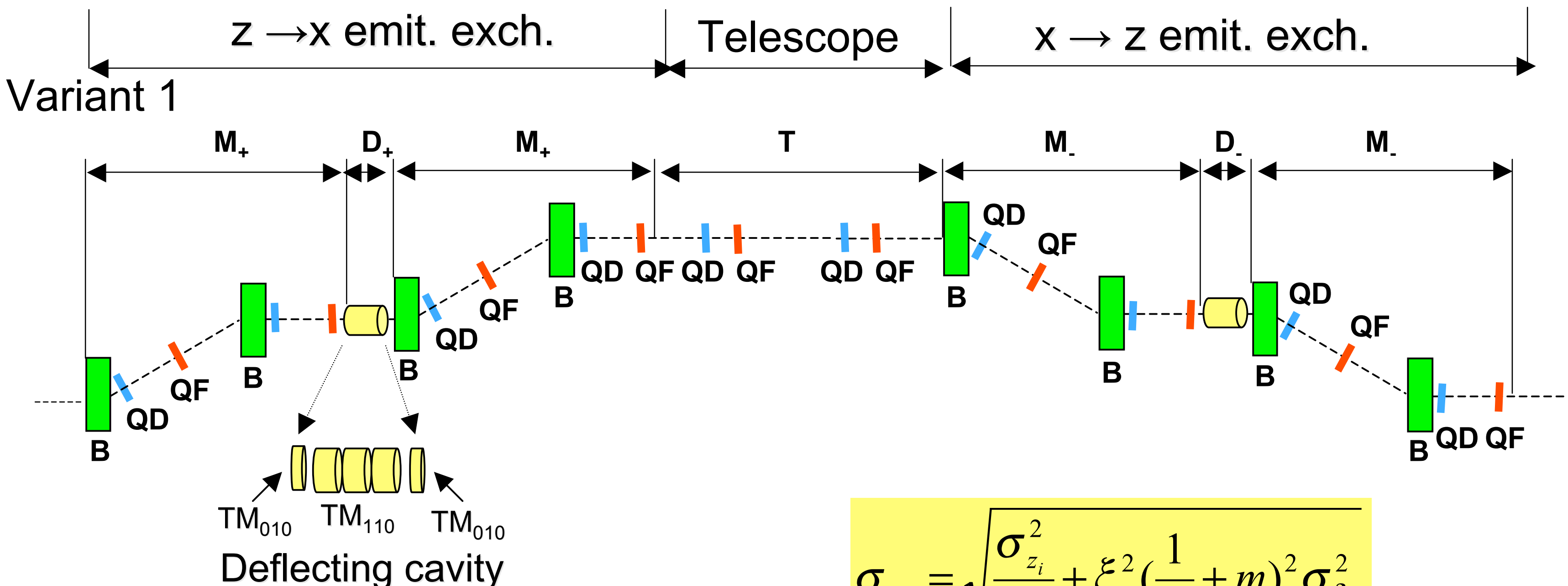
EE 3: **L**=0 μm
$\varepsilon_x \sim 0.14$ μm
$\varepsilon_y \sim 0.14$ μm
$\varepsilon_z \sim 100$ μm

0.25 nC
(prebunched)

Zholents & Zolotorev
A schematic of the bunch compressor

(manipulate longitudinal phase space
with ease of a transverse phase space)

Focusing properties of individual sections



No chirp case,
M= magnification, ξ=R$_{56}$

$$\sigma_{z_f} = \sqrt{\frac{\sigma_{z_i}^2}{m^2} + \xi^2 \left(\frac{1}{m} + m\right)^2 \sigma_{\delta_i}^2}$$

$$\sigma_{\delta_f} = m \sigma_{\delta_i}$$

Conclusions

- The dam has broken
- The fish are free
- Magic is in
- New ideas abound
- Thanks Slava

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