CSR Effects in Beam Dynamics

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A Special Beam Physics Symposium in Honor of Yaroslav Derbenev's 70th Birthday
Thomas Jefferson National Accelerator Facility
Newport News, VA
August 2-3, 2010

Outline of the talk

- Radiative reaction force for a point charge, singularities and infinities
- Coherent radiation of bunches and CSR force
- Transverse self-forces in relativistic beams
- Applications: CSR instabilities in rings and bunch compressors
- Conclusion

Quotation

"A good theory is like a caricature: it neglects unimportant and it exaggerates essential traits".
I. Pomeranchuk

DRSS

TESLA FEL-Report 1995-05

Microbunch Radiative Tail-Head Interaction

Ya.S.Derbenev, J.Rossbach, E.L.Saldin, *
and V.D.Shiltsev †
DESY, Notkestrasse 85, 22603, Hamburg, GERMANY

September 28, 1995

DS

SLAC-PUB-7181
June 1996
( )

Transverse Effects of Microbunch Radiative Interaction

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V. D. Shiltsev
FNAL, AD/Physics, M.S.345, P.O.Box 500, Batavia, IL, 60510

June 3, 1996

Radiative reaction force for a point charge

An accelerated electron radiates electromagnetic waves and loses energy. The energy balance equation involves a *radiative reaction force* (RRF). In non-relativistic limit

$$F_{\mathrm{rad}} = \frac{2}{3}\frac{e^2}{c^3}\ddot{\boldsymbol{v}}$$

Ultra-relativistic RRF ($\gamma \gg 1$)

$$F_{\mathrm{rad}} = \frac{2e^4}{3m^2c^4}(F_{kl}u^l)(F^{km}u_m)\frac{\boldsymbol{v}}{v}$$

Runaway solutions

$$\dot{\boldsymbol{v}} \propto e^{3mc^3t/2e^2}$$

Schwinger derives in 1949 most of the properties of the synchrotron radiation from analysis of RRF.

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Coherent radiation and RRF in beams—bit of history

What is the RRF for a coherently radiated bunch of charged particles?

- For historical review see J. Murphy, Beam Dynamics Newsletter No. 35, p. 21(2005).
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Coherent radiation and RRF

There are two approaches to calculate RRF in a bunch.

- Consider interaction of two electrons at a small distance moving in a circular orbit (Murphy et al.).
- Consider the beam as continuous medium and use retarded potentials technique (DRSS).

$$\phi(\mathbf{r},t) = \int \frac{\rho(\mathbf{r}',t_{\mathrm{ret}})}{|\mathbf{r}-\mathbf{r}'|} d^3r', \quad \mathbf{A}(\mathbf{r},t) = \frac{1}{c}\int \frac{\mathbf{j}(\mathbf{r}',t_{\mathrm{ret}})}{|\mathbf{r}-\mathbf{r}'|} d^3r'$$

$$\mathbf{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}, \qquad \mathbf{B} = \nabla \times \mathbf{A}$$

with $t_{\mathrm{ret}}(\mathbf{r},\mathbf{r}',t) = t - \frac{1}{c}|\mathbf{r}-\mathbf{r}'|$. The singularities in the integrands are integrable in 3D (and 2D). However, not much can be done analytically in general case of 3D.

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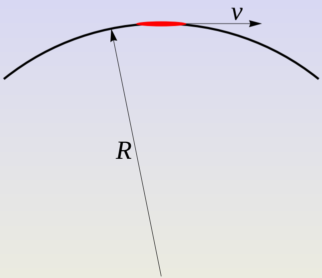
Line-charge beam



We would like to solve the problem in 1D (line charge beam), however the integrals for potentials diverge logarithmically in this case. Why does this happen?

Space charge field of line-charge beam

The divergence has nothing to do with RRF; it is due to the space charge field.

For a long thin beam ($\sigma_r/\gamma\sigma_z \ll 1$), the space charge longitudinal electric field

$$E_{\parallel} \sim \frac{Q}{\sigma_z^2\gamma^2} \log \frac{\sigma_z\gamma}{\sigma_r}$$

This field diverges when $\sigma_r \to \infty$, but it is a very weak singularity. For ultra-relativistic beam it is also small as γ^{-2}.

Solution: take simultaneously the limits $\sigma_r \to 0$ and $\gamma \to \infty$ ($v = c$) ["exaggerate essential traits"]. The singularity disappears in this limit and the problem then becomes remarkably simple.

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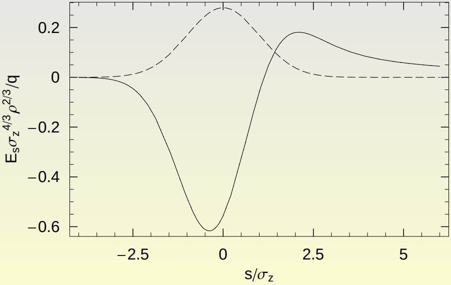
CSR for a line-charge beam

CSR (RRF) longitudinal force in bunch with $\lambda(s)$ distribution function

$$\mathcal{E}_s(s) = -\frac{2Q}{R^{2/3}3^{1/3}} \int_{-\infty}^{s} \frac{1}{(s-s')^{1/3}} \frac{\partial \lambda(s')}{\partial s'} \, ds'$$

For Gaussian distribution



Further development of the theory

- Generalization for a bend of finite length (Saldin, Schneidmiller, Yurkov, 1997; Stupakov, Emma, 2002)
- CSR force in an undulator (Saldin, Schneidmiller, Yurkov, 1998)

These works still assume an infinitely thin beam with all the particles moving along the same trajectory.

Development of computer codes that treat CSR forces: TRAFIC[4], `elegant`, CSRTrack, TREDI, etc. Code development requires computational algorithms devoid of singularities. Recent publications: Talman (PRST-AB, 2004); Warnock et al. (NIM-A, 2006); Mayes and Hoffstaetter (PRST-AB, 2009).

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CSR in toroidal pipe

Warnock and Morton, PA, (1990); K. Y. Ng (1988) solved the problem of particle radiation in a metallic toroid. Warnock used to make plots of CSR wakefields for various application in 1990.

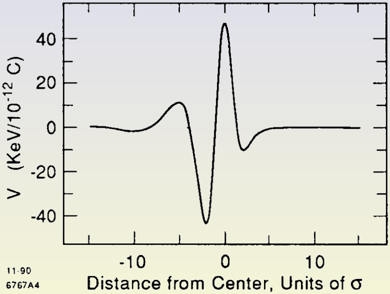

Figure 5: Wake voltage per turn for the parallel-plate model, $R = 149.5$m, $h = 2$cm, bunch length $\sigma = 61\mu$m.

Transverse self-force in relativistic beam

Ya. Derbenev, V. Shiltsev, "Transverse effects of microbunch *radiative* interaction", SLAC-PUB-7181. The dominant part of the transverse force in relativistic beams has nothing to do with radiation. The subject has a history, starting from 80s, riddled with errors.

Can we find transverse forces for a line-charge beam? No, they are infinite again.

It can be studied in configuration where there is no radiation. In $z = 0$ plane

$$x'' + Kx = \frac{e}{E}\left(F_r - \frac{\phi}{R}\right)$$

$F_r = E_r + (v_\theta/c)B_z$ is the Lorentz force per unit charge, $x = r - R$.

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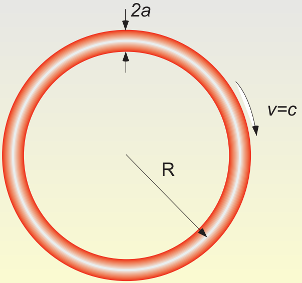


Transverse self-force in relativistic beam

The fields are time-independent and can be solved using a small parameter a/R. For $z = 0$ (λ - charge per unit length)

$$\phi = \lambda \left(1 - \frac{x^2}{a^2}\right) + 2\lambda \ln \frac{8R}{a},$$

$$F_r = \frac{\lambda}{R} \left(2 \ln \frac{8R}{a} - 1 - \frac{x^2}{a^2}\right).$$

The fields are singular in the limit $a \to 0$, but the RHS is not

$$F_r - \frac{\phi}{R} = -\frac{2\lambda}{R}$$

This is *centripetal* force found by Derbenev and Shiltsev for the general case of a line-charge beam.

Comments of further development

Surprisingly, this field is still considered somewhat controversial. There were many discussions in literature of so called "cancellation effect" problem.

Transverse self-fields within an electron bunch moving in an arc of a circle

G.A. Geloni[a], J.I. Botman[a], O.J. Luiten[a], M.J. van der Wiel[a], M. Dohlus[b], E.L. Saldin[b,*], E.A. Schneidmiller[b], M.V. Yurkov[c]

DISCUSSIONS ON THE CANCELLATION EFFECT ON A CIRCULAR ORBIT*

R. Li and Ya. S. Derbenev, Jefferson Lab, 12000 Jefferson Ave., Newport News, VA 23606, USA

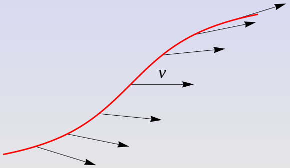
Misconceptions regarding the cancellation of self-forces in the transverse equation of motion for an electron in a bunch

G. Geloni [a] E. L. Saldin [b] E. A. Schneidmiller [b] M. V. Yurkov [c]

Modern CSR codes include the transverse self-force.
It seems, though, that it does not play a big role in the beam dynamics.

Generalization of line-charge dynamics for arbitrary motion

In general, the velocity of particles is not tangential to the beam line. The self-field of such a motion for a line-charge beam was studied in [Stupakov, FEL08]. Again, it is assumed that $\beta = 1$, but γ is kept on the LHS if equations of motion

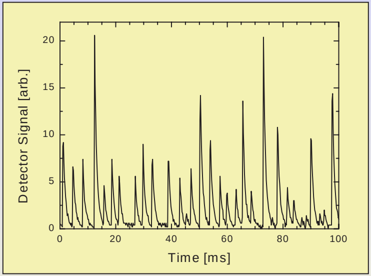


$$V = \phi - \boldsymbol{\beta} \cdot \mathbf{A}$$

$$\frac{d(mc\gamma + e\phi/c)}{dt} = \frac{e}{c}\frac{\partial V}{\partial t}$$

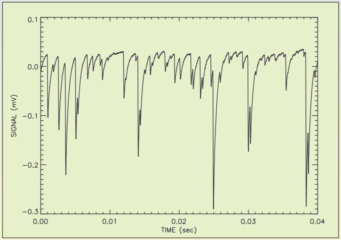
$$\left(mc\gamma + \frac{e\phi}{c} - \frac{eV}{c}\right)\frac{d\boldsymbol{\beta}}{dt} = -e\frac{d\mathbf{A}_\perp}{dct}\bigg|_\perp - e\nabla_\perp V$$

Applications: CSR instability in rings



Bursts of CSR (far infrared) in NSLS VUV ring [Carr et al., NIM, 387, (2001)]. Frequency range from ~ 6 to ~ 60 GHz.

Bursts of CSR in BESSY II [G. Wustefeld et al.]. Typical wavelength ~ 0.5 mm.

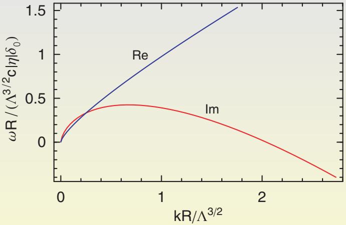


CSR instability in rings – theory

Use Keil-Schnell theory (Stupakov and Heifets, 2002). The dispersion relation

$$\frac{i r_0 c^2 Z_{CSR}(k)}{\gamma} \int \frac{d\delta\,(df/d\delta)}{\omega + ck\eta\delta} = 1$$

$r_0 = e^2/mc^2$, $f(\delta)$ – energy distribution function.



The beam is unstable for such wavelengths that

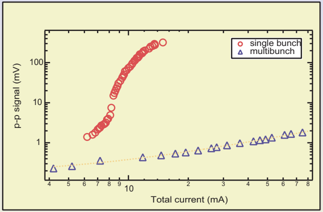
$$kR < 2.0\Lambda^{3/2}.$$

$$\Lambda = \frac{1}{|\eta|\gamma\delta_0^2}\frac{I}{I_A}\frac{R}{\langle R\rangle}$$

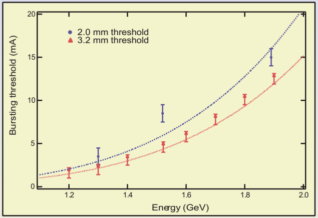
Energy spread introduces Landau damping and stabilizes short wavelengths.

CSR instability in rings – ALS experiment

J. Byrd et al., PRL, 224801 (2002). Beam energy varied from 1.2 to 2 GeV. Spectrum was measured by 94 GHz microwave detector and Si bolometer (up to $\lambda = 100$ µm).



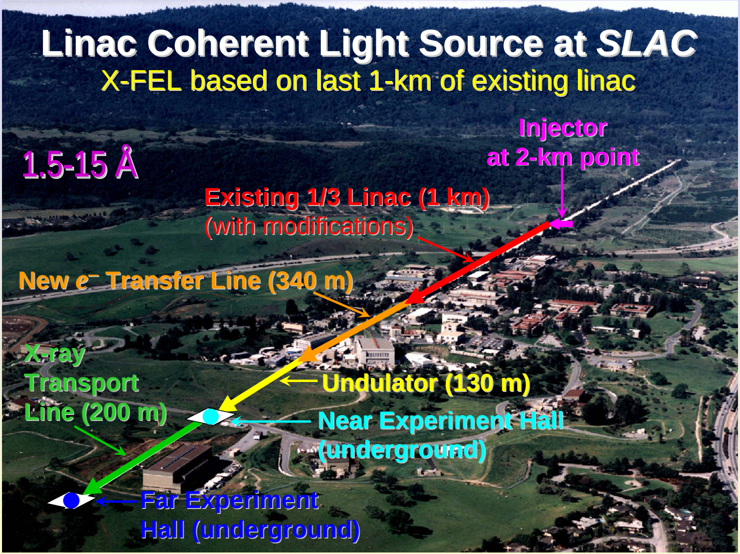
Bolometer signal as a function of total current for single and multibunch operation (300 bunches).



Bursting threshold as a function of electron beam energy at 3.2 and 2 mm wavelength.

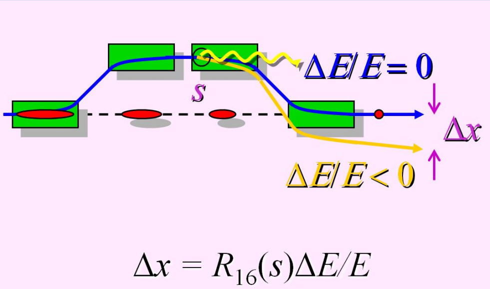
Very good agreement with theory for threshold of the instability. Subsequent experiments on BESSY, SPEAR and other rings confirmed the theory. Simulations by Warnock, Venturini.

Linac Coherent Light Source at SLAC



CSR in bunch compressors

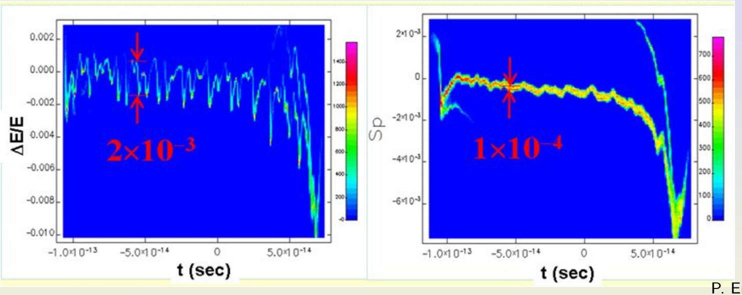
CSR effects can cause beam emittance growth in bunch compressors.



In addition to the emittance growth, in 2001 M. Borland observed in simulations a microbunching instability in a BC due to CSR. It is similar to the microbunching instability in rings, but develops on in one passage of the system.

Microbunching in bunch compressors

Simulation of CSR induced microbunching instability for the LCLS beam (P. Emma).

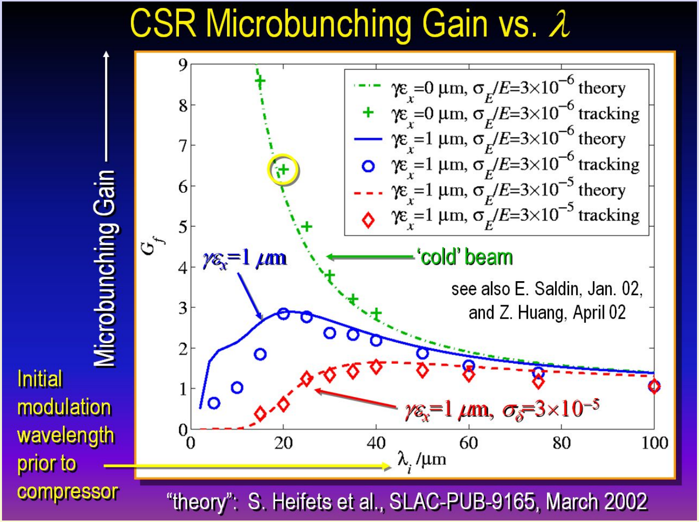


P. Emma

The theory of the instability was developed by 2002 by Heifets et al., Huang and Kim, and the DESY group.

LCLS design has a laser heater to increase the energy spread in the beam to suppress this instability.

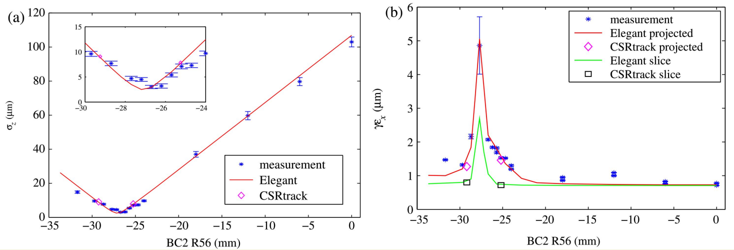
Microbunching in bunch compressors



P. Emma

CSR instability in the LCLS BC2

Experiment on the LCLS showed a good agreement with simulations using `elegant`. Notice that `elegant` uses a simple 1D model for the CSR longitudinal force and neglects the transverse self-force of the beam!



From Bane et al., PRST-AB, 12, p. 030704 (2009) .

Conclusion

- The two papers, DDRS and DS, marked an important step in the development of the theory of both longitudinal and transverse CSR wakefields. The method developed in these papers allows generalization for more complicated problem. The results are surprisingly simple, and are widely used in analysis of the effect.
- The field of CSR effects in beam dynamics has been under constant development since mid-90. It is an important element of the beam physics of short bunches. I presented two examples: CSR instability in rings, and bunch compressors.
- The original 1D model of DRSS, generalized by Saldin et al. for the case of short magnets, shows remarkable agreement with the experimental data from the LCLS.