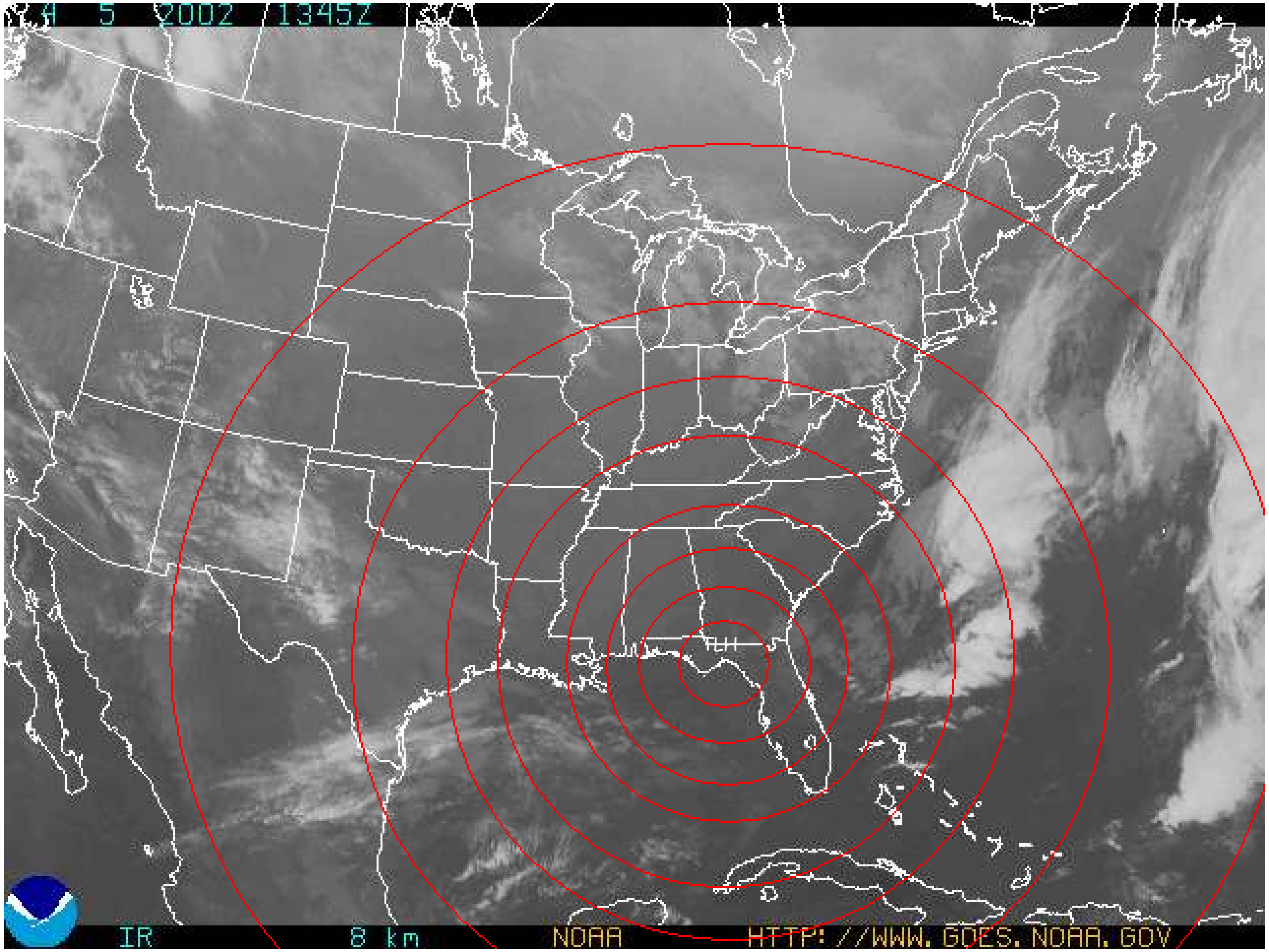
Cascades in the Constituent Quark Model

How can you find Tallahassee?

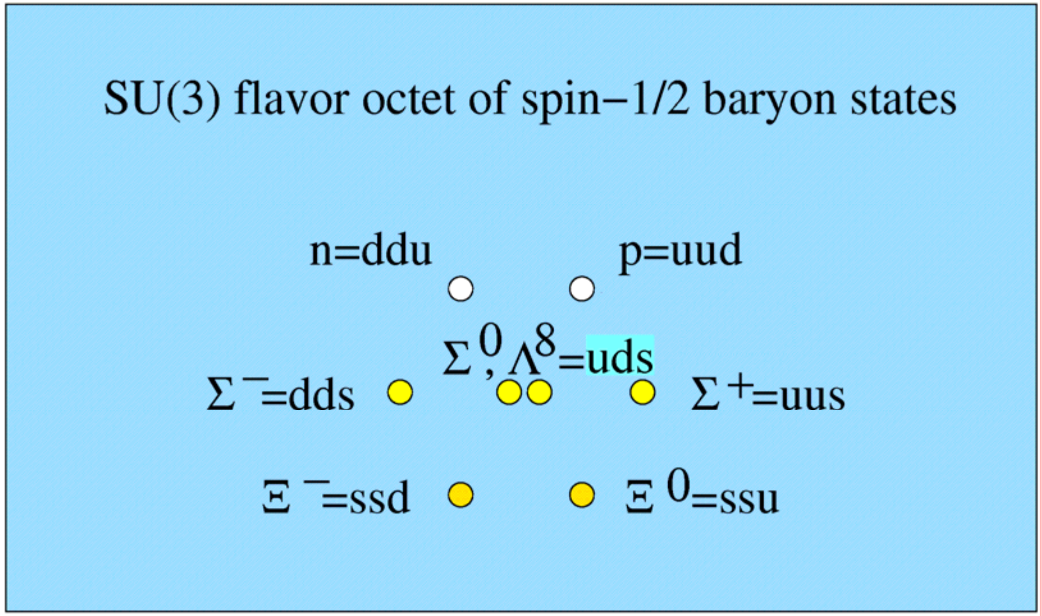
- Head SSW
- Stop when summer humidity(%) = T(°F) = wind speed (mph) = 95-100
- Try not to make jokes about elections, or Bushes



Cascade Baryons in the Constituent Quark Model

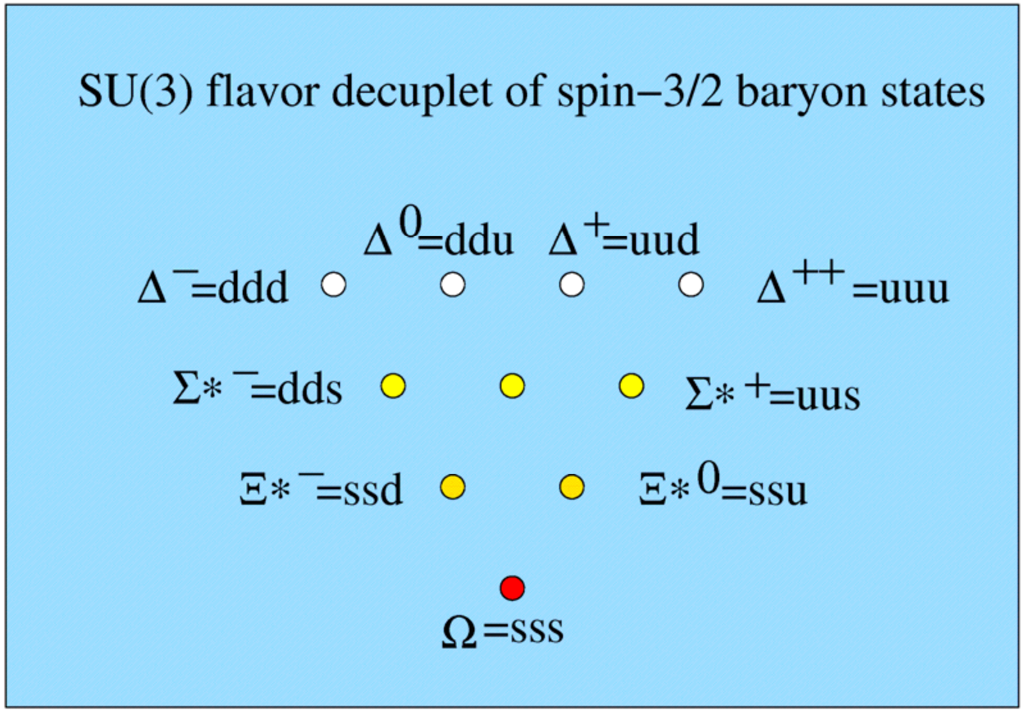
- $SU(3)_f$ symmetry
  - Corrections substantial
- What is special about Cascades?
  - Lightest excited Ξ^* states may have narrow widths, why?
  - Configuration mixing weaker than in N/Δ
- QM predictions for spectrum and decays
  - Chao, Isgur & Karl: NR model, EME decay model
  - Relativized model, $^3P_0$ pair creation decay model
    - First $\Xi^*(\frac{1}{2}^+)$ $[P_{11}]$ may be isolated and narrow
- Conclusions

Cascade baryons



- Three light (u,d,s) quarks have 27 possible flavor combinations
  - $3 \times 3 \times 3 = 3 \times (\bar{3} + 6) = 1 + 8 + 8' + 10$
  - (8,8') is flavor octet containing neutron and proton (mixed exchange symmetry) with spin $\frac{1}{2}$ ground states

Cascade baryons…



10 is flavor exchange symmetric

- Requires quark spin 3/2 for total anti-symmetry (ground states)
- All but Ω are higher-spin versions of octet states

$SU(3)_f$ symmetry

- $SU(3)_f$ symmetry is broken by the strange-light quark mass difference
  - Possible to describe strange baryons using an $SU(3)_f$ symmetric basis
    - Not ideal, everything is strongly mixed
  - Better choice is "uds" basis where you (anti)symmetrize only in u,d quark degrees of freedom
    - Isospin much better symmetry than $SU(3)_f$
  - E.g.: $\phi_\Lambda = (ud-du)s/\sqrt{2}$
    $\phi_\Sigma = uus, (ud+du)s/\sqrt{2}, dds$
    - In Ξ symmetrize only in ss pair, ϕ_Ξ = ssu, ssd

What is special about Cascades?

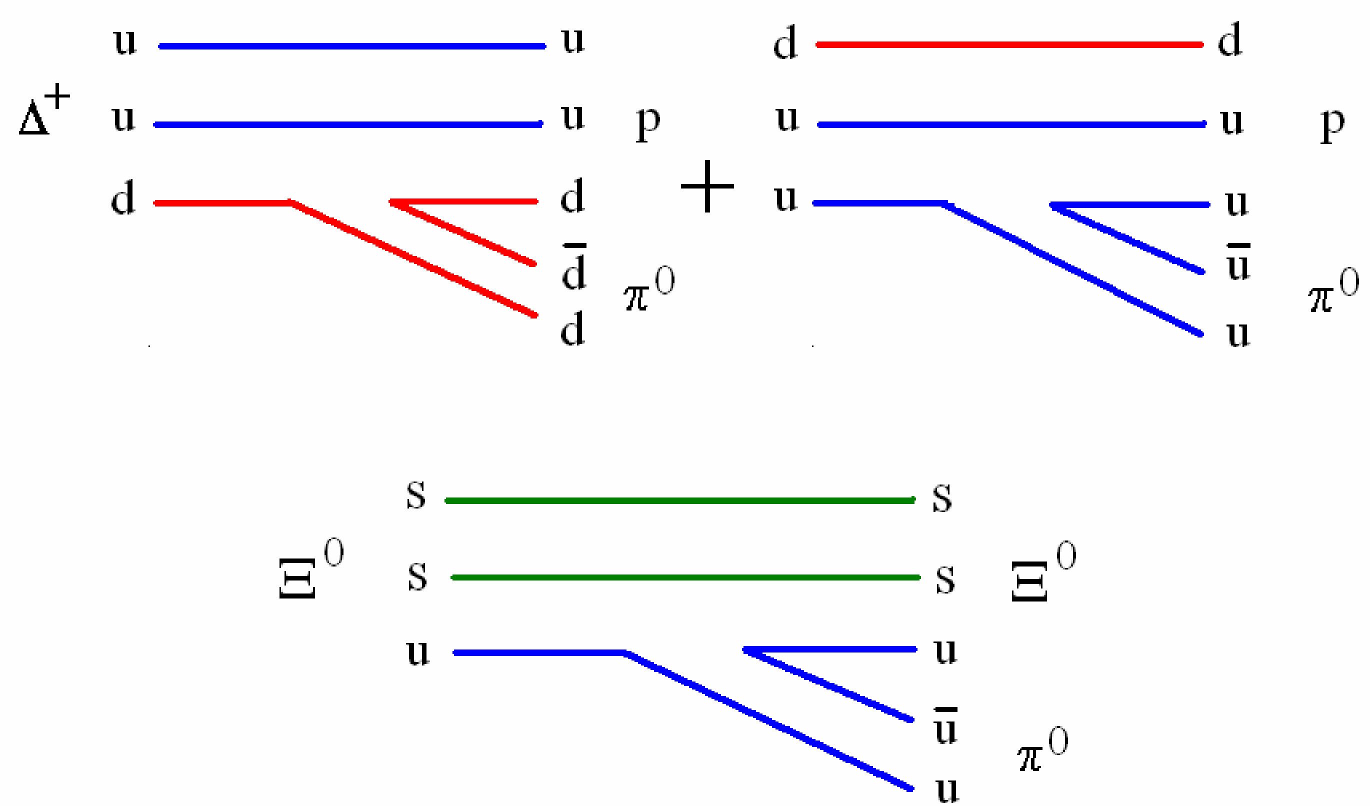
- Decays $\Xi^* \to \Xi\pi$ are suppressed relative to N, $\Delta \to N\,\pi$
  - Other channels involve K, which cuts down the available phase space
  - Leads to the possibility of narrow excited states

Why are some Cascades narrow?

- Example: $\Delta(1232)^{+} \to p\pi^{0}$ has width 120 ± 5 MeV
  - But $\Xi^{*0}(1530) \to \Xi^{0}\pi^{0}$ has width 9-10 MeV
- Some of this is phase space:
  - Decay momentum for $\Delta \to N\pi$ (P-wave) decay is 227 MeV
  - For $\Xi^{*}(1533) \to \Xi\pi$ (also P-wave) is 152 MeV

What is special about Cascades?

- $|\text{Flavor overlap factor}|^2$ smaller by 2



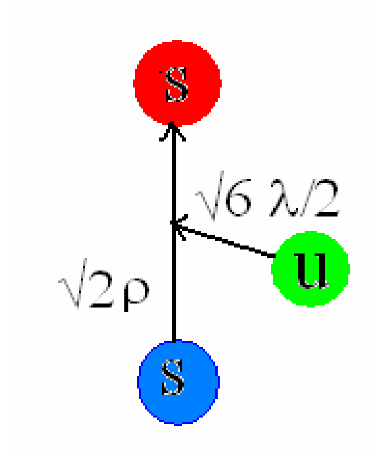
Fewer degeneracies

- Three-body system has two 3-D relative coordinates

  $\rho = (\mathbf{r}_1 - \mathbf{r}_2)/\sqrt{2}$

  $\lambda = (\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3)/\sqrt{6}$

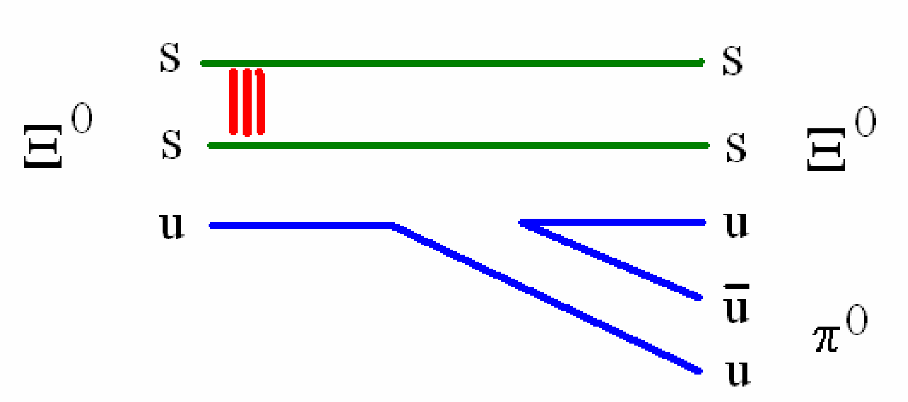
  If all quark masses equal (N, Δ, Ω), excitation costs same energy for both



- Exchange symmetry requires two strange quarks at ends of ρ coordinate in Ξ states
  - Excitation of ρ costs less energy than λ **if** confinement potential flavor independent
    - E.g. in 3-D H.O. $\omega^2 = 3K/\mu$, so $\omega_\rho < \omega_\lambda$

Decay selection rules

- Ground state Ξ has $n_\rho = l_\rho = 0$ ($=n_\lambda=l_\lambda$)
  - If two strange quarks are spectators in a strong decay like $\Xi^* \to \Xi\pi$



    - Exited state can't have n_ρ or l_ρ different from ground state (no overlap)
    - Those Ξ^* states with ρ excited should decouple from $\Xi\pi$, which has the largest phase space
  - Spoiled by mixing, but hyperfine small

Weaker hyperfine interactions

- Assume flavor-dependent short-range (contact) interactions
  - E.g. one-gluon exchange (DeRujula, Georgi, Glashow)

$$M = \sum_{i=1}^{3} m_i + \frac{2\alpha_s}{3}\frac{8\pi}{3}\langle\delta^3(\mathbf{r})\rangle \sum_{i<j=1}^{3} \frac{\mathbf{S}_i \cdot \mathbf{S}_j}{m_i m_j}$$

- Hyperfine interactions progressively weaker because of explicit flavor dependence (seen in spectrum):

  $1/(m_{u,d}\, m_{u,d})$ (all three quark pairs) in N, Δ

  $1/(m_{u,d}\, m_s)$ (2 pairs), $1/(m_{u,d}\, m_{u,d})$ (1 pair) in Λ,Σ

  $1/(m_{u,d}\, m_s)$ (2 pairs), $1/(m_s\, m_s)$ (1 pair) in Ξ

Patterns in Ξ^* spectrum and decays

- E.g. lightest $J^P = 7/2^+$ Ξ^* states (F_{17})
  - Should have L=2 from $l_\rho=2$ ($D_{\rho\rho}$) or from $l_\lambda=2$ ($D_{\lambda\lambda}$)
  - Lightest state should be $l_\rho=2$ ($\omega_\rho < \omega_\lambda$)
  - Splitting should be substantial

- Lightest state should decouple from $\Xi\pi$, and so have small widths

Is this decoupling mechanism realistic?

- Examine stretched Λ and Σ excited states

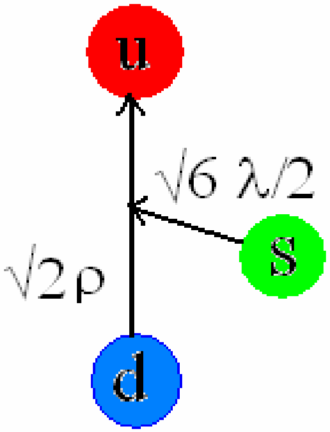
$\Lambda\ 5/2^- = [(ud-du)s/\sqrt{2}]\ \chi^S_{3/2}\ \Psi^\rho_{11}$

$\Sigma\ 5/2^- = [(ud+du)s/\sqrt{2}]\ \chi^S_{3/2}\ \Psi^\lambda_{11}$

(Suppressed totally AS color wvfn.)

$\Psi^\rho_{11} = \alpha_\rho \rho_+\ \Psi_{00}, \quad \Psi^\lambda_{11} = \alpha_\lambda \lambda_+\ \Psi_{00}$

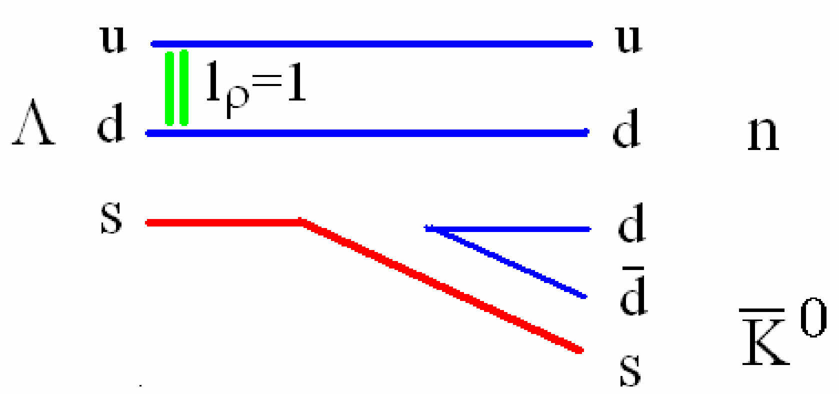
$\Psi_{00} = [\alpha_\rho^{3/2} \alpha_\lambda^{3/2} / \pi^{3/2}]\ \exp\{-(\alpha_\rho^2 \rho^2 + \alpha_\lambda^2 \lambda^2)/2\}$



- s quark heavier, so $\omega_\rho > \omega_\lambda$ and $m_{\Lambda\ 5/2-} > m_{\Sigma\ 5/2-}$
- Expt: $\Lambda(1830)D_{05} > \Sigma(1775)D_{15}$

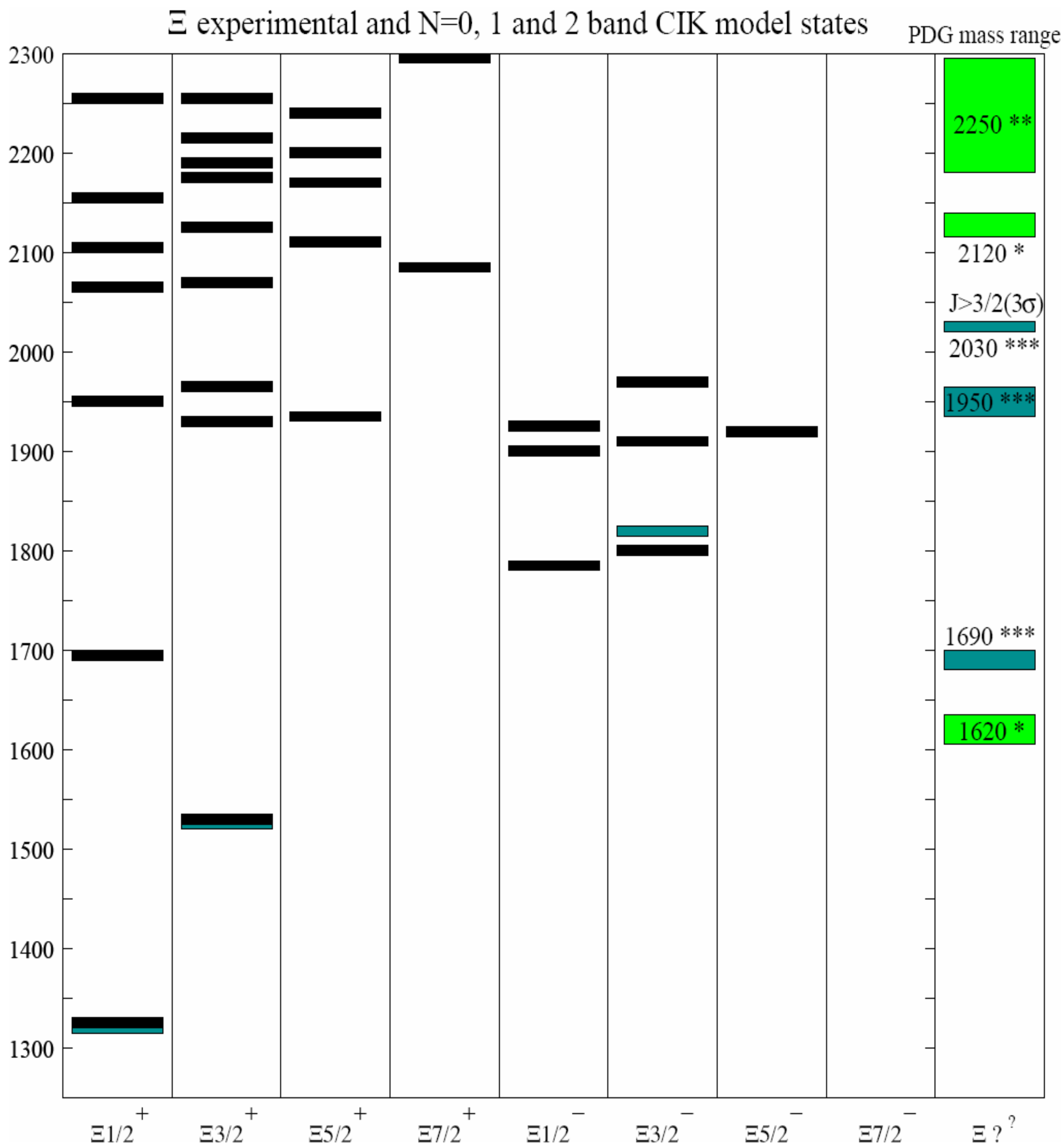
Decoupling mechanism…

- ρ-type spatial excitations in Λ should largely decouple from the NK state:
  - $\Lambda(1830)D_{05}$ has 3-10% BR to NK
- λ-type should couple:
  - $\Sigma(1775)D_{15}$ has 37-43% branch to NK



Chao, Isgur and Karl

- Use H.O. basis to N=2
  - 1st order perturbation theory in anharmonic terms (linear, Coulomb) in spin-independent potential
  - NR kinetic energy
  - 1st order perturbation theory in:
    - Hyperfine (spin-dipole/spin-dipole) interaction
      - Contact term (Δ-N, Σ-Λ,…)
      - Tensor term (important mixings)
      - Spin-orbit term should be present, too large
        - » Some cancellations


Ξ experimental and N=0, 1 and 2 band CIK model states
PDG mass range
2300
2200
2100
2000
1900
1800
1700
1600
1500
1400
1300
2250 **
2120 *
J>3/2(3σ)
2030 ***
1950 ***
1690 ***
1620 *
Ξ1/2 +
Ξ3/2 +
Ξ5/2 +
Ξ7/2 +
Ξ1/2 −
Ξ3/2 −
Ξ5/2 −
Ξ7/2 −
Ξ ? ?

CIK strong decays

- Use elementary-meson emission model
  - Pseudoscalar mesons emitted directly from quark lines
  - Parameters fixed by fit to N, Δ, Λ, Σ strong decays

Chao Isgur and Karl Ξ states (expt)

mass	state	J^P	wvfn	$\|A_{\Xi\pi}\|$ $MeV^{\frac{1}{2}}$	$\|A_{\Lambda K}\|$	$\|A_{\Sigma K}\|$	$\Sigma\|A_i\|^2$ (MeV)
1785	$[S_{11}]_1$	$1/2^-$	$^2P_\rho$	3.6	4.2	3.7	44
1890	$[S_{11}]_2$	$1/2^-$	$^4P_\lambda$	5.5	0.8	2.2	36
1925	$[S_{11}]_3$	$1/2^-$	$^2P_\lambda$	1.5	1.4	5.4	33
1800 1820	$[D_{13}]_1$	$3/2^-$	$^2P_\rho$	1.6 1.5	3.6 2.7	3.9 2.7	31 24
1910	$[D_{13}]_2$	$3/2^-$	$^2P_\lambda$	4.3	0.9	4.6	40
1970	$[D_{13}]_3$	$3/2^-$	$^4P_\lambda$	3.7	1.8	3.2	27
1920	$[D_{15}]_1$	$5/2^-$	$^4P_\lambda$	9.8	4.7	3.4	130

Chao Isgur and Karl Ξ states (expt)

mass	state	J^P	wvfn	$\|A_{\Xi\pi}\|$ $MeV^{\frac{1}{2}}$	$\|A_{\Lambda K}\|$	$\|A_{\Sigma K}\|$	$\Sigma\|A_i\|^2$ (MeV)
1695	$[P_{11}]_2$	$1/2^+$	mixed	1.0	0.7	0.2	1.5
1950	$[P_{11}]_3$	$1/2^+$	mixed	1.8	2.6	3.4	22
1530	Ξ^*	$3/2^+$		4.5 3.2			20 10
1930	$[P_{13}]_2$	$3/2^+$	$^2D_{\rho\rho}$	0.1	2.1	4.3	23
1965	$[P_{13}]_3$	$3/2^+$	$\sim {}^2S_{\rho\rho}$	1.6	3.0	4.1	57
1935	$[F_{15}]_1$	$5/2^+$	$^2D_{\rho\rho}$	0.5	1.9	4.9	28
2110	$[F_{15}]_2$	$5/2^+$	$^4D_{\rho\rho}$	0.8	3.0	2.5	16
2085	$[F_{17}]_1$	$7/2^+$	$^4D_{\rho\rho}$	1.0	5.4	4.2	48
2195	$[F_{17}]_2$	$7/2^+$	$^4D_{\lambda\lambda}$	8.6	1.1	1.0	76

Chao Isgur and Karl Ξ states…

- Recall:
  - Roper at 1440, width ~350 MeV
  - N(1710), width ~100 MeV
- Exciting prospect:
  - Roper-like state $\Xi[P_{11}]_2$ at ~ 1700 MeV
    - Isolated from negative-parity states at ~1800-1900 MeV, which have Γ = 25-50 MeV
    - Widely separated from its $\Xi[P_{11}]_3$ partner at ~1950 MeV (width ~ 25 MeV)
  - Width of a few MeV:
    - Phase space; also $\Xi\pi\pi$ contribution small

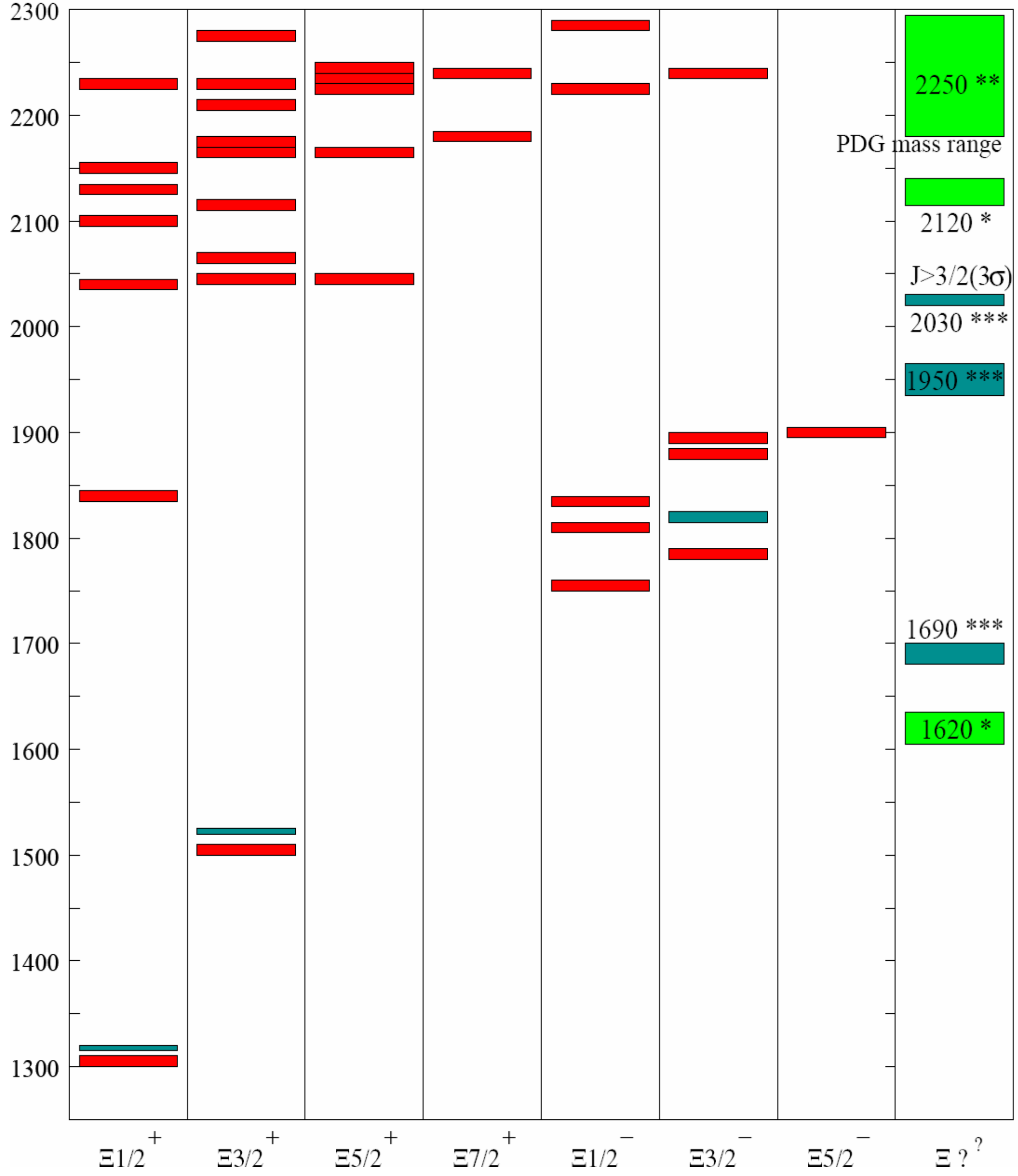
Relativized model

- Variational calculation in large H.O. basis (with N. Isgur)
  - Flux-tube confinement, plus associated spin-orbit
  - Include OGE Coulomb, contact, tensor, spin-orbit
  - Relativistic KE, relativistic corrections in potentials, e.g.

$$\left(\frac{m_i m_j}{E_i E_j}\right)^{\frac{1}{2}+\epsilon_{\rm cont}} \frac{8\pi}{3}\alpha_s(r_{ij})\frac{2}{3}\frac{\mathbf{S}_i\cdot\mathbf{S}_j}{m_i m_j}\left[\frac{\sigma_{ij}^3}{\pi^{\frac{3}{2}}}e^{-\sigma_{ij}^2 r_{ij}^2}\right]\left(\frac{m_i m_j}{E_i E_j}\right)^{\frac{1}{2}+\epsilon_{\rm cont}}$$

- Strong decays calculated in pair creation ($^3P_0$) model
  - $^3P_0$ is popular phenomenological decay model
  - Has advantage that emitted mesons have structure
    - Can correlate many decays with very few parameters
- **No** new parameters for Ξ decays
  (recent calculation with W. Roberts)

Ξ experimental and CI model states below 2300 MeV



Relativized-model Ξ states

- N/Δ, Λ/Σ results show:
  - Average mass of light -ve parity excitations too light by ~50 MeV
  - Average mass of light +ve parity excitations too heavy by ~50 MeV
  - Splittings in -ve parity states too small when introduce strange quark
    - Effects of strange-quark mass on kinetic energy diluted by $(p^2+m^2)^{\frac{1}{2}}$

CI/CR Ξ states/decays (expt)

mass	state	J^P	$\|A_{\Xi\pi}\|$ $MeV^{\frac{1}{2}}$	$\|A_{\Lambda K}\|$	$\|A_{\Sigma K}\|$	$\Sigma\|A_i\|^2$ (MeV)
1755	$[S_{11}]_1$	$1/2^-$	9.3	13.6	15.3	506
1810	$[S_{11}]_2$	$1/2^-$	15.1	4.0	10.0	344
1835	$[S_{11}]_3$	$1/2^-$	2.7	4.7	12.5	186
1785	$[D_{13}]_1$	$3/2^-$	1.5 1.5	3.1 2.7	3.3 2.7	23 24
1880	$[D_{13}]_2$	$3/2^-$	2.3	1.7	2.3	13
1895	$[D_{13}]_3$	$3/2^-$	2.7	2.0	3.0	20
1900	$[D_{15}]_1$	$5/2^-$	6.5	3.3	2.7	60

CI/CR Ξ states/decays (expt)

mass	state	J^P	$\lvert A_{\Xi\pi}\rvert$ $MeV^{\frac{1}{2}}$	$\lvert A_{\Lambda K}\rvert$	$\lvert A_{\Sigma K}\rvert$	$\Sigma\lvert A_i\rvert^2$ (MeV)
1840	$[P_{11}]_2$	$1/2^+$	1.9	2.8	2.1	16
2040	$[P_{11}]_3$	$1/2^+$	5.2	5.3	5.1	81
1530	Ξ^*	$3/2^+$	3.2 3.2			10 10
2045	$[P_{13}]_2$	$3/2^+$	4.9	7.6	6.7	127
2065	$[P_{13}]_3$	$3/2^+$	1.7	5.1	10.5	110
2045	$[F_{15}]_1$	$5/2^+$	0.3	0.9	2.6	8
2165	$[F_{15}]_2$	$5/2^+$	1.4	2.1	0.2	6
2180	$[F_{17}]_1$	$7/2^+$	1.5	3.1	2.3	17
2240	$[F_{17}]_2$	$7/2^+$	5.0	0.2	0.0	25

Relativized-model Ξ states…

- Roper-like state $\Xi[P_{11}]_2$ may be at ~ 1790 MeV
  - May be close to lightest negative-parity states at ~1800-1900 MeV, lightest of which are still likely to have Γ = 25-50 MeV
  - Widely separated (200 MeV) from its $\Xi[P_{11}]_3$ partner at ~1990 MeV (width ~ 80 MeV)
  - Width 10-15 MeV

Conclusions and outlook

- It is likely that there are excited Ξ baryons that are relatively narrow
  - We need to calculate $\Xi^*(1530)\pi$ decays i.e. $\Xi\pi\pi$ final state
  - CIK did this already in elementary meson emission model
- It would be very interesting to see the lightest excited states in certain partial waves decoupling from the $\Xi\pi$ channel
  - Confirm flavor independence of confinement
- Semi-leptonic decays in progress
  - **Muslema Pervin**, W. Roberts & SC